An Experimental Problem-Based Approach in Mathematics Teaching and Mathematical Problem-Solving Performance in Grade 3 Primary School Pupils

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Abstract: Problem-solving is a key aspect of mathematics instruction. Although it is part of several national curricula, its implementation is still being vastly explored. The present research, which is part of a larger project, aimed to investigate the effect of using problem-oriented instruction in grade 3 of primary school on students' mathematical abilities of solving problems in arithmetic, geometry, and logic. The study used a pedagogical experiment, with a sample of 240 pupils in Grade 3 from four randomly selected Slovenian primary schools. Students were divided into two groups: 100 in the experimental group and 140 in the control group. Two tests of mathematical knowledge, constructed by the authors, were used to assess potential differences between the groups before and after the experiment took place. The results showed that the use of an experimental model of problem-based learning had a statistically significant effect on pupils' mathematics problem-solving achievement in all the domains of mathematics (i.e., arithmetic, geometry, logic & set theory). The study suggests that modern school reforms should focus on problem-based mathematics instruction as a means of achieving meaningful and lasting knowledge in students. Additional research is needed to replicate the results due to the limited number of participants in the present research.

Keywords: achievements, control group, elementary, experimental group, problems

INTRODUCTION

Mathematics education is a rapidly evolving field, as new trends and methodologies are continually being demonstrated to be more effective than previously employed ones. For instance, problem-solving is gaining more and more attention in mathematics education (Siagian et al., 2019). It provides students with opportunities to engage with mathematical concepts and ideas in meaningful and authentic contexts (English & Gainsburg, 2015; Jurdak, 2006). Through problem-solving, students can develop their mathematical reasoning, critical thinking, and analytical skills (Belecina & Ocampo Jr., 2018). They learn how to formulate problems, identify relevant information, and apply appropriate strategies and techniques to solve them. Moreover, problem-solving promotes active self-regulated learning (Muis, 2008), as students take responsibility for their own learning and monitor their progress and performance. By using problem-solving in mathematics education, teachers can create a student-centered (Gezim & Xhomara, 2020; Saragih & Napitupulu, 2015) and inquiry-based learning environment that fosters creativity, curiosity, and collaboration (Divrik et al., 2020). Furthermore, problem-solving can help students develop a positive attitude toward mathematics (Albay, 2019; Russo & Minas, 2020) and appreciate its relevance and applicability in various domains of life.

Teaching students to solve problems in mathematics means promoting students' learning and understanding of mathematics (Albay, 2019; Ali et al., 2010). This approach involves presenting students with problems that require them to engage in mathematical thinking and reasoning to arrive at a solution (Francisco & Maher, 2005). Problem-solving teaching encourages students to use higher-order thinking skills, such as analysis, synthesis, evaluation, and creativity, to explore and investigate mathematical concepts and relationships (Collins, 2014; Widana et al., 2018). It also promotes the development of metacognitive and affective competencies (Lai et al., 2015), as students learn how to monitor their own thinking and emotions and how to persist and persevere in the face of challenges and setbacks (Mariano-Dolesh et al., 2022). Specifically, a student's mindset is indicative of their degree of conceptual understanding in problem-solving, with those possessing a growth mindset demonstrating greater potential for successfully solving mathematical problems.

Problem-solving is particularly important in the early stages of mathematics learning (Tarim, 2009; van Bommel & Palmér, 2018). Through problem-solving, children develop their critical thinking skills and acquire mathematical knowledge that is applicable to real-life situations (Hartmann et al., 2021). Research shows that incorporating problem-solving activities in early math education can enhance children's conceptual understanding, improve their problem-solving abilities, and increase their motivation and interest in mathematics (Cheung & Kwan, 2021). Thus, it is essential to incorporate problem-solving in early math education to help children develop the necessary skills and knowledge needed for future academic success (van Bommel & Palmér, 2018).

Despite the importance of using problem-solving-based instruction, several studies have found that teachers lack competencies and knowledge about problem-solving in mathematics (Barham, 2020; Chapman, 2015). In particular, teachers' math anxiety and math self-efficacy are correlated to teachers' ability to solve problems in mathematics (Akinsola, 2008). Moreover, teachers' beliefs about mathematics and problem-solving do also affect the extent to which they use this strategy in class (Saadati et al., 2019). Teachers' attitudes toward problem-solving may also influence students' beliefs about it (Yorulmaz et al., 2021).

Considering the Slovenian context, it might be noticed that in the national mathematics curriculum for primary school (Žakelj et al., 2011), several goals regarding problem-solving are mentioned. In particular, it is stated that students need to develop reasoning, generalizing, abstracting, investigating, and problem-solving. Considering, for instance, arithmetic, students should be able to solve arithmetic problems that are also related to real life. Nevertheless, the model of teaching is still traditional, i.e. behavioristic. Teachers do include elements of the national curriculum involving problem-solving, but the teaching model has not yet shifted from a behavioristic one to a problem-based approach. Therefore, it is unclear whether students exposed to a problem-based approach would be better problem solvers than those exposed to traditional lessons.

The aim of the present research is, therefore, to evaluate the efficacy of the problem-based approach with regard to the more traditional (i.e., behaviorist) model of teaching mathematics in primary school. In particular, we expect students that are exposed to a more problem-solving-oriented mathematics lesson to have higher achievements in tasks of all taxonomic levels and from all mathematics domains (i.e., arithmetic, geometry, and logic).

LITERATURE REVIEW

Mathematical problems

A mathematical problem is one in which a person attempts to overcome a problem situation by using mathematical tools (Phonopichat et al., 2014; Zhu, 2007). In the broadest sense, Magajna (2003) defines a mathematical problem as a situation that initially creates a sense of the unknown and discomfort because one is unable to explain a certain fact to oneself or is unable to achieve a desired goal, which will be resolved upon solving the problem. Problems in math can be classified according to different criteria. In general, they can be distinguished by the visibility of the solution path. Magajna (2003) distinguishes between (Kolovou et al., 2011; Mwei, 2017):

- *Routine Problems* (which are not even problems in the usual sense, but rather exercises), where the solution path is clear to the solver in advance, either because he has already been through it several times or because the path is evident from the formulation of the problem or from the context;
- *Non-Routine Problems* where the solution path is unclear and the problem is encountered for the first time. The state of the solution routine depends to a large extent on the knowledge and experience of the solver.

Problems in mathematics are also distinguished by the specificity of the goal. When the goal of the problem is well defined, we speak of a closed problem, and when the goal is not defined, we speak of an open problem, where the solver sets his own goals and tries to achieve them by investigating them (Bahar & Maker, 2015). Cotič and Valenčič Zuljan (2009) distinguish mathematical problems at the classroom level mainly according to the path and the goal and classifies them into:

- Problems with a closed path and a closed goal;
- Problems with an open path and a closed goal;
- Open-path and open-goal problems, noting that closed-path and closed-goal problems dominate mathematics teaching at this level.

This can give students the misconception that most mathematical problems have a single path and a single solution. If we want to achieve higher levels of proficiency in students, then we need to offer them other types of problems that stimulate their thinking in finding paths and different solution strategies.

Rupnik Vec and Kompare (2006) note that most problems (that students encounter in school mathematics, e.g. arithmetic problems, and problems in science) are closed (structured). When solving closed problems in school, we usually use domain-specific strategies to solve a particular type of problem within that domain. However, in contrast to school problems, most life problems are open-ended and at least partly unstructured. Teaching, therefore, requires the design of learning situations that are characterized by similarities to life problem situations. As a rule, these problems are open-ended problems in which students go through all the stages of problem-solving, from identifying and interpreting the problem to finding, testing, and judging solutions. These types of problems can be used at all stages of the learning process: they can provide a realistic context in which to build and deepen knowledge or to build on it.

Problem-Solving in Mathematics

Mathematical problem-solving is the process of employing mathematical ideas, know-how, and techniques to resolve a mathematical issue (Schoenfeld, 1985). It entails locating the issue, comprehending it, coming up with a solution plan or strategy, putting the plan into action, and evaluating the outcome (Polya, 1945; Schoenfeld, 1987; Voskoglou, 2011). In order to foster critical thinking, reasoning, and mathematical literacy, problem-solving is a crucial component of mathematics education (Daulay & Ruhaimah, 2019; Lee, 2016). Additionally, it assists students in gaining a deeper comprehension of mathematical ideas and techniques as well as in applying what they have learned in practical settings (Carson, 2007; Stacey, 2005).

Problem-solving, therefore, emphasizes independent solving problems and overcoming the learned constraints that make it difficult to move from the initial to the final state. Thus, we solve a problem when we want to answer a question or achieve a goal, and we cannot easily recall the answer from long-term memory or we encounter obstacles to solving it.

According to Schoenfeld (1985), in order to effectively solve problems, individuals must possess and proficiently use appropriate resources (such as mathematical concepts and procedures), heuristic strategies (both general and specific), metacognitive control (the ability to oversee and monitor the entire problem-solving process), and appropriate beliefs (in terms of one's perspective, motivation, and confidence).

If we take a closer look at this and look at our teaching practice, we can quickly see that there are few situations in our teaching practice where students can develop these thought processes. Schoenfeld (1985) mentions several times that it is necessary to change the conception of mathematics in general from learning formulas to finding different solutions, from memorizing formulas to exploring patterns, from solving problems to formulating research questions, making connections, and solving problems. If teachers pay more attention to this, then students will have the opportunity to experience mathematics as a changing, dynamic, evolving discipline, rather than the rigid, absolute, and closed discipline that is currently imposed.

Problem-Based Learning

Problem-based learning is defined as a way to foster more creative forms of thinking, experiencing, and evaluating (Seibert, 2021), which is also the basis for problem-solving (Merritt et al., 2017; Nurlaily et al., 2019). It should therefore not be seen only from a narrow methodological perspective as a teaching method. Nevertheless, we can agree that this principle cannot be subordinated to others, because it alone cannot be used to realize all the curriculum objectives and at the same time to observe all the principles of teaching, but learning is more effective if we introduce problem-orientation and problem logic (Strmčnik, 2001). In particular, problem-based learning focuses on problems in which students can construct their knowledge (Mulyanto et al., 2018).

Strmčnik (2001) classifies problem-based learning as a teaching principle. The principle of problem-oriented teaching is applied when problem orientation is present in all phases of the learning process. On the other hand, problem-solving is a teaching method. Problem-based learning extends to the whole classroom, to all its content and process dimensions, while problem-solving has a narrower meaning and covers only part of the learning activity. Nevertheless, problem-solving is where problem-based learning innovations are most directly expressed (see Aslan, 2021).

Problem-based learning is oriented towards both exploring the problem situation and acquiring new knowledge, as well as investigating and reflecting on one's own way of learning (Downing et al., 2009). In doing so, students build on and acquire both contextual and procedural knowledge as well as metacognitive knowledge (Downing et al., 2009; Downing et al., 2011; Siagan et al., 2019). Problem-based learning is characterized by a problem situation, which can be defined by the teacher or by the teacher together with the pupils. In doing so, pupils acquire new knowledge through exploration, argumentation, verification, and taking a standpoint while being mentally active. In this way, problem-based learning to a large extent balances the relationship between reproductive and creative knowledge acquisition (Strmčnik, 2001).

A slightly different view of problem-based learning is offered by Wood (2003), who specifically analyses both the role of the learner and the role of the teacher, as both are involved in problem-based learning. From the learner's point of view, she mentions the important advantages of problem-based learning, which are reflected in (Wood, 2003):

- The active role of the learner in solving the problems themselves, which allows for the understanding of the knowledge acquired, while at the same time developing collaborative skills that are important for further learning;
- Facilitating cross-curricular links and knowledge transfer between subjects;
- High motivation of students to formulate and solve problems;
- Activating prior knowledge and the acquisition of related knowledge, and the development of skills (thought processes).

The effectiveness of problem-based learning can be evaluated in relation to: the teaching style or teacher guidance/orientation, the characteristics of the learners and classroom interaction, the individual learning goals of the learners, as well as the problem itself. It is therefore a rather complex teaching method with several interrelated success factors. Problem-based learning is learner-centered. It assumes that learners will explore, and link theory to practice, apply experience and knowledge to formulate the problem, and find solutions (Savery, 2006; 2015; 2019). Poorly structured problems that relate to an authentic life situation and adequate support from the teacher are crucial for the effectiveness of problem-based learning.

Previous Studies

The problem-based teaching model has been widely employed to enhance students' conceptual and problem-solving knowledge. A previous study within the same research project demonstrated the effectiveness of this model in fostering conceptual understanding in mathematics among primary school students (Doz et al., 2024). Compared to a traditional, procedural approach, problem-based instruction proved superior in developing students' ability to grasp mathematical concepts, particularly in arithmetic, logic, and geometry. These findings suggest that problem-solving skills are fundamental across all mathematical domains and that real-world problem contexts foster a deeper understanding. In particular, problem-based instruction encourages students to actively engage with problems rather than passively receive information, promoting a deeper understanding as they explore, justify, and verify their solutions. Furthermore, by incorporating real-life scenarios, this form of active learning helps students relate mathematical concepts to practical applications, making the learning process more meaningful and aiding conceptual retention.

Numerous studies have shown that problem-based learning significantly enhances primary school pupils' problem-solving abilities. Problem-based learning fosters critical thinking and problem-solving skills by engaging students in complex, real-world tasks (Hendriana et al., 2018). Primary school pupils exposed to problem-based learning demonstrated superior problem-solving abilities compared to those taught through traditional methods (Ali et al., 2010). Problem-based learning not only improves problem-solving performance but also promotes pupils' self-confidence in tackling mathematical challenges (Merritt et al., 2017). Therefore, integrating problem-based strategies into mathematics instruction leads to a deeper understanding of mathematical concepts, thereby enhancing problem-solving skills. These findings align with the notion that problem-based learning emphasis on active learning, critical thinking, and contextual problem solving supports the development of essential mathematical competencies. Overall, the literature consistently underscores the positive impact of problem-based learning on primary school pupils' ability to approach and solve mathematical problems.

Research conducted by Li & Tsai (2017) implemented a year-long problem-based learning intervention in a fifth-grade mathematics classroom in Taiwan. Students engaged in complex, real-world problems requiring them to apply mathematical concepts collaboratively. The study observed that students developed a deeper understanding of mathematical concepts and demonstrated improved problem-solving skills over time. Additionally, the problem-based learning approach fostered long-term knowledge retention and a positive attitude toward mathematics.

Aims of the Research

Modern school reforms around the world aim to achieve meaningful and lasting knowledge in students, and this is the reason why curricula, syllabuses, and, consequently, mathematics teaching are being updated. We decided to carry out this research in order to investigate and update the existing educational process in mathematics teaching in primary education. The research problem was focused on the design and evaluation of an experimental model of problem-based mathematics education.

The present study is part of a larger research project (Doz et al., 2024). Previous research has highlighted that a problem-based approach can help students develop better conceptual knowledge; for instance, Grade 3 pupils who followed problem-based instruction demonstrated superior performance in understanding mathematical concepts. In this study, we aimed to explore Grade 3 students' problem-solving abilities, particularly when addressing more complex problems.

An experiment was carried out in school practice with Grade 3 pupils to determine whether the use of an experimental model of problem-based learning had a statistically significant effect on pupils' mathematics achievement. Our general research hypothesis is therefore the following:

General Hypothesis: Students receiving an experimental model of problem-based mathematics instruction will be more successful in solving mathematical problems in all mathematical domains (arithmetic and algebra, geometry with measurement, and logic and language) than students receiving classical (behavioristic) mathematics instruction.

METHODS

Participants

The study was carried out on a sample of 240 pupils in the 3rd grade in four randomly selected Slovenian primary schools from the Coastal Region, 100 pupils were included in the experimental group (57.0% females) and 140 pupils in the control group (52.1% females). All the selected primary schools were urban schools with equally solid working conditions. The socioeconomic status of students of both groups has shown that pupils are mainly from middle-class families.

In the current study, grade 3 students were selected in accordance with the objectives outlined in the National Curriculum for Mathematics (Žakelj et al., 2011), which emphasizes the importance of fostering students' problem-solving skills. Consequently, this age is well-suited for an exploration of instructional methods that hold the potential to enrich these critical skills.

The sampling procedure adhered to a systematic process aimed at ensuring the acquisition of a representative and unbiased sample. To designate the schools, a random sampling method was employed. All primary schools within the Coastal Region of Slovenia were cataloged, and a random number generator was deployed to designate four schools from this comprehensive list. Subsequently, we sought and obtained the permissions from the school principals to conduct our study. Within each of the four selected schools, all grade 3 classes were encompassed within the study. Classes from two of these schools were randomly assigned to the control group, while classes from the remaining two schools were assigned to the experimental group, based on a random coin flip to determine each school's group assignment.

Once approval was obtained from school principals and teachers, all participants and their parents were thoroughly informed about the research objectives, duration, and potential benefits. Informed parental consent was obtained. The study was conducted in full compliance with the European Code of Conduct for Research Integrity.

Instruments

The study used a pedagogical experiment as part of the empirical research approach. Quantitatively, we analyzed the data by means of knowledge tests (two knowledge tests in mathematical problems).

Test of Knowledge

For the purposes of the research, two knowledge tests were designed (Appendix A and B). The initial and final mathematical knowledge of the experimental and control groups was assessed by means of an initial and final knowledge test. The tasks in both knowledge tests were designed taking into account Gagné's taxonomy (Cotič & Žakelj, 2004) and the mathematical content of grade 3 according to the mathematics curriculum (Žakelj et al., 2011).

The pre-test (Appendix D) consisted of 17 questions, including 10 from the domain of arithmetic, 4 concerning logic and sets, and 3 regarding geometry and measurements. The post-test (Appendix E) consisted of 22 questions, including 15 from arithmetic, 3 from logic, and 4 from geometry. The distribution of the questions among these three mathematics domains was proportional to the topics in the national curriculum for the 1st, 2nd, and 3rd years of primary schools in Slovenia (Žakelj et al., 2011).

For the pre-test, students had 1 school hour (50 minutes) to complete it; to complete the post-test, students had 2 school hours on two different days. The test was divided into two smaller tests of 11 questions each (first part: questions from 1 to 11; second part: question from 12 to 22). Most of the tasks were open-ended, but in some exercises, the pupils needed to circle one of the four given answers.

Each exercise in the pre-test was worth 1 point (Appendix D): the pupils got the point if the exercise was solved correctly, otherwise, they got 0. The post-test (Appendix E) had a variable number of points, with one point for each sub-task the exercise presented.

Regarding the taxonomic levels of the test, they were made following Gangé's taxonomy (Gagné, 1985), which is generally used to classify mathematics problems and tasks, and it is used also in the Slovenian National Assessment (Nacionalni Preizkus Znanja; RIC, 2023). Therefore, tasks regarded (1) the understanding of the concepts and definitions (conceptual knowledge), (2) the solving of simple problems (procedural knowledge), and (3) the solving of complex problems (problem-solving). In the latter, students had to decide the most suitable procedure to apply to solve the problem and needed to think about whether the tasks could be solved. For the purpose of the present research, problems from levels I and II are considered "elementary" problems, while problems from level III are considered "complex" problems.

The instrument we developed tests students' knowledge of (1) arithmetic (A), (2) geometry and measuring (G), and (3) logic and set theory (L). The decision to incorporate these mathematical topics is based on the fact that they correspond to the same subjects evaluated in the Slovenian National Assessment (RIC, 2023). In the test, problems concerning arithmetic were preferred, since it is the mathematical topic that pupils are the most used to in the first grades of elementary school, similarly as in the Slovenian National Assessment (RIC, 2023).

The characteristics of both knowledge tests (objectivity, reliability, and validity) were demonstrated on a pilot sample of 102 Grade 3 pupils from two randomly selected Coastal schools.

The validity of the test was determined through qualitative analysis. The contents of the test, which aimed to measure students' mathematical knowledge (and, specifically, problem-solving abilities) of grade 3 pupils, were analyzed by a group of 8 experts (3 professors of mathematics education, 2 mathematicians, and 3 primary school teachers). Based on their critical analysis of the instrument, we improved its contents and structure.

The objectivity of the instrument regarded two factors, i.e., (1) the objectivity of test-taking, and (2) the objectivity of the evaluation of pupils' answers. Regarding the first, researchers gave pupils detailed information about the test itself, thus limiting the possible researchers' influence on pupils. Concerning the second, the test was structured in such a way that there were many closed-type questions. Moreover, the tests were corrected and graded by three independent people (Hallgreen, 2012): a researcher (R) and two mathematics teachers (T1, T2), who demonstrated almost perfect inter-raters' agreement.

The reliability of the instrument was assessed using the method of parallel tests (Mueller & Knapp, 2018). The same students were tested two times; the second test was not the same test as the first, as it was a parallel version of it, i.e. a test that was composed in such a way that it tested the same contents and taxonomic levels as the first. The measure of reliability was obtained considering the correlation between the two results. The correlation is positive, high, and statistically significant ($r = 0.85$; $p < 0.001$), indicating excellent reliability of the instrument.

Moreover, we also considered the difficulty of the test and its items (Mahjabeen et al., 2017). In particular, we maintained only the test items that had the index of difficulty, i.e. the ratio between correct answers and all answers, between 10% ("difficult" questions) and 90% ("easy" questions).

Among the questions present in the test there were examples of problems:

- *Problems with More Data Than Necessary for a Solution*: When students first encounter such problems, the teacher assists them with questions and instructions (e.g., "Don't you think Tina's task is confusing?", "Are all the data necessary to solve the problem?", "Which data are not necessary to solve the problem? Why? Cross them out", "Which data are necessary to solve the problem? Why? Underline them"). An example of this kind of problem in the test: *Tina says: "Yesterday I went to the swimming pool. There were 8 of us: 4 girls, 3 boys, and my dog. We ate sixteen candies, one ice cream, and two lollipops there. How many sweets did we eat at the swimming pool?"*

- *Problems with Multiple Solutions*: To teach a student to apply mathematics to real-life problems, it is essential to introduce them to problems with multiple solutions. Everyday problems almost never have just one solution. Sometimes there is no solution, other times there are several different solutions, from which the best or most acceptable one must be chosen according to the situation. Thus, the "best" solution changes depending on the circumstances or the person in the specific situation. Furthermore, such problems offer the student the opportunity to realize that mathematics is not a dogmatic discipline where every situation has precisely one predetermined solution. An example of this kind of problem in the test: *Tina is looking at school supplies in the store window. She has 10 EUR. Which school supplies can she buy? Write down several solutions. What would you buy if you had 10 EUR?*
    - [notebook] □ 2.50 €
    - [scissors] □ 5 €
    - [folder] □ 4.50 €
    - [pencil] □ 3 €
- *Problems Without Enough Data for a Solution*: When solving problems with students that do not have enough data for a solution for the first time, we help them with the following questions: "Can you solve the problem? Why not?", "Which data are missing?", "Can you find the missing data?", "Where or how will you find the missing data? (If you can't find it, determine it yourself.)". An example of this kind of problem in the test: *Jan went to his grandmother's place in the countryside with his family on Sunday. The road they took to get to his grandmother's place is 80 km long. On the way back home, Jan's father chose a shorter route. How many kilometers did they travel in both directions? Can you solve the problem? If not, why not?*

Procedure

In the present research, the experimental method of pedagogical research was used. The experiment was carried out in existing primary school departments. This means that the sections were not equated to chance differences before the experiment. Pupils in the experimental group were exposed to a problem-based learning environment, while students from the control group continued studying in a traditional model. The experimental and control groups included female teachers, equal in terms of educational level. The experiment lasted 8 months.

In this intervention, students in the experimental group were exposed to a learning environment that promotes active learning, critical thinking, and collaborative problem-solving, potentially leading to a deeper understanding of mathematics (Doz et al., 2024). This environment included various instructional strategies and activities designed to engage students actively in the learning process. Teachers utilized (also) real-world problem scenarios to stimulate students' curiosity and encourage them to think critically about mathematical concepts. Students in the experimental group worked also in small groups to solve mathematical problems. The intervention aimed to move beyond rote memorization and procedural understanding; instead, it sought to develop student' ability to apply mathematical reasoning in various contexts, thereby deepening their overall comprehension of mathematics. During regular mathematics lessons, the teachers presented students of the experimental group with problems that were accessible to grade 3 students. These problems primarily involved arithmetic, geometry, and logic. Some problems were solved together with the teacher, who promoted critical thinking, while other problems were solved by students working in small groups cooperatively. All the problems were then reviewed by the teacher and discussed with the whole class. Each pupil had the opportunity to contribute to the solving process. Additionally, students from the experimental group received, alongside the traditional homework from the workbook, additional problems. Those problems also included ones that have multiple solutions, no solutions, or several ways to solve them and that required reasoning. Some examples of each problem solved by teachers or by students are presented in Appendix C. The exercises given to the teachers as additional material before the experiment started were developed by the researchers and printed out for the teachers, who then gave their students some photocopies of the problems. All problems had some proposed solutions that were worked out by the researchers and/or included commentaries about how to implement the exercises in their regular mathematics classes (e.g., at what stage should some problems be given, what questions might the teacher ask the students to stimulate their reasoning, etc.), which followed the directions of Polya (1945).

Meanwhile, students in the control group continued with the traditional educational model, characterized by more passive learning and teacher-centered instruction. The fact that both the experimental and control groups had female teachers with comparable educational qualifications ensured that any potential differences in outcomes were not attributed to variations in teaching expertise. The experiment's duration permitted a comprehensive assessment of the impact of the problem-based learning environment on students' performance and conceptual understanding in mathematics. This research design facilitated the exploration of potential benefits associated with this innovative pedagogical approach in comparison to the traditional teaching method.

The initial knowledge test was administered to the control (CG) and experimental (EG) groups before the start of the experiment, and the final knowledge test was administered after the experiment under the same conditions and with the same tester. The study was carried out over a period of 8 months. The year before the start of the experiment, teachers from the EG were instructed to carry out the experiment. During these sessions, teachers acquired theoretical and practical knowledge about problem-solving, received additional material (workbooks, worksheets, etc.), and were instructed not to use a problem-oriented model of teaching during the current year. During the study itself, the EG was taught mathematics in grade 3 using the experimental model of problem-based learning, while the CG was taught mathematics in grade 3 using the (traditional) behavioral model.

Data Analysis

The data in the empirical part were analyzed using the statistical analysis software SPSS 26.0. The following statistical procedures were used:

- *Analysis of Covariance (ANCOVA)*: it was used to check for differences between the CG and EG in the final test by controlling for the pre-test;
- *Descriptive Statistics*: mean (*M*), standard deviation (*SD*), frequencies, minimum (*min*), and maximum (*max*); they were used to describe the achievements and the demographics;
- *Kolmogorov-Smirnov Test for Normality of Distribution*: it was used to determine whether the usage of parametric statistical tests was possible. All data were normally distributed;
- *t-test (Cochran-Cox approximate t-test method)* to determine the differences in knowledge of mathematical problems in mathematical content (arithmetic; geometry with measurement; logic and language) between the experimental and control group students at the beginning and at the end of the experiment.

RESULTS

Pre-test

In Table 1 we present the descriptive statistics for the pre-test. In addition, we report the results of the *t*-test, which was used to check for possible differences between the experimental and control groups. For the purposes of the research, we calculated the statistics for easier (I) and more complex problems (II). Easier problems are those that are from the second Gagné taxonomy scale, while more complex problems are those from the third Gagné taxonomy. A graphic representation of the test achievements is in Figure 1. Differences between the EG and CG might be found in the pre-test, in favor of the EG. In particular, students in the EG showed better performance than students in the CG in both easier (p = 0.001) and more complex (p = 0.002) problems. Regarding geometry, the EG demonstrated superior performance compared to the CG in easier problems (p = 0.014), but there was no significant difference in more complex problems (p = 0.945). In terms of logic, there was no significant difference between the EG and the CG in easier problems (p = 0.448), but statistically significant differences were found in favor of the EG in more complex problems (p = 0.004). Overall, the results of the *t*-tests indicated that students in the EG initially achieved better results than students in the CG in most of the tasks.

Test	**Group**	***M***	***Max points***	***SD***	***min***	***max***	***t***
A I	EG	3.34	5	1.46	0	5	3.27**
	CG	2.63		1.44	0	5	
A II	EG	2.94	5	1.40	0	5	3.11**
	CG	2.30		1.35	0	5	
G I	EG	1.43	2	0.74	0	2	2.49*
	CG	1.16		0.75	0	2	
G II	EG	0.46	1	0.50	0	1	0.069
	CG	0.46		0.50	0	1	
L I	EG	0.35	2	0.55	0	2	-0.76
	CG	0.41		0.56	0	2	
L II	EG	0.62	2	0.63	0	2	2.90**
	CG	0.36		0.61	0	2	

Note. A = arithmetic; G = geometry; L = logic; EG = experimental group; CG = control group.
* $p < 0.05$; ** $p < 0.01$
Table 1: Descriptive statistics and the results of the *t*-test for the pre-test.

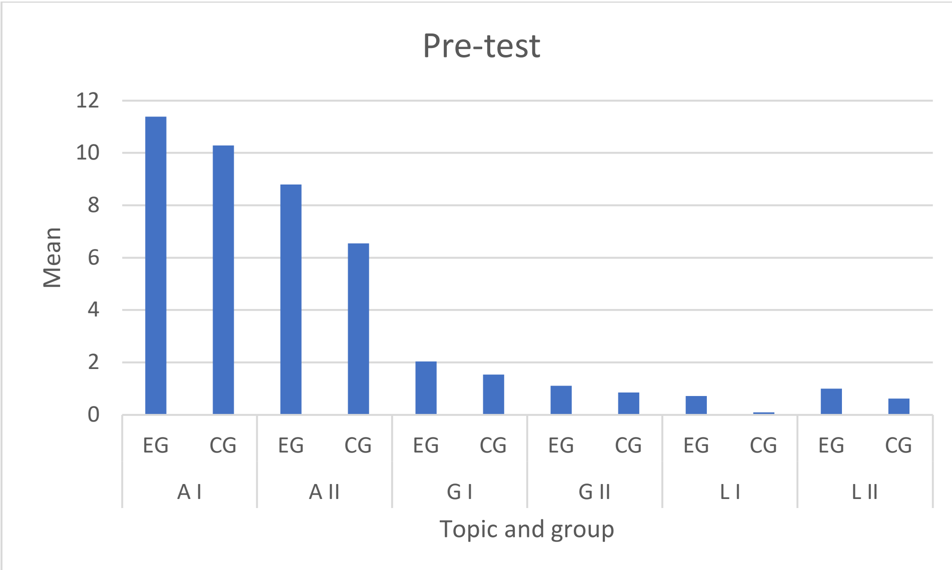


Figure 1: The means of the pre-test.

Post-test

In Table 2 we present the descriptive statistics for the post-test. In addition, we report the results of the ANCOVA test with one variable (the results of the initial tests), which was used to check for possible differences between the experimental and control group with regard to the initial test. For the purposes of the research, we calculated the statistics for easier (I) and more complex problems (II). A graphic representation of the test achievements is in Figure 2.

Test	**Group**	***Max points***	***M***	***SD***	***min***	***max***	***F***
A I	EG	13	11.38	2.00	5	13	1.17
	CG		10.29	2.99	1	13	
A II	EG	17	8.79	4.88	0	17	1.50
	CG		6.54	4.86	0	16	
G I	EG	3	2.03	0.96	0	3	6.02*
	CG		1.53	1.11	0	3	
G II	EG	2	1.10	0.94	0	2	4.01*
	CG		0.85	0.96	0	2	

L I	EG	1	0.71	0.46	0	1	125.98***
	CG		0.09	0.29	0	1	
L II	EG	3	0.99	0.88	0	2	4.07*
	CG		0.62	0.82	0	2	

Note. A = arithmetic; G = geometry; L = logic; EG = experimental group; CG = control group; F=ANCOVA results.

* $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$

Table 2: Descriptive statistics and results of the ANCOVA for the post-test.

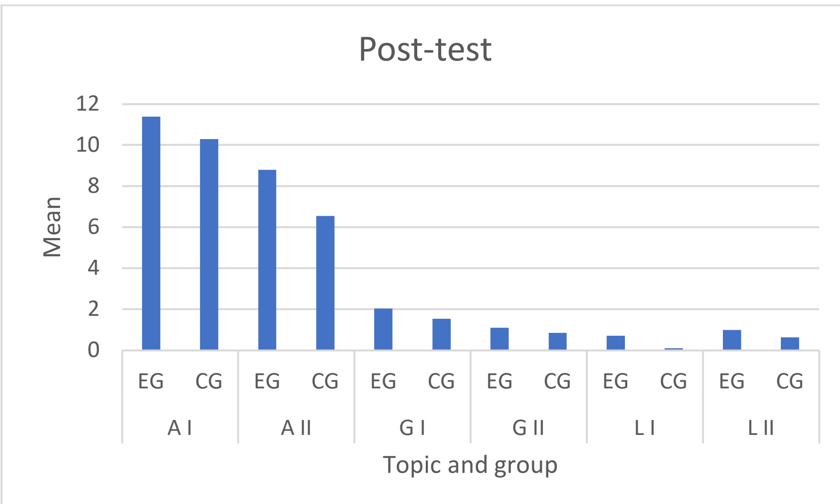


Figure 2: The means in the post-test.

From Table 2, we understand that the EG had higher achievements than the CG in solving both easier and more complex problems. The statistically higher achievements are present in the domain of geometry and logic. In particular, despite having a statistically significant advantage in the pre-test, the EG demonstrated a statistically significant difference from the CG in geometry and logic. No statistically significant differences between the EG and CG were detected in easier arithmetic problems (p = 0.280) and more complex arithmetic problems (p = 0.222), indicating that the experimental approach did not significantly change the achievements of the EG and CG when initial knowledge is considered.

On the other hand, despite having relative advantages in the pre-test, the EG showed better achievements in easier (p = 0.015) and more complex (p = 0.047) geometry problems, as well as in easier ($p < 0.001$) and more complex (p = 0.045) logic problems. Therefore, the EG generally performed better than the CG in two of the three mathematics topics. As noted in Table 4, the EG also generally outperformed the CG in arithmetic; however, considering the initial knowledge advantage of the EG over the CG, these differences might be attributed to the EG starting with a very good baseline knowledge.

Comparison Between the EG and CG

Students in both groups were good at solving simple arithmetic problems (e.g., "*Ana's mother is 31 years old. Her daughter Anna is 20 years younger. How old is Ana?*"), as these types of problems appear most frequently in our textbooks and workbooks, and are also the ones we teachers ask most often. These types of problems were solved correctly by 87.5% of EG students and 79.2% of CG students. Even simple problems in logic and geometry were solved more successfully by EG students. It was evident how narrow a range of problems was solved by CG students, as they rarely solved logic and geometry problems by measurement. In simple logic problems, pupils had to sort the elements of a given set according to two properties, using different tables (e.g. Carroll diagrams). There were 70.8% of pupils in EG were successful in these problems, while only 9% of pupils in CG were successful, even though such problems also appear in grade 3 prescribed mathematics textbook. In the EG, students solved these mathematical problems through three levels of presentation (i.e. enactive, iconic, and symbolic). The CG students solved the problem only in the workbook at the iconic or symbolic level. They did not do the concrete-experiential activity, which is, however, at the level of the first three grades, one of the obligatory steps towards the development of cognitive processes. At the same time, they did not integrate spreadsheets in a meaningful way in solving various problems, both mathematical and non-mathematical.

In simple geometry problems with measurement, 67.8% of EG pupils and 51.1% of CG pupils were successful. Although the problems were very simple, they were alien to the CG pupils, who themselves stated that they were different from the ones they solve at school. In our schools, pupils are used to solving problems in examinations that are very similar (or even identical) to those they solve in class. Since mathematics lessons do not develop different strategies for solving mathematical problems, students are then helpless in solving "different" problems.

Even with more challenging problems, the EG students were more successful. The average achievement in solving more difficult problems in arithmetic was 51.78% in EG and 38.50% in CG. In geometry problems, the average achievement was 55% in the EG and 42.70% in the CG. There was also a large difference in the knowledge of logic problems (EG achieved 49.43%, CG 31.11%). The more difficult problems included compound (guided and unguided) problems and:

- Problems that do not have sufficient data to solve;
- Problems with more data than needed for the solution;

- Problems with multiple solutions;
- Problems in which the given data are read from a spreadsheet.

In addition, the test asked students to solve the following problems:

- To formulate a meaningful problem for a given illustration and then solve it;
- To formulate a meaningful question to the text of the problem and then solve the problem;
- Choose a mathematical problem that goes with the given calculation.

Of these problems, CG pupils in mathematics lessons solved mainly composite problems and fewer problems where they read the given data from a table and problems where they formulated a meaningful problem to a given illustration. In the EG, however, all of these problems were solved in the classroom. Interestingly, students from both groups performed the worst on the composite problems. In EG, 18.5% of students solved these problems correctly, while in CG 14.4% of students did so. Although these problems are often solved in mathematics lessons, only some students are able to decompose a compound problem into sub-problems on their own, without teacher guidance, asking intermediate questions that lead them to the solution.

In Table 3 the frequencies (in %) of how well students from EG and CG solved more challenging problems are presented.

Problems	**EG**	**CG**
Problems with more data than required	59.5	41.6
Problems with fewer data than required	82.0	63.3
Problems with multiple correct answers	20.8	10.0
Problems where data are presented in tables	80.5	71.1
Formulate a problem based on a picture	78.4	62.6
Formulate a question based on a text	54.3	37.4
Choose a mathematical problem that can be solved with a given operation	71.6	44.4

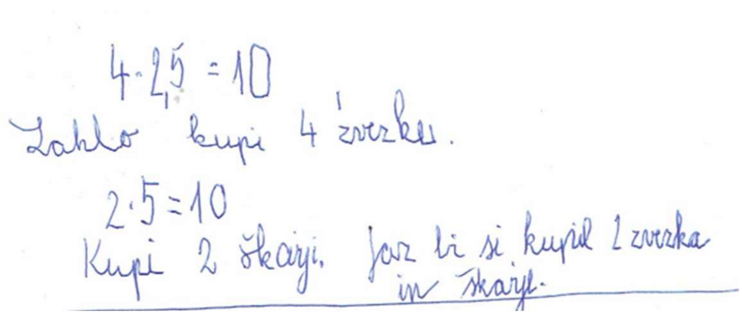
Table 3: The frequencies of correctly solved problems in EG and CG.

Table 3 shows that EG students outperformed CG students on all types of problems. Both did poorly on problems with multiple solutions (20.8% in EG and 10% in CG), as in most cases they settled for one solution, even though they were told in the task to write down all possible solutions.

Most students believe that if you find one solution, there is no need to look for others because they have "solved" the problem correctly (Figure 3). The EG teachers also found that these were the problems that caused the most problems for the pupils. Only the ablest students searched for or found more solutions even if the teacher did not guide them during the problem-solving (Figure 4).

Problem: Tina is looking at school supplies in the store window. She has 10 EUR. Which school supplies can she buy? Write down several solutions. What would you buy if you had 10 EUR?

- [notebook] □ 2.50 €
- [scissors] □ 5 €
- [folder] □ 4.50 €
- [pencil] □ 3 €



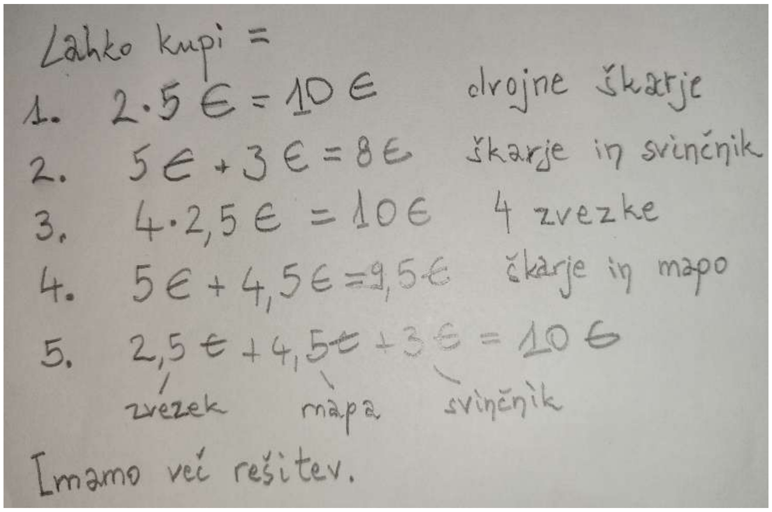
$4 \cdot 2{,}5 = 10$

He could buy 4 notebooks.

$2 \cdot 5 = 10$

He can buy 2 scissors. I would buy 2 notebooks and scissors.

Figure 3: A boy (CG33) from the CG solving incorrectly the problem with multiple correct answers.



He can buy =

1. 2 · 5€ = 10€ two scissors
2. 5€ + 3 = 8€ scissors and a pencil
3. 4 · 2,5€ = 10€ 4 notebooks
4. 5€ + 4,5€ = 9,5€ scissors and a folder
5. 2,5€ + 4,5€ + 3€ = 10€ notebook, folder, pencil

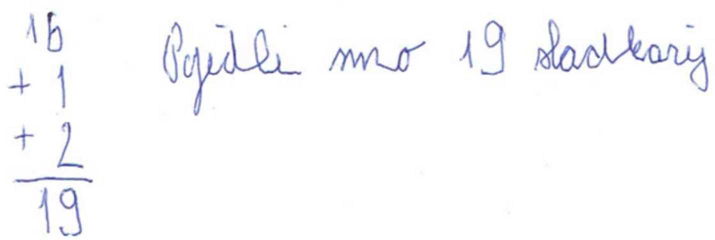
We have multiple solutions

Figure 4: A boy (EG61) from the EG solving correctly the same problem with multiple correct answers.

Pupils who were in the EG in third grade are now in fourth grade. In these classes, mathematics is still taught according to the concept we developed in our experiment. The teachers of these four fourth grades have been trained in the new concept of mathematics education through continuous training and study groups. On the final test for the fourth grade, 42.2% of the students solved these types of problems correctly, which confirms the findings of some contemporary research (Ali et al., 2010; Lessani et al., 2017) that the ability to encode the components of a problem and to plan the solution of different types of problems progresses with age and with appropriate teaching.

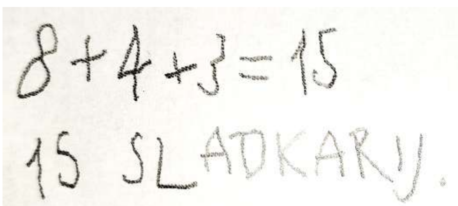
For problems with more data than needed for the solution, it was found that 59.5% of the EG students were able to read the problem text with comprehension and were able to find the data needed for the solution (Figure 5). In CG, 41.65% of students solved the problem correctly. The incorrect solutions were mainly due to the fact that they used all the data, including unnecessary data, in the solution, as they always “had to” use all the data in the problems they encountered in class (Figure 6).

Problem: Tina says: “Yesterday I went to the swimming pool. There were 8 of us: 4 girls, 3 boys, and my dog. We ate sixteen candies, one ice cream, and two lollipops there. How many sweets did we eat at the swimming pool?



16 We ate 19 sweets
+1
+2
___
19

Figure 5: A girl (EG85) from the EG solving correctly the problem with more data than needed.



8+4+3 = 15

15 sweets

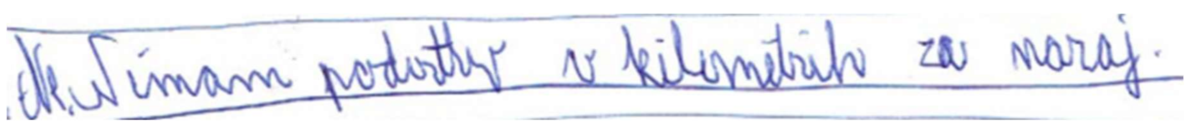
Figure 6: A boy (CG02) from the CG solving incorrectly the same problem with more data than needed.

Students were very successful in solving problems that did not have enough data to solve, with 82% of EG students and 63.3% of CG students successfully solving this problem (Figures 7, 8, and 9). Most students stated that they liked this type of problem the most, as they found it that they could determine the value of the missing data themselves. Teachers also confirmed that pupils were very motivated to solve these types of problems, but were surprised that pupils were able to find the missing data very quickly and then assign very meaningful values to it.

Problem: Jan went to his grandmother's place in the countryside with his family on Sunday. The road they took to get to his grandmother's place is 80 km long. On the way back home, Jan's father chose a shorter route. How many kilometers did they travel in both directions? Can you solve the problem? If not, why not?

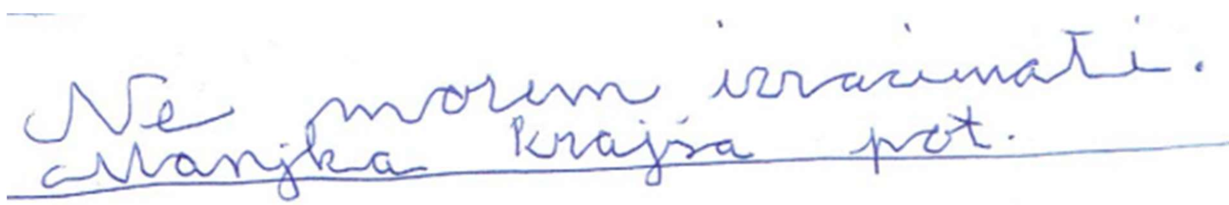
I can solve it.
The second road [grammatical error] is shorter than 80 km.
The road is long less than 160 km.

Figure 7: A girl (EG97) from the EG solving incorrectly the problem with not enough data.



No. I don't have data for the kilometers back.

Figure 8: A boy (CG09) from CG solving correctly the same problem with not enough data.



I can't calculate it.
The shorter road is missing.

Figure 9: A boy (CG74) from EG solving correctly the same problem with not enough data.

In addition to these problems, the favorite problem for students from both groups was to create a meaningful problem to go with a given illustration. Additionally, 80.5% of the EG students and 62.6% of the CG students solved this problem correctly. We also analyzed how original the students were in formulating their problems. The EG pupils formulated a wide variety of problems:

- Problems using multiplication, division, addition, and subtraction to solve them;

- Problems in geometry;
- Problems in the data processing.

In CG, students only designed problems that they solved using multiplication and addition. At the same time, the problems were constructed exclusively according to the model in the workbook. This points to the fact that our current mathematics teaching provides little opportunity for students to solve problems in their own way and that creativity is not sufficiently encouraged. However, in the EG, the teachers developed flexible and creative thinking. The problems that the pupils constructed show that pupils at this age level are able to construct mathematical problems in a creative and meaningful way if properly encouraged by the teacher.

In the next problem, students were asked to add one or more meaningful questions to the text of the problem and then solve the problem. This problem was successfully solved by 54.3% of the EG pupils and only 37.4% of the CG pupils. The problem text was made "harder" by writing the given numerical data in words rather than numbers. Research (Moghadam et al., 2012) has shown that writing a quantity with a number or with a word has a great influence on the comprehension of the text. Therefore, the teacher needs to be aware of all the factors that influence the understanding of the problem and to guide the pupil skillfully in solving it. In the EG, teachers instructed students to underline information that is written in both number and word form.

Students in both groups did very well on problems in which they read the given data from a table. In the EG group, 80.5% of the students solved the problem correctly, and in the CG group, 71.1% of the students solved the problem correctly. In recent years, reading and writing in various forms of spreadsheets have been integrated more into classroom teaching in both mathematics and science (Isiksal & Askar, 2005). This is confirmed by the results obtained.

In the test, pupils were also given a problem where they had to find the problem that the given calculation solves among given problems. This was a twofold task: to understand the text and to have a good grasp of the basic arithmetic operations. The same numbers "appeared" in all the given problems, so the students had to read all three problems very carefully and then decide on the correct solution. In most cases, CG students chose the first problem, as only 44.4% of the students solved the problem correctly. This confirms the findings of previous studies (Bernardo, 1999; Phonapichat et al., 2014) that students do not read the whole text, but in most cases rewrite the given data and perform the computational operation that is being discussed in the mathematics lesson at the time. The EG pupils solved the given problem very well, with 71.6% of the solutions being correct. In the EG mathematics lessons, we have been putting a lot of emphasis on the first stage of problem-solving: understanding the problem.

DISCUSSION AND CONCLUSIONS

The main purpose of our study was to test the hypothesis that EG students who received a new model of mathematics instruction, in which problem posing and problem-solving were the leading didactic-mathematical activity, would be more successful in solving all types of mathematical problems than CG students who received traditional mathematics instruction, in which the main emphasis is on training in arithmetic operations.

To check the validity of the hypothesis, a pedagogical experiment was developed. Students from the EG and CG took an initial (pre-)test to check for possible differences in the initial knowledge. Results have shown that students from EG had better achievements than students of CG in the majority of the mathematics domains. As a possible explanation for why this phenomenon occurred, teachers' experiences might be considered. For instance, the teachers from the EG were prepared for the experiment throughout the year through various forms of training (workshops, lectures, seminars, etc.) when they were teaching in the second grade. They were also given materials (method manual, task book, workbook, etc.) which they then used in the experiment when students were in the third grade. The new concept makes the problem the central content. In other words, mathematics is not taught just "for the numbers and the operations between them", but for the "problems". Thus, all fundamental mathematical concepts were built from problem situations that arose from the pupils' experience. Teachers emphasized generalized knowledge and problem-based learning, in contrast to the prevailing concept of mathematics education in this country, which is based primarily on the transfer of skills and knowledge. Instruction based on such transfer, however, "implements" knowledge and techniques immediately after the introduction of new content, regardless of whether the mental connections have been made so that the learner understands the content and can apply it. This transfer can also include the transfer of rules, principles, and methods to similar tasks without the learners understanding and being able to apply them. The scope of this transfer is relatively narrow. For example, the technique of computational operations falls within this range.

Based on all the results obtained and their analysis, we can conclude that the model of mathematics teaching implemented in the EG, in which problem posing and problem solving were the leading didactic-mathematical activity, is a success. This confirms our research hypothesis: EG will be more successful than CG in solving both simple and more complex problems in the context of arithmetic, geometry with measurement, and logic with sets.

The study suggests some important theoretical and practical implications. Firstly, the results support the effectiveness of a problem-based learning approach in mathematics teaching. This finding aligns with existing theories that emphasize the value of active learning, critical thinking, and problem-solving in developing students' mathematical skills and conceptual understanding (Chairuddin & Farman, 2019; Hu et al., 2018; Meke et al., 2018). Findings therefore suggest that when problem-posing and problem-solving become central components of mathematics instruction, students are more successful in tackling both simple and complex mathematical problems (Darhim et al., 2020). This bolsters the theoretical foundation for the integration of problem-based learning strategies in mathematics education. Secondly, one practical implication is that educators and curriculum designers (policymakers) should consider incorporating problem-based learning approaches into the mathematics curriculum, especially for primary school students. This approach may enhance students' problem-solving abilities and overall math performance. Nevertheless, teachers should receive training and support in implementing problem-based learning effectively (Ali et al., 2010). They need strategies to create an environment that encourages problem posing and problem-solving activities in the classroom.

LIMITATIONS AND FUTURE DIRECTIONS

There are several possible limitations of our study. Firstly, the sample size is still small, so the results cannot be generalized to the whole population. Additionally, the study only focuses on students from one particular region. Future research could explore whether problem-based mathematics instruction is effective in other countries and with other curricula. This could help to determine whether the results of this study are generalizable to other contexts.

Secondly, the study only examines the impact of a specific teaching method on student performance in math, and it does not take into account other factors that may affect student performance, such as motivation (Gilbert et al., 2014), beliefs about mathematics (Mason, 2003), mathematics anxiety (Ashcraft & Ridley, 2005; Wu et al., 2012), and others.

Thirdly, the study only examines the short-term impact of the teaching method, and it is unclear whether the effects would persist over the long term. While this study showed positive results in the short term, it would be interesting to see if the effects of problem-based mathematics instruction persist over a longer period of time. Future research could follow up with the students from the experimental and control groups to see if the differences in achievement continue to hold true in later grades or even in adulthood.

This study found that teacher training was an important factor in the success of problem-based mathematics instruction. Future research could explore different types of teacher training and their effectiveness in implementing problem-based mathematics instruction. This could help to identify the most effective ways to train teachers for this type of instruction.

While this study showed that problem-based mathematics instruction was effective in improving student achievement, it would be interesting to explore whether this type of instruction also increases student engagement and motivation. Future research could measure student engagement and motivation and compare them between the experimental and control groups. Moreover, problem-based mathematics instruction could potentially be enhanced through the use of technology (Jacinto & Carreira, 2017; Lee & Hollebrands, 2006). Future research could explore how technology can be used to support problem-based mathematics instruction and whether this enhances student achievement and engagement. Additionally, further research should explore the long-term effects and scalability of the mentioned approach in different educational settings and age groups. Investigating the transferability of our results to other cultural settings and educational systems could provide additional valuable insights.

Based on the findings and insights derived from the study, several recommendations can be made, both for educators and policymakers. Firstly, policymakers should review and potentially revise the mathematics curriculum to align it with problem-based learning principles. This should involve the development of curriculum materials, task books, and resources that facilitate problem-posing and problem-solving activities. Ensuring that the curriculum encourages critical thinking and application of mathematics in problem-solving contexts is essential. Secondly, results have shown the importance of prioritizing conceptual understanding over rote memorization and algorithmic approaches in mathematics education. Educators should therefore emphasize the application of mathematical concepts in real-world problem-solving scenarios can foster deeper comprehension among students. Also, teachers should encourage an active learning environment where students engage in critical thinking and collaborative problem-solving.

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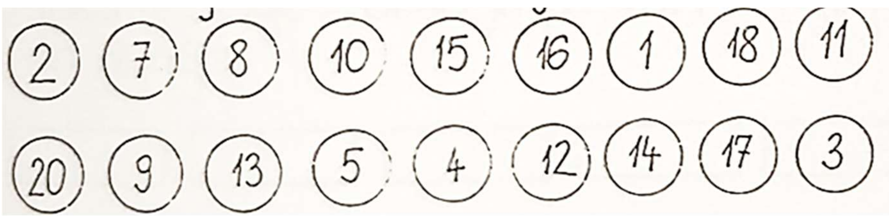
APPENDIX

Appendix A - Pretest

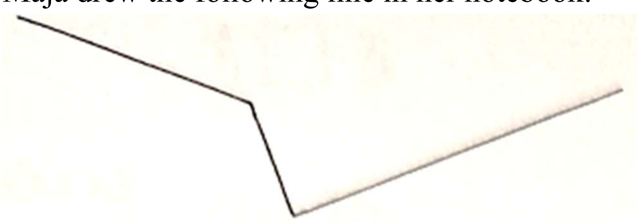
1. Complete it in such a way that the left-hand side will be equal to the right-hand side.

   15 - 6 = 8 + ___

2. Color the circles that contain numbers greater than 7 and less than 13.



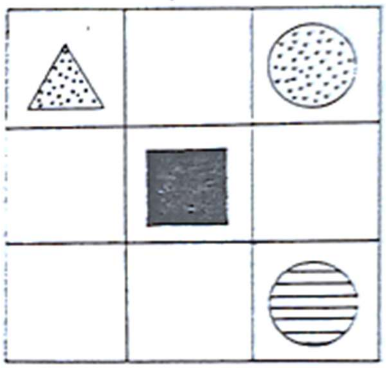
3. Tina read 8 pages of her book on the first day. On the second day, she read 6 pages more than the first day. How many pages did Tina read on the second day?
4. There were 12 cars parked in the parking lot. Then, 4 cars left, and 3 cars arrived. How many cars are in the parking lot now?
5. Tina and Rok have a total of 90 €. How much money can Tina and Rok have?
6. Maja drew the following line in her notebook:



   What is the length of the drawn line?

7. Create a problem based on the following picture and solve it.

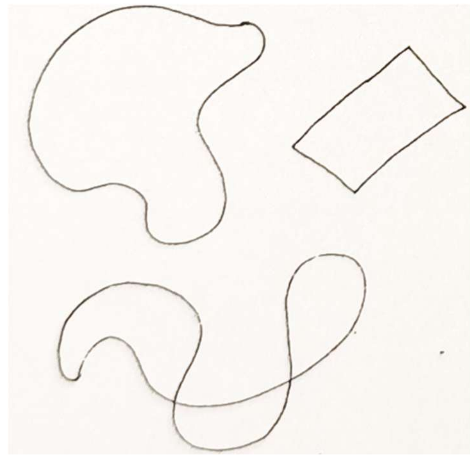
8. Look at the following table. Find the rule that was used to order the following shapes and complete the table.



9. Maja, Jana, Nina, Tina, and Ana live in the same block of apartments. Nina is taller than Maja, Jana, and Tina, but she is not the tallest. Jana is the shortest and is right after Maja. Sort the heights of the five girls from tallest to shortest.
10. Jaka bought 2 pencils and 3 notebooks at the stationery store. Which information do you need to know how much money Jaka spent at the stationery store? Circle the correct answer:
    a. name of the stationery store;
    b. I don't need anything;
    c. price of the pencil and notebook;
    d. age of the saleswoman;
    e. only the price of the pencil.
11. Mira has . Tine has  . Who has more money?
12. Metka is 12 years old. Her brother is 5 years old. How many years older is Metka than her brother?
13. In the orchard, they cut down 16 trees: 5 apple trees, 4 peach trees, and some plum trees. How many plum trees did they cut down?
14. Martina and Matej were drawing lines.

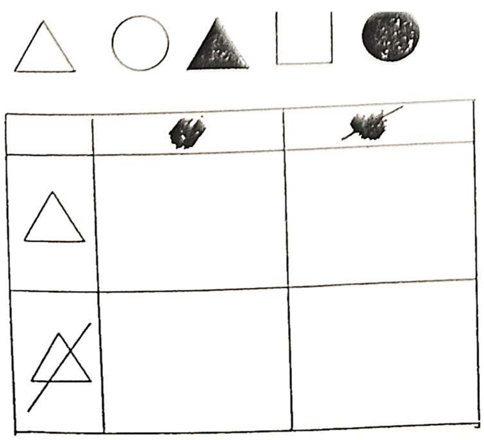
Martina

Matej



How do Martina's and Matej's lines differ?

15. Place each of the following shapes in the correct location in the table.



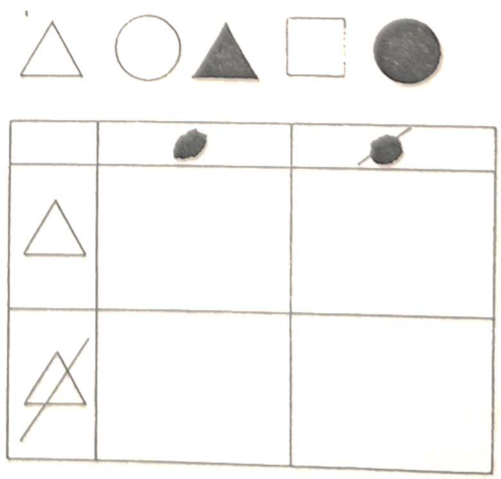
16. A class of 20 students is going on a trip to Piran. The bus ticket costs 2 €. On the day of the trip, Tine, Iva, and Maja fell ill, while Urška and Žiga went to an athletics competition. How many children went on the trip?
17. Which word problem matches the calculation $5 + 3 = 8$? Circle the correct answer.
    a. Tina has 8 books, and Jan has 3 books. How many books do they have together?
    b. There are 5 flowers in each bouquet. How many flowers are there in 3 bouquets?
    c. Tina is five years old. Her sister is three years old. How many years old are they together?

Appendix B - Post-test

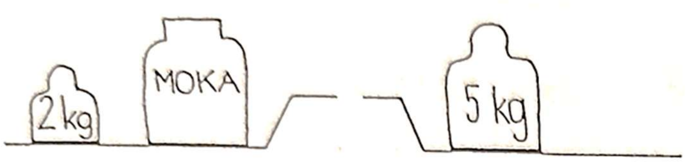
1. The computation $9 \cdot 8 = 72$ is correct, Which of the following computations is also correct?
    a. $9:8 = 72$
    b. $72 \cdot 8 = 9$
    c. $72:8 = 9$
    d. $72 \cdot 9 = 8$
2. There were 136 boys and 128 girls who signed up for the competition. How many children competed?
3. The number has the following properties: it is less than 30, and you say it when you count by multiples of 3 and 8. What is this number?
4. Lena and her two friends put a bird feeder on the tree. Immediately, 4 sparrows, 8 great tits, and 9 blackbirds flew in. How many great tit legs were on the tree?
5. Matjaž has 23 blocks. Metka has 12 fewer than Matjaž, and Marko has three times as many as Metka. How many blocks do all three of them have together?
6. The price of a theater performance is 50 euros. All the seats in the theater were sold. Which of the following information do you need to determine the total price of all the tickets sold?
    a. The number of actors on stage
    b. The number of seats in the theater
    c. The age of the audience
    d. I don't need anything
7. Ana's mother is 31 years old. Her daughter Anna is 20 years younger. How old is Ana?
8. Maja has 114 sweets. She wants to divide them among her 6 friends. How many sweets do each friend get?
9. Tina is looking at school supplies in the store window. She has 10 EUR. Which school supplies can she buy? Write down several solutions. What would you buy if you had 10 EUR?
    a. [notebook] 2.50 €
    b. [scissors] 5 €
    c. [folder] 4.50 €
    d. [pencil] 3 €
10. Read the text and write a question based on it, then solve the problem. In ten desks, there are 2 students each, and in four desks, there are 4 students each.
11. Jan went to his grandmother's place in the countryside with his family on Sunday. The road they took to get to his grandmother's place is 80 km long. On the way back home,

Jan's father chose a shorter route. How many kilometers did they travel in both directions? Can you solve the problem? If not, why not?

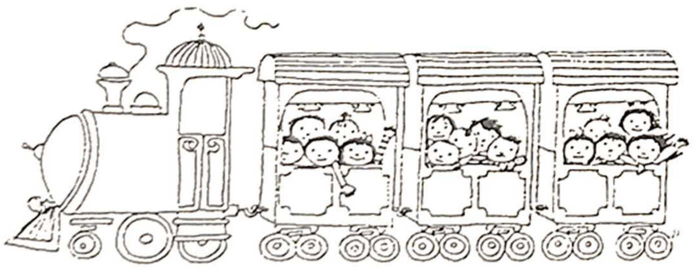
12. Tina says: "Yesterday I went to the swimming pool. There were 8 of us: 4 girls, 3 boys, and my dog. We ate sixteen candies, one ice cream, and two lollipops there. How many sweets did we eat at the swimming pool?"
13. Place each of the following shapes in the correct location in the table.



14. Rok, Jan, Ivo, Bor, and Tine are friends. Ivo is taller than Rok, Jan, and Bor, but he is not the tallest. Jan is the shortest and is just before Bor. Arrange them in order of height from shortest to tallest.
15. How much flour is there if the drawn scale is in balance?



16. Mom buys 2 liters of milk every day. How many liters of milk does she buy in a month that has four weeks and 2 days?
17. Tina has 24 candies, Maja has 19. Nastja has fewer candies than Tina but more than Maja. How many candies could Nastja have?
18. Create a word problem based on the drawing and solve it.



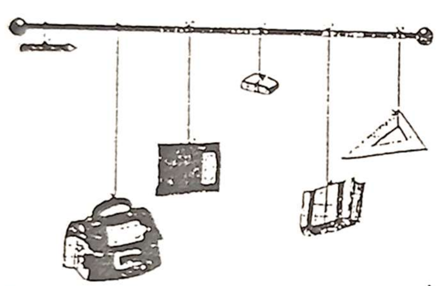
19. Tina and Rok are enthusiastic photographers. They particularly enjoy photographing nature and people. Look at the table to see how many photos they took in one month. [We present also the original picture of the table in the test.]

	Photos	
	Nature	People
Tanja	14	18
Rok	19	21

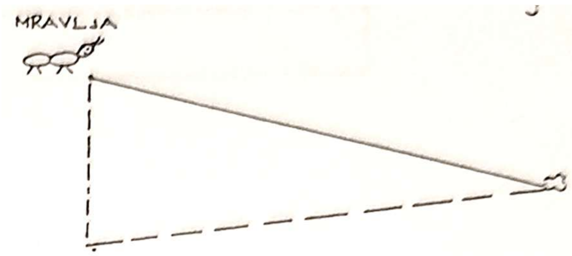
	FOTOGRAFIJE	
	NARAVE	LJUDI
TANJA	14	18
ROK	19	21

a. How many photos did Tanja take?
b. How many more nature photos are there?
c. How many photos did both of them take together?

20. A painter painted 12 paintings. He displayed them on panels. He hung the same number of paintings on each panel. How many paintings could be on each panel?
21. The objects are hanging on identical elastic bands. Arrange them from the heaviest to the lightest (write the names of the objects).



22. The picture below shows two different paths from the ant to the bread crumb. How long is the longest path? [mravlja = ant]

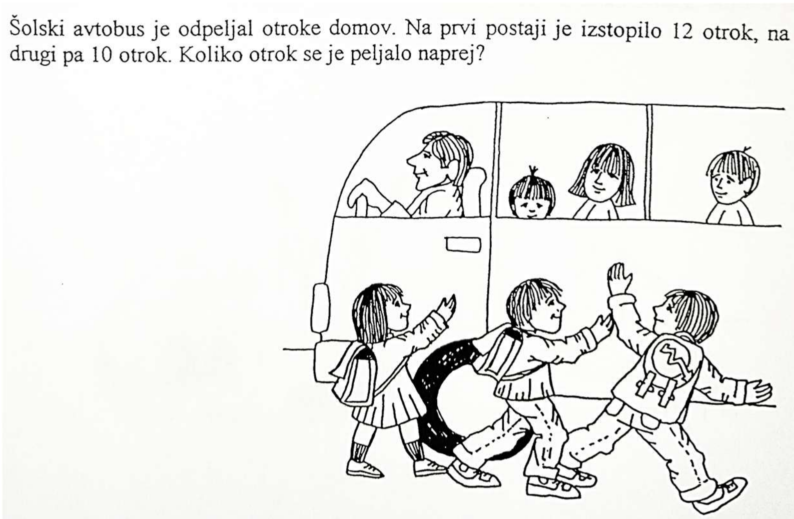


23. Which word problem matches the calculation $5 \cdot 3 = 15$? Circle the correct answer.
   a. Tina has 5 balls, and Jan has 3 balls. How many balls do they have together?
   b. There are 5 flowers in each bouquet. How many flowers are there in 3 bouquets?
   c. Tina is 5 years old. Her sister is 3 years old. How many years old are they together?

Appendix C – Examples of problems solved in class

A dragon with six heads has brought his sons on a walk. His sons each have half the number of heads that the father has. How many heads are there altogether (father and sons)?

Figure C1: Example of an arithmetic problem without enough data.

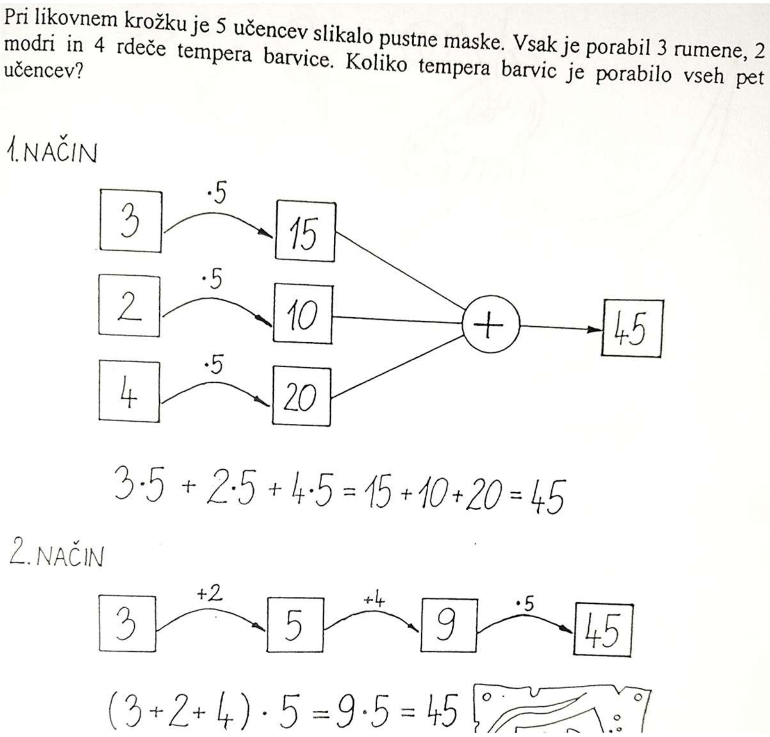


The school bus has taken the children home. At the first bus stop, 12 children got off, and at the second stop, 10 children got off. How many children are still on the bus?

Figure C2: Example of an arithmetic problem without enough data.

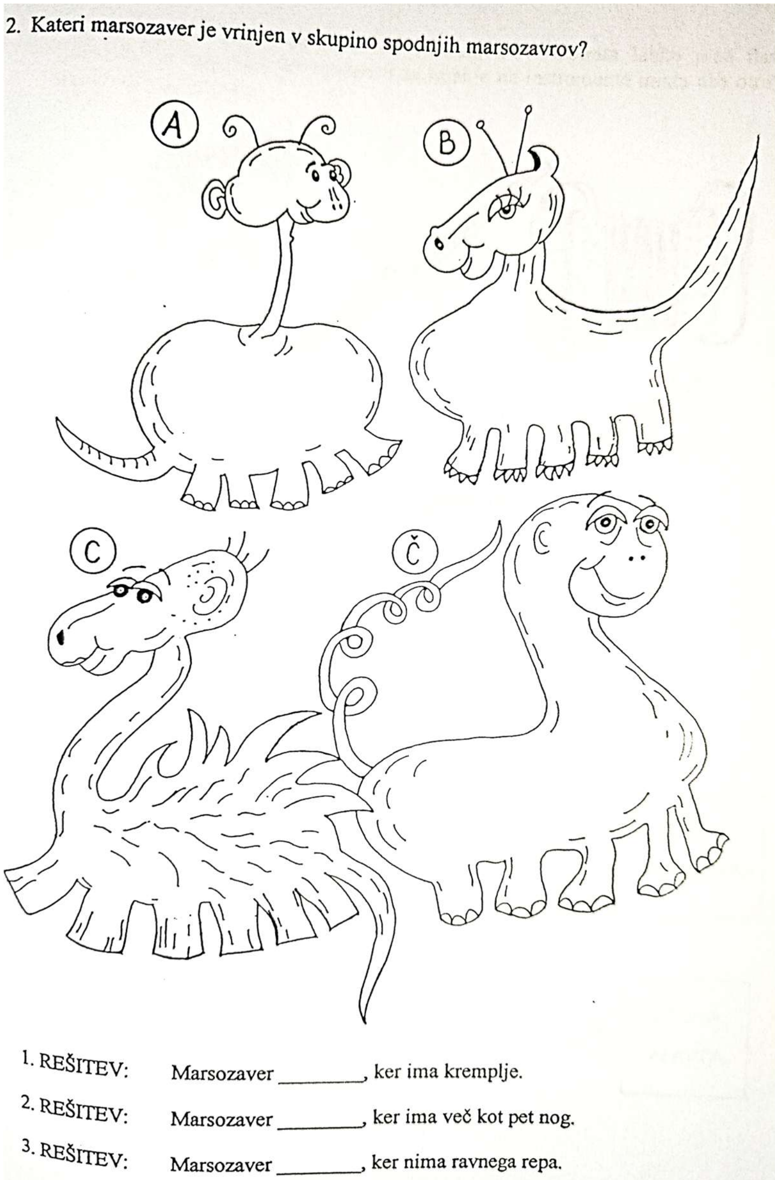
Metka placed a bird feeder on the tree. Immediately, 3 sparrows, 7 tits, and 9 blackbirds arrived. How many tit legs are on the tree?

Figure C3: Example of an arithmetic problem with more data than required to solve it.



In the art club, 5 students were painting carnival masks. Each one used 3 yellow, 2 blue, and 4 red tempera paints. How many tempera paints did all five students use in total?

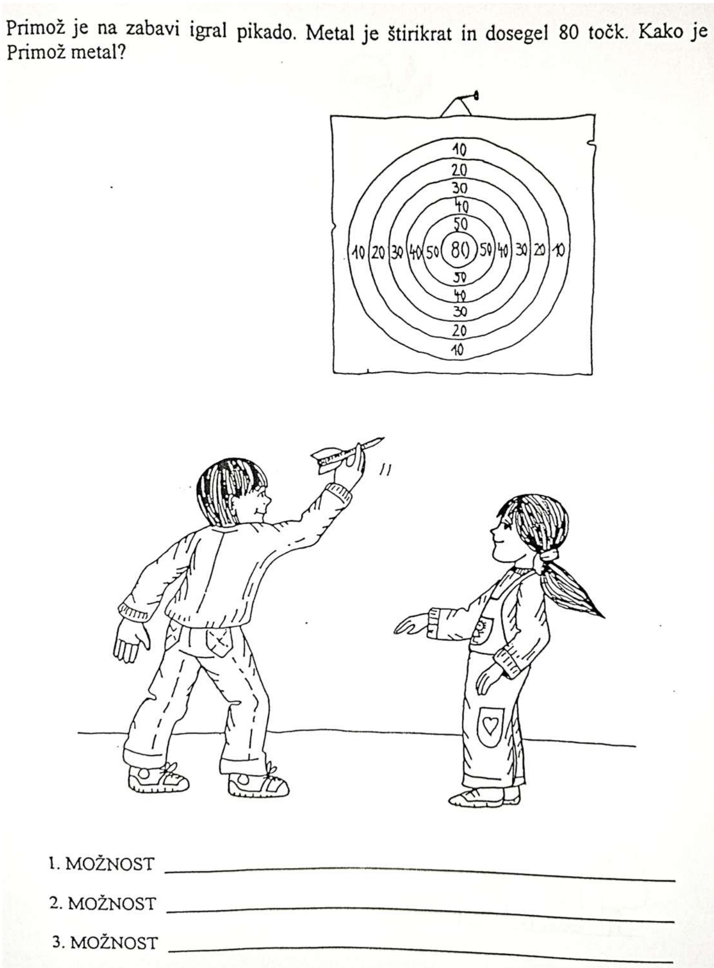
Figure C4: Example of an arithmetic problem with multiple ways to solve it.



Which Martian dinosaur is inserted [i.e., it does not belong] into the following group of Martian dinosaurs?

1st solution: Martian dinosaur _______, because it has claws.
2nd solution: Martian dinosaur _______, because it has more than five legs.
3rd solution: Martian dinosaur _______, because it does not have a straight tail.

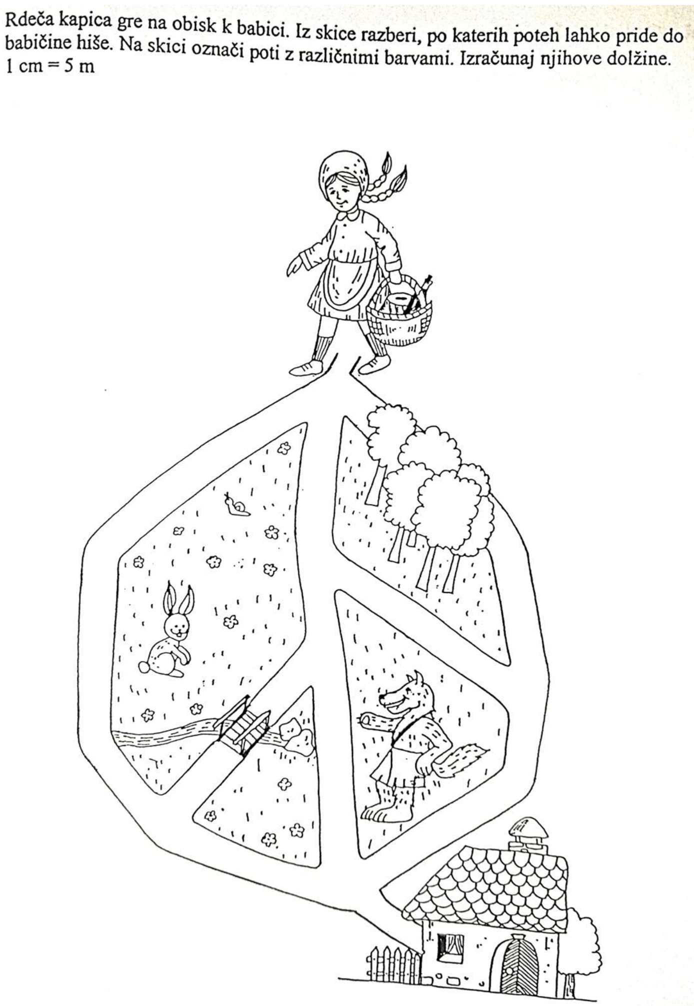
Figure C5: Example of a logic (and sets) problem with multiple ways to solve it.



Primož played darts at the party. He threw four times and scored 80 points. How did Primož throw?

1st possibility: _____________
2nd possibility: _____________
3rd possibility: _____________

Figure C6: Example of an arithmetic problem with multiple ways to solve it.



Little Red Riding Hood is going to visit her grandmother. From the sketch, determine which paths she can take to get to her grandmother's house. Mark the paths on the sketch with different colors. Calculate their lengths. 1 cm = 5 m.

Figure C7: Example of a geometry (and measurement) problem with multiple ways to solve it.

Appendix D – Pre-test points, mathematics domains, and taxonomy

Exercise	Points	Mathematics domain	Taxonomy
1	1	A	I
2	1	L	I
3	1	A	I
4	1	A	I
5	1	A	II
6	1	G	I
7	1	A	II

8	1	L	II
9	1	L	II
10	1	A	II
11	1	G	I
12	1	A	I
13	1	A	I
14	1	G	II
15	1	L	I
16	1	A	II
17	1	A	II

Note. A = arithmetic; G = geometry; L = logic; I = easier problems; II = more complex problems.

Table D: The structure of the pre-test.

Appendix E – Post-test Points, Mathematics Domains, and Taxonomy

Exercise	Points	Mathematics domain	Taxonomy
1	1	A	I
2	2	A	I
3	1	A	II
4	2	A	II
5	3	A	II
6	1	A	I
7	2	A	I
8	2	A	I
9	3	A	II
10	2	G	II
11	2	A	II
12	1	L	I
13	1	L	II
14	1	G	I
15	2	A	I
16	1	L	II
17	3	A	II
18	3	A	I
19	2	A	II
20	1	G	I
21	1	G	I
22	1	A	II

Note. A = arithmetic; G = geometry; L = logic; I = easier problems; II = more complex problems.

Table E: The structure of the post-test.