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Active Learning in Developmental Classes of Mathematics.

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Abstract

Background*: Active learning is described as a process where students engage actively in problem solving that promotes analysis and synthesis of the class topics. In the light of recent findings and publications, the active learning style of teaching is more efficient in STEM fields than the traditional lecturing style, where students listen passively.* ***Purpose:*** *This article describes a transition process from lecture-oriented classroom into activities-oriented classroom for an Algebra and Trigonometry class at a large urban community college.* ***Methodology/Approach:*** *The presentation includes sample worksheets and explains methods of implementing them in the classroom. The most important aspect of this study presents motivations and methods of revising the worksheets based on observation of the classroom, students' performance, students' reactions, quiz results, and class assessment. The last one appears in both of its aspects: formal and informal.* ***Findings/Conclusions****: Students' suggestions and feedback are key factors in designing a successful class worksheet. The kind that not only assists in learning the material but exposes gaps in students' knowledge and errors in learning procedures.* ***Implications****: Active learning based on class worksheets offers numerous benefits. Among which, an opportunity for students to participate actively in the revision of class materials, seems to be the most significant.*

Keywords: active learning, developmental mathematics, class worksheets, revision of class worksheets

INTRODUCTION

Active learning is defined in [1] to be a process where students actively engage in activities such as problem solving and discussion that promote analysis, synthesis, and evaluation of class content. It emphasizes higher-order thinking and often involves group work.

The study was performed in a large, urban community college that runs on a schedule of the quarter system with two 12-week and two 6-week semesters. According to the college statistical data, more that 60% of students live on less than average household financial resources. Many students are overwhelmed with multiple jobs, or family issues, often taking too many classes yet not graduating within five years.

Developmental (or remedial) mathematics classes are defined as academic services that prepare students for college level courses. At LaGuardia Community College, they are taught in highly heterogeneous groups, usually between 23 and 27 students of different cultures and attitudes, and who have significantly different previous knowledge in the subject. In addition, some students may have language difficulties, in particular with understanding spoken language. Most students were previously exposed to a majority of the material, but did not reach a desired skill level, or their knowledge may be fragmentary. The main issue in class might be students' attitudes: either a student is overly certain of their skills, or is exhibiting an anxiety. A typical student has a combination of both. Other problems that students might have are: notoriously late arrivals, skipping classes, coming to class directly from night shift and being very tired. Personality traits and low social skills might at times cause issues, for example students refuse to collaborate with a group, make jokes about other students, consistently refuse to present their work on the board, are overly shy or overly talkative. Low attendance is very common, especially during times of difficult weather since many students have to rely on public transportation.

CLASS WOKSHEETS

Class worksheets for encouraging active learning are designed to address issues mentioned above. They are implemented in a form of two-sided handouts and contain problems assigned for students to reinforce lectures. This pedagogical concept is based on the observation that lecture material is more likely to be absorbed if students practice it and reflect on it immediately after the lectures.

Class worksheets improve students' attention and focus during lectures, which is visible through the semester. As the time passes, students' work becomes more clear and diligent. The average number of errors per worksheet decreases significantly during the semester from 10 on the beginning to 0 after just four weeks. In addition, students with class anxiety develop new learning strategies that are free from previous issues.

From the perspective of the teacher, it is valuable to have an immediate insight into students' notes, particularly repetitive incorrect solutions that students submit. For both students and the teacher, the worksheets provide immediate feedback about students' learning outcomes. Often students realize how much material they have missed only after receiving a quantity of class worksheets. At the same time the worksheets provide a clear outline of study with material for tutoring and for self-reflection.

The strength of the class worksheets lies in the flexibility of their implementation in the classroom, and in the fact that they can be combined with other learning modalities, such as quizzes, homeworks, projects, exams etc. In addition, they can contain an assessment of the lectures and labs. The most important aspect of the class worksheets is that they are graded and returned to students in a timely manner for corrections. Ideally, students apply the correction, resubmit the worksheets which are then re-graded and returned to students. In practice, students may not be even willing to look at their own mistakes. This behavior exposes anxiety towards negative feedback and should be addressed in class early during the semester before it accumulates into a serious issue.

The study described in this article was a trial and is still a work in progress in terms of revising the worksheets and revising the means of their implementation in the classroom. The assessment was based on students' comments on anonymous surveys, on informal

discussions and on students' performance on the worksheets. The feedback was collected over three 12-week semesters in three different sections of two different courses. One of them was designed in an accelerated format.

Sample lesson design is suggested in [2] for calculus class but the description does not contain details how to revise the first submission of worksheets, and how to assess them effectively for the purpose of expanding this concept to other topics.

Class worksheets usually contain a sequence of examples of increasing difficulty that help students reflect on the lecture material. Preparing and revising class worksheets usually takes few semesters and begins with designing just few lectures and analyzing the outcomes, students' questions, and reactions on difficult concepts. For the first submission of class worksheets, I found it valuable reviewing old exams and checking repetitive errors or common misunderstandings, and addressing them. The first topic chosen was among the easiest in terms of pedagogy and the least challenging for students.

CLASS ORGANIZATION

Classes meet twice a week for 120-minutes except the accelerated class that meets twice a week, once for 240-minutes and once for 180 minutes. All courses have a tutor who attends most class meetings and is available to students if they have questions. Students can meet the tutor outside of the class during their convenient time. Every class begins with a mini-lecture followed by few sample problems. Then students work on questions from the worksheet that reflect that topic. As I walk around the classroom, checking students' work for errors and answering questions, other students present their solution on the board. Sometimes several students volunteer to go the board, but sometimes no one solves the problems correctly, and I have to review the material again and solve the problems. After each class the worksheets are collected and graded for credit. If the topic goes smoothly, the worksheets may be collected on the same day after class, but if the topic proves to be challenging, the worksheets may be collected with a delay. During the

next class, they are returned to students for corrections and resubmissions. At times, it took several resubmissions before everything on the activity sheet was solved correctly. Every week students took a quiz on the recent material and I checked students' quiz errors to address topics that they misunderstood. Often, I had to respond to common complaints that the quiz contained problems not covered in class by pointing out similar questions from the class worksheets.

Usually students work in a very uneven fashion, because some students missed many classes or simply work slowly and are behind, and some others work faster and are ahead. It is good to keep in mind that allowing students to work at their own speed will eventually make the lecturing for the entire class highly inconvenient or even impossible. During the second semester with the worksheets I was tempted to allow self-paced work for all students in class. Several students raced through the material and completed all work four weeks before the end of the 12-week semester just to realize that they do not recall the topics from the beginning. At the same time, other students were half way through the program and did not have any motivation to speed up their work.

EXAMPLE OF REVISIONS OF A WORKSHEET: FACTORING AND FACTORING BY GROUPING

Factoring and factoring by grouping are the main lesson objectives for this class. The first part of the worksheet is addressing the issue of students misreading the question and giving as the answer the greatest common denominator in place of factoring it from an expression. The second part provides few examples that introduce students to factoring by grouping. It responds to common errors, where students misplace addition and multiplication signs in factoring by grouping.

Analyzing the samples of the final exams from previous semesters I observed that students misread the question about factoring which asked for the greatest common factor

itself, or for factoring it from an expression. It seemed to be a relatively small issue but along the same lines several students had difficulties writing correctly the answers for factoring by grouping. The errors appeared to be small typos, but I thought that they might be caused by some deeper misunderstanding, thus I decided to address this problem.

The first version of the class worksheet addresses the issue with factoring by separating questions about finding the greatest common divisor and factoring it. Unfortunately, to my surprise, several students did not realize that they were working on the same sequence of examples in problem 1 and in problem 2. Thus, motivated by students' behavior and performance, I combined these two problems after the revision.

Factoring $3x$ from $12x + 9x^2$, or factoring $3x^2y$ from $12x^2y + 9x^2y^3$ does not seem to cause any confusion among students. But factoring $3(x+1)$ from $12(x+1) + 9(x+1)^2$ is a big step in thinking for many students. Most commonly students try to distribute and simplify the expression rather than apply factoring. Apparently, the sign "+" in the parenthesis between "x" and "1" confuses the process of finding the greatest common factor (GCD). After observing students' confusion, I realized that this topic should be addressed during a lecture that ought to be delivered before students begin working on this problem. After factoring $3(x+1)$ from $12(x+1) + 9(x+1)^2$, and $3(p+1)$ from $12(p+1)q - 3(p+1)$, and working on few more examples following these questions, students should be ready for factoring by grouping. But while observing students' progress in class, I realized that many of them did not even bother to solve these problems and went directly to factoring by grouping. Just as during the exams, commonly they tried to "guess" the answers and were usually misplacing multiplication and addition signs. The revised version of the worksheet addresses this issue by adding more questions to factoring by grouping. At the same time, I realized that a mini-lecture could curtail this behavior if it is delivered as soon as students complete the part of the worksheet about the GCD.

Revisions suggested by students

Revisions of the worksheet were based on informal discussions with students and a survey at the end of the class that asked for suggestions to improve the worksheets.

Students suggested adding a section number and the title of the topics. The request was motivated by need of being able to relate the worksheet to a book section and the homework. Some students prefer to search online for additional study material and they need the title of the topic for that purpose. As reported by students, Khan Academy is the most popular source of video tutorials.

Another suggestion made by students was related to sample problems worked out on the sheet. As students pointed out, the easier problems of finding GCD of two numbers were covered in the past, and thus were easy to recover from the lectures. But more advanced problems, like factoring expressions containing addition, as $(p + 1)$, would easily be forgotten. Thus, students suggested including sample solved problems of more advanced questions.

Students suggested using fewer problems of lower difficulty level and more problems of higher difficulty level, so that the practice in the classroom becomes more effective. It may sound contradictory that students voluntarily requested more problems with higher level of difficulty for the class worksheet. According to my experience, students who participate in active learning classes and are asked for advice how to improve them, become truthfully supportive and offer their best judgement free of selfish point of view.

As a result of students' request the problem for finding the GCD of 12 and 9 was removed from the worksheet, but is still presented during the lecture to connect with students' previous knowledge.

Students claimed that having separate problems for indicating and factoring GCD was confusing because it made them think that these two topics were not related to each other. Consequently, these two problems were merged in the revised version of the worksheet.

The new version provides more problems for factoring by grouping. The first examples contain numerical not letter coefficients.

Sample revision of a worksheet based on students' suggestions

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Comments are indicated in *cursive blue.* Deleted questions are indicated in purple, added questions are indicated in red.

WORKSHEET BEFORE REVISIONS

Class activity sheet 8 Name_____

Number of the textbook section, titles of the topics, and tutorials are not included

1. Find the greatest common divisor (GCD) of
   a) 12 and 9
   b) x^2y and y^2
   c) $12x$ and $9y$
   d) $12x$ and $9x^2$
   e) $12x^2y$ and $9x^2y^3$
   f) $12(x+1)$ and $9(x+1)^2$
2. Factor the greatest common divisor:

a) $x^2y + y^2$
b) $12x + 9y$
c) $12x + 9x^2$
d) $12x^2y + 9x^2y^3$
e) $12(x+1) + 9(x+1)^2$

3. Find the greatest common divisor and factor it from the expression:
   a) $-12pq - 3p$
   b) $p^3q^4 - p^4q^3 + pq$
   c) $-12(p+1)q - 3(p+1)$
   d) $-12(p+1)^3q - 3(p+1)^3$
   e) $-12(p+1)^2q - 3(p+1)^5$

f) $-12(p+1)^2x^2 - 3(p+1)^5x$

4. Factor by grouping

The simplest problem should contain numerical coefficients. The first two problems are in a format that allows a transparent procedure of factoring by grouping

a) $a + ax + b + bx$

b) $5x + 10y + ax + 2a$

c) $3ab^2 + 6b^2 - 12ab - 24b$

WORKSHEET AFTER REVISIONS

Class worksheet, section 13.1 Name_____

Factoring the greatest common denominator. Factoring by grouping.

Example of factoring a monomial from a polynomial expression:

Since $GCD(x^2y, xy^2) = \boldsymbol{xy}$ thus $x^2y + xy^2$ factors in the following way:

$x^2y + xy^2 = \boldsymbol{xy}\left(\frac{x^2y}{\boldsymbol{xy}} + \frac{xy^2}{\boldsymbol{xy}}\right) = \boldsymbol{xy}\,(x+y)$,

Example of factoring a binomial from a polynomial expression:

Since $GCD((p+4)^2y, (p+4)y^2) = (\boldsymbol{p+4})\boldsymbol{y}$, thus $(p+4)^2y + (p+4)y^2$ factors in the following way:

$(p+4)^2y + (p+4)y^2 = (p+4)\boldsymbol{y}\left(\frac{(p+4)^2y}{(p+4)\boldsymbol{y}} + \frac{(p+4)y^2}{(p+4)\boldsymbol{y}}\right) = (p+4)\,\boldsymbol{y}\,((p+4)+y)$

1. Find the greatest common divisor (GCD) of the terms and factor it from the expression

The simplest problem was removed but is still part of the lecture

a) $12x^2y + 9xy^2$

b) $12(p+4)^2y + 9(p+4)y^2$

c) $12(p+4)^2(q-1) + 9(p+4)(q-1)^2$

d) $p^3q^4 - p^4q^3 + pq$

e) $-12(p+1)q - 3(p+1)$

f) $-12(p+1)^3q - 3(p+1)^3$

g) $-12(p+1)^2q - 3(p+1)^5$

h) $-12(p+1)^2x^2 - 3(p+1)^5x$

5. Find the greatest common divisor and factor it from the expression:

g) $-12pq - 3p$

h) $p^3q^4 - p^4q^3 + pq$

i) $-12(p+1)q - 3(p+1)$

j) $-12(p+1)^3q - 3(p+1)^3$

k) $-12(p+1)^2q - 3(p+1)^5$

l) $-12(p+1)^2x^2 - 3(p+1)^5x$

Questions 1, 2, and 3 were merged into one question

2. Factor by grouping
d) $x^2 + x + x + 1$

e) $25x^2 - 30px + 30px - 36p^2$

f) $a + ax + b + bx$

g) $5x + 10y + ax + 2ay$

h) $3ab^2 + 6b^2 - 12ab - 24b$

This question was expanded and new problems use different letters and variables.

ASSESSMENT

At the end of two of my courses based on class worksheets I proctored a short survey asking students about their experience with this type of learning. Regardless of the small samples, I could already see varied responses to the following question: *"Do you have a feeling that you learned more in a classroom with worksheets than in a traditional classroom? Or was it the other way around?"* About 40% of the students were enthusiastic about this style of learning and claimed than they learned more with worksheets that without them, about 30% were neutral, saying that they learned about the same. About 30% students claimed that they learn more from lectures and preferred them as the main class module.

SUMMARY

Class worksheets offer a powerful tool for engaging students in class, however they involve a lot of challenges for the teacher in handling students' questions individually. As well they offer many observations about students' learning habits and issues. In particular, resubmissions of the worksheets provided additional information about students' attitudes related to correcting their own errors, or being evaluated. It became clear that students are much more interested in starting a new class worksheet than correcting an older one. Often students felt resistance towards correcting their errors and requested a clean copy of the worksheet. Students who miss a class, or arrive late, have a chance to catch up quickly with the group, which is impossible for lecture-based teaching. Students who miss few days of classes due to illness have the advantage of a clear outline of work and can take the worksheets to a tutoring room. After observing active learning classes for two semesters I concluded that class worksheets are a successful tool if properly blended with the course work.

Active learning based on class worksheets offers numerous benefits. Among which, an opportunity for students to participate actively in revision of class materials, seems to be the most significant

REFERENCES

[1] Bonwell CC, Eison JA, (1991) Active Learning: Creating Excitement in the Classroom, Washington DC: George Washington University

[2] Freeman S., Eddy S.L., McDonough M., Smith M.K., Okoroafor N., Jordt H., Pat-Wenderot M., (2014) Active learning increases student performance in science, engineering, and mathematics, Proc Natl Acad. USA 2014 Jun 10, 111(23): 8410-8415

[3] Code, W., Piccolo C., Kohler D., MacLean, M., (2014) Teaching Methods comparison in a large calculus class, ZDM Mathematics Education 46: 589-601

[4] English L.D., Halford G.S. (1995). Mathematics education: Models and processes. Mahwah, NJ: Lawrence Erlbaum Associates.