Book Review for *BIRTH OF A THEOREM by Cédric Villani*

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Title: Birth of a Theorem

Author: Cédric Villani (translated from French by Malcolm DeBevoise)

Place: New York, NY

Publisher: Farrar, Straus & Giroux

Publication Date: 2016

Pages: 250

Special Features: none

Price: $16.00

ISBN: 978-0-374-53667-1

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By means of his own personal experience the author Cédric Villani clearly succeeds in answering key questions, such as what life is like being a mathematician, and how the work of mathematics gets done. That is what makes the reading of this book such an exciting journey. It is definitely not necessary to wade through the technical details of the interpolated mathematics; these may be ignored, as I did in reading the book.

For the author's background and credentials, I quote from the front piece of the book: "Cédric Villani is the director of the *Institut Henri Poincaré* in Paris and a professor of mathematics at the *Universite de Lyon*. His work on partial differential equations and various topics in mathematical physics has been honored by a number of awards, including the Fermat Prize and the Henri Poincaré Prize. He received the Fields Medal in 2010 for results concerning Landau damping and the Boltzmann equation."

Cedric Villani begins our journey by relating a work session with his colleague Clement Mouhot on the regularity condition for the Boltzmann equation. The Boltzmann equation models the evolution of a rarefied gas made up of billions of particles that collide with one another, the statistical distribution of the positions and velocities at time t indicating the density of particles

whose position is x and whose velocity is v. The author takes us through his logical discussion and comes back to Landau damping.

The first step in mathematics research is knowing exactly what you are working on, and what you are trying to do. Speaking with various mathematicians in his group, the author garners ideas from a variety of different disciplines that he hopes will be useful in his research. Papers mentioned, although having apparently no connection with his problem, are food for thought in the search for answers.

Digression seems to be beneficial to the problem solving process, because then he digresses to a vignette on Joseph Fourier and the decomposition of vibrations. Villani remarks on the usefulness of dreams to the intellect. Sometimes dreams help to clarify the nature of the research problem: the means to the solution is often more important than the result. Recalling that other researchers have solved part of the problem, or a related problem, he realizes that a new approach with new tools will be necessary to solve his problem.

Villani describes his arrival at the Institute of Advanced Study at Princeton (IAS), and the planned work schedule. He muses about how and what it takes to win the Fields Medal. Using email and phone calls, he continues to work on his problem along with Clement in France.

Villani laments the fate of great luminaries, such as Bobby Fischer, Paul Erdős, Grigori Perelman, Alexander Grothendieck, Kurt Gödel, and John Nash as he struggles with his problem on Landau damping. Talking with a colleague at IAS about the colleague's own work helps to clarify the issues. This does not yet bring a solution to Landau damping, because the proof is not working. Also, he must decide whether to stay at IAS in Princeton versus going back to France to be director of the Institute Haute Poincaré (IHP), but ultimately decides to return to France.

Clement Mouhot is discouraged and practically wants to give up on the insurmountable problems they are confronted with in the proof and is about to email Villani, but finally he sees a way to solve the difficulties. The two mathematicians continue to work together to patch the holes in the proof, and finally put all the pieces in place to complete the proof. Exaltation is the dominant emotion.

When the theorem is presented at Princeton there are in the audience some critical physicists finding faults with the argument. This criticism stimulates Villani to work faster than normal in resolving the difficulties in the proof. Discovery in one area suggests solutions in other areas as well. Though the corrections are not finished, he goes to bed exhausted and somewhat discouraged.

However, waking early the next morning the solution immediately pops into his head. While he was unconscious during sleep, his subconscious mind continued to work on the problem. Then he sits down to work, discovers that the solution works and must be written out. Triumph! And thus it goes: between problems that crop up and the solutions, back and forth. Finally, illumination and euphoria!

As in music, mathematics is something you are struck by, enamored, until nothing else matters. Music and mathematics are known to support each other. Confronted in mathematics with the proof that remains just out of reach, happiness mixed with pain. Villani relates incidents in the life of Mikhail Gromov, who developed the theory of convex integration.

While out walking around IAS and enjoying the natural beauty of the landscape Villani encounters Vladimir Voevodsky and they discuss their respective work and the prospect of the computer checking of proofs. Villani is also reflecting on the Landau damping problem and having to say good-bye to Princeton and IAS. As often happens in thinking about mathematics, unpredictable encounters lead from one problem to another.

Villani describes his prepared address to the upcoming Congress of Mathematical Physics in Prague. But he is dominated by concern over the non-acceptance of his research in Acta Mathematica, yet there is consolation for recently having won the Fermat Prize. Again, he digresses to Mittag-Leffler, as founder and first editor of Acta Mathematica, with the recollection of Poincaré as Mittag-Leffler's favorite author, who founded dynamical systems while studying the stability of the solar system. Villani obsesses over assumed failure to publish in Acta Mathematica. There follows a consultation with another colleague who points out a weakness in the proof. When Villani tries to write a justification for the steps, he discovers a careless oversight, and sets about to repair it.

Receives momentous news that he has won the Fields Medal, which leads to elation. However, this must be kept a secret until the International Mathematical Union announces it. Villani describes the origin of the Fields Medal. Returning to Paris from a trip to Cairo he reflects on his entire career. At long last he receives the confirmation letter advising his award of the Fields Medal.

Attends the funeral of Paul Malliavin (Malliavin Calculus), and fondly recalls the strong early encouragement from his older mentor. Villani discusses the Poincaré Conjecture with Gregori Perelman's solution and proof, and subsequent refusal of prizes and honors.

The fevered excitement at the presentation of his Fields Medal is recalled: the grandeur, meeting distinguished friends and colleagues, partying and reveling; the participants then returning to their usual lives in universities and research centers. Villani contemplates the aftermath of fame for having received the Fields Medal. This is the process of mathematical discovery: struggle with a difficult problem, success, adoration and fame, dejection that it's over, then going on to a new problem. There are always the nagging doubts: what if the proof is wrong? At last he receives word from Acta Mathematica that his proof on Landau damping is accepted. A theorem is truly born at last. The process goes on unceasingly.