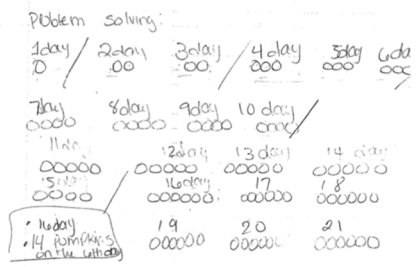
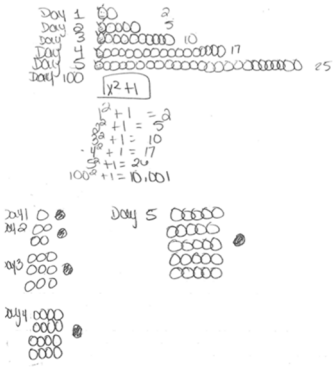


5. Growing Caterpillar



Book Review of "THE OUTER LIMITS OF REASON" by Noson Yanofsky

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Title: The Outer Limits of Reason. What Science, Mathematics, and Logic Cannot Tell Us
Author: Noson S. Yanofsky
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Even if we do not admit it, or not aware of it, our daily existence is determined by the never-ending flows among conscious and subconscious, passive and active, yin and yang or what other names one may call them. In this constant flow of well-posed and ill-posed inquiries we often experience confusions, paradoxes, dilemmas and limitations, but do not pay attention to them for

the sake of the united experience of the composite mind. However, when our mind constructions are used for the purpose of highly formalized practices, such as science or mathematics, we are forced to face these paradoxes to understand the limitations of our work.

Noson S. Yanofsky in his book "The Outer Limits of Reason" makes an attempt to define the limitations of human and computer languages, calculations, and procedures. He is successful in creating a text that can be read many times to offer a manifold of topics and their interconnections.

I found the book incredibly inspiring, intriguing and stimulating, and it seems that philosophers and thinkers of all kinds would benefit from reading it. The book contains carefully crafted discussions of which tasks can and which cannot be performed within the limitations imposed on the performance by the performers. In mathematics these limitations have been changing throughout the centuries, beginning with the ruler and compass for the ancient Greeks and ending with programming for modern scientists. Regardless of the times and the set ups, the limits of reason are always present and affect the way we think, live and work.

The book has a structure of an academic publication and is divided into ten chapters with each chapter being divided into several sections and subsections. Every chapter begins with generous mottos that reflect on the concepts to be discussed in the following text. Every chapter closes with a section that provides a follow up of the themes touched upon in a form of a further reading. It is not just a list of volumes related to the topic, but each paragraph gives a connection and detailed description of exactly which themes are expanded. Since the book is used as a textbook, this additional section encourages students to read beyond the volume, but the way this encouragement is performed, appears to be highly effective.

"The Outer Limits of Reason" begins with analysis of a problem of covering the chess board with domino tiles, which is a classic in the world of puzzles and games. Chapter 2 is a collection of various well known or less known paradoxes including Epimenides' paradox, Quine's sentence, barber paradox, heterological paradox, Russell's paradox, Yablo's paradox, "interesting number" paradox, Berry phrase, etc. Chapter 3 discusses dilemmas of identity for objects and humans with an example of the ship of Theseus, asking whether a ship of Theseus can remain under its name after replacing its planks? Similarly, do humans retain their identity after seven years even if all cells of the body are replaced? This is quite engaging topic studied intensely by western and eastern philosophers. For example, the dilemma of existence was intensely studied by Buddhist masters who introduced the Middle Way saying that the composite things (as we are, and the ship of Theseus is) neither exist nor non-exist and making a distinction between the "inherent existence" and "ultimate existence."

What is particular about the book is that the author does not simply present and discuss these paradoxes but makes them into a starting point of a discussion about a nature of things. For example, the Zeno's paradox about taking a step that is equal to a half of a given distance is a starting point of a discussion about ability to measure distances of the lengths not smaller than the

Planck's length. Examples of modern science and popular culture are discussed throughout the book. A game of the Monty Hall Show gives rise to a dispute about the strategy of knowing others' strategy and using it for creating our own strategy (The situation resembles the "tit for tat" strategy or the prisoners dilemma). Fuzzy logic is motivated by examples of vague definitions of being bald or having uncertainty of how many grains can be taken away from a heap of grains, so it remains a heap? Chapter 4 contains proofs that the set of rational numbers is equinumerous to the set of natural numbers. At the same time, it discusses the original proof of George Cantor that the set of rational numbers is not equinumerous with these sets. The author brings to this chapter the axioms of the Zermelo-Frenkel set theory and the axiom of choice, and poses the questions whether mathematical theorems are discovered or invented?

Classical problems from computer science are presented in chapter 5: computing complexities, finding paths on Konigsberg bridges, traveling salesman problem, Hamiltonian cycle problem, set partition, etc. Here the author goes one step beyond the standard disputes and presents superexponential problem and PSPACE problems that are becoming even more relevant during the times of the Big Data. The importance of the dilemma of the complexity of a problem "P vs. NP" was recognized by mathematicians and listed as one of the seven Millennium Problems (http://www.claymath.org/millennium-problems). Computing impossibilities such as Halting Problem or Zero Program Problem and their interconnections are analyzed in chapter 6. Here the author discusses touchy topic whether machines will ever be able to mimic the actions of the human mind, pointing out that humans have self-reference and consciousness, which are currently not programmable, but the future may bring surprises in that matter.

Chapter 7 carries elements of quantum mechanics and relativity theory and discusses whether the two theories can be united into one, but the main theme of this chapter is predictability of a system. With an example of the three-body problem the book unfolds the problem of chaotic systems which simply do not follow the same arrangements as the tame systems such as the two-body problem.

A clear distinction between science and non-science is discussed in Chapter 8 together with connections among the mind, science, mathematics, and the universe. Here three fundamental questions are brought to the surface and analyzed carefully:

1. Why is there any structure in the universe?

2. Why is the structure that exists capable of sustaining life?

3. Why did this life sustaining structure generate a creature with enough intelligence to understand the structure?

In the light of the issues with the observer touched upon in chapter 7 these questions pose quite a mind dilemma whether humans as evolving creatures are on the path of reaching the enlightening purpose of observing the universe. But it seems that we are still far from understanding it sufficiently. Especially because in the presence of an intelligent observer who can observe the

universe, who would observe the observer? The chapter concludes with a discussion of metascience and its conservation laws: momentum, angular momentum, energy that correspond to symmetries of the system: place, orientation, and time. Limitations of mathematics are presented in Chapter 9. Often self-imposed such as already mentioned Greek School of Mathematics' constructions with a ruler and a compass but sometimes generated by the limitations of its structure as in case of arithmetic. Thus, Gödel incompleteness theorem is carefully presented with its implication of proving consistency of one system in a another, stronger system.

With a large quantity of topics touched upon in the book the author makes in the last chapter a brilliant decision to provide a summary of previous divagations. Here all limitations are classified in terms of the kind of limitations they admit. Paradoxes are placed in a table and characterized in terms of objects of self-reference and the type of consequences. At the end of the book the definition of reason is provided and discussed in detail.

Everybody who is interested in philosophy of science will benefit from reading this book, which is packed with exciting topics. Choosing just a few to reflect on the entire content is really hard.

To close, I would suggest that the author includes exercises and open problems related to the topic in the future editions of the book. The readers will highly benefit from having their own walk of mind parallel to the dispute presented in the book.