Practice Does Not Always Make You Perfect

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Abstract: *The study investigated the impact that conceptual understanding has on mathematics achievement of remedial mathematics students in a community college located in New York City. The study sample consisted of 105 remedial mathematics students from four elementary algebra sections. Two of these four sections were under Conceptual treatment. The other two sections were under procedural treatment and served as the control group. To measure subjects' conceptual and procedural knowledge, the participants completed two quizzes (a conceptual quiz and a procedural quiz) a week before the final exam. Students' mathematics achievements were measured using the conceptual quiz score, the procedural quiz score and the final examination score. The study found that the conceptual treatment group performed better on both quizzes, despite the fact that the procedural group practiced more procedural problems. The final examination score mean for the conceptual group was higher than the one for the procedural group.*

Keywords: Mathematics anxiety, Procedural knowledge, Conceptual knowledge

INTRODUCTION

Remedial mathematics has become an academic and career obstacle for many students, particularly community college students. In fact, it has become the largest single barrier to student advancement. Approximately 24 % of students who entered community college in the academic year of 2007-2008, the less likely that student is to ever complete college English or Math (Bailey, 2009). While the courses often do not qualify for college credit, students must nonetheless pay tuition for them.

A central goal for mathematics educators is to help students nurture their mathematics understanding. However, community colleges' teaching of Algebra is mostly procedural. In mathematics, it is important to know both the basic concepts and the correct procedures for problem solving. To overcome "negative attitudes" toward mathematics, educators should use "concrete manipulative materials" to form the connection between concrete learning and abstract thought (Taylor & Brooks, 1986, p. 10). Although The National Council of Teachers of Mathematics (1989) recommended that teachers emphasize the use of conceptual problems to help students understand mathematics subjects, to date, little research has been conducted regarding the impact that conceptual understanding in mathematics has on students' achievement at the community college level (NCTM, 1989). Rittle-Johnson & Alibali (1999) investigated the relationship between conceptual and procedural knowledge in children's learning. Their findings suggest that conceptual knowledge influences students' procedural knowledge and vice versa. However, the impact that conceptual knowledge has on procedural knowledge is greater than the reverse.

***Research Question*:** Are there significant differences in mathematics achievements between students who have been exposed to a conceptual treatment and those taught in ways that emphasize procedures?

METHODOLOGY

Setting and Participants: In this study, the population consists of remedial mathematics students at LaGuardia Community College (LaGcc) located in New York City. In fall 2015, 10% of the students at LaGcc were white non-Hispanic, 37% were Hispanic, and 15% were Black and 38% other races. The Fall 2015 enrollment was about 18,623, of which 58% was female and 42% male. Among the 18,623 students in academic programs, 50% of them were non-native born students. LaGcc was selected because of my familiarity with the environment. However, the author did not teach any elementary section in which the data was gathered. The study was done throughout the second session of Fall 2016 that started from January 4 to February 16. This was a 6 weeks session including a final week. The elementary algebra courses including the ones that participated in this study met Monday to Thursday for 2 hours per day lecture, 2 hours computer lab and 2 hours tutoring lab per week. The instructors led the lecture and the computer lab sessions. Students used a mathematics platform, "educosoft", to complete their homeworks in the computer labs. These homeworks were mainly procedural. College assistants –students who were in their final college year- led the tutoring labs using worksheets that were prepared by the course coordinator.

The study sample consisted of 105 remedial mathematics students from four elementary algebra sections. Thus, the sampling frame met the following criteria: (a) potential subjects were elementary algebra (MAT 096) students in LaGcc (b) they were students enrolled in the four sections selected to participate in this study. The mean enrollment for each elementary algebra section was 30.

Research Design: For this study, we randomly selected two elementary algebra sections to a conceptual treatment and the other two sections were under procedural treatment. Participants' assignment to groups was not randomized. This means that a quasi-experimental design was used in this study. Because of lack of randomized design, we tried to select groups that were as similar as possible, so we could fairly compare them. For instance, participants in all groups were not repeating elementary algebra course. One procedural treatment section and one conceptual section were scheduled in the morning between 8am to 12pm. The other two sections were given in the afternoon between 12pm and 4 pm. Each of these sections met 2 hours a day from Monday to Thursday. The computer labs homeworks and tutoring lab worksheets were the same for all four sections. The instructors in all four sections were adjuncts from the Mathematics department and each has less than 2 years of teaching experiences. Two of the four sections were under conceptual treatment: instructors in these sections followed lesson plans that focused on concepts rather than procedures (figure 1). The other two sections were under procedural treatment. Instructors in the procedural treatment courses followed lesson plans that were focused on procedures rather than concepts (figure 2). The researcher prepared 7 conceptual lesson plans for the conceptual groups and 7 procedural lesson plans for the procedural groups. These lessons plans were split into 9 sessions throughout the semester. The researcher fully observed all four groups' sections – procedural groups sections and conceptual group sections - when instructors taught the lesson plans that we prepared. Before each class meeting, I met with the instructor for about 15 minutes to go over the lesson plan. This was done with all groups.

Figure 1: Conceptual Approach to Teaching Slope

Objective: The objective of this lesson plan is to help students gain a deeper understanding of slope and to be able to quantify it from a conceptual approach. This objective can be achieved through real life settings or experiences. By gaining a clear understanding of slopes, students will be able to appreciate how a concept such as slope is useful in understanding the world around us.

Methodology: A real life setting of skiing resorts is used to illustrate the concept. For the computational part, a triangle made of blocks is used. This lesson plan includes 6 stages.

Stage 1: Students are introduced to the rating level of ski runs. The ratings are: easy, moderate, steep, and vertical; a horizontal aspect is added here for completeness. The class is put into groups, and each group is asked to place the 6 different hills into the categories listed above by the ratings.
Stage 2: The class comes together and discusses why students placed the hills in each category. The idea of measuring steepness is introduced, and the teacher asks the groups to develop a way to measure steepness.
Stage 3: The groups are given a worksheet (See worksheet below) ….and …. to help them develop a formula to measure the steepness of given lines (using rise over run). They are asked to consider what lines have in common, what their differences are, and how a formula for steepness might be developed. Groups develop their own formula and then share them with the class.
Stage 4: The groups share their formulas with the class. The formulas are then tested on lines that go in different directions from which the class has been using.

Math Topics

Geometric context: Slope analysis and interpretation, calculation of slopes, ordered pairs, graphing lines.

Tools

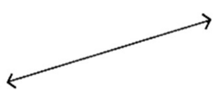
Multimedia projector, Computer, educational blocks, Picture of Ski resorts

Time 60 minutes

When to Introduce

This approach should be introduced to students at their first exposure to equation of a line.

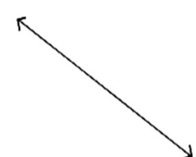
Teacher's response: Teacher brings out a triangle made of blocks to illustrate how slopes differ in size, using the pictures, the lines and the number line to show how the sign of the slope could be generated. At this point, the notion of first quadrant and second quadrant of the Cartesian plane may be used. Teacher will then generalize students answers to come up with the formula: Slope = $(y_2-y_1) / (x_2-x_1)$ given two points (x_1, y_1) and (x_2,y_2).
Mini-Experiment 1: Now that you've developed a formula for finding the slope of the line, you are to use your transparent graph chart to find the slope of the lines below.



Slope is________

Slope is ________

Slope is _________



Slope is _________

Mini-Experiment 2: Now that you've learned how to find the slope of a line, we're going to switch things up. You are going to be given a slope, and you need to graph a line with that given slope. There are infinite number of lines that all have the same slope, so there's no one-way to do this. Instructors will ask some students to share their answers with the class.
Stage 5 (practice): Students get to practice the following problem using the slope formula.
Problem: Find the slope of the line passing through:
a. (1, 2) and (3, 3) b. (3, 4) and (-3,-6) c. (-4, -2) and (-1,2) d. (3,2) and (3,7)
Have volunteer students share their answers with the class
Stage 6: Students will be asked to find two points from the given equations to find the slope a. $y = 2x+1$ b. $y = 2$ c. $x=3$
Students share their findings and discuss about easier ways to find the slope (given an equation) without using points. The goal of this assignment is to come up with the slope intercept form $y = mx +b$ (m is the slope and b is the y-intercept).

Figure 2: Procedural Approach to Teaching Slope

Objective: Students will be able to develop an accurate formula for finding the slope of line. **Methodology:** The given formula of slope is used to solve problem. This lesson plan includes 6 stages. **Stage 1** (10 minutes): The students are introduced to definition of slope: In coordinate plane, the slope of a straight line is defined by the change in y divided by the change in x. Slope = Change in y / Change in X = $(y_2 - y_1)/(x_2 - x_1)$ Give an example applying the formula above **Stage 2** (15minutes): Students are put into groups to practice using the slope formula. Practice problem: Graph each pair of given points below then find their slopes. (use separate graphs) a. (1,2) and (3,3) b. (3,4) and (-3,-6) c. (-1,-2) and (-1,2) d. (-3,2) and (0,7) e. (1,2) and (0,2) f. (1,2) and (1,7) **Stage 3**: (10 minutes) Students are asked to go the board to share their results with the rest of the class. **Stage 4:** (10 minutes) Now that we have a slope and a line from each given pair, let's have a discussion about when the slope is positive, negative , zero or undefined. **Stage 5** (7 minutes) Instructor introduces the slope intercepts formula: y = mx + b and how to find the slope using the slope-intercept form. Example: Instructor solves the followings: a. y= 2x+1 b. y=2 c. x =3 **Stage 6** (8 minutes) Students solve the following problem and share their results with the class. Practice Problem: Find the slope of the following equations: a. y=3x+1 b. 2y=x-1 c. 3x+2y =2 d. x+2=2 e. 2y+1=0	**Math Topics** **Geometric context:** Slope formula and interpretation, calculation of slopes, ordered pairs, graphing lines. **Tools** Multimedia projector, Computer. **Time** 60 minutes **When to Introduce** This approach should be introduced to students at their first exposure to equation of a line.

Figure 3: Conceptual and Procedural Quiz

Conceptual Quiz (25 min)	Procedural Quiz (25 min)
1. Explain why the equation x+ 1 = x+3 has no solution. 2. Explain the difference (in terms of the solutions) between x+1 =3 and x+1>3 3. Do you agree or disagree with the following statement? $x^2 = -1$ has no solution. Explain your answer 4. x (x-1) = 1 implies x=1 and x-1=1. Do you agree or disagree ? Explain your answer. 5. Explain the following statement: The graph of a function can have infinite x intercepts and at most one y intercept. 6. Explain the following: | x | < - 2 has no solution. 7. The system of equations 2x+5y = 6 and 2x+5y = 5 has no solution. 8. The equation y = 2 has a slope of 0 (m = 0) and x = 2 has an undefined slope. Explain both cases. 9. You want to rent a car for your coming vacation. One rental agency charges a flat fee of $55 per day, while another charge $10 per day plus 20 cents for each mile driven. You expect to drive an average of 150 miles a day during your vacation. How much more money will you spend per day if you use the first rental agency? 10. –x < 3 implies that x > -3. Explain the change of the symbol < to >.	1. Solve the equation x+ 1 = x+3 2. Solve the inequality 2x + 1 < 3x-1 3. Solve $2x^2 = 18$ 4. Solve $x^2-2x-1=0$ 5. Find the intercepts of the equation: 3x+5y = 15 6. Solve and graph the solution: | x +1 | < 2 7. Solve the system of equations: x+5y = 6 and 2x+5y = 5 8. Find the slope of the equation 3x+5y = 15 9. Find the equation of the line containing the points (2, 3) and (4, -1). 10. Evaluate f (–1) for the function $f(x) = x^2-2x+1$

Note: Each quiz is worth 10 pts and 25minutes were allowed to complete each one of them

Instruments: The participants in both groups completed a conceptual quiz and a procedural quiz (Table 3) a week before the final Exam. One is approached conceptually and the other was focused on procedural skills. The purpose of the conceptual quiz was to explore how well students understand the subject matter. On the other hand, the procedural quiz was to determine how well students know the rules or procedures for solving mathematic problems. Participants were given 50 minutes to complete both the procedural and conceptual quiz. Each quiz was worth 10 points. To measure achievement, participants' final exam scores were also collected from course instructors. The final exam is a departmental exam that has 25 questions which are more procedural questions. Students are given 2 hours to complete the final exam. The researcher graded all quizzes and the final exams were grade by the instructors.

A T-test was used to explore whether the final exam score mean differed significantly between the conceptual and procedural group. The final exam score was set to be the response variable for the analysis of the variance (ANOVA).

RESULTS AND DATA ANALYSIS

First, summary data (Table 1) on quiz scores (conceptual and procedural quizzes), and final exam scores for all respondents are described. The factor analyses of these variables were reported in order to compare the factor structure of these tests for both the conceptual and procedural group.

Table 1: Descriptive Statistics for all students' final examination score and quizzes (N=105)

	Minimum	Maximum	Mean	Std. Deviation
Conceptual-quiz	2	10	5.69	1.867
Procedural-quiz	3	10	6.47	2.024
Final-Exam-Score	6	96	65.00	13.516

**Quizzes are out of 10 and the final exam score is out of 100.*

Table 2: Descriptive Statistics for Conceptual treatment group (N=53)

	Minimum	Maximum	Mean	Std. Deviation
Conceptual-quiz	4	10	6.81	1.532
Procedural-quiz	3	10	7.08	2.111
Final-Exam-Score	35	96	69.09	12.023

**Quizzes are out of 10 and the final exam score is out of 100.*

The results of Table 3 below show that subjects in the procedural treatment had a final exam score mean of 60.83 which was lower than the final exam score average of all respondents (65) shown in Table 1. For this group, the conceptual quiz average (4.54) and procedural quiz average (5.85) were also lower.

Table 2 shows that subjects in the conceptual treatment averages on conceptual quiz (6.81), procedural quiz (7.08) and final examination (69.09) are higher than the averages of all respondents shown on table 1.

Table 2 and Table 3 show that students under conceptual treatment performed better on both quizzes and the final examination.

A set of tests of normality (Normal QQ plot, Kolmogorov-Smirnov test for normality) was performed on each group and all variables appear to be normally distributed (figure 4 and figure 5).

Figure 4 Normal Q-Q Plot for conceptual group using final exam score

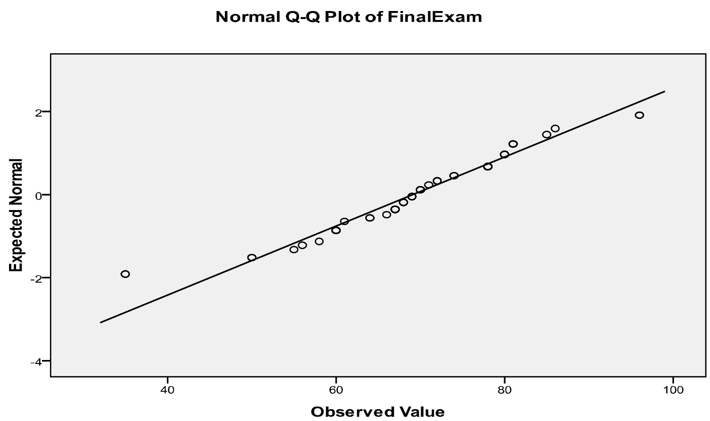


Figure 5 Normal Q-Q Plot for procedural group using final exam score

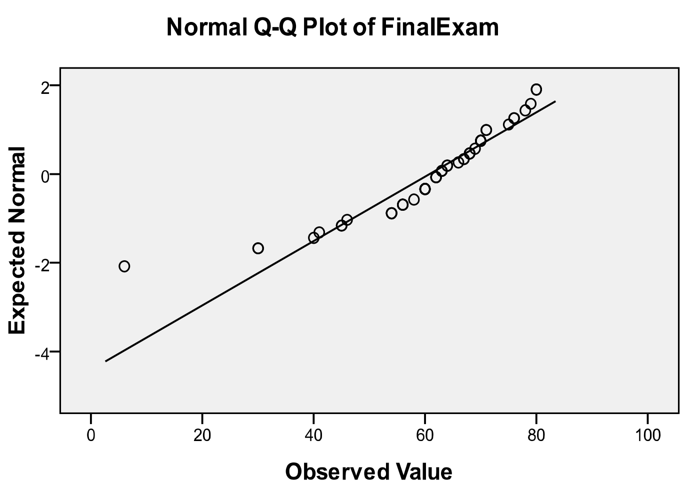


Table 3: Descriptive Statistics for Procedural treatment group (N=53)

	Minimum	Maximum	Mean	Std. Deviation
Conceptual-quiz	2	8	4.54	1.434
Procedural-quiz	3	10	5.85	1.742
Final-Exam-Score	6	80	60.83	13.784

**Quizzes are out of 10 and the final exam score is out of 100.*

First, The Levene's variance test shown in Table 4 for equality of variance displays a significance level of 0.666, which is greater than 0.05. Here, the Null hypothesis of equal variance is not rejected. There is enough evidence to say that the variance on the final exam score achievement for the two groups is statistically different. This is rather expected because scores on the final examination are dependent on how students have been taught throughout the semester and are relatively predictable.

Table 4: Independent sample test for Final Exam Score

	Levene's Test for Equality of Variance		T.test for Equality of Means						
								95% Confidence Interval of the Difference	
Final-Exam Score	F	Sig.	t	df	Sig.	Mean Diff	Std.Err Diff	Lower	Upper
Equal Variance	.188	.666	3.277	103	.001	8.267	2.523	3.264	13.271
Unequal Variance			3.273	100.58	.001	8.267	2.526	3.256	13.279

Second, the significance level of the test for equality of means (Table 4) is 0.001, which is less than 0.05. Here, the Null hypothesis will be rejected. There is sufficient evidence to say that there is a difference between the mean final examinations of the two groups. In other words, the teaching methods to which students are exposed, whether they emphasize conceptual understanding or use a procedural approach, affected the outcome of students' final exam scores.

Table 5: Group Statistics using Final exam score

	Group	N	Mean	Std. Deviation	Std. Error Mean
Final-Exam Score	Conc	53	69.09	12.023	1.651
	Proc	52	60.83	13.784	1.911

As it can be seen in the group's statistics tables (Table 5), students exposed to teaching methods that emphasized conceptual understanding performed better than students who were taught in ways that emphasized procedures.

The specific group is retained as expected in the model with a p-value of less than 0.05. The relevance of the group has been established since the beginning, and with the coefficients, to estimate a final exam score, the formula* stated below Table 6 was used.

Table 6. Coefficients of the final-exam-score model

Model		Unstandardized Coef		Standardized Coef		
		B	Std. Error	Beta	T	Sig.
1	(Constant)	27.296	4.219		6.470	.000
	conceptual-quiz	3.602	.886	.498	4.066	.000
	Procedual-quiz	2.923	.517	.438	5.657	.000
	Groupdummy	-2.880	6.565	-.107	-.439	.662
	Conceptconc	-.362	1.105	-.096	-.328	.744
	Conceptconc	-.362	1.105	-.096	-.328	.744

*Estimated-FinalExam = 28.157+2.918*proceduralquiz + 3.419* conceptualquiz - 4.888*groupdummy

CONCLUSION

The performance of community college students in the subject of mathematics has raised concerns for decades. Several measures have been taken to address this problem. Mathematics departments and remedial course instructors are constantly working to teach these courses in ways that will better help students understand the subject matter. At LaGuardia Community College, remedial non-credit courses constitute nearly 65% of all courses offered by the Mathematics department. Placement in a specific level in these courses is based on a university placement test. Unfortunately, it is not uncommon for students to repeatedly fail these remedial courses, sometimes up to three times. Unfortunately, these courses are mandatory and can prevent students from obtaining a college degree.

The study explored the relationship between mathematics achievement and conceptual understanding. It examined the mathematics achievement difference between conceptual method and procedural method of teaching. . Four variables were used to answer this question: the final examination score, the procedural quiz, the conceptual quiz, and the conceptual groups (groupdummy). The final examination score was the response variable.

The ANOVA test revealed that the relationship between conceptual quiz, procedural quiz, group (groupdummy) and final examination (response variable) was statistically significant. The results revealed that the conceptual groups outperformed the procedural groups on the conceptual quiz. The conceptual quiz average score for the conceptual groups (6.81 out of 10) was higher than the one for the procedural (4.54 out of 10). The conceptual quiz questions were not based on problem solving. These questions were designed to test students' knowledge of the subject matter. As the results illustrated, the conceptual groups had a better understanding of the subject matter. The conceptual groups also performed better on the procedural quiz, despite the fact that the procedural groups practiced more procedural problems than the conceptual groups and was exposed to a procedural treatment. The procedural quiz average score for the conceptual group was 7.08/10 and the procedural group's average was 5.85/10. The procedural groups' lower average score on the procedural quiz indicates that the conceptual treatment provided a more flexible understanding of mathematics, which allowed them to utilize their knowledge as a tool to solve problems. In other words, conceptual groups were more able to reason logically, formulate, represent, and solve mathematical problems. This finding supports Brownell's (1973) idea: "the greater the degree of understanding, the less the amount of practice necessary to promote and to fix learning" (p.188). These findings also support the NCTM's reforms (1989) in mathematics education, which argued that teachers should inculcate conceptual understanding before approaching procedural knowledge.

For the same reasons, the final examination score mean for the conceptual groups was higher than the one for the procedural groups by a margin of approximately 10%. This should not be a surprise, since the final examination format was similar to the procedural quiz. These results demonstrate that conceptual understanding is an important component of proficiency, along with factual knowledge and procedural facility.

Overall, this study statistically demonstrates that the use of conceptual method of teaching plays greater role on improving students' mathematics achievement and understanding of the subject matter. The rote memorization common in the traditional method of teaching mathematics focuses mainly on mastering rules. The procedural method, while sacrificing attention to concepts and when applied alone, is easy to forget or hard to remember; therefore, it is often associated with pain and frustration for students. While taking the examination, students must recall their lessons and the material they studied, a technique that generates rote learning, disabling students from performing well.

This study reveals that students achieve higher scores in mathematics when they are engaged in exploring and thinking rather than engaging only in rote learning of rules and procedures. In fact, these conceptual and active methods help students build the necessary confidence to learn new mathematical concepts. The question then still remains: In what way can these initiatives and instructional strategies be implemented in remedial mathematics classes in order to improve students' mathematics achievement.

REFERENCES

Brownell, W. A. (1973). *Meaning and skill—Maintaining the balance*. In F. J. Crosswhite, J. L.

Bailey, T. (2009). Challenge and opportunity: Rethinking the role and function of developmental education in community college.New Directions for Community Colleges,145, 1130.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Rittle-Johnson, B., & Alibali, M.W. (1999). *Conceptual and Procedural Knowledge of Mathematics:Does One Lead to the Other?* Journal of Educational Psychology, Vol.91, 175-189

Taylor, L., & Brooks, K. (1986). *Building Math Confidence by Overcoming Math anxiety*. Adult Literacy and basic Education, v10.