Aha!Moment from Korea.
Personal communication
Bronislaw Czarnocha

I met the team from Korea: ***Yoon, Sangjoon*;Oh, Kookhwan; Oh, Yaerin; Bae, Mi Seon; Kim, Doyen; Kwon, Oh Nam*** in Szeged, Hungary' PME 40 during their presentation of the paper that caught my attention: **ANALYSIS ON THE MENTAL STRUCTURE OF STUDENTS LEARNING GEOMETRY: Based on APOS Theory.** I know a bit about APOS theory; so I listened and, since from their talk it was clear they were describing the formation of the new schema, I asked whether they did in fact noticed an Aha!Moment in the process
Below is the answer to my question

We entirely concur with your opinion about beginning of the Schema construction.

We could convince that the student did further operation on object.

And Yes! we observed Aha! moments by students.

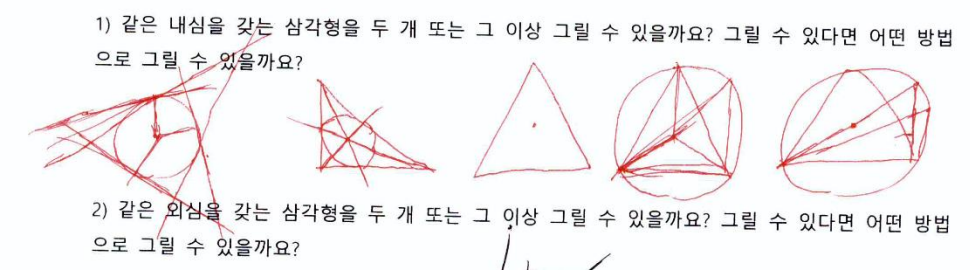
Especially, at the begining, student-1 didn't know the solution of interview question : finding two more triangles with the same incenter or circumcenter.(ppt # 9, solution with 'red')

Finding solution processes of student-1 are as follows

1) He had no ideas about the question.

2) He recalled that all the tringles in a certain circle, of which one side be a diameter of a certain circle, are right triangles.

3) He associated these with "a" circumscribed circle of right triangles.



4) He found that it is

possible to draw triangles in a circle with not only right angle but also acute angle and obtuse angle. (Aha!)

5) He extended this ideal!!! But he could answer the question about 'only' circumcenter. (So the triangles with same circumcener, drawn by student-1 in ppt #9, have always a common side.)

6) And then, he applied the idea to the question about inscribed circle.(Aha!!) (As you see in my attached file of 'answering of student-1', the triangles with same incener have also a common side)

7) Then, "I didn't know the solution before, but I JUST find it! Wow, (it is) really wonderful (for me to solve it like this way!) ", he said.

Aha!Moment from Poland
An elephant – or what use can be made of metonymy?
Celina Kadej, Matematyka #2, 1999

Linear equations with one unknown can be solved already by students in the elementary school. Those are simple equations and students often formulate them by themselves while solving word problems. Sometimes the problems lead to equations a bit more complex than the elementary additive equations of the type x + a = b.

I have had an opportunity to listen to the discussion of two enthusiastic students solving a standard word problem: *The sum of two numbers is 76. One of the numbers is 12 more than the other. Find both numbers.* It was a problem from Semadeni's set of problems for the 3rd grade and one had to solve it using equations and that's where the difficulty appeared:

Przemek (read Pshemik) wrote the equation: x + (x+12) = 76. To solve it was a bit of a problem for him, but still he dealt with it. He drew an interval and then a following dialog had taken place [between him and his friend Bart]:
P: *That is that number*: he extended this interval by almost the same length, and the another one like that.
And this is that number plus 12
B*: and this all together is equal to 76…*
P*: No, this is an equation, d'you understand…*
B could not accept it…
B: *Why did you draw this interval? You don't know yet what it's supposed to be?*
P: *That's not important.*
B: *Why 76?*

P: '*cause that's what is in the problem*
B*: that iks, that iks add 12 and that's supposed to be 76..?*
P: *Look instead of iks there is a little square in the book* – P showed the little square in the book.
B: *Aha, but here, here is written something else*
P: *But it could be as here. And now I am inputting a number into this square*.
B: *A number?! Why into the square?*
P: *No, it's into the window. Into this window I input the number which comes out here.*
B: *But here is a square* – B insisted.
P: *It's not a square but a window, and one inputs the numbers into that window*.
B: *How so*?:
P: *Two windows are equal 64, one window is equal 32. Well, now, you subtract 12 from both sides,* and *you see that the two windows are equal to 64*.
B: *But are there numbers in the windows*?
P: *Two windows are 64, so one window is 32*
B: *Window!?*
P: *That's right, a window. Look here:* ***an elephant*** *and* ***an elephant*** *is equal* ***64***. *Therefore what is* ***one elephant*** *equal to?* ***Two elephants*** *are equal* ***64****. So,* ***one elephant*** *is* ***equal*** *to what*?
B: ***An elephant****? Hmm, I see.* ***One elephant equals 32****. I understand now… so now the equation…*
P: *If* ***two elephants*** *are equal* ***60****, then* ***one elephant*** *is* ***equal*** *what?*
B: An elephant?, ok, one elephant equals 30. I see it now…..Now equation……..**aaaaaaa**