Book Review for *MÖBIUS AND HIS BAND* edited by J. Fauvel, R. Flood, and R. Wilson

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Title: Möbius and His Band

Author: (edited by) John Fauvel, Raymond Flood and Robin Wilson

Place: New York, NY

Publisher: Oxford University Press

Publication Date: 1993

Pages: 172

Special Features: none

Price: $29.95

ISBN: 0-19-853969-X

Reviewer: Roy Berglund

Although this informative book is a collection of essays by six authors, each with a different perspective of Möbius's contributions, there is no disruption in the historical theme, rather a consistency that is quite seamless; the chapters can be read in any order without loss of continuity. The final chapter contains research done by others in subsequent years, such as optimization in Morse Theory, dynamical systems, chaos, relativity and quantum mechanics, which were not yet discovered in the age of Möbius.

August Ferdinand Möbius was born on 17 November 1790 and died on 26 September 1868 in an area of Saxony between Leipzig and Jena, between Prussia to the North and the Hapsburg Empire to the South. Thus, Möbius considered himself a Saxon and not a German. At this time the development of mathematics to higher levels paralleled the rise of Germany from independent states, through invasions, wars and revolutions, to an empire under the military might of Prussia of Frederick the Great, while collaterally France was engaged in a revolution that utilized all of its resources, with little emphasis on the arts and mathematics.

This was an age of transition in arts, sciences and politics throughout most of Europe. Mozart (an Austrian), Goethe, Gauss and Beethoven were flourishing, the French revolution standardized weights and measures into the Metric system, as Napoleon Bonaparte came to power, and most of Europe was at war.

Not much is known about Möbius's early life and education. However, he entered the University of Leipzig at age 18, and first studied law but changed to mathematics, physics and astronomy (under Karl Mollweide). Möbius visited Göttingen to study astronomy under Gauss, who was the director of the observatory there. He later visited Halle, where he studied mathematics under Johann Pfaff, who had been the teacher of Gauss.

Möbius considered joining the Prussian army, because his life as a teacher met with little success. Students attended only if the lectures were free, and also because in his unsatisfying role as a mathematics teacher he worked only with low-level students on calculations. Because he was not Prussian, he bolstered his Saxon pride, avoided military service and returned to finish his habilitation thesis. In 1816 was appointed Professor of Astronomy at the University of Leipzig, where he remained for the rest of his life. In 1848 he was promoted to Director of the Observatory. Thus, Möbius spent most of his life as an astronomer, although he is remembered for his mathematics.

At the turn of the 19th century France promoted mathematics significantly, both for social advancement and as service to the state. Lagrange at the Ecole Polytechnique, along with Laplace, Monge, and Legendre were doing everything from teaching to research. Germany had nothing like this, but after Napoleon's defeat of Germany at the Battle of Jena in 1806, Germany developed a new national pride mainly through the University of Berlin, where the research-oriented professional philosophy emerged, especially for mathematics, and constituted a new unique approach to professional mathematics enduring into contemporary times that involved teaching, research and graduate seminars, thereby creating new knowledge as well as teaching it, and with research taking prominence. Prior to this time teaching and research were separate and distinct disciplines that did not interact. It is important to note the constant alternation in mathematics evolution of France, then Germany, as a result of war and politics.

In the 19th century Germany took over the mathematical leadership from France, the most important German mathematician being Gauss. The increased rigor in German schools, with teachers doing research as well as teaching, resulted in mathematics becoming a subject in its own right and as an independent discipline in the university. High schools, or *Gymnasien,* followed the same protocol: the professionalization of teachers, and the institutionalization of mathematics as an independent or autonomous discipline. Student seminars were held once a week, and provided students the opportunity to present a topic, to be criticized and guided by their professors. This introduced students to research publications and research techniques, resulting in increased emphasis on pure mathematics.

The *Schulprogramme* stimulated teachers in the lower schools to participate by publishing material on basic topics. A return to basic synthetic geometry (J. Steiner) began the resurgence of geometry versus arithmetization, with emphasis on descriptive and projective geometry. The priority in English and Scottish universities was to teach, not to conduct research, whereas in German universities, research by means of the Ph.D. programs was to give all candidates some research exposure. Hence, German replaced Latin as the academic language, due to the proliferation of German science periodicals beginning with the appearance of Crelle's *Journal for Pure and Applied Mathematics* in the 1820s, followed by other journals to make research results more accessible.

Halley's comet appeared once again in 1835 and spurred Möbius to produce two popular treatises which analyzed the comet's orbit and the wider laws of astronomy in order to perfect the knowledge of solar system dynamics, including a more accurate positioning based on Newton's laws of motion. Möbius's work *Mechanik des Himmels* (1843) presented basic information of celestial mechanics to the amateur. Heinrich Olbers of Bremen was the foremost comet expert and discovered several comets as an amateur astronomer. The observatory at Königsberg was to be headed by Friedrich Bessel, who was both a superlative theoretician and a meticulous observer, and these observatories were state established by the local regent, or king of Prussia. Möbius was appointed astronomer (observer), and later director in 1848 at Leipzig University.

Although Möbius was not an astronomer by vocation, his own mathematical ideas and researches were conducted in a context of intense astronomical activity, taking place in the discipline in which he held a professorial chair. Thus, Möbius had access to the latest developments in astronomy and was able to relate this to his teaching This confluence of talented astronomers, scientists and instrument makers continued up to 1914 in which Germany's role was paramount.

In his day Möbius was most known for astronomy, but the work that established him among mathematicians was his work in geometry and mechanics. Geometry diminished in importance in the 18th century, eclipsed by algebra and calculus; Lagrange codified mechanics in terms of calculus. However, Monge, as head of the Ecole Polytechnique and a geometer, brought geometry back as the core of the curriculum. Jean Poncelet created projective geometry. Louis Poinsot gave a geometrical description of the way a body rotates. Möbius independently studied mechanics by means of geometry and algebra. German geometers Plücker and Hesse championed the geometric approach to mechanics.

Möbius is best known for his Möbius band or strip, conceived from his study and prize for the geometric theory of polyhedrons. His other discoveries include the Möbius function, inversion formula, transformation and the Möbius net.

In 1827 Möbius published a book on barycentric calculus, introducing barycentric coordinates, to describe lines and conics and their transformations. Möbius solved the center of gravity problems by recourse to law of the lever, which in three dimensions becomes the barycenter of the object.

In the 1830s Möbius studied statics, considering questions such as "what effect do certain forces have on the behavior of a rigid body?", and then producing a book on the subject.

Möbius had influence and contributions to topology, although in his time, there was no such discipline, the Möbius band notwithstanding. In the 18th century Euler related the vertices, edges and faces of pyramids, prisms, crystals of various kinds to produce the formula V-E+F=2. However, the Swiss mathematician Simon Lhuillier showed that in some cases Euler's formula is incorrect, since this depends on the number of 'holes' in a solid. Möbius described and explained the one-sided notion of his band as non-orientability. Another mathematician, Johann Listing, who studied and worked with Gauss, and from whom his ideas of topology derived, consequently published a book on the generalization of Euler's theorem on polyhedrons in 1861, in which he described the Möbius band. Riemann's study of many-valued functions, described geometrically as Riemann surfaces, led to Felix Klein's concept of automorphic functions. Thus, two main threads of topology emerged: the analytic and the algebraic. The prominent ideas that interested Möbius were topology, symmetry, and celestial mechanics: topology by means of the Möbius strip; symmetry in crystallography; celestial mechanics as in planetary motion. Möbius's contribution was doing mathematics effectively and concentrating on what's important, which is the essence of his modern legacy.

This review has tried to demonstrate the interdependency of scientific research and the conditions of the society within which it occurs, since the scientist and his society do not operate in mutually exclusive realms. This book should be of particular interest to scientists and sociologists pursuing the history of science and technology.