

Book Review for *PRIME OBSESSION* by John Derbyshire

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Title: Prime Obsession

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The book's title *PRIME OBSESSION* might mislead the eye of mystery buffs, thinking it is an adventure fiction. Only the mathematically aware would discern that it has something to do with prime numbers, and be worthy of perusal. This is how I acquired this intriguing book, rapidly gleaning that it is indeed about the mysteries of prime numbers and the Riemann Hypothesis, with which I was acquainted. The book engages the interested reader with a penchant for mathematical topics, those such as teachers, students, and anyone trained beyond basic mathematics, who seek a friendly but informative view into number theory and analysis, and presented at a level of the average intelligent reader, by way of biography and the methods attempted to solve the problem.

The book's organization into alternating chapters devoted to the biography of Bernhard Riemann, and the other half to an exposition of his Hypothesis, tends to separate the mathematician Riemann from his work. Reading other available accounts could be enlightening. The author's suggestion for reading all of the biography chapters followed by reading all of the technical chapters is a useful adjunct.

Presentation of the hero of the narrative, the nineteenth century mathematical phenomenon Bernhard Riemann, is interesting in terms of scientific biography. Most of what is known of Riemann's personal life has been garnered from his letters to his family. It is difficult to know how much of his mathematical inspiration was from his own ingenuity or from the influence of his teachers of mathematics. The technical development of Riemann's ideas culminating in the Hypothesis is quite good.

His doctorate from Göttingen, containing the Cauchy-Riemann equations of complex analysis as well as the beginnings of Riemann surfaces demonstrates his emerging mastery of the subject.

Riemann's election as a corresponding member of the Berlin Academy necessitated the presentation of a second research (habilitation) thesis to the faculty, in which paper entitled "On the Number of Prime Numbers Less Than a Given Quantity" is the landmark work that includes the Riemann Hypothesis involving the zeta function. We note critically that Riemann evaded any attempt at solution of the Hypothesis in his paper; he claimed that it was not central to his purpose. This would not be acceptable in the contemporary mathematics journals. He died on July 20, 1866, just short of his fortieth birthday. During the nineteenth century such deaths from tuberculosis were common.

The Riemann Hypothesis (RH), the statement of which is repeated as a *leitmotiv* throughout the book, simply says ***all the non-trivial zeros of the of the zeta function have real part one-half.*** Over a century and a half have elapsed and no one has been able to prove, or disprove, this assertion. All that was accomplished over this time are approximation methods to a solution and features of a solution, but no formula (closed form solution) is offered. For reference the Riemann zeta function $\zeta(s)$ is written

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \dots$$

It is known that the distribution of primes depends intimately on the zeros of the zeta function.

The author goes into considerable detail concerning power series, and why they converge to a finite number, or not. This might not appeal to readers who have not studied these. Even less familiar are experiments approximating the zeta function by setting the infinite product (Euler product) to an infinite sum (Dirichlet sum). The diagram of the zeta function graph, noting points where it is zero within the critical strip, shows the non-trivial zeros in the middle of the critical strip. This is definitely mysterious to the uninitiated.

The discussions of field extensions to rings by E. Artin, A. Weil, H. Hasse, and P. Deligne become even more esoteric, as is operator theory, relating the non-trivial zeros of the Riemann zeta function to the eigenvalues of some Hermitian operator. Realizing that the author is attempting to provide a conspectus of many such attempts at proving, or disproving, the Riemann Hypothesis, most readers would be tempted to skip over these parts of the book.

The author John Derbyshire is a mathematician, linguist, and writer on many topics. His other book, *UNKNOWN QUANTITY*, is about the history of algebra. In *PRIME OBSESSION* he has created an interesting, enjoyable and informative book to enlighten the reader about the nature of the Riemann Hypothesis, and perhaps inspire someone to try and prove it.