MATHEMATICS TEACHING-RESEARCH JOURNAL ONLINE
VOL 8, N 3-4
Fall and Winter 2016/17

BRONISLAW CZARNOCHA

CHAPTER 1.1 TEACHING-RESEARCH NEW YORK CITY MODEL (TR/NY CITY)

TR/NYCity Model is the methodology for classroom investigations of learning, which synthetizes educational research with teaching practice. It is conducted simultaneously with teaching and the aim of improvement the learning by the teacher of the class in the same classroom, and beyond.

INTRODUCTION

TR/NYCity Model is based on the careful composition of ideas centred around Action Research (Lewin, 1946) with the ideas centred around the concept of the Teaching Experiment of the Vygotskian school in Russia, where it "grew out of the need to study changes occurring in mental structures under the influence of instruction" (Hunting, 1983). From Action Research we take its focus on the improvement of classroom practice by the classroom teacher and its cyclical instruction/analysis methodology, and from Vygotsky's teaching experiment we take the idea of the large-scale experimental design based on a theory of learning and involving many sites – different classrooms (B. Czarnocha, 1999, Czarnocha and Prabhu, 2006). Vygotsky teaching experiment methodology introduced the possibility of viewing the classroom teacher as a member of a collaborative research team investigating the usefulness of research based classroom integration. The integration of these two distinct frameworks re-defines the profile of a teacher-researcher:

1. as an education professional whose classrooms are scientific laboratories, the overriding priority of which is to understand students' mathematical development in order to utilize it for the betterment of the particular teaching and learning process;

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

2. who as a teacher can have the full intellectual access to the newest theoretical and practical advances in the educational field, knows how to apply, utilize and assess them in the classroom with the purpose of improving the level of students' understanding and mastery of the subject;
3. who as a researcher has a direct view of, and the contact with the raw material of the process of learning and development in the classroom, acts as a researcher in the context of the daily work and uses that process to design classroom improvement and derive new hypotheses and general theories on that basis.

The implicit vision underlying the profile is the conceptual and practical balance between researches and teaching, where both components of the educational profession are given equal value and significance; both the research knowledge of the researcher and the craft knowledge of the teacher are resources for the teacher-researcher.

Admittedly, the proposed profile is ambitious, yet it's doable, especially in the context of community colleges whose full time mathematics faculty have PhD level experience in mathematics, physics or engineering research and can relatively easily transfer those skills into classroom-based investigations of learning. On the other hand, given the progressing collapse of public education in US, the majority (80%) of freshman students who enter every semester into our colleges require remediation to be able to get to college level courses. The remediation starts on the level of arithmetic through algebra it constitutes 80% of our "bread and butter" courses. The placement into, and exit from remediation is decided by the university wide – standard exam. Consequently, the mathematics faculty of community colleges are intimately familiar with the issues of school mathematics. The composition of research skills with the craft knowledge of teaching elementary mathematics is at the basis of the formulation of TR/NYCity Model.

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

MATHEMATICS TEACHING-RESEARCH JOURNAL ONLINE
VOL 8, N 3-4
Fall and Winter 2016/17

HISTORICAL BACKGROUND AND DEVELOPMENT OF TR/NYCITY MODEL.

Stenhouse TR Acts

TR/NYCity owns its formal origins to Action Research of Kurt Lewin (1946) and Teaching Experiment methodology of Vygotsky. TR/NYCity model finds its completion in the bisociation of Koestler (1964) leading to the Stenhouse TR acts (Rudduck and Hopkins, 1985).

Lewin proposed the Action Research methodology in the context of the quest for improvement of "group relations", a euphemism for interracial relations in US of 30ties and 40ties. He saw it as "…a comparative research on the conditions and effects of various forms of social action, and research leading to social action." His Action Research cycle consisted of the stages (or steps) of diagnosis with plan for action, implementation of action, its assessment providing at the same time the basis for "modifying overall plan" and leading to the next cycle. It was however Stenhouse who introduced Action Research methodology into education profession as teaching-research in the inaugural lecture at the University of East Anglia in 1979 presentation "Research as basis for teaching" – a theme whose importance has steadily grown till contemporary times. Already in early seventies of the last century he recognized that one of the possible explanations for the failure of research

> "…to contribute effectively to the growth of professional understanding and to the improvement of professional practice… was the reluctance of educational researchers to engage teachers as partners in, and critics of, the research results." (Rudduck and Hopkins, 1985).

The extracts from the transcripts of seminars with the part-time MA students reveal his understanding of Action Research in terms closely related to TR/NYCity model arrived at spontaneously through our work. He understood Action Research primarily as "the type of research in which the research act is necessarily a substantive act; that is an act of finding out has to be undertaken with an obligation to benefit others than research community" (p.57), in our case, students in ours, and other classrooms. However, it's the concept of "*an act* [which is] *at once an educational act and a research act*" (p.57), that completes a stage in our development of thinking technology, that is the process of integration of

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

research and learning theories with the craft knowledge of the profession anchored in practice. The bisociative framework (see below) of TR acts produces new mental conceptions, the product of thinking technology. These conceptions (e.g. schema, ZPD, hidden analogy, bisociation) become part of the discourse within the community of teacher-researchers, tools to design methodology for improvement of classroom craft and for deepening one's research interest.

It is surprising Stenhouse did not utilize Action Research cycles. It could be because the curriculum research he envisioned as conducted by teachers, apart from case studies, was to test hypotheses arrived at by curriculum research outside of the teacher's classrooms (p.50).

The second root of our methodology is anchored in the methodology of the Teaching Experiment of Vygotsky, which had a professional research team together with teachers investigate the classroom and was conducted "…*to study changes occurring in mental structures under the influence of instruction*" (Hunting, 1983). Interestingly, introduction of professional research into classroom by Vygotsky and his co-workers in the thirties was the fulfilment of the first part of the Stenhouse's vision of the seventies who demanded "*In short, (1) real classrooms have to be our laboratories, and (2) they are in command of teachers, not researchers*" (p.127). For the second part of Stenhouse vision we propose classrooms, which are in the command of teacher-researchers as the synthesis of both methodological efforts.

The Teaching Experiment methodology reappeared in the work of Steffe and Cobb (1983) as a constructivist teaching experiment, which was appropriated by Czarnocha (1999) for teaching purposes in high school class of mathematics, already as a tool of a teacher. Czarnocha (1999) realized that the constructive teaching experiment can easily become a teacher's powerful didactic instrument when transformed into guided discovery method of teaching.

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

Design Science

The interest in the work of the professional practitioner of whom teacher is but one particular example has been steadily increasing in the second half of the previous century since the work of Herb Simon (1970), the Design of the Artificial. His work proposes the design as the "*principal mark that distinguishes the professions from sciences"* (p.55-58). Kemmis and McTaggart (2000) developed the principles of Action Research, while Schon (1983) investigated the concept of a Reflective Practitioner through the process of reflection-in-action. Both frameworks had found applications in the work of teachers and researchers through joint collaborations, however the research/practice gap hasn't been yet bridged.

The terms Design Experiment, Design Research or the Science of Design are often interchangeable and they refer to the professional design in different domains of human activities. It was introduced into research in Math Education by Ann Brown (1992), Collins (1992), and Whittmann (1995). Anne Brown had realized during her exceptional career that psychological laboratory can't provide the conditions of learning present in the complex environment of a classroom and transformed her activity as a researcher directly into that very classroom as the leading co-designer and investigator of the design in the complex classroom setting. In her own words: "As a design scientist in my field, I attempt to engineer innovative classroom environments and simultaneously conduct empirical studies of these innovations" (A. Brown, 1992). She provided this way one of the first prototypes of design experiments which, theoretically generalized by Cobb et al. (2003), "entail both "engineering" particular forms of learning and systematically studying those forms of learning within the context defined by means of supporting them…". The profession has followed her lead seeing the classroom design experiments as theory based and theory producing. Paul Cobb et al. (2003) assert that Design Experiments are conducted to develop theories, not merely to empirically tune what works. Design research paradigm treats design as a strategy for developing and refining theories (Edelson, 2002). Even Gravemeyer (2009) who defines "the general goal of Design Research to investigate the possibilities for educational improvement by bringing about and studying new forms of learning" hence stating it closer to substantive quality formulated by Stenhouse, yet he warns us that "great care has to be taken to ensure that the design experiment is based on prior research…" eliminating this way the designs anchored in prior practice.

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

Unfortunately, the educational research profession cuts itself off by these restrictions from the source of profound knowledge contained in the tacit and intuitive craft knowledge of the teachers. Clearly, if the goal is improvement of learning, a more general framework is needed which recognizes both education research and teaching practice as two approaches of comparable significance, value and status.

Frameworks of Inquiry and the Unity of Educational and Research Acts

We find such a framework within the three frameworks of inquiry identified by Margaret Eisenhart (1991): theoretical, practical, and conceptual (Lester, 2010). Following Eisenhart, Lester (2010) posits three types of frameworks used in Math Education, first, the theoretical framework based upon theory i.e. the constructivist, radical constructivist and social constructivist theories discussed second, a practical framework, "… which guides research by using 'what works' … this kind of research is not guided by formal theory but by the accumulated practice knowledge of practitioners and administrators, the findings of previous research, and often the viewpoints offered by public opinion" (p. 72). The third is a conceptual framework that can pull from various theories as well as educational practice.

The theoretical framework guides research activities by its reliance on a formal theory; that is, a theory that has been developed "on the theoretical, conceptual, and philosophical foundations" (Lester, 2010) by using an established, coherent explanation of certain sorts of phenomena and relationships—Piaget's theory of intellectual development and Vygotsky's theory. However, as soon as such a theory- based design undergoes a TR cycle, the initial determinative role of theory changes into the JiTR-approach (Just-in Time-Research; see below), which allows for the participation of craft knowledge based on the teaching experience in equally significant manner.

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

The Practical Framework is employed in what we refer to as 'action research' and as discussed, it has some common components with teaching-research.

> "For Scriven, [quoted in Lester (2010)] a practical framework guides research by using "what works" in the experience of doing something by those directly involved in it. This kind of framework is not informed by formal theory but by the accumulated practice knowledge of practitioners and administrators, the findings of previous research, and often the viewpoints offered by public opinion. Research questions are derived from this knowledge base and research results are used to support, extend, or revise the practice." (Lester 2010)

However, the distinction that we make with Lester's description of a practical framework and a framework for teaching research is that we, as researchers, view the goal of teaching-research to inquire into how theory and models of learning reflect upon what the teacher and student experience in the classroom. The question for the teacher researcher and supportive TR community is what needs to be transformed or changed in the existing theories or models in order to improve the fit between these frameworks and classroom practice?

The third and final framework considered by Lester is that of

> "a conceptual framework [that] is an argument that the concepts chosen for investigation, and any anticipated relationships among them, will be appropriate and useful given the research problem under investigation. Like theoretical frameworks, conceptual frameworks are based on previous research, but conceptual frameworks are built from an array of current and possibly far-ranging sources. The framework used may be based on different theories and various aspects of practitioner knowledge" (Lester, 2010).

We argue that amongst the three frameworks for research present in philosophy of education research only the conceptual framework allows for the possibility of bisociative synthesis between teaching and research through Stenhouse TR acts.

Of special importance in working with conceptual frameworks is the notion of *justification*. A conceptual framework is an argument including different points of view

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

and culminating in a series of reasons for adopting some points and not others. The adopted ideas or concepts then serve as guides: to collecting data, and/ or to ways in which the data from a particular study will be analysed and explained (Eisenhart, 1991). According to Lester (2010) "…too often educational researchers are concerned with coming up with "good explanations" but are not concerned enough with justifying why are they doing what they are doing…" (p.73).

Our insistence on the balance between research and teaching practice, the basis for the unified Stenhouse TR acts, finds its justification and fulfilment in the bisociation of Koestler (1964) that is in "*a spontaneous leap of insight which connects previously unconnected matrices of experience*" (p. 45). A bisociative framework is the framework composed of "two unconnected matrices of experience" where one may find a "hidden analogy" – the content of insight (Chapter 1.2). Given the persistent divide and absence of deep connections between research and teaching practice, TR/NYCity constitutes a bisociative framework composed of "unconnected [in general] matrices of experience" of teaching and research, within which one can expect high degree of creativity on the part of the teacher-researcher through leaps of insight leading to the unified Stenhouse acts defined above. The process of coordination of TR/NYCity with Koestler bisociation theory is the guiding theme of Unit 2: Creative Learning Environment. Unit 2 presents the search for classroom creativity by Vrunda Prabhu during which this coordination has taken place revealing "hidden analogy" between Koestler theory and Prabhu's teaching practice.

We can state now a new definition of TR/NYCity methodology:

TR/NYCity Model is the conceptual bisociative framework of Design Research conducted by the classroom teacher, whose aim is to improve the process of learning in the classroom, and beyond – the characteristic of its "substantive nature".

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

TR bisociative framework facilitates integration or, still better, synthesis of practice and research through instances or sequences of instances of Stenhouse acts which are "at once an educational act and a research acts" (Rudduck and Hopkins, p.57). In what follows we will call them Stenhouse TR acts. The Stenhouse TR acts are the foundation stones of "thinking technology" discussed below within which their unity is naturally positioned. The facilitation of longer or shorter instances of Stenhouse TR acts can be reached from either teaching practice or from application of research to practice, as well as from both simultaneously. The "skeletal structure" (Eisenhart, 1991) of the TR/NYCity conceptual framework can be obtained as requirements and conclusions from the definition.

We discuss different designs of teaching experiments and TR investigations in Unit 4, The Teacher as a Designer of Instruction: TR Design, while in Chapter 3.2 we discuss "nuts and bolts" of classroom teaching experiment. The Introduction to Unit 4 develops the "skeletal structure" of TR/NYCity as the consequence of the definition.

TEACHING-RESEARCH CYCLE (TR CYCLE)

Just-in-Time Teaching (JiTT) and Just-in-Time Research (JiTR)

Teaching-Research cycle is the fundamental instrument in our work, which allows for the smooth integration of research and teaching practice within our conceptual framework. The difference from other similar cycles of Action Research or of the Design Experiment (Cobb et al., 2003) is simple: it allows the teacher-researcher to enter the classroom investigation from either of both directions, from research and from teaching. There is however, an important methodological trade off: whereas a Design Experiment researcher prepares the design of classroom intervention on the basis of prior research, the teacher-research uses Just-in-Time approach, that is research literature consultation takes place during the TR cycle, generally at the Analysis and Refinement nodes, when we either compare the results to assumed theory of learning, or when we search for

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

adequate theoretical framework to understand the learning situation, or in any other unclear classroom situation.

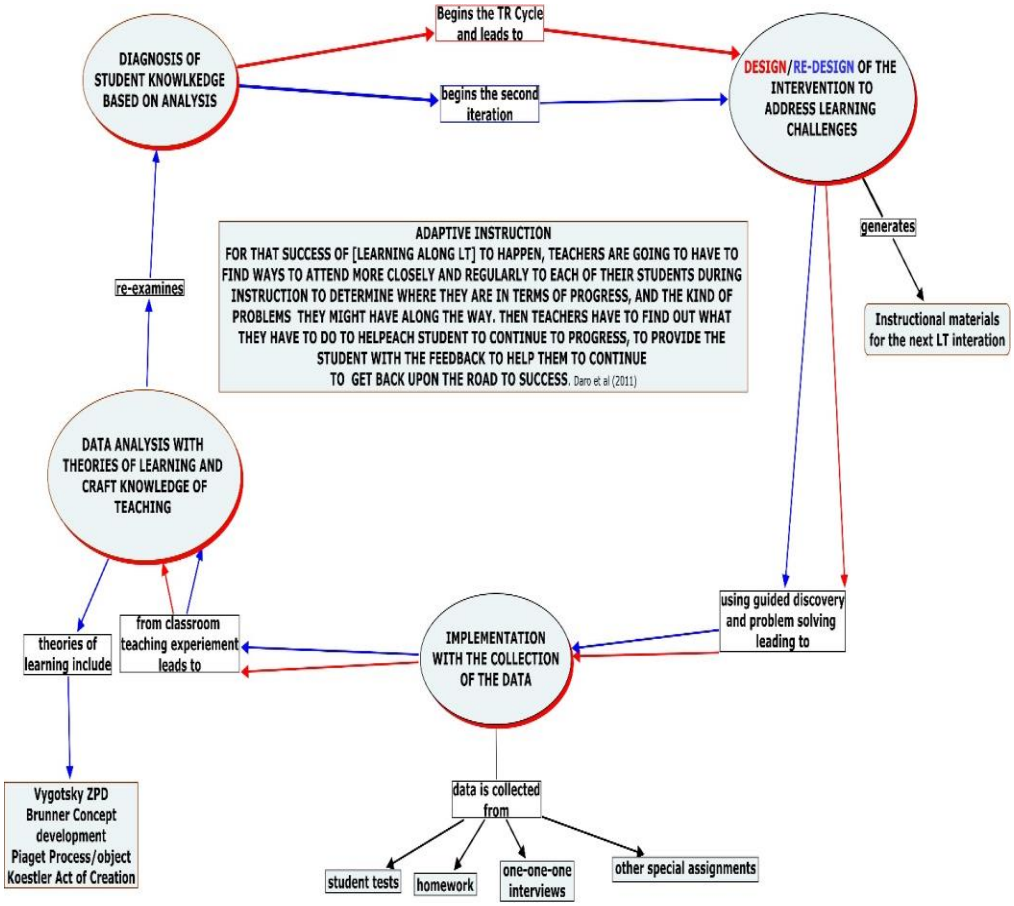
Just-in-Time Teaching (JiTT) as expressed by Novak et al. (1999) is a teaching and learning strategy based on the interaction between web-based study assignments and an active learner classroom. Students respond electronically to carefully constructed web-based assignments which are due shortly before class, and the instructor reads the student submissions "just-in-time" to adjust the classroom lesson to suit the students' needs. Thus, the heart of JiTT is the "feedback loop" formed by the students' outside-of-class preparation that fundamentally affects what happens during the subsequent in-class time together. JiTT has been used well together with Peer Leader methodology (Mazur and Watkins, 2009).

Analogically, Just-in-Time Research (JiTR) is research and teaching strategy based on the "feedback loop" formed between the didactic difficulties in the classroom encountered by a teacher who turns to educational research results that may throw light into the nature of these difficulties. At this moment, the classroom teacher makes contact with the bisociative framework of TR/NYCity model.

Anchoring TR in TR cycle.

Fig. 1. The TR Cycle

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj



It is in the introduction of educational research into the classroom that we differ from Action Research. The JiTR approach differs from standard educational research in that theory is repositioned from being a required foundation to the Just –in-Time solution for didactic difficulties in the mathematics classroom.

William J. Harrington, describing his work of a teacher-as-researcher in Laura R. Van Zoest (2006) states that, "Teachers do informal research in their classrooms all the time. We try a new lesson activity, form of evaluation, seating arrangement,

grouping of students, or style of teaching. We assess, reflect, modify, and try again, as we consider the perceived consequences of changes we made." Hence, there is a natural pathway that extends these informal activities into systematic research, offered by the TR/NYCity model that successively progresses along *Teaching-Research* (TR) cycles of diagnosis, design of instruction in response to diagnosis, collection of relevant data and its analysis, and, ultimately, with the help of relevant external research

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

results through JiTR approach, the redesign of interventions. The TR cycle, the explicit generalization of Action Research principles in the classroom, is particularly well fit into our work because of our work's naturally cyclic structure via semesters or academic years. Since every teacher has an option of repeating to teach the same course to a new cohort of students, the TR cycle allows for the continuous process of classroom investigations of the same research question during consecutive semesters. The sequential iteration of TR cycles is one of the main methodological research tools of the TR/NYCity Model facilitating the process of integration of teaching and research into a new unit of professional classroom activity, teaching-research.

TR/NYCity requires a minimum of two full TR cycles within a context of a single teaching experiment to fulfil the requirement of improvement of instruction. In its insistence on the improvement of learning through cycle iteration, TR/NYCity incorporates and generalizes the principles of Japanese and Chinese Lesson studies (Huang and Bao, 2006).

Consequently, every teaching experiment of the TR/NYCity Model has a main teaching-research question, composed of two sub-questions:

– What is the state of the students' knowledge under the impact of the new intervention?
– How to improve that state of knowledge?

The duration of the TR cycle can vary depending on intervention. In can last a year, a semester, and a couple of days or even one class. In its rudimentary form we can find it even in teacher-student inquiry dialogs (see example in Chapter 4.1).

The bisociative creativity of the teacher reaches its fulfilment during this period of reflection and redesign spurred by the simultaneous consideration of data analysis results, relevant teaching experience, relevant JiTR results from professional literature and appropriate theories of learning or conceptual development. It is precisely at this moment when the new teaching-research hypotheses are formed, leading to new theories and investigations. The focus of this teaching-research activity is the investigation of student learning followed by the design of teaching, whose effectiveness is often investigated in the subsequent TR cycle.

Instructional Adaptability of the TR/NYCity Model via TR Cycle

The increased degree of flexibility created by this integration of teaching and research within a single "tool box" helps teachers reach new levels of instructional adaptability to student learning needs. In fact, the

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

comparison of the adaptive instruction described by (Daro et al., 2011) with the TR cycle reveals a very high degree of correspondence:

> For that [success of LT framework] to happen, teachers are going to have to find ways to attend more closely and regularly to each of their students during instruction to determine where they are in their progress toward meeting the standards, and the kinds of problems they might be having along the way. Then teachers must use that information to decide what to do to help each student continue to progress, to provide students with feedback, and help them overcome their particular problems to get back on a path toward success. This is what is known as adaptive instruction and it is what practice must look like in a standards-based system.

Every TR cycle consists of the following components:

- (1) The design of the instruction/intervention, in response to the diagnosis of student knowledge,
- (2) Classroom implementation during an adequate instructional period and collection of data; this incorporates problem-solving, guided discovery classroom discourse and design of interventions for diagnosed difficulties,
- (3) Analysis of the data, in reference to existing experimental classroom data, appealing to the general theory of learning through J-i-T approach and the teacher-researcher's professional craft knowledge,
- (4→1) Design of the refined instruction based on the analysis of the data obtained in steps 1 through 3, leading to the hypothesized improvement of learning. The symbol "4→1" is intended to convey that the 4^{th} step in the cycle is equivalent to going back to the 1^{st} step in the cycle.

As a result, every such 1→2→3→4→1 is an instance of adaptive instruction– finding the level of students' understanding through tests, homework assignments and one-on-one interviews, responding to the difficulties by the re-design of the intervention, implementation and assessment. Consequently, the TR cycle is called for, as the theoretical framework of the teacher's work in a mathematics classroom driven by the Common Core Standards. Transformations of the teacher's pedagogy and improvements, based on research and evidence, have to take place exactly within such a framework. Chapters 4.2, 4.4 and 4.5 provide detailed examples of two (or more) full cycles of such an approach.

Generalization in TR/NYCity Model .

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

One of the central questions asked of frameworks related to action research is the question about the generality of our assertions. How general is TR/NYCity? Why and how that what we understand in the Bronx, has any bearing anywhere else? In terms of the original definition at the beginning of the chapter, what is the nature of the word "beyond" in that definition? TR/NYCity has three ways to generalize its findings: By coordination with a theory whose correctness has been asserted in the profession. If we coordinate our findings with a theory, then they acquire degree of generality afforded to the theory, that is one can draw conclusions from the findings in terms of the coordinated theory of learning. These conclusions might be relevant, with proper modifications to any classroom situation to which that theory applies. By running an artefact used in a TR investigation through many iterations with different cohorts of students. As a result, the artefact acquires large degree of generality, which provides the basis for its application to different new situations (Chapter 2.2).A special window of generalizations opens up when we consider student populations with similar socio-economic status to the one in the Bronx. The similarity of the socio-economic status results in similar cognitive/affective challenges experienced by students to which similar adaptive interventions are needed (Kitchen et al.) The successful generalization of TR/NYCity artefacts has been reached amongst Indian Dalits (downtrodden) of Tamil Nadu (Chapters 2.2 and 5.3.1) and in Poland amongst rural students of Southern Poland (Czarnocha, 2008). The discussion of artefacts in the context of Design Research (Unit 4) brings forth an important clarification that its generalization can be obtained by expanding its application to similar student populations.

Thinking Technology

The dictionary definition of technology is "the application of scientific knowledge for practical purposes, especially in industry." Thinking technology in TR/NYCity model is

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

the process of integration of research results and framework with craft knowledge of the teacher. This spontaneous process inherent for TR/NYCity model finds its elegant expression in Koestler bisociation theory and Stenhouse TR acts.

It is a very subtle process, in which scientific concepts such as "hidden analogy" of Koestler become the critical tools, metaphors with the help of which we start to identify classroom situations, the term becomes a phrase with the help of which we, members of the TR team start communicate with each other in our own new language. In fact, by making the connection between scientific meaning and classroom situation we create the analogy between two generally separate matrices of thinking – hence the connection itself is a new bisociation, a possibility of new meaning.

One could conjecture that any process of coordination (as distinct from application) of a theory of learning with elements of teaching practice is the bisociative creative process during which new connections and therefore new meanings are made.

The process of coordinating research and teaching practice is facilitated by the duality inherent in the teacher-researcher work (Malara and Zan, 2002). The practice of teaching-research duality creates a new mental attitude promoting a novel design of instructional methodologies while, at the same time, requiring an investigative probe into student thinking, on the basis of which consequential teaching and research decisions are made. This duality is explored deeper in Units 2 and 4. The exploration together with utilization of the duality is conducted by the classroom teacher-researcher. In this process, teachers are not solely engaged in research on learning, they are also engaged in the transformation of teaching on the basis of, and through that research. This means that they do not simply incorporate the results of research into their teaching practice but rather allow methods of research to become the methods of teaching leading to Stenhouse TR acts. Thus the route towards Stenhause TR acts is through the process of integrating research knowledge and craft knowledge in practice of teaching. In this process, teachers

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

do not switch into a role of researcher, instead, they oscillate between the role of a teacher and the role of a researcher and fuse their efforts toward a new unit of professional activity – bisociative teaching-research with its Stenhouse TR acts.

TR/NYCITY AND THE DISCOVERY METHOD OF TEACHING.

The discovery method of teaching has been the preferred instructional method by the teacher-research team working with and developing TR/NYCity methodology since its inception. The Discovery method of teaching has a fundamental role in the TR/NYCity model. This method was introduced into TR/NYCity via the Texan Discovery method created and formulated by R. L. Moore, a topologist brought up by the Chicago school of mathematical thought of the thirties. B. Czarnocha and V. Prabhu adopted this method during their NSF grant in calculus 2002-2006. However, our understanding of its role in TR classrooms came with time through many TR investigations and teaching experiments. Using different approaches such a "guided discovery method", "inquiry method" or "inquiry leading to discovery", it has appealed to our imagination and practice as teacher-researchers because with its help we could lay bare student authentic thinking for our investigations.

On the one hand, from the educational aspect Discovery method provides learning environment best suitable for facilitation of bisociation. According to Koestler (1964) subjective, individual bisociation are more often encountered in the condition of "untutored learning". The Discovery method is one of the closest classroom approximations of this condition. This approach to teaching relies on designing situations and using techniques, which allow the student to participate in the discovery of mathematical knowledge. These are authentic moments of discovery with respect to student's own knowledge, which in the further development of methodology are related to subjective Aha! Moments of Arthur Koestler (Chapter 1.2).

On the other hand, from the research point of view, it is the best instrument, which opens the nature of student thinking to us, teacher-researchers for investigation through careful

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

interaction. It allows us to investigate and to extend the scope of students' ZPD, to help in eliminating misconception as well as in facilitating bisociations. Thus the process of TR together with Discovery method of teaching constitutes an extended in time Stenhouse TR act.

Creativity: From Bathos to Pathos – From Habit to Originality

The institution of creativity as the structural component generated within the learning environment provided by teaching-research has significant consequences beyond its cognitive importance.

Vrunda Prabhu has found out (Chapter 2.4) that student success in her classroom depended on three closely connected components of (i) cognition, (ii) motivation and (iii) self-regulated student learning (Prabhu, 2006). More specifically, when creativity is explicitly nurtured and facilitated in a mathematics classroom in the context of such an integrated learning environment, it can transform the habit of distaste toward mathematics into mathematical originality supporting Koestler's assertion that "creativity means breaking up habits and joining the fragments into new synthesis" (p. 619). Moreover, according to Koestler:

> The creative act, by connecting previously unrelated dimensions of experience, enables him [the inquirer] to attain a higher level of mental evolution. It is an act of liberation – the defeat of habit by originality.

Habitual dislike of mathematics is, at present, one of the main student obstacles for success in mathematics learning that could be eliminated with the help of that "act of liberation" providing a pathway from Bathos to Pathos, using Koestler metaphor (p. 96).

Summary of the argument

To summarize the argument, TR/NYCity is the generalization of Action research and of the Design experiment methodology (Design experiment methodology is seen here as the

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

further development of the Teaching Experiment of Vygotsky school in Russia). In its original vision it was seen as the bridge between the two methodologies, which eliminates the limitations of both – a new integrative conceptual framework. By the same token, TR/NYCity is designed specifically to bridge the gap between research and teaching practice – one of the fundamental obstacles in the effective transformation of mathematics education. The need for such a bridge was indicated by the report of US National Research Council, How People Learn-Bridging Research and Practice (Donovan et al., 1999). We review below essential components of the research/teaching practice gap in our profession as seen by contemporary reports.

GAP BETWEEN RESEARCH AND PRACTICE

English (2010a) notes that the complexity of educational theory and philosophy, has lead to a gap between educators and researcher based upon concerns about the relevancy of such philosophies to educational practice,

> "The elevation of theory and philosophy in mathematics education scholarship could be considered somewhat contradictory to the growing concerns for enhancing the relevance and usefulness of research in mathematics education. These concerns reflect an apparent scepticism that theory-driven research can be relevant to and improve the teaching and learning of mathematics in the classroom. Such scepticism is not surprising...claims that theoretical considerations have limited application in the reality of the classroom or other learning contexts have been numerous...it remains one of our many challenges to demonstrate how theoretical and philosophical considerations can enhance the teaching and learning of mathematics in the classroom..." (p.66).

Harel (2010) and Lester (2010) both note that government funding agencies and panels created to direct government research efforts are increasing restricting their attention to quasi experimental-control group efforts with a goal of what works i.e. action research. They advance the hypothesis that more attention to research frameworks would perhaps counter the ideology that all research should be practical-statistical i.e. scientific based methodology based upon a p value indicating success or failure i.e. 'what works.' Harel's

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

(2010) claim that attention to frameworks is lacking in educational research is due in part to his belief that there exists "...a feeling on the part of many researchers that they are not qualified to engage in work involving theoretical and philosophical considerations."(p.88-89) The issue that arises for those of us advocating for a more active role of teachers in integrating educational research and craft is that, if researchers feel they are not qualified then how much more likely those teachers feel unqualified. That is, how can practical research methodology such as that used in action research be expected to integrate theory and practice in a meaningful way when its practitioners may feel unqualified to engage in theoretical considerations? This question is particularly relevant to us because we strongly believe in order for reform efforts, indeed, any research based pedagogy to actually improve education there must be a sustained effort in the school and that any such effort must involve the teacher and the researcher working together or a teacher-researcher to determine what works as well as to reflect upon why it does or does not work from both a practical craft level as well as through the lens of theoretical framework.

Another reason reform effort to improve mathematics education through theoretical considerations has floundered is that mathematical education theories are often appear impractical to the craft practitioner to implement i.e. theories that provide little guidance for instructional design but within the research community there is often contradictory positions about such efforts. The result is that reform efforts and counter reactionary movements tend to arise and disappear like last year's fashion statements. Sriramen and English (2010) comment on an early attempt by mathematicians to change traditional mathematics called New Math which in the 50's and 60's tried to change the rigidity of traditional mathematic through a top down approach to pedagogical change. "One must understand that the intentions of mathematicians such as Max Beberman and Edward Begle was to change the mindless rigidity of traditional mathematics. They did so by emphasizing the whys and the deeper structures of mathematics rather than the how's but

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

in hindsight…it seems futile to impose a top-down approach to the implementation of the New Math approach…" (p.21). Goldin (2003) notes how behaviourism led to a back to basics counter movement within mathematics education: "behaviourism was fuelling the 'back to basics' counterrevolution to the 'new mathematics', which had been largely a mathematician-led movement. School curricular objectives were being rewritten across the USA to decompose them into discrete, testable behaviours" (p.192). Goldin (2003) also notes that constructivism has more recently displaced this back to basic reactionary movement. "Radical constructivism helped overthrow dismissive behaviourism, rendering not only legitimate but highly desirable the qualitative study of students' individual reasoning processes and discussions of their internal cognitions" (p.196). Yet he warns that the excessive of radical constructivism will render it impractical and unsuitable "Constructivists excluded the very possibility of 'objective' knowledge about the real world, focusing solely on individuals' 'experiential world'" (p.193).

The point being that a top-down approach to educational reform by research experts has not succeeded and we venture will never succeed without first teacher buy in, but this is not near enough, in order for the craft practitioner to continue to implement reform methodology and to design instruction based upon theory, when the researcher goes back to academia the teacher must internalize the theory and even more how such theory relates to design of instruction. Yet we consider that even this is not enough to sustain reform efforts especially with underserved populations that demonstrate serious negative affect with mathematics. The approach to educational research in which experiments have a beginning and an end is founded upon an underlying assumption that some truth can be found that will dramatically change educational practice. This assumption needs to be re-evaluated if educational craft practice is to actualize the benefits of research. We consider that a constant collaboration between educational researchers and teachers is needed and provides the best hope of actualizing change in educational practice to close widening gap between research and theory and the scepticism it has caused. Boote (2010)

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

comments on the need for continual teacher development based upon design research in improving educational practice: "Indeed, the professional development of all participants may be more important and sustaining than the educational practices developed or the artefacts and knowledge gained" (p.164). Examples of such an international professional development of teacher-researchers based on TR/NYCity methodology are discussed in the Unit 5.

THE COMPARISON BETWEEN TEACHING-RESEARCH AND DESIGN-BASED RESEARCH

The discussion in this section is the continuation of the theme found in the section *Frameworks of Inquiry and the Unity of Educational and Research Acts*, which gets further clarification in the Introduction to Unit 4. Our aim here is to provide a detailed comparison between theoretical and practical frameworks as seen from the point of view of TR/NYCity, which we see as the conceptual framework creating the bridge between the two via TR cycle.

Research, in particular, design-based research	**Teaching-Research, in particular TR/NYCity Model**
Theory driven: (EDUCATIONAL PSYCHOLOGIST, 39(4), 199–201 Copyright © 2004, Lawrence Erlbaum Associates, Inc. William A. Sandoval, Philip Bell Design-Based Research Methods for Studying Learning in Context: Introduction.) Design-based research can contribute to theoretical understanding of learning in complex settings. Each of the articles by Sandoval, Tabak, and Joseph reveal how the design of complex interventions is an explicitly theory-driven activity.	**Practice driven:** (*Professional Development of Teacher-Researchers*, Rzeszow University, Poland, 2008) (*Teaching Experiment NYCity Method*. 2004) Teaching-research is grounded in the craft knowledge of teachers that provides the initial source and motivation for classroom research; it then leads to the practice-based design. Its aim is the improvement of learning in the classroom as well as beyond.
Use of Theories of Learning in Design-Based Research: (Educational Researcher, Vol. 32, No. 1,	**Use of Theories of Learning in Teaching-Research:** (Dydaktyka Matematyki, 2006, v.29,

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

pp. 5–8), (Design-Based Research: An Emerging Paradigm for Educational Inquiry by The Design-Based Research Collective, 2003) In addition, the design of innovations enables us to create learning conditions that learning theory suggests are productive, but that are not commonly practiced or are not well understood.	Poland, Teaching-Research NYCity Model. B. Czarnocha, V. Prabhu) The design of innovation enables the teacher-researcher to create the Creative Learning Environment based on teacher's craft knowledge, which improves learning in the classroom and transforms habits such as misconceptions, into student originality (Koestler, 1964). Learning theories are used as needed to support teachers' craft knowledge.
Focus of the Teaching Experiment in Design-Based Research: (Journal for Research in Mathematics Education. 14(2) pp.83-94, 1983, Cobb, P. and Steffe, L. P., The Constructivist Researcher as Teacher and Mod el Builder) Cobb and Steffe assert that the interest of a researcher during the teaching experiment in the classroom is "in hypothesizing what the child might learn and finding [as a teacher] ways and means of fostering that learning".	**Focus of the Teaching Experiment in Teaching-Research:** Proceedings of the epiSTEME Conference, Bombay, Homi Bhabha Institute, 2007, B. Czarnocha, V. Prabhu Teaching-Research and Design Experiment – Two Methodologies of Integrating Research and Classroom Practice) …The interest of a teacher-researcher is to formulate ways and means to foster what a student needs to learn in order to reach a particular moment of discovery or to master a particular concept of the curriculum (Czarnocha, 1999). Since, however, "such moments occur only within students' autonomous cognitive structures, the [constructivist] teacher has to investigate these structures during a particular instructional sequence [in order to be of help to the students]. In this capacity, he or she acts as a researcher".
Use of Iteration in design-based research: (ICLS, 1, pp.968-975, 2010, Confrey, J., Maloney, A., The	Use of Iteration in TR/NYCity model: Step 1: Process of iteration, starting with the first iteration designed on the basis of teaching practice.

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

construction, refinement and early validation of the equi-partitioning Learning Trajectory) …articulating, refining and validating is an "iterative process of research synthesis and empirical investigations involving" many types of evidence. Step 1: Meta-research of the concept to create the prototype. Step 2: Iterative refinement of the prototype	Step 2: Incorporation of research results as needed in between consecutive iterations. It is the concept of iteration of the design from semester to semester together with the related refinement that can bring in now relevant research results illuminating the classroom situation or providing help in the design of appropriate set of assignments.

The TR cycle through its natural iteration of teacher's activity from semester to semester provides the opportunity to move beyond the narrow " chicken or the egg" question of "What is the primary, or the more important realm, — research or practice?" and to creatively integrate design-based practice and design based research (see Unit 4).

REFERENCES

Bishop, A. (2000). Research, Effectiveness, and the Practitioners' World. In R. Lesh and A. E. Kelly (Eds.), *Handbook of Research Design in Mathematics and Science Education*. Hillsdale, NJ: Lawrence Erlbaum.

Boote, D., N. (2010) Commentary 3 on Re-Conceptualizing Mathematics Education as a Design Science In. B. Sriramen and L. English (Eds.) *Theories of Mathematics Education: Seeking New Frontiers* (pp.121-122) Spring Verlag, Berlin Heidelberg.

Brown, A.L. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom setting. *The Journal of Learning Sciences*, 2 (2), pp. 141-178**.**

Collins, A. (1992) Towards a design Science of Education In E. Scanlon and T. O'Shea (eds) *New Directions in Educational technology*. Berlin: Springer-Verlag, 1992.

Confrey, J. and Maloney, A. (2010). The construction, refinement and early validation of the equi-partitioning Learning Trajectory. *ICLS*, 1, pp. 968-975**.**

Cobb, P. and Steffe, L. P. (1983). The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education*, 14(2), pp. 83-94.

Czarnocha, B. (2014) On the Culture of Creativity in Mathematics Education, in *Special Issue Journal of Teaching Innovations*, vol.27, N 3. University of Belgrade, Serbia

Czarnocha, B. (1999). El Maestro Constructivista Como Investigador. In *Educacion Matematica*, 11(2) pp. 52-63.

Czarnocha, B. (2008). Ethics of Teacher-Researchers. In Bronislaw Czarnocha (Ed.), *Handbook of Mathematics Teaching-Research – A tool for Teachers-Researchers*. Poland: University of Reszów.

Czarnocha, B., Prabhu, V. (2004). Design Experiment and Teaching-Research – two Methodologies to Integrate research and Teaching Practice, *epiSTEME*, 1, Goa, India. Retrieved from http://www.hbcse.tifr.res.in/episteme

Czarnocha, B. and Prabhu, V. (2006) Teaching-Research NYCity Model. *Dydaktyka Matematyki*, 29, Poland.

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

Daro, P., Mosher, F. and Corcoran, T. (2011). Learning Trajectories in Mathematics. *Consortium for Policy Research in Education*. Columbia University: Teachers College.

Design-Based Research Collective. (2003) Design-Based Research: An Emerging Paradigm for Educational Inquiry. *Educational Researcher*, 32 (1), pp. 5–8.

Donovan, S., Bransford, J.D., Pellegrino, J.W. (1999) How *People Learn: Bridging Research and Practice.* National Academy Press

Duval, R. (July 23-27, 2000). Basic issues for research in mathematics education. In Tadao Nakahara, Masataka Koyama (Eds.) *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education* (PME 24), Hiroshima, Japan.

Edelson, D. (2002) Design Research: What we Learn When we Engage in Design. *The Journal of the Learning Sciences*, vol.11, No.1 pp.105-121

Eisenhart, 1991 Conceptual Frameworks for Research circa 1991: Ideas from a cultural Anthropologist; Implications for Mathematics Education Researchers in Underhill, Robert G. (Ed) *North American Chapter of the International Group for the Psychology of Mathematics Education, Proceedings of the Annual Meeting* (13th, Blacksburg, Virginia, October 16-19, 1991)

English L. D. (2010) Preface to Part III: Theoretical, Conceptual, and Philosophical Foundations for Mathematics Education Research: Timeless Necessities, In. B. Sriramen and L. English (Eds.) *Theories of Mathematics Education: Seeking New Frontiers* (pp.121-122) Spring Verlag, Berlin Heidelberg.

Feldman, A. and Minstrel, J. (2000). Action Research as a Research Methodology for the Study of the Teaching and Learning of Science. In A.E. Kelly and R.E. Lesh (Eds.*) Handbook of Research Design in Mathematics and Science Education*. Mahwah, NJ: Lawrence Erlbaum Associates.

Goldin, G.A. (2003) Developing Complex Understanding: On the relation of mathematical education research to mathematics, *Educational Studies in Mathematics*, 54, pp. 171-202.

Gravemeijer, K., van Erde, D. (2009) Design Research as means of building Knowledge Base for Teachers and Teaching in Mathematics Education. *The Elementary School Journal*, Vol.109, No.5 pp.510-524

Harel G. (2010) Commentary on: On the Theoretical, Conceptual, and Philosophical Foundations for Mathematics Education Research in Mathematics Education, In B. Sriramen and L. English (Eds.). *Theories of Mathematics Education: Seeking New Frontiers* (pp.87-96), Springer Verlag, Dordrecht Heidelberg London New York.

Rongjin Huang and Jiamsheng Bao (2006) Towarsd a Model for Teacher Professional Development in China: Introducing Keli, *Journal of Mathematics Teacher Education* (2006) 9:279–298© Springer 2006

Hunting, R**. P. (1983)**. Emerging methodologies for understanding internal processes governing children's mathematical behaviour**.** ***Aust****ralian Journal of Education*, 27(1), pp. **45-61.**

Kemmis, S., and McTaggart, R. (2000). Participatory action research. In N. Denzin and Y. Lincoln (Eds.), *Handbook of qualitative research* (2nd ed., pp. 567–605). Thousand Oaks, CA: Sage.

Kieran, C., Krainer, K., and Shaughnessy, J. M. (2013). Linking Research to Practice: Teachers as Key Stakeholders in Mathematics Education Research. In Clements, M. A., Bishop, A., Keitel, C., Kilpatrick, J., and Leung, F. (Eds.), *Third International Handbook of Mathematics Education, Vol. B*. Netherlands: Springer.

Koestler, A. (1964). *The act of creation*. London: Hutchinson and Co.

Lester Jr., F. K. (2010) On the Theoretical, Conceptual, and Philosophical Foundations for Mathematics Education Research in Mathematics Education, In *Theories of Mathematics Education: Seeking New Frontiers, B. Sriramen and L. English (Eds.) (pp.67-86), Spring Verlag, Berlin Heidelberg.*

Lewin, K. (1946). Action research and minority problems. *J. Soc. Issues*, 2(4), pp. 34-46.

Malara, N. A. and Zan, R. (2002). The Problematic relationship between theory and practice in English, L. (Ed.) *Handbook of international research in mathematics education*, NJ: Lawrence Erlbaum Associates, Inc., pp. 553-580.

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj

Mazur and Watkins, 2009 Using JiTT with Peer Instruction Jessica Watkins and Eric Mazur in *Just in Time Teaching Across the Disciplines*, Ed. Scott Simkins and Mark Maier, pp. 39-62 (Stylus Publishing, Sterling, VA, 2009)

Mohr, M. (1996). *Ethics and Standards for Teacher Research: Drafts and Decisions*, conference paper delivered at AERA conference, New York.

Novak, G, Patterson, E.T., Gavrin, A.D., and Christian, W. (1999). *Just-In-Time Teaching: Blending Active Learning with Web Technology*, Upper Saddle River, NJ: Prentice Hall.

Pawlowski, M. (2001). Badać jak uczyć. *Nauczyciele i Matematyka*, 38, pp. 22-25.

Prabhu, V. and Czarnocha, B. (2015). Problem-posing and problem-solving dynamics in the context of Teaching-Research and Discovery Method. In Singer, M., Cai, J. and Ellerton, N. *Problem Posing*, Springer.

Rudduck and Hopkins (1985*) Research as the Basis for Teaching. Readings from the work of Lawrence Stenhouse.*Heinemann Educational Books.

Sandoval, W. and Bell, P. (2004). Design-Based Research Methods for Studying Learning in Context: Introduction. *Educational Researcher*, 32 (1), pp. 5–8.

Shulman, L. S., and Keislar, E. R. (Eds.). (1966). *Learning by discovery: A critical appraisal*. Chicago: Rand McNally and Company.

Simon, Herb (1970) *the Science of the Artificial*, MIT Press, Cambridge, Mass.

Sriramen and English (2010) Preliminary Remarks: Surveying Theories and Philosophies of Mathematics Education, In *Theories of Mathematics Education: Seeking New Frontiers*, B. Sriramen and L. English (Eds.) (p.7) Springer Verlag, Berlin Heidelberg.

Whittman, E. (1995) Mathematics Education as a Design Science. *Educational Studies in Mathematics*, Vol.29, No. 4 (Dec., 1995), pp.355-374 Springer

Van Zoest, L. (2006). *Teachers engaged in research: Inquiry in mathematics classrooms, Grades 9-12* (PB), NCTM.

AFFILIATIONS

Bronislaw Czarnocha, Ph.D.

Mathematics Department

Eugenio Maria de Hostos Community College

City University of New York

Readers are free to copy, display, and distribute this article, as long as the work is attributed to the author(s) and Mathematics Teaching-Research Journal On-Line, it is distributed for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MT-RJoL. MT-RJoL is published jointly by the Bronx Colleges of the City University of New York.
www.hostos.cuny.edu/departments/math/mtrj