# A Procedural Mindset in a Conceptual World of Mathematics

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**Abstract:** A procedural mindset is one of the most significant obstacles mathematics teachers confront when teaching for conceptual understanding. This educational criticism and connoisseurship responds to one first-year teacher's experience shifting to a more conceptual mindset to mirror the Common Core State Standards and examines three elementary teachers observed over an eight-week time frame before and after the administration of high stakes testing. Based on the analysis of 168 activities that were included in the lessons but were **not** part of the intended lesson, 68 percent of the activities taught were procedural, and 31 percent were conceptual. This article discusses the implications for students by these pedagogical choices made by the teachers.

Keywords: procedural, conceptual, common core standards, pedagogical choices

## A Procedural Mindset in a Conceptual World of Mathematics

Mathematics teachers today face many obstacles when teaching mathematics conceptually, but the most substantial obstacle is mostly around the planned, and unplanned pedagogical decisions teachers make in every mathematics lesson. This article responds to one first-year elementary teacher's experience shifting to a more conceptual mindset to mirror the Common Core State Standards in an article titled Letting Go by Kevin Junod (2017). Additionally, the qualitative research presented elaborates on Junod's (2017) perspectives using educational criticism and connoisseurship. As this teacher admitted,

> During my first year of teaching, my teammates and I taught our students computations. We thought having our students simply solve a calculation correctly was true understanding of mathematics. Teaching just computations to rote memory was easy to teach and felt rewarding (Junod, 2017).

To better understand a teacher's mathematical practices, it is essential to understand a teacher's knowledge base and where a teacher's pedagogical decisions originate. This educational criticism addresses the following research questions:

*Research Question 1*: What pedagogical choices do third-grade mathematics teachers make to prepare their students for high stakes testing under the Common Core State Standards?
*Research Question 2*: What implications are there for students by these pedagogical choices made by the teachers?
*Research Question 3*: What are the challenges and opportunities of a teacher's pedagogical choices?

**Grounded in Literature**

Teachers are now being asked to instruct students in ways that were neither taught during preservice training nor learned as a student themselves in early grades (Heitin, 2014). Therefore, Beckmann (2010) contends that this lack of scholarly ability sometimes makes mathematics teachers uninterested in considering new ideas or instructional strategies. School districts/counties are now rewriting their curriculums or adopting curriculums like *Bridges* to help teachers mirror the expectations of the Common Core State Standards (CCSS) which has been adopted verbatim, partially, or with modifications by forty-one states, the District of Columbia, four territories and the Department of Defense Education Activity (Do DEA). However, the states Indiana, Oklahoma, and South Carolina withdrew from adopting the CCSS in 2014 alone. Osborne (2015) asserts that what was once merely a research-based explanation for the present transformation of the public school educational system has progressed into an outright demand for a shift to the Common Core State Standards. Teachers are the key to enacting the standards as they are designed and, for this reason, teachers need to know the mathematics well and how to teach the standards in captivating and compelling ways, but they are not given the support or the direction to be successful in this endeavor (Beckmann, 2010). Therefore, the CCSS for Mathematics are arduous and put high demands on the teachers due to their conceptual nature; as a result, teaching becomes more routine than authentic where students are learning through exercise-oriented lessons versus inquiry-based lessons due to high demands related to test scores and the time-intensive nature of teaching conceptually (Beckmann, 2010). As Junod (2017) admits, "I failed my students when they solved story problems or any problem that did not have an obvious equation to solve. My students were unsuccessful at these problems because the problems were too complex for them to identify the process to the solution". Teaching mathematics conceptually requires attending to student's questions, anticipating obstacles, capitalizing on opportunities, making connections, and providing enrichment beyond the immediate tasks (Rowland & Zazkis, 2013). As such, teachers do not feel they have the time to allow their students to persevere in problems solving, so the alternative is to teach the mathematics procedurally. In essence, scripts, tests, and textbooks cannot continue to take a teacher's place in the classroom instruction, but a teacher must apply freedom and inventiveness that the teaching profession demands (Vecellio, 2013). Dewey (1938) contends that teachers must have a sympathetic understanding of students as individuals giving the teacher an idea of what is actually going on in the minds of those learning, Therefore, a teacher's interest shifts towards a more profound knowledge of a student's mathematical understanding and also more towards concrete mathematics lessons (Ticha & Hospesova, 2006).

In two publications by the National Council of Teachers of Mathematics (NCTM): Improving practice, Improving Student Learning (NCTM, 2007) and the Principles and Standards of School Mathematics (NCTM, 2000), the council provides a vision for mathematics pedagogy, however, there is a disparity between NCTM's vision of mathematics pedagogy and what is actually occurring during mathematics instruction across the United States (Walkowiak et al., 2014). According to Osborne (2015), the new standards, both Common Core and NCTM, go deeper requiring a more refined skill set for teaching based on relevant pedagogical content knowledge. A teacher's motivation to refine their teaching methods can be fueled by the support they receive to create sociomathematical norms or mathematical discussions that are specific to students' mathematical activity (Kazemi & Stipek, 2001). These norms are what regulate mathematical argumentation and influence the potential learning opportunities for both the students and the teacher (Yackel & Cobb, 1996). In the study conducted by Wilson and Downs (2014), many of the teachers pushed for the use of gradual release and modeling to support students in learning mathematical concepts and grappled with slowing releasing students to work on their own and modeling to support their students with learning mathematical concepts to differentiate their instruction. However, these strategies will most likely be ineffective at advancing attainment of mathematical knowledge and claim that these approaches provide only narrow opportunities for students to gain competence in math standards (Wilson & Downs, 2014). As Junod (2017) eventually realized is that he needed to let go of the ideology that students needed to learn procedurally through repeated practice of problems and that he needed to focus on problem-solving skills to succeed on high-stakes testing. He had to learn that it was okay to let his students fail on problems repeatedly to the point of frustration allowing them an opportunity to improve on their problem-solving skills. Junod (2017) concluded that allowing his students to struggle was not enough, but he had to anticipate their struggles and create guiding questions that would support them in solving the problems presented to be successful in solving unfamiliar problems presented on the high-stakes test. Vogler and Burton (2010) question whether or not teachers are genuinely providing enough standard-based instruction practices to allow students to make sense of mathematics in meaningful ways and the ability to apply higher-order thinking strategies to solve mathematical problems. Vecellio (2013) asserts that educators need to prepare students for this world which requires teachers to take on interdisciplinary issues, projects and problem situations that present themselves during mathematics instruction. For this reason, it is critical to understand the dynamics of pedagogical practices in mathematics and how to find a balance between conceptual and procedural mathematics instruction with effective questioning.

**Pedagogical Practices in Mathematics**

Shulman (1984) conceptualizes pedagogical content knowledge and defines it as a "blending of content and pedagogy into an understanding of how particular topics, problems, issues are organized, represented, and adapted to the diverse interests and abilities of learners and presented for instruction" (p. 8). Vecellio (2013) suggests that this is part of what means to become a pedagogue. The pedagogue examines all the standards to see how they figure into problems and processes of the world, and then teach those standards in rich, powerful, and authentic ways which include using a variety of texts and other media during instruction. (Vecellio, 2013). Mason and Davis (2013) attest that a teacher who is mindful not only of the subject matter but the pedagogical facets of mathematics is in a better position to direct a student's attention to what is essential, to choices available, and to criteria that might be applied when working with students on exercises and worked examples.

A teacher may have their personal theories of learning, interpretation of their own brand of mathematics, teaching styles, and so forth, but the major factors in a teachers' capacities to engage flexibly and productively with their students is their capacity for in-the-moment pedagogy (Mason & Davis, 2013). In-the-moment pedagogy is described by Mason and Davis (2013) as the scope and range of a teacher's mathematical thinking, the pedagogical strategies associated with this thinking, and the academic procedures that come to mind in the moment. Similarly, Dewey (1938) describes his principle of continuity of experience which rests upon the fact of habit. The basic characteristic of habit is that every experience enacted and undergone modifies the one who acts and undergoes, whether we wish it or not, the quality of subsequent experiences (Dewey, 1938). To put it another way, the principle of continuity of experience means that every experience in the classroom both takes up something from prior activities or discussions and modifies the quality of those activities or discussions that follow it (Dewey, 1938). This in-the-moment pedagogy often causes confusion and tension for teachers as they try to navigate the selection of instructional approaches to address particular mathematical standards (Wilson & Downs, 2014). Persevering in problem-solving and critiquing others' mathematical arguments are some behaviors and skills that would be utilized in these classroom practices to grow pedagogical competence (Wilson & Downs, 2014). In either case, the Department of Education (2008) concludes that the sweeping recommendation that instruction be entirely student-centered or teacher directed is not supported by research and should be rescinded if such research does exist. "High-quality research does not support the exclusive use of either approach" (Department of Education, 2008, p. xxii). Therefore, there would be a pedagogical struggle among teachers to find balance among procedural and conceptual learning in mathematics.

**Procedural and Conceptual Understanding in Mathematics**

Procedural knowledge is made up of two specific parts: One part is composed of the formal language of mathematics which includes an awareness of symbols used to represent mathematical ideas and the syntactic rules for writing symbols in an appropriate form; the second part consists of rules, algorithms, and procedures used to solve mathematical problems (Hiebert & Lefevre, 1986). According to Hiebert and Lefevre (1986), the key feature to procedural knowledge is that the procedures are executed in a predetermined linear sequence. When teaching for procedural knowledge, the teacher ensures that students get to automaticity (Willingham, Winter 2009-2010). The teacher explains to students that memorizing procedures and facts is necessary because it frees the mind to think about concepts (Willingham, Winter 2009-2010).

Conceptual understanding, on the other hand, is achieved by the building of relationships between pieces of information (Hiebert & Lefevre, 1986). The joining process emerges between two pieces of information that are already stored in a memory or between an actual piece of knowledge and one that is just learned (Hiebert & Lefevre, 1986). These relationships encumber the individual facts and schemes so that all the pieces of information are now linked to some network (Hiebert & Lefevre, 1986). Sociomathematical norms that characterize or promote reasoning based upon conceptual knowledge include: (a) the explanation that a student gives for a problem consists of a mathematical argument and not just a procedural description; (b) the mathematical thinking demonstrates understanding relationships among multiple strategies; (c) errors made give students opportunities to reconceptualize a problem, explore disparities in solutions, and attempt different strategies, and (d) working collaboratively employs individual accountability and reaching agreement through mathematical discussions. Between conceptual knowledge and procedural knowledge, conceptual knowledge is the most difficult to obtain because this knowledge is not easily acquired since a teacher cannot pour concepts directly into a student's head, but instead, new concepts must build upon knowledge that already exists within a student (Willingham, Winter 2009-2010). Drawing connections among mathematical topics, as Hiebert & Lefevere (1986) describes, deepens conceptual knowledge, but, unfortunately, it is one of the desired outcomes that is rarely met in the mathematics classrooms across the United States (Willingham, Winter 2009-2010). This outcome is primarily due to the conflict between the Common Core and NCTM standards requiring students to persevere in problem solving and the procedural mindset of teachers who feel that in order for their students to succeed on the high-stakes testing their only choice is to push for procedural versus conceptual understanding.

**Finding Balance: Experience that Leads to Growth**

According to the Department of Education (2008), if mathematical ideas are taught using real-world situations, then students will improve their performance on assessments that involve similar real-world problems. Vecellio (2013) asserts that educators need to prepare students for this world which requires teachers to take on interdisciplinary issues, projects and problem situations that present themselves during mathematics instruction daily. Therefore, it is the responsibility of the educator to see the direction an experience is heading and follow it (Dewey, 1938). Being aware that students often become more engaged with the real-world aspects of math problems rather than the mathematical concept intended helps teachers anticipate student responses and prepares a mechanism to embody the line of thinking into the math concept being considered (Inoue & Buczynski, 2010). Therefore, students are encouraged to learn, discover, understand or solve problems on their own by experimenting and evaluating possible answers by trial and error, so they have a higher problem-solving performance than those students taught only one way to solve a problem (Bruun, 2013). Rowland and Zazkis (2013) believe that this extended exposure to mathematics serves as a support structure for a teacher's readiness to hypothesize, experiment, to take risks, and, most importantly, take advantage of unforeseen opportunities that arise during instruction. One of the primary responsibilities of an educator is that they are not only aware of the main essence of shaping the experience by the surrounding conditions, but that the teachers also recognize what surroundings are conducive to having experiences that lead to growth (Dewey, 1938). Dewey (1938) asserts that, above all, teachers should know how to use their surroundings, both physical and social, that exist during a lesson so that teachers can acquire all that students have to share in order to build up experiences that are worthwhile. To understand the lived experiences of students, teachers must explore more deeply into what "knowing-in-the-moment can be like in order to suggest how teachers can develop that knowing, without losing the essential complexity of the phenomenon" (Mason & Davis, 2013, p. 188). Mostly, this has to do with teachers being mathematical with and in front of their students (Mason & Davis, 2013). Therefore, the most crucial attitude a teacher can form with students is the desire to go on learning (Dewey, 1938). For this reason, it is essential for mathematics education shift from the basic transfer of information, instructions, and algorithms to grasping, acting, experiencing, and developing a desire for life-long learning (Ticha & Hospesova, 2006). This requires a change in the teacher's role in promoting new schemes and demanding more from students through differing strategies.

**Summary**

The Common Core State Standards go more in-depth and require a more refined skill set based on relevant pedagogical content knowledge (Osborne, 2015). It is optimal for a teacher to be cognizant of the mathematical subject matter as well as the pedagogical facets of mathematics to direct students to what is essential and the varying ways problems can be solved (Mason & Davis, 2013). Therefore, a teacher's motivation can be fueled by the support they receive to create sociomathematical norms in their classroom (Kazemi & Stipek, 2001). This is where conceptual versus procedural understanding of mathematics is pivotal. Procedural knowledge is when procedures are followed in a predetermined linear sequence ensuring automaticity. However, conceptual understanding is gained by the students building relationships between pieces of information acquired over time. Drawing connections among mathematical concepts are what deepens conceptual knowledge. The Common Core and NCTM standards require this connection, but teachers do not have the training or support to be successful even when given a scripted curriculum. It is essential to understand the specifics of a student learning procedurally and conceptually in a mathematics classroom. More importantly, there are several lessons that can be learned from conducting an educational criticism and connoisseurship to study the pedagogical decisions teachers make during mathematics instruction and how these decisions affect a student's ability to conceptualize math concepts.

**Method**

In the 1960s, Eisner (1991) developed a method of qualitative research that he named "educational criticism and connoisseurship". Using the arts as the basis for his thinking, Eisner came to discern connoisseurship as the art of appreciation and criticism as the art of disclosure (p xi). He formulated a manner in which one can disclose what one learned through his and her connoisseurship as description, interpretation, evaluation and thematics. Thus, an educational criticism and connoisseurship was used to examine mathematics instruction to develop an in-depth understanding of the pedagogical choices third-grade mathematics teachers make to prepare their students for high stakes testing in three different suburban school settings within a large district in one of the states of the Rocky Mountain West. The purpose was to provide a vivid picture of how teachers were challenged with making pedagogical decisions while teaching within a conceptual framework and how students' learning was impacted.

**Four Dimensions of Educational Criticism and Connoisseurship**

Educational Criticism has four dimensions through which the critic disseminates his/her observations: *description, interpretation, evaluation, and thematics* (Eisner, 1991). When *describing*, the critic uses narrative to portray what is essential from the profound qualities of the experience. The goal is to express what it would feel like to be in the environment in which the researcher is trying to portray, and in doing so helps the reader know the environment (Eisner, 1991). The critic cannot attend to everything in an educational setting, but instead provides those elements that help the reader participate vicariously in the experience to understand and critique the interpretations made by the critic based upon these observations. Through *interpretation*, the critic explores the meanings and consequences of the educational setting observed. According to Eisner (1991), the goal is to illuminate "the potential consequences of practices observed and reasons that account for what's been seen" (p. 95). As mentioned earlier, theory, experience, and various viewpoints influence how the critic interprets the educational events described; therefore, there is not a right or wrong interpretation. Instead, the interpretation provides a means of exchange between the critic and the reader to develop a concept of reality. The *evaluative* dimension examines the educational significance of the description and interpretation. The aim of education "is not merely to change students, but to enhance their lives" (Eisner, 1991). The evaluation piece helps to discern if the educational experience observed has met this objective. Since the researcher makes choices in what he/she pays attention to, the values that guide the observation also inform the evaluative dimension and imbue what is written. Lastly, *thematics* afford the reader with the larger lessons a criticism has to offer (Eisner, 2002). “That is, every particular is also a sample of a larger class. In this sense, what has been learned about a particular can have relevance for the class to which it belongs” (Eisner, 1991). The researcher’s aim is to provide readers with novel theories or guides to help them understand and appraise the pedagogical choices third-grade teachers make to prepare their students for high stakes teasing under the Common Core Standards. By studying the practices of three teachers in this study, the researcher has developed a powerful understanding of the implications there are for students by these pedagogical choices. The researcher’s goal is to provide a vivid description, interpretation, and analysis of these pedagogical decisions to guide stakeholders to understand these pedagogical decisions in hopes of improving mathematical practice.

### Validity in Educational Criticism and Connoisseurship

According to Eisner (1991), validity in qualitative research and educational criticism and connoisseurship categorically relies on three different criteria: (a) structural corroboration; the “means through which multiple types of data are related to each other to support or contradict the interpretation and evaluation of a state of affairs” (p. 110); (b) consensual validation: “at base, agreement among competent others that the description, interpretation, evaluation, and thematics of an educational situation are right” (p. 112); and (c) referential adequacy: the “extent to which a reader is able to locate in its subject matter the qualities the critic addressed and the meanings he or she ascribes to them…when readers are able to see what they would have missed without the critic’s observation” (p. 114). Therefore, it was essential to have observations of mathematics lessons, but also interviews before and after the observation window, as well as teacher reflections of the lessons observed each offering additional details and clarifications to the observations.

### Research Setting

Observing three third-grade classroom teachers at three different elementary schools for a total of 30 hours allowed for an in-depth understanding of the decisions teachers made in preparing students for high stakes testing under the Common Core State Standards and the factors that led to these decisions. Criterion sampling was used so that three third-grade teachers in the Red River District (pseudonym) could be studied. The reasoning behind studying third-grade, in particular, was because the pressure is more significant in third-grade classrooms versus second-grade classrooms due to second-grade scores not being counted when determining a school’s accountability rating (Plank & Condliffe, 2013).

## Data Collection Methods

Within the three schools in which the third-grade teachers reside, the researcher conducted a pre and post interview with each teacher around the observation window and also reviewed lesson plans weekly. Most importantly, the researcher collected data related to the training and professional development provided to prepare the teachers for the teaching of the Common Core State Standards and, in particular, the new conceptual mathematics curriculum, Bridges in Mathematics, introduced this year. The observation window was six weeks prior to the administration of the high stakes assessment and two weeks following the completion of the high stakes assessment. Each teacher was observed weekly to record descriptive and reflective notes during the mathematics lesson. Seeing student reactions to lessons taught and how they interacted with other students and the teacher allowed the researcher to answer the research question around implications for the students by these decisions and how these pedagogical choices influenced the students understanding of mathematical concepts.

## Data Analysis Methods

Once written and recorded observations were collected up to the point of the administration of high stakes testing and also collected a few weeks after the assessment, observation notes, lesson plans from the actual curriculum, documents used during each lesson and journal entries were put in chronological order and grouped by teacher. The key was to have a complete picture of the lesson observation from what was actually taught to what was the intended lesson from the curriculum. Once the observations, curriculum lesson plans, supplemental lesson documents, journal entries, lesson plans were put in chronological order, they were read four times using a different type of lens for each reading. During the first reading, the researcher focused on items/activities observed during the lesson that directly correlated to the intended lesson plan in the curriculum. The researcher then focused on items that were observed that were **not** part of the intended lesson plan in the curriculum. For the second reading, the researcher analyzed those items/activities that were **not** part of the intended lesson plan from the intended curriculum to distinguish if the item/activity was procedural or conceptual in nature. As a result, those items/activities that were procedural in nature that were not part of the intended curriculum and those items/activities that were conceptual in nature that were not part of the intended curriculum became apparent. This made it very easy for the researcher to see what pedagogical decisions the teacher was making outside of the intended curriculum and if they were procedural or conceptual in nature. Also, analyzing how much the teacher followed the intended curriculum or modified it to her own pedagogy was equally important. The researched determined how many items/activities were not part of the intended curriculum and determined the percentage of those activities that were procedural and conceptual in nature based on the teacher's pedagogical decisions. The third reading focused on if the pedagogical decisions made by the teacher throughout the entire lesson impacted the intended skills or concepts from the intended curriculum lesson in the teacher handbook. The researcher determined what pedagogical decisions made by the teacher during the observations impacted the intended skills/objectives by highlighting the skills/objectives that were covered by the observed lesson on the curriculum lesson plan and those skills/objectives **not** meet by the pedagogical decisions of the teacher during the observation. The fourth reading involved highlighting those activities/items that were actually left out of the intended curriculum lesson. Those items/activities that were left out of the intended curriculum that were procedural and conceptual in nature conceptual were noted. This gave the researcher a visible picture of those pedagogical decisions made by the teacher during the observations that were procedural or conceptual in nature. While reading the interviews, it was essential to disregard the actual question asked so the researcher could focus on what is actually said. Interviews were kept separate and used to analyze for teacher reflection to get an overarching picture of the teachers' feelings about high-stakes testing and the pressures of teaching a new curriculum this year. The journals filled out by the teachers for each lesson observed were read in conjunction with the lesson plans and observations.

## Findings

After identifying 186 items/activities that were **not** part of the intended lesson plan provided in the teacher handbook, the researcher determined which of those items/activities were procedural or conceptual in nature. After analyzing each item/activity, 68 percent of these items/activities were deemed procedural in nature, and 31 percent of these items/activities were deemed conceptual in nature. Based on this vast percentage difference between those items/activities added by the teacher that was procedural versus conceptual illustrates a teacher's insecurities with teaching conceptually or even their preference for teaching procedurally. With the new curriculum mainly using strategies to build conceptual understanding, it was not surprising to see such a gap in percentages. If a teacher is not confident in her ability to teach a concept conceptually, she will resort to what she already knows. This idea is reinforced by Mason and Davis (2013) in that teachers seem to misunderstand the vital connection between mathematical awareness and in-the-moment pedagogy to be successful in teaching mathematics for conceptual understanding. The vignette below illustrates a lesson that advocates for a conceptual approach through a lesson to be followed in the teacher handbook, but she finds herself teaching the lesson strictly for procedural understanding focusing strictly on the algorithm for area and perimeter.

> The teacher stands by the pocket chart in 'number corner' as the students sit on the United States carpet with their eyes glued to the teacher as she reviews the terms area and perimeter and asks the students to tell her how they are alike and different. A large number of students raise their hands in hopes that the teacher calls on them. As the teacher calls on one student to give an attribute, the students anxiously wait for her to call on another student to add to his response. The teacher reads the term on the card and begins to chant, "Perimeter is—you add the sides up!" The students repeat

> the chant with her as she puts the card back in the pocket chart. She quickly pulls the next card out of the pocket chart and displays it to the students. Immediately hands bolt up to the ceiling as the teacher reads the term on the card, "Area." The students immediately start performing motions with their hands with their palms up as if they are lifting a box and then immediately makes a cross with their forearms and then raises the imaginary box to the ceiling. The teacher then repeats the chant as the students follow, "Area is—base times height." The students follow along as the teacher repeats the chant and makes the hand motions to match. She repeats this three more times in hopes that the students would repeat the chant in sync but is greatly disappointed. She then says, "Perimeter is—you add the sides up." The students continue to mimic the teacher's hand motions as they sing chorally the algorithms to find the area and perimeter of a geometric figure. The students are then asked to return to their desks as the teacher saunters over to the Smartboard data projector to project a square centimeter. The students slowly begin to settle in their desks as chairs begin clanking against desks as they get comfortable. The teacher writes on the paper projected on the Smartboard—Perimeter: 1 + 1 + 1 + 1 = 4 cm. She then proceeds by writing the algorithm for finding the area of the square centimeter—Area: 1 x 1= 1 $cm^2$. The students watch as the teacher displays her calculations on the Smartboard.

It was interesting to observe the teacher using the notation $cm^2$ versus writing square centimeters. The students do not learn about exponents in third-grade, but the teacher made a pedagogical choice to display this notation. What was even more interesting was that the students did not even question what it meant. This indicated that the students were used to being fed information instead of discovery.

> She continues by writing the word 'dimensions:' with 1 by 1 written underneath it. She places an arrow pointing to the 1 and writes width under it and then does the same for the other numeral 1 and writes height below the arrow. The students continue to watch the teacher display writing on the Smartboard in utter silence. You could almost hear the gears moving in their heads as they try to make sense of what the teacher is displaying on the Smartboard. No hands are raised as she proceeds to put up a strip that is 10 centimeters long. She asks the students, "How would we find the perimeter of this rectangle?" The students sit in silence as four hands raise timidly to be called upon. The teacher calls on one student to answer her question. The student answers, "You would add 10 + 1 and 10 + 1." The teacher quickly writes this down to be displayed on the Smartboard and then moves on to ask the students how to find the area of the rectangle. A very excited boy leaps from his chair to raise his hand. The teacher patiently waits to call on him in hopes that other students would raise their hands. She calls on the student, and he responds, "1 x 10 = 10". The teacher immediately asks, "10 what? We don't want a naked number!" Then several students chuckle as the student answers, "square centimeters." Most of the students look bewildered as the teacher asks, "Could we use this to measure something if we did not have a ruler?" Most of the class chorally answers, "Yes!" The teacher immediately asks, "Why?" She then answers her own question without calling on the students, "Because we know that each square is a centimeter." The students are then instructed to open their student workbooks to page 206.

The teacher began the lesson conceptually as intended by the teacher handbook but quickly strayed teaching procedurally by focusing on the algorithm for area and perimeter. The teacher was to ask student pairs to compare and contrast the two terms by how they are alike and different. In introducing these terms, the teacher immediately felt the need to sing the chants associated with the algorithms. Next, the students were to share their ideas with the class than with their partner. From there, the students were to get out a ruler and a set of base ten pieces setting aside one small square unit. The teacher was to pose the following questions about the square unit and have students use the centimeter side of their ruler to help find the answers: What are its dimensions? What is its perimeter? What is its area? From there, everyone was to proceed to measure the base ten strip using his or her rulers to help find the dimensions, perimeter, and area of the strip in metric units. Per the observation, no manipulatives were passed out. The next activity in the handbook instructed the teacher to ask the students what they notice about measuring area as compared to measuring perimeter determining how they are similar and different. As indicated by the lesson plan in the teacher handbook, the students were to discover the difference between area and perimeter by measuring a square unit and base ten strips on their own. The teacher chose to use procedural activities to move the lesson along not allowing the students to conceptually understand area and perimeter through measurement. At the very beginning of the lesson, the teacher's intentions were going in the right direction, but feeding the students the information was how she chose to continue the lesson. This could have been a lesson within a series of lessons discovering area and perimeter. However, if the teacher introduced the algorithm during the very first lesson of the series, it makes sense as to why she did not expand the discussion in this particular lesson since it would be redundant. This is another downfall of a teacher not following a curriculum as instructed. If the lessons were written to build upon each other each day to build conceptual understanding, and the teacher makes pedagogical choices to change the lesson to be procedural in nature, then the lessons that follow would not serve their purpose. Figuratively speaking—she would be putting the cart before the horse. The following vignette illustrates how a teacher's intentions were sincere in fostering conceptual understanding of area and perimeter, but, in actuality, she was narrowing the scope of strategies to a few students' ideas.

> The teacher has already displayed an example of a table in the shape of a backwards L on the Smartboard (see figure 1) that has an area of thirty square centimeters. The problem at the top of the page reads—*One day when Emery was in town doing errands, the Goat Twins, Zachary and Whackery, decided to surprise him. They got some of Emery's small square tables out of the shed and arranged them in an unusual way. When Emery got home, he laughed at the twins and said, "OK, if the two of you are so smart, can you tell me the area of this big new table you've arranged?" Help the twins. How can they find the area of the table they made without having to count all the small squares?* The teacher allows one student to share a way to divide the table into sections to calculate its area. The teacher now asks the students to get out their journal and label the page 'Goat Twins Table'. The teacher asks, "Does someone have a different way?" The teacher calls on Taylor

and asks the students to write 'Taylor's Way' in their journal. The teacher is modeling this on the Smartboard. The student then proceeds to tell the class that he shaded this part (pointing to the 6x3 section of the L shape) and this square (pointing to the 3x4 section of the L shape) [see figure 2]. The students rush madly to replicate the teacher's notations from the Smartboard into their own journals. The teacher asks, "What is your equation?" The student replies reluctantly, "3x6=18, 4x3=12, so 18+12= 30 units $^2$. The teacher is working ahead of Calvin as the students rush to write down Calvin's way. The teacher responds to Calvin, "Good, we got the same answer, University so that way works! Who wants to show another way? Who has another way?" Several hands raise in the air in hopes of seeing their name in lights up on the Smartboard. This time Jordan is called upon to answer. She jumps out of her seat rushing over to stand next to the teacher who is displaying the same L shaped table with no markings on the Smartboard (see figure 1). She excitedly tells the class, "I did the bottom 2x3, the middle 2x3, the top left 1x3, then the top middle 2x3 and then the far right 2x3 and then finally 1x3 for this last section." The teacher colors in each section identified by Julie in a different color separating each with a thick dark line (see figure 3). The teacher coordinates the color of each section with the multiplication sentence to match by the diagram. The teacher says, "There are many ways to split this table up. There may be a more efficient way of splitting it up instead of counting each one. So, what is the equation for Jordan's way?" She continues with Jordan's way by finding the product of each multiplication sentence. She asks Jordan, "What do we do next?" She responds, "Well, I added them up as I went." The teacher replied, "But you actually added them all up, right?" Jordan replies, "Yes!" Suddenly, the teacher announces, "Well, I am going to do it this way. 3+3=6 and 6x4=24 so 6+24=30 units$^2$. The teacher explored one more student's way to split up the table finding the area and then abruptly asks the students to complete page 212 in their student book.

Figure 1

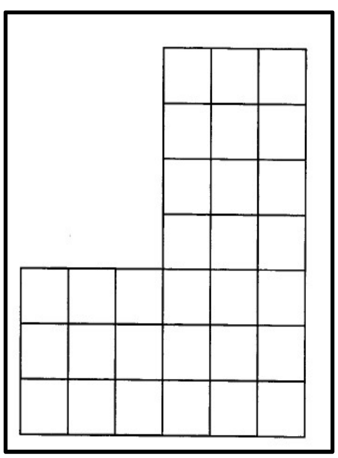


Figure 2

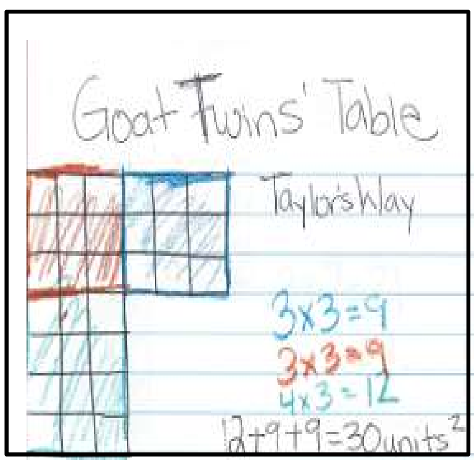
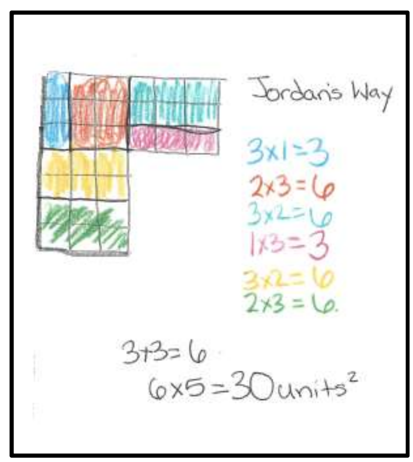



Figure 3




The teacher's intention was to explore different strategies for finding the area of an irregular shape. She shared with the students four different ways to split up the table to find its area given by four students. The crucial aspect missing out of this discussion was allowing students to disagree with the division of the shape and a discussion about a better alternative. Instead, the teacher made a pedagogical choice to keep the discussion between her and one other student at all times. Also, the teacher handbook instructed the teachers to give the students time to solve the problem on their own and share in pairs before sharing with the class. As each strategy was shared by a student, the students should have been encouraged to disagree and share a potentially more straightforward way. Instead, the teacher shared **her** way of solving the problem without any further discussion. Also, she did not give Jordan any validation of her strategy of adding as she went as if Jordan's way was discouraged. The teacher was so set on having an equation to solve, that she could not see that Jordan's way was just as efficient. In the teacher's reflection of this lesson collected the following week, she enjoyed watching the students figure out the area of the table as many were as possible, but she would have prepared the table array in advance figuring that her student could not draw an accurate array which caused disorganization and lack of flow. The teacher also felt that the majority of her students could come up with at least one way of finding the area, however, they struggled with the lack of organization. These reflections are collected and not discussed. This reflection highlights the teacher's inability to allow students to persevere in problem solving and how the accuracy and organization of her students' arrays overpowered the need for students to have a conceptual understanding of area and perimeter.

**Discussion**

Based on this analysis of 168 activities/items that were included in the lessons but were **not** part of the intended lesson provided in the teacher handbook, 68 percent of the items/activities taught to students during observations were procedural in nature and 31 percent of these items/activities were conceptual in nature. This leads to the question: What implications are there for students by these pedagogical choices made by the teachers? According to Junod (2017), the implications center around a teacher's ability to let go of their procedural mindset allowing students the time and space to persevere in problem solving without supplying a procedure to follow. When teachers make a pedagogical decision to merely push for solving a computation correctly to help students understand mathematics, they have taken away a student's ability to solve real-world problems out of the context of a number sentence. The result is that students find word problems to be too complex and are unable to identify the process to the correct solution on their own. They also struggle with the perseverance to problem solve. The reality is that many teachers find themselves switching to telling and explaining when things are not going as planned during a mathematics lesson (Mason & Davis, 2013). The teachers in this study found it difficult to let students think and problem solve on their own and were insistent on teaching the most efficient strategy to solve the problem.

According to the new standards, students should be taught to persevere in problem-solving enabling them to come up with varying solutions to problems. Without this struggle, students lose the opportunity to discuss their thinking and reason with other students when conceptual items/activities are not incorporated. Learning new mathematical concepts hinges on what a student already knows, so as students advance, new concepts will depend more on conceptual knowledge (Willingham, Winter 2009-2010). For instance, correct conceptual understanding of the equal sign in a number sentence depends on understanding algebraic equations, so if students fail to gain conceptual understanding, it becomes more difficult for students to catch up since new conceptual knowledge depends on the old. Thus, students become more likely to just memorize algorithms and apply them without understanding (Willingham, Winter 2009-2010). This was clearly evident in this study with the teachers' use of chants in order for students to memorize the algorithms when conceptual understanding was not gained first. Perhaps chants would be acceptable once algorithms are introduced at the very end of a series of lessons around a single mathematical concept, but not before. One explanation for this pedagogical decision might be that the teachers did not understand the importance of the item/activity presented in the teacher's handbook and, therefore, chose to adapt it or skip the activity altogether. However, according to the teacher reflections collected after each observation, the teachers did not mention why they skipped activities and reiterated that they thought their lessons were successful. Mathematics educators need to know whether or not an item/activity measures conceptual knowledge, but in order for the teacher to be successful, the teacher needs to know whether the students understand the definition of a concept or if they just memorized it (Star & Stylianides, 2013). Another possibility that would lead to this pedagogical mindset is that teaching procedurally has just become habit, especially for the more experienced teachers. The basic characteristic of habit is that every experience enacted and undergone modifies the one who acts and undergoes, whether we wish it or not, the quality of subsequent experiences. Therefore, every experience in the classroom both takes up something from prior activities or discussions and modifies the quality of those activities or discussions that follow it (Dewey, 1938). This in-the-moment pedagogy often causes confusion and tension for teachers as they try to navigate the selection of instructional approaches to address particular mathematical standards (Wilson & Downs, 2014). According to survey data collected by Vogler and Burton (2010), the teachers indicated they use a balance of standards-based and traditional instructional practices, but it was unclear the amount of time and focus used on both. However, these practices tend to lend themselves to either traditional or standards-based instruction which, in reality, lies within the intention of the teacher and cannot be measured by a survey (Vogler & Burton, 2010). This is the main reason the researcher chose observation of mathematics lessons being taught as being critical for this study. The researcher found discrepancies between what the teacher described in their reflections they were doing during instruction either conceptually or procedurally and what was reality. One explanation might be that these teachers were not allowed opportunities to observe mathematics' lessons taught conceptually for this curriculum during their professional development and what it looks like for the teacher and the students. According to the participants in the study, they only received a two-day training for this new curriculum which included only a review of the teacher's manuals and not videos of lessons being taught conceptually with students. With this new mathematics program having a conceptual framework, the teachers would need time in the classroom to conduct lessons ensuring a follow-up session with the trainers to address teachers' questions or concerns regarding the program. According to the teachers in the study, they did not receive the help or support necessary to implement this new time demanding technique of guide discovery or conceptual understanding. Additionally, if the teacher does not have an understanding of the way these ideas build across grades, it is possible for a teacher with good intentions to prematurely introduce an algorithm, thus, preventing the development of a deeper understanding of the concept (Wilson & Downs, 2014). Therefore, following a conceptual lesson intended in the teacher handbook is recommended until a teacher feels confident that it will not change future objectives or activities within the curriculum. The researcher cannot conclusively state that the suggestions above will rectify the lack of conceptual learning during mathematics lessons, but it would lead teachers to challenge a procedural mindset in order for students to find a balance when modeling strategies to students. The wait time necessary to allow students to persevere in their problem-solving skills is important to comprehend. As asserted by Rowland and Zazkis (2013), teaching mathematics conceptually requires attending to students' questions, anticipating obstacles, capitalizing on opportunities, making connections and providing enrichment beyond the immediate tasks. Ultimately, there is no place for a procedural mindset in a mathematics classroom when the new standards require a balance of both procedural fluency and conceptual understanding.

**Contribution to the Field**

This educational criticism and connoisseurship highlights how a teacher's procedural mindset hinders the ability to embrace conceptual learning of mathematics and, therefore, does not release a student to learn mathematics conceptually. This is compounded by the pressures of high-stakes testing. Teachers must resist the urge to teach mathematics "the way it has always been done." In order to reach all students in the classroom, the teacher must allow students to persevere in problem solving and explore strategies on their own. Only then will students be able to build skills necessary for success with unknown problems presented during high-stakes testing. This goes against everything experienced teachers have been taught and requires trust in the process of the struggle. This does not require a teacher to abandon their mathematics curriculum assigned by their district or school but modify group and independent practice with the use of real-world problems. For example, if a teacher is working in an urban school where a worksheet to be assigned involves the addition of three-digit numbers, several different word problems can be created on chart paper diversifying each problem to meet the needs of every student or group in the classroom. Groups would then solve the problems using appropriate manipulatives for the skill and would display their strategy on the chart paper. The teacher can monitor each group's progress using a rubric to score their problem solving, reasoning, communication, connections, and representations ensuring accountability. Rubrics can be customized to focus on certain skills and understandings. Group work allows students to work with others to persevere in problem solving and argue constructively their strategies they experiment with to solve the problems. The key is allowing students time to problem solve on their own versus telling them how to solve problems procedurally. Teaching students strategies would be appropriate if they cannot offer any strategies on their own. Gauging what students know through these types of activities before teaching a mathematics concept procedurally is optimal. In a whole group setting, you can introduce a word problem and work on it as a group having individual students share their strategies on the document camera using manipulatives, drawings or computations until all strategies are exhausted by the students. This allows the teacher to ask essential questions and encourage students to argue their answer without the correct answer being given away at this point. The Common Core Standards were developed to enhance a mathematics curriculum to instill a desire for students to problem solve thus increasing their success on unfamiliar problems given on high-stakes testing. Teachers must have faith in their students and allow them to persevere without being told the steps necessary to solve problems. It has become habit to "feed" students strategies and steps needed to solve problems. The greatest gift that can be given to students is the ability to think and problem solve. This is not only a 21st-century skill but what all future employers look for in their applicants. We are doing our students a disservice if we do not teach them to think. A conceptual curriculum can be given to teachers along with weeks of training, but if the procedural mindset remains, there will be no change in thinking. This study highlights that teaching procedurally, in most cases, is second nature and perseveres even after teachers are given a conceptual mathematics curriculum and training; it is almost a subconscious act. Recognizing that a procedural mindset exists is the first step towards embracing the conceptual teaching of mathematics.

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