Exploring students' discoveries based on Inquiry-Based Learning Strategy

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INTRODUCTION

Inquiry-Based Learning (IBL) strategy is a student-centered learning method (Lee 2011). Recent studies show that student-centered learning methods are effective at motivating students and enhancing their learning experience (Boaler 2016; Kim & Lee, 2011). The IBL strategy increases students' curiosity, and it helps students explore problem-solving strategies (Von Reness & Ecke, 2017). Thus, researchers acknowledge that the IBL strategy leads to many positive student outcomes in undergraduate mathematics (Kuster, G., Johnson, E., Keene, K., & Andrews-Larson, C 2018). In addition, students' self-led exploration allows them to discover various approaches to solving problems. However, most of the IBL research was conducted at four-year institutions with upper-level mathematics courses (Von Reness & Ecke 2017). This paper applies the same conceptual approach to learning in a community college setting, an experiment which has not done before. This experiment demonstrates how the IBL setting helps community college students to develop and enhance their learning. Also, it demonstrates how community college students get engaged and discover their own problem-solving strategies in IBL setting class. This study was conducted in a college credit bearing class precalculus, focusing on the topic of developing approaches to simplify powers of i.

THE PURPOSE OF THE STUDY

The main purpose of the study is to explore if the IBL strategy encourages students' learning in a community college. Although various researchers demonstrated that student learning significantly improved in IBL setting, it is not a well-known or widely known instructional strategy at community colleges. Therefore, this study seeks to apply the IBL setting in a community college to assess whether or not a similar result can be expected compared to other existing studies. The following research questions support the aim of the study:

1. Can it be possible to have similar expectations, such of that developing curiosity, and enhancing student learning both at four-year institutions as well as at community colleges?

2. Can the IBL setting enhance students' appetite for self-led learning, and incentivize them to find varied approaches to finding solutions?

3. Can the IBL method increase student engagement with other students and increase collaboration?

Simplifying Powers of *i*

The imaginary number is defined to be of the form $a+bi$, where $i=\sqrt{-1}$, and a,b ∈ ℝ. Thus, any non-real values, such as $\sqrt{-3}$, $\sqrt{-5}$, and $\sqrt{-9}$ are indicated as $i\sqrt{3}$, $i\sqrt{5}$, and $3i$. It is because all those values can be expressed as $\sqrt{3}\cdot\sqrt{-1}$, $\sqrt{5}\cdot\sqrt{-1}$ and $\sqrt{9}\cdot\sqrt{-1}$. (Miller and Gerken2015; Larson, 2017). One of the topics in the college algebra or precalculus is simplifying the powers of i. The concept is basically defined based on $i^2=-1$, and developed others:

$$i=i$$
$$i^2=-1$$
$$i^3=-i$$
$$i^4=1$$
$$i^5=i$$
$$i^6=-1$$
$$i^7=-i$$
$$i^8=1$$
$$\vdots$$

Figure 1: Patterns of simplifying power of *i*.

Based on these patterns, students are asked to simplify higher power of i and define equivalent value. For example, $i^{11}=-i$, and $i^{10}=-1$. The main aim of this experiment is to explore how many different approaches to simplifying the power of i community college students can find. Does IBL setting increase students learning appetite, and assist them at finding multiple approaches for problem-solving?

TYPICAL APPROACHES TO SIMPLIFYING POWERS OF *i*

Based on the pattern of the power of i, the most well-known approach is dividing the exponent by four. It is because the pattern is rotated with four different values; i ,-1, $-i$, and 1. Hence, if the remainder is 0, then the answer is 1; if the remainder is 1, then the answer is i; if the remainder is 2, then the answer is -1; and, finally, if the remainder is 3, then the answer is -*i*.

Another well-known approach is dividing the exponent by two. In this case, the remainder is always either 0 or -1. Thus, we need to look at the dividend. If the dividend is odd, and the remainder is zero, then the answer is -1, but if the remainder is 1, then the answer is -*i*. If the dividend is even, and no remainder, then the answer is 1, and if the remainder is 1, then the answer is *i*.

Another less frequently taught approach is to use the multiple of 100. Whenever the exponent is more than a three-digit number, the last two digits need to be analyzed. For example, i^{23411} , the exponent is more than a three-digit number. The exponent 23411 can be re-written as 23400+11. Now, the expression becomes $i^{23400} \cdot i^{11}$. Since 23400 is a multiple of 4, i^{23400} is equal to 1. It implies that $i^{23400} \cdot i^{11} = (i^{100})^{234} \cdot i^{11}. = ((i^4)^{25})^{234} \cdot i^{11} = 1 \cdot i^{11} = i^8 \cdot i^3 = -1.$

METHODOLOGY

The study demonstrates how the instructor taught the topic "Complex Number" based on IBL strategy. The researcher takes the role of the instructor and teaches the topic. There are twenty-five students participating in the study: thirteen males and twelve females. It is the first time IBL research is being applied in a mathematics class at a community college, so it doesn't compare students learning between the IBL group and an ordinary null group.

A new IBL activity needs to be created because the existing IBL activities are based on upper-level college mathematics classes. The researcher created this IBL experiment to suit a community college-level mathematics course. Among the complex topics, this article focuses on the topic "Simplifying Powers of *i*."

Teaching Simplifying Powers of *i* based on IBL Strategy

The instructor prepares two different IBL-based experiments, one; individual and the other; group activities.

Before the beginning of the first activity, the class discussed the pattern of the power of *i*. As shown in Figure 2, the instructor provided various flash cards, so that each student received a unique problem to simplifying the power of *i*.

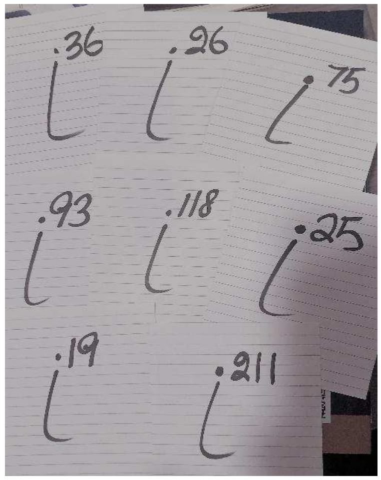

Figure 2: Activity cards for simplifying power of *i*

The instructor provided four quadrant slues *i,* -1*, -i, and* 1. Each student evaluated his/her assigned problem and found the right quadrant and answer. All students were asked to discuss with their quadrant peers who picked the same answers and verified whether each of them found the correct answer or not. If not, they should re-evaluate their questions, and move to another place where their modified answer is located. As soon as all the students chose the correct position, the instructor asked students to discuss their approach to problem-solving.

Quadrant 1: i	Quadrant 2: -1
Quadrant 3: $-i$	Quadrant 4: 1

Figure 3: Four Quadrant Slues *i*, -1,. –*i*, and 1

The second activity was to evaluate the multi-digits power of *i*. At this time, students were grouped and discussed the problem with their group members. The instructor set up a speaker for each group, and the speaker is the only person to share their group answer. For more active group collaboration, the instructor clarified that the speaker of each group present and discuss his/her group discussion with other groups.

STUDENT DISCOVERIES

Activity 1

All twenty-five students received different cards. Cards were randomly distributed, and students were asked to evaluate the higher power of *i*. Based on this activity, students independently found two different ways to solve the problem. The first approach was based on dividing the exponent by four and analyzed the remainder. Hence, the given power of *i*: i^n, students divide the exponent n by four, and if there is no remainder, then the answer is 1, the remainder is 1, then *i*, the remainder is 2, then -1, and the remainder is 3 then –*i*. Another approach divided the power n by two. In this case, if the quotient is odd, and no remainder, then the answer is -1; if the remainder is 1, then the answer is -*i*. If the quotient is even, and no remainder, then the answer is 1 and if the remainder is 1, then answer is *i*. Those are typical approaches that are introduced in various textbooks. However, no other approaches were found based on this activity.

Activity 2

In the second activity, students were grouped by five or six people and discussed the problem. The instructor randomly selected the speaker from each group, and he strictly communicated with those speakers. Each group must share and verify their answer with the speaker. Each speaker presents his/her group answer. Students in other groups can ask questions to other groups, but only the speaker from each group can answer a given question.

The instructor provided a multi-digit power of *i* for the second activity. For example: $i^{20123420}$, $i^{1985401}$. Most of the students used either dividing the exponent by four or by two. Among those approaches, the students that used the divisor four struggled to figure out the answer. But, the students' groups, who divided *n* by the divisor two, got the answer more promptly. The following dialogue demonstrates how they showed the work.

Group A: If the power is odd, then the exponent is needed to subtract 1 from the total power and rewrite the expression as $i \cdot i^n$.

For example, $i^{1985401} = i \cdot i^{1985400}$. *After that, just divide the exponent value 1985400 by 2, then it is 992700 which is even, so the answer is i. Another example,* $i^{20123420}$, *This problem is already even power, so they just divided n by 2. 20123420/2=10061710. Since the division is even, the answer is 1.*

However, the groups that divided the power by four couldn't get the answer promptly. So, comparing to the algorithm between dividing the power by two and four, dividing by two seemed easier to solve.

In a moment later, one student suggested a simpler property based on dividing the exponent *n* by four. Here is how he describe to simplifying the power of *i*.

Student B: If the power n is more than a three-digit number, we only care about the last two digits. For example, $i^{1985401}$*. The exponent is 1985401. Although it is a big number, we only need to analyze the last two digits which is 01. Since 01 is not divisible by 4, and just remainder is 1, so the answer is i.*

However, other students were confused and asked him again:

Other students: How come? Why?

Students B: Because the exponent 1985401, we can rewrite is as 1985400+01, and 1985400 is a multiple of 100, and 100 is a multiple of 4, so it must be divided by 4 anyway. So, we only worry about two digits: 01.

The instructor: What is the reason that you wrote 1985401 as 1985400+01? Why not 1985400+1?

Students B: Because I want to leave out two digits.

Other students were still confused and started debating this question with him.

Other students: What do you mean 01 divides by 4? Isn't it 1 divide by 4?

Students B: Okay. Let me show a different example: $i^{10845631}$*. As you see the exponent is 10845631. This exponent can be expressed as 10845600+31. Since 10845600 is a multiple of 100, and 100 is a multiple of 4, it can be written as* $1 \cdot i^{31}$.

The instructor: You need to clarify what the reason is that you can re-write $i^{10845631}$*as* $1 \cdot i^{31}$.

Student B: $i^{10845631} = (i^{100})^{108456} \cdot i^{31} = ((i^4)^{25})^{108456} \cdot i^{31}$*. Since* $i^4 = 1$*, it can be expressed as* $1 \cdot i^{31}$*. When you divide 31 by 4, the remainder is 3. So the answer is -i.*

It is apparent that the student understands the exponent property: $a^m \cdot a^n = a^{m+n}$ clearly. Soon after other students tried the other question: $i^{20123420}$

Student C: So, you meant that we only divide 20 by 4 because other digits 20123400 is already a multiple of 4. Since the number 20 is a multiple of 4, so the answer is 1.

Student B: That's right.

Other students: That's awesome!!

Based on this activity, students were dedicated to working on simplifying the power of *i* and developed various ways to solve the problems.

OBSERVATIONS FROM THE EXPERIMENT

Based on the IBL activities of simplifying power of *i*, the experiment provides the following observations.

First of all, the IBL instructional strategy motivated students and increased their curiosity. Their curiosity encouraged them to collaborate with each other voluntarily, and all the students discussed the problems. During the discussion, students understood different methods for evaluating the power of *i*. and found dividing the power *n* by two is simpler than divide by four. Secondly, the study also found that the IBL strategy develops a deeper understanding of the topic. If the instructor provided a lecture on simplifying the power of *i,* it would be possible to instruct one method, i.e., either dividing the power *n* by two or four. It is because instructing various methodology may cause confusion for the particular student group; it becomes a reason that the class progress slows down in the lecture-based classroom environment. Therefore, instructors usually choose one method to teach the students. However, in the IBL class, students have developed up to three different approaches based on their own collaboration. One of the students, who found a new approach witnessed that he found the method while he was discussing with his groupmates. Overall, all students shared various approaches that they are developed together.

Finally, the IBL strategy allowed the instructor to cover more topics. Besides covering the topic of simplifying the power of *i*, the class covered combining, multiplying, and dividing complex numbers. Although those topics were not intended to be explored in this IBL experiment, students were already exploring topics outside of the scope of this experiment students were eager to volunteer to share their work with peers, and students were collaborating with each other to figure out the problems.

FURTHER STUDY

This IBL experiment demonstrated that IBL strategy develops students' curiosity and enhances their learning experience. So, it would be worthwhile for instructors to consider create lessons based on IBL strategies. In addition, it would be valuable to compare IBL setting and ordinary lecture-style classes to document students learning outcomes and further study in this area.

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