The Broadwell-Burch Four-stage Model of Competence Development and the Mathematics Teaching Profession

Fanni Biró, Csaba Csíkos

Institute of Education, University of Szeged, Hungary;

MTA-SZTE Metacognition Research Group

fanni.biro@gmail.com, csikoscs@edpsy.u-szeged.hu

Abstract: This study aims to reveal the possible benefits of incorporating the Broadwell-Burch four-stage model of competence development into mathematics teacher training. First, a historical overview is given about the appearance of the four-stage model starting from the original sources by Broadwell and Burch arriving at today's widespread use in the medical and other professions. The case of the teaching profession and teacher education has almost been untouched in the literature; consequently, it is the right time to raise questions and dilemmas about the strength, usefulness, and applicability of the four-stage competence model. We conclude that mathematics teacher education and teachers' continuous professional development training can benefit from considering and taking note of the phases described in the four-stage competence development model. A hypothetical "reflective competence" stage seems to be especially important when describing the pedagogical content knowledge required from math educators

Keywords: mathematics teacher education research, competence development, continuous professional development, pyramid model, pedagogical content knowledge

INTRODUCTION

The Broadwell-Burch Four-stage Model of Competence Development: Historical Roots, Theoretical Rationale and Visualization

Historical emergence of the model

Several decades ago, a speculative model of skill development was published. Broadwell (1969) proposed a list of sequential developmental stages by which the development of teaching skills could be described. The four stages and their brief descriptions were as follows. (1) Unconscious Incompetent – poor teaching while being unaware of it. (2) Conscious Incompetent – poor teaching while being aware of it and seeking help. (3) Conscious Competent – starting to get results in teaching while knowing the reason for it. Finally, at the (4) Unconscious Competent level, the

teacher is a "good teacher by nature" (Broadwell, 1969). According to Broadwell, the Unconscious Competent level applies solely to a small subset of teachers. The fact that these teachers are unaware of what makes them effective and unable to articulate their decision-making and instructional practices presents significant challenges.

The descriptions and the labels of the four stages in Broadwell's model form a combinatorically complete, 2x2 system with two dimensions and two values in both dimensions. Consciousness and unconsciousness are two possible values with respect to the self-referential or explicit knowledge the teachers may possess about themselves and their teaching practices. Nevertheless, using the term awareness would be a better way to name this dimension since, according to Hacker (1998), awareness refers to potentially reportable knowledge, and in Broadwell's model, the dimension of consciousness or unconsciousness is defined in this sense, i.e., referring to potentially available, verbalized, or potentially verbalizable knowledge. The other dimension is the success of teaching measured in some way.

The success or effectiveness of teaching has already been widely studied. One of the highly important studies (van de Grift et al., 2014) explicitly advocated a theoretical model that covers the range of observable teaching skills from simple to complex. In their conceptualization, the teaching competence – a term often used generously and sketchily in the literature – can be evaluated by means of measuring a range of teaching skills that build on each other and are classified into hierarchical levels. A further, general assumption about the development of teachers' competence is that the evaluation of teachers' professional development may be described by the principle of "sequential outcomes", i.e., thinking and doing, affecting and strengthening each other (see Thurlings and de Brok, 2017).

The Counter-intuitive Nature of the Model

One of the strengths of the sequential four-stage model was probably the surprisingly counterintuitive order of the stages. When it comes to the first two stages, it seems evident that understanding the reasons behind poor performance is advantageous. However, opinions may differ regarding the significance of awareness associated with successful completion. Why should one perceive it as a higher level of performance when individuals can no longer verbalize their decisions and lack conscious control over ongoing processes? Given the direct and immediate relevance of this question to the teaching professions, it is important to acknowledge that Broadwell (1969) employed the term skill in the broad, everyday sense: Teaching is indeed a skill.

Throughout the previous decades, the meaning of the term "skill" has evolved in many ways and directions in different content domains. In line with the aims of the current paper, we recall a definition that comes from the field of reading research. Afflerbach et al. (2008) paid off an old debt by defining the difference between skills and strategies. Although they focused on the difference between reading skills and reading strategies, the essence of the distinction can be easily

transferred to other domains where rich systems of skills develop. Their definition was the following (Afflerbach et al., 2008, p. 368): "Reading skills are automatic actions that result in decoding and comprehension with speed, efficiency, and fluency and usually occur without awareness of the components or control involved." In other words, properly functioning reading skills are characterized by automaticity. Other skill systems at the heart of school curricula, like writing and arithmetic, can also be characterized by quick and efficient functioning without much awareness.

The sequence of levels 3 and 4 in the four-stage model becomes understandable from the idea that similar automatisms apply to fairly different skills. As Afflerbach et al. (2008) emphasized in the case of reading skills, conscious and deliberate strategies may become skills with months of practice. In the area of arithmetic skills, Csíkos (2022) presented a model about the possibly diminishing role of strategic (metacognitive) components. Effortless and automatic operation can be the key to development in many (or almost all) essential areas of human performance, as argued by Leahey and Harris (1993, p. 284), "what had formerly required conscious thought becomes intuitive, and an important question concerns what happened to the rules followed consciously by the novice".

The four-stage model proposed by Broadwell fell on fertile ground, and the Gordon Training Institution had great merit in dissemination and propagation. In addition to making the four-stage model widely known, there is a relevant content innovation attributed to Noel Burch (1970). While the original Broadwell-model was about teaching skills, the potential of generalizability made a conceptual extension of the model to learning any new skill. Later, Adams (2011) gave a summary of the generalized model studded with personal experiences, and she claimed that it was Noel Burch (1970) who developed the so-called "Learning Stages Model". In this generalized model, the labels for each stage are as follows: (1) Unconsciously unskilled, (2) Consciously unskilled, (3) Consciously skilled, and (4) Unconsciously skilled.

Consistent with the common trend observed in models that hold the promise of extensive applicability, the four-stage model has been applied in various training fields, reflecting the spirit that nothing is more practical than a good theory. The counterintuitive order of levels 3 and 4 merited particular attention. Across different domains, it has become evident that the role of awareness (consciousness, or metacognition) serves as a temporary support mechanism, akin to scaffolding. While conscious planning, monitoring, and evaluation can undoubtedly play crucial roles at certain stages of development or in response to specific tasks or contexts, it is not inherently valuable to consistently engage in conscious planning, monitoring, and evaluating for their own sake.

Indeed, the model's popularity started to rise in the 10s of this century, and in recent years a real re-recognition or renaissance of the Burch- or Broadwell-model has taken place. In scientific databases, a spectacular increase in the number of citations can be observed. It seems that the power of the model was recognized in several different fields of training and coaching. Various visu-

alizations were developed to better convey the model's message since the original sources (either Broadwell's or Burch's) provided a purely verbal and linearly sequential structure.

Different Types of Visualizations of the Model

Various two-dimensional visualizations appeared, with the four stages allocated to different quadrants of the plane. One of the earliest examples of such visual representation in scientific literature is a quadrant model that lacked labelled axes (Kent 2010). Peel and Nolan (2015) offered a visualization featuring two axes, each with dichotomous values. Focusing on medical residents' learning, they positioned the axis of instruction and feedback horizontally (whether its intensity is high or low), and the axis of needed support vertically (high or low). Incompetence was linked to a high level of instruction and feedback, while consciousness was associated with support residents might require. The consecutive levels of competence were thus associated with four quadrants in a two-dimensional plane (see Figure 1).

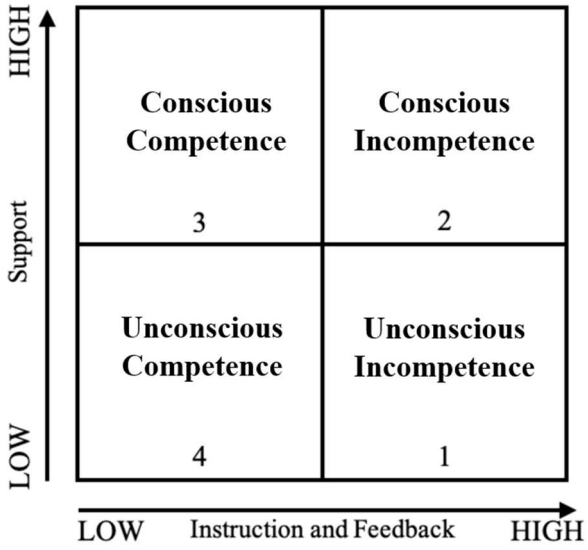

Figure 1: Two-dimensional arrangement of the model (adapted from Peel and Nolan 2015, p. 2)

Esterle and Brown (2020) suggested a two-dimensional quadrant model where confidence in one's ability is presented as a function of experience (Figure 2). With the four consecutive stages, experience grows, while confidence draws a U-shaped curve, i.e., greater confidence can be observed in the unconscious first and fourth phases.

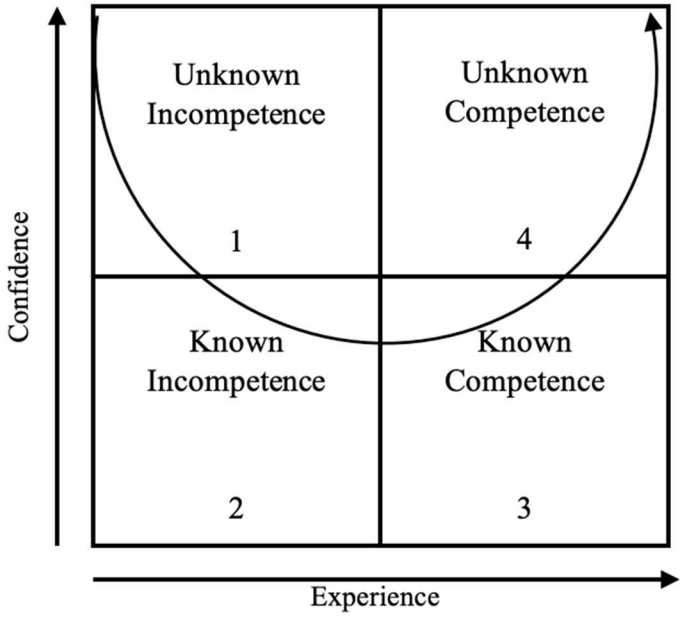

Figure 2: Two-dimensional arrangement of the stages of the model enriched by further variables (adapted from Esterle and Brown 2020, p. 1)

Interestingly, some researchers discuss the concept of cycles or wheels of competence development. They propose that the four quadrants of the plane represent consecutive stages, where the level of unconscious competence can lead back to unconscious incompetence once again (see Bhutalazi & Singh, 2019). While it might be argued that reaching a very high level of competence in a particular domain or skill system could unveil the recognition of unconscious incompetence, from an educational perspective, particularly in formal and non-formal training settings, the assumption of such self-recurring cycles might be discouraging or ambiguous. Nevertheless, Bierwolf and Frijns (2019) claim that a simple two-dimensional quadrant model may be misleading, so it is better to talk about a kind of spiral, i.e., a series of continuous learning cycles (Figure 3). Burgess et al. (2020) warn that there is a long-term potential (danger) arriving once again at the stage of unconscious incompetency, like drivers who overestimate their skills.

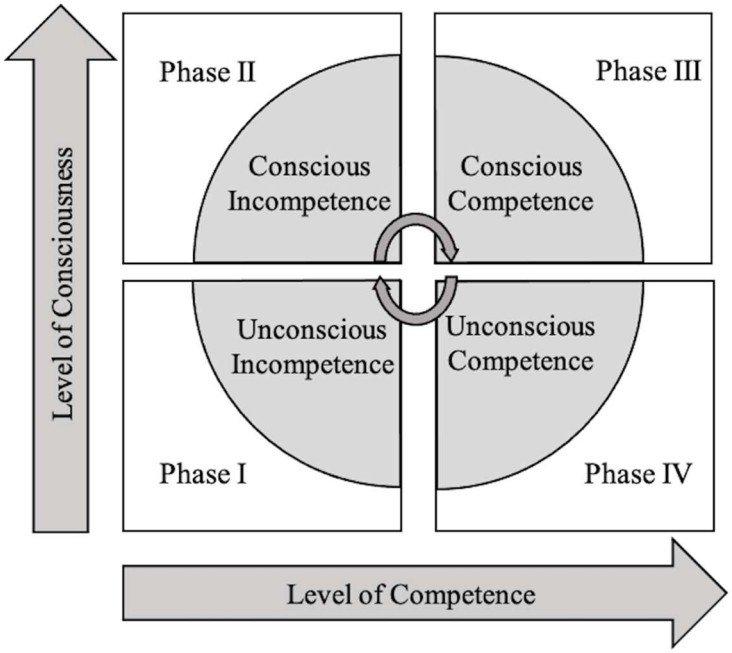

Figure 3: The recurring cycles model of the four phases (adapted from Bierwolf and Frijns 2019, p. 33)

As Neves (2022) provocatively highlighted in the context of psychotherapists, becoming aware of the unconscious incompetence can be a distressing experience when a therapist encounters unexpected information. Neves emphasizes the co-existence and multi-layered nature of conscious and unconscious competencies within the same individual. Furthermore, it is understood that different stages of competence can coexist within one person. This possible co-existence of different levels of competencies within the teaching profession poses challenges when attempting to apply the four-stage model in teacher education.

Another kind of two-dimensional representation used the horizontal line as a timeline, depicting the four stages as consecutive while incorporating additional characteristics. It is worth considering that the original four stages of the development of teaching skills were not inherently tied to specific time periods or a linear timeline. Keeley (2021) illustrated the four developmental stages using a timeline-like graph with the four stages following each other, and performance was depicted on the vertical axis (Figure 4). In this kind of visualization, the course of the performance curve resembles the logistic curve model widely used in developmental psychology.

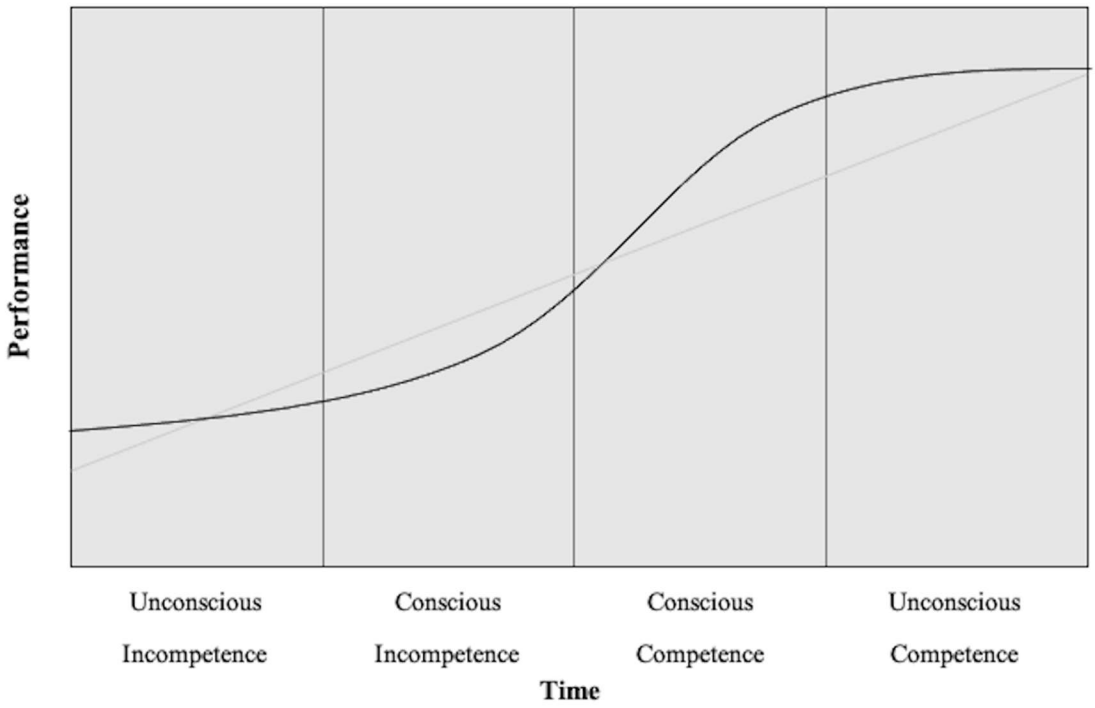

Figure 4: Timeline-based arrangement of the four levels of the model (adapted from Keeley 2021, p. 2)

The steeply rising section of the performance curve is, of course, between the second and third stages. Luczak and Kubo (2022) found neural correlates represented on the timelines for the four-stage model. Their model reassures the steep developmental phase, but not between the second and third, but between the first two stages. Indeed, considering the pedagogical relevance, we can confirm the importance of this first transition phase, which was also reassured by an empirical work on environmental awareness (Permana et al., 2020).

A pyramid-like visualization of the four-stage model appeared in Moore and Durham's (2011) work (Figure 5). In the field of dental education, they showed how the four stages are built on each other within a rectangular-shaped figure.

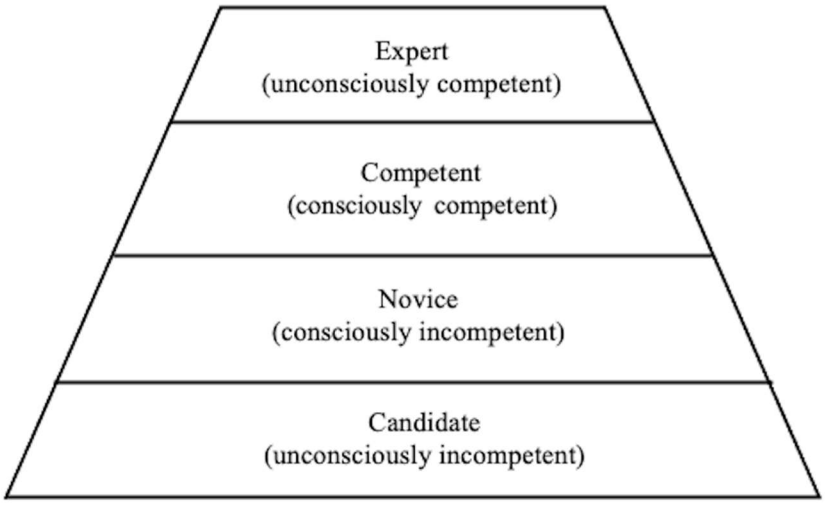

Figure 5: The first pyramid-like arrangement of the four stages labeling them with terms widely used in the cognitive science literature (adapted from Moore and Durham 2011, p. 54)

Drawing from the cognitive science literature, Moore and Durham (2011) extrapolated that the ultimate stage of unconscious competency would require approximately ten years of dedicated practice. This notion of a pivotal "10-year" period aligns with the findings of numerous expert-novice studies conducted among musicians or chess players (see, e.g., Ericsson et al., 1993; Schneider et al., 1993).

LITERATURE REVIEW

Applications in Several Disciplines: Widespread Recognition of the Broadwell-Burch Model

Applications in Psychology and the Social Sciences

The initial applications of the four-stage model encompassed coaching and adult training, and established a significant connection with university courses in teacher training and continuous professional development, particularly within the domains of medical and dental trainings and CPD programs. Curtiss and Warren (1973) succinctly captured the essence of the four-stage model in the context of adult training practices, without explicitly referencing either Broadwell or Burch. They introduced a broader term, "competence," replacing the previous concepts of teaching or other specific skills. The stages are now referred to as the typical sequence for acquiring competence. As for the need to proceed to the third to the fourth stage, they argue:

> *With practice, you feel "conscious competence" where the skills are painfully obvious and undeveloped, and you feel embarrassed at your attempts and the lack of smooth and polished performance. (p. 89)*

Jarvis and Baloyi (2020) explicitly connected the second and third stages (conscious incompetence and conscious competence) to Vygotsky's theory of the proximal zone of development. When the student is already aware of being unsuccessful, a more experienced tutor can help achieve successful functioning while maintaining awareness. One possible description of this kind of help uses the term metacognitive scaffolding (Wright, 2018). As for the connections of the four-stage model with another important and recognized model of cognitive development, Syroyid (2021) found connections between the four stages and the Piagetian stages of general cognitive development. Therefore, the four-stage competence development model is compatible with relevant developmental theories and models, and provides the hope for effective yet understandable generalization for many areas of training development.

In search for the power of the model not only as a description of what happens in consecutive phases of skill development but as a suggestion and explanation at the same time, there were several attempts to describe how to teach, how to intervene in order to help students proceed from one level to the next. In the context of scaffolding PhD students, Elliot (2022) suggests different kinds of support along the four stages in both the formal and the hidden curriculum. The transition from the first to the second stage can be considered a period of purely scientific (academic) support. In later stages, it is the emotional, social, and psychological support tools that will play a leading role. In an action plan devised for librarians, Van Camp (2019, p. 99) emphasized that "the first stages are inherently frustrating". This is in line with what Elliot suggested about the importance of emotional support at the beginning. Another aspect of the specialty of the first transition phase (between the first and second stages) was revealed when students from liberal arts and sciences took part in an investigation. De Krom (2023) collected empirical evidence on how students express themselves in a writing assignment at the conscious incompetence stage of development. Some of them believed that a demonstration of effort such as indicating "I have no idea what you mean here" (p. 70) is required to get a better grade.

According to Wisniewska et al. (2021), the practical activities of organizational learning can be associated with the second stage of the competence development model. In other words, it is crucial to reach at least the stage of conscious incompetence to feel the need for self-study or self-reflection. Kortmann and Scholten (2020) attributed considerable explanatory power to the four-stage model in cooperative game-based learning in the sense that the importance of the second stage en route to conscious competence was also supported by empirical data.

The Broadwell-Burch Model in the Medical Professions

The application of the four-stage competence development model extends to the field of medical education as well, as it has proven to be fertile ground and a significant area of application in the past decade. In an empirical investigation focusing on a specific technique, Asadoorian et al. (2022) revealed a potential connection between a decreasing level of confidence in ultrasonic instrumentation and participants' shift from unconscious incompetence to conscious incompetence. It becomes evident that although the experience of diminishing confidence may be regarded as a negative emotion, it is an unavoidable aspect of the developmental process.

A reasonably general model for clinical training was developed by Lane and Roberts (2022). They warn that stage 4 can be misinterpreted as the final mastery of a skill, where the learner has nothing more to learn. They also emphasize the contextualized nature of the model. Lane and Roberts raise some other concerns as well, which are instructive when the model is generalized to the field of teacher training. One is considering the possible effects of preparatory reading on the actual performance. Another issue is the measurability of performance in terms of both quantitative and qualitative indicators.

The work of Lee et al. (2017, p. 132) provides a direct and immediate source of inspiration for educational science: "Most competent endoscopists fall into the unconsciously competent category, where they can perform a procedure skillfully, but not necessarily describe or verbalize how or why they do it, making them poor instructors." Burgess et al. (2020) suggest that continuous maintenance of acquired skills through deliberate practice can be beneficial. Deliberate practice of a skill already mastered may even enable the attainment of a hypothetical fifth stage (Jensen, 2013) of competence development (Figure 6).

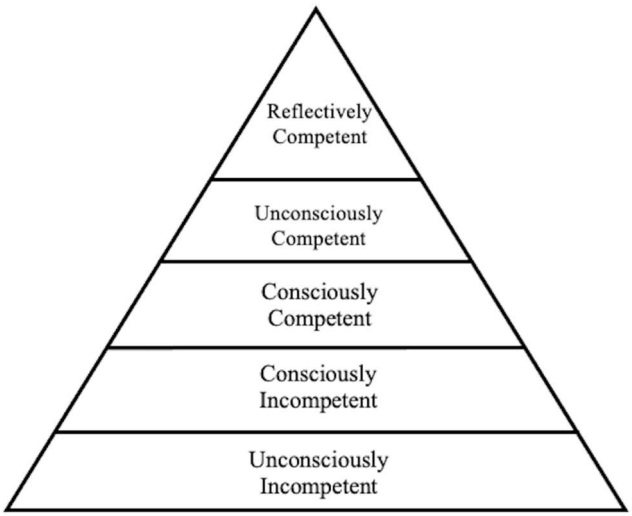

Figure 6: Pyramid-type arrangement of the five stages of competence development

Jensen's (2013) research report for the Karolinska Institute was the first to propose a fifth stage: reflective competence. This stage has since been adopted by others to describe the "conscious competence of unconscious competence"—a phase in which unconsciously competent individuals, particularly when tasked with teaching others, must become aware of and able to articulate their previously automatic practices, effectively stepping out of "auto-pilot" mode (Jensen, 2013, p. 11). In their review of coaching as a form of continuous professional development for surgeons, Willemot et al. (2023) focused on the first three stages of the competence development model. They argue that effective coaching should support the coachee in progressing from unconscious incompetence through conscious incompetence to conscious competence. While this does not imply that the fourth stage is overlooked, the authors emphasize that, for the purposes of designing coaching interventions, the initial stages are the most relevant.

The Broadwell-Burch Model and the Teaching Profession

Differences and Similarities Between the Teaching and the Medical Professions

Continuous professional development is essential not only in medicine but also in education. Given the growing number of studies applying the competence stage model in medical contexts, it is timely to explore its potential in teacher education. While the model's relevance is clear for foundational skills across educational settings, its applicability to more complex competencies, such as teaching, may be less immediately apparent. Building on Darling-Hammond's (2006) observation that teacher training lags nearly a century behind medical education, it is worth considering how the four-stage competence model could inform the development of teaching expertise. The system of skills necessary for being an effective teacher can be labelled as teaching skills. In this sense, we return to the origins of the model, as Broadwell's (1969) original four-stage framework was developed with the teaching profession in mind. Without further operationalization or elaboration of what the "teaching skills" exactly are, we may rightly propose that this is a system of several psychological constructs involving factual knowledge, more or less automatized skills, and the capability to make decisions at the strategic (planning, monitoring and evaluation) level. From this point of view, there is a striking similarity between the medical and teaching professions.

While considering the similarities between teaching and the health professions, our attention turns to the meaning of the different developmental levels and how teacher educators can utilize this knowledge effectively. To facilitate this exploration, we will simplify and dichotomize the two aspects of the model and assume that teachers can be efficient in their work (as an overall, yes-or-no aspect), and they can be either capable or not capable of reflecting on their work (as possibly judged by experts of the field). Additionally, our focus encompasses both pre-service teachers and in-service teachers as learners. Furthermore, we aim to delve into the description of

the hypothetical fifth stage of the model, specifically examining what reflective competence entails in the teaching profession (see Figure 6).

The Pyramid Model and the Teaching Profession

At the initial stage, pre-service teachers may be considered ineffective and they lack the ability to engage in professional reflection on their activities. However, as they progress through teacher training courses, their competence in reflecting on their own activities, as well as those of their peers, gradually increases. It is important to note that this growth in reflective competence does not automatically translate into becoming effective teachers. For university-based teacher trainers, it presents a significant yet attainable challenge to provide guidance and support to pre-service teachers as they progress to this second stage, i.e., the level of conscious incompetence.

According to popular belief and naïve hypotheses, "practice makes perfect," and the teaching profession is no exception. In this maxim, there is often a limited perspective on the process of learning from experience, as if the mere act of doing something would automatically elevate the level of competence through accumulated experience. However, the role of practice and experience is fairly different for novice teachers and for their experienced peers. Based on McIntyre's maxims, Hagger et al. (2008) identified two distinct types of reflective activities. For novice teachers, the essential tools of reflective practice involve actively reflecting and consciously considering ideas shared by their more experienced peers, as well as educational researchers. In contrast, experienced teachers engage in a different form of reflective practice, primarily centered on bringing their deeply embedded beliefs rooted in their practice to conscious awareness.

Becoming an effective and reflective teacher requires long years of deliberate practice. We use the term "deliberate practice" in line with Ericsson et al.'s (1993) formulation, which distinguished between work, play, and deliberate practice. Teacher training programs designed for continuous professional development may facilitate the process. However, an intriguing question arises: Is it genuinely possible for teachers to reach the fourth stage in the model, i.e., becoming efficient while simultaneously incapable of reflecting or verbally reporting on their teaching skills? Equally important is the exploration of how (and whether) the capability of reporting on strategic decisions may diminish or be lost.

There is another critical aspect of the model to consider before we can reasonably propose it for further consideration and as a useful starting point for empirical research. In addition to the so-called teaching skills, there exists another essential component of teaching competence consisting of domain-specific or content-related processes. This component can be identified as pedagogical content knowledge (PCK) and it necessarily contains the knowledge and acquisition of various skills that are not purely teaching skills, but different skills to be taught to the learners. A key question arises: Which stage of these skills to be taught to children should teachers reach in order to be effective in their profession?

An Example From Basic Skills Development

Let us consider some examples from the realm of basic skills and some other systems of skills. The classic three Rs – reading, writing, arithmetic – must be thoroughly mastered by those who teach these basic skills. We would even allow the assumption that elementary teachers must reach the fourth stage in the sense that their reading, writing and calculation skills must be effortless and fluent. Other examples of more complex skills include playing an instrument or driving a vehicle. Those who undertake to teach these complex skills to others are expected to possess a reasonably high level of proficiency. A piano teacher must be able to play masterpieces, although not necessarily at the level of the top artists going on world tours or even performing daily as their livelihood. A driving instructor must be a very experienced driver, but it is unlikely that they consciously accomplish the decisions and those high amplitude movements (e.g., clearly perceptible turning of the head in a desired direction) that they demand from their students. Similarly, master chefs, synchronous interpreters, masons, and experts in most occupations exhibit high level of efficiency in their work, while they may engage in consciously planning, monitoring and evaluations of their actions at times, it is not feasible for these conscious efforts to be present throughout their daily activities. What is the key condition for a professional to be able to teach others?

According to a cynical opinion, as cited from G. B. Shaw by Shulman (1986, p. 4), “He who can, does. He who cannot, teaches.” Although one could argue that this often-mentioned phrase is not even worth talking about, the low prestige of the teaching profession is a real problem, with a significant number of teachers feeling that their work and profession are undervalued by society (OECD, 2019). In order to better understand the message of this ill-famed maxim, we can use the four-stage model. According to one possible interpretation, many professionals are more competent and more effective in a given field than an instructor might be. Car racers, piano virtuosos, or, to stick to basic skills, professional writers, world-class mathematicians, or the best stage actors are likely to be on a higher level of achievement in their very own field of expertise. However, if we would ask questions about their reflective competence, they may be less skilled in talking about the decisions they make during performance. Only some of the greatest artists and scientists are capable of reporting on the processes they are executing. In light of the four-stage model of competence development, they are indeed at level 4, with excellent efficiency, but with limited consciousness, i.e., overwhelmingly relying on quick and intuitive decisions, having probably already lost the ability to provide introspection about the ongoing processes.

Bloom's Taxonomy and the Broadwell-Burch Model

Finally, the already powerful four-stage developmental model may be enhanced by showing its connection to Bloom's taxonomy: according to Shtaltovna (2021), the four stages not only resemble but essentially connect to the widely known taxonomy of educational objectives originally proposed by Bloom (1956). Shtaltovna provided a comparison of the revised Bloom taxonomy of educational objectives and the levels of the model, which she named the Burch/Bartlett-model. In essence, the six levels of the revised Bloom-taxonomy can be matched with the four levels of the Broadwell-Burch model (in her paper, she used the label 'Burch and Bartlett model'). The two highest levels of Bloom's taxonomy, i.e., Evaluation and Creation, can be matched with the level of unconscious competence, since the already mastered skills enable automatic and easy performance. The third and fourth levels of the revised Bloom-taxonomy, i.e., Application and Analysing (the latter is labelled Experimenting in Shtaltovna's work), represent the level of conscious competence. Finally, the first and second stages of Bloom's revised taxonomy can be matched with the stages of unconscious and conscious incompetence. Since the Bloom-taxonomy has been the basis for curriculum development for long decades in many education systems and, therefore, a basis for operationalizing learning targets, the correspondence and parallels between the two models are noteworthy but still speculative in nature.

METHOD

The Broadwell-Burch Model and the Mathematics Teaching Profession

An Example: Adding Fractions

It is time to address the applicability of the four- or five-stage model in the field of mathematics teaching. As a first step, we apply the model to the topic of adding fractions. There are at least two reasons for choosing this content. First, as emphasized by prominent scholars and the National Mathematics Advisory Panel (2008, p. 18), "The most important foundational skill not presently developed appears to be proficiency with fractions (including decimals, percents, and negative fractions)." Second, the handling of incorrect examples plays a crucial role in learning fractions (see Heemsoth & Heinze, 2013). This requires a high level of pedagogical content knowledge in order to discuss possible misconceptions while also taking advantage of the benefits of learning from errors.

<u>*Unconscious Incompetence*</u>: A well-known classical example in the literature was given by Borasi (1987). She gave the example of a mistake often made by students: $2/3 + 5/7 = 7/10$. Many children simply add the two numerators and the two denominators making the new numerator and denominator from their sums. This error is labeled a "rational error" (Ben-Zeev, 1995), since obviously there is a kind of rational and conscious strategy behind the decision of using an otherwise faulty algorithm. Therefore, unconscious incompetence in this case does not refer to the lack of any kind of consciousness, but to the lack of consciousness concerning the actual prob-

lem to be solved. The children who use the algorithm presented above are usually not aware of why the procedure leads to bad results.

Conscious Incompetence: On the way towards mastering the algorithm of adding fractions, a next step may be that the students are aware of the above algorithm's flaw, i.e., they know that they must not automatically add the numerators and the denominators, but for some reason they are still not capable of getting the right answer. This might be due to a calculation error, or they may forget how to determine the least common multiple, and further additional problems may hinder achieving the correct result. In this stage, a student usually knows that they must do something different from simply adding the 'upper' and the 'lower' figures, but are still unable to perform well.

Conscious Competence: This is the stage we expect from good problem solvers. Students are not only expected to perform well, but we also expect them to be aware of the steps of the algorithm. Moreover, we also expect conscious and precise use of concepts like numerators, denominators, and least common multiple. Since pre-service teachers proved to be good performers in mathematics, this is also the level that we can assume as the competence level already acquired by prospective mathematics teachers.

Unconscious Competence: Imagine an upper secondary student who is capable to add fractions quite quickly and flawlessly, but unable to name the parts of the fractions, or cannot recall the concept of the least common multiple. They may perform well, but would certainly have some difficulties when trying to provide scaffolding to struggling learners. Only a part of the difficulties would come from the lack of verbally fluent use of the different concepts. These may be replaced by simpler substitution concepts (e.g., "upper figure"), but the main difficulty would come from the lack of introspection. In order to help others, we should first know well our own mental processes, and above this, the long-term accumulated pedagogical content knowledge components should be activated.

We may rightly claim for mathematics teacher training that, above the expected high level of performance (unconscious performance), a fifth level of knowledge should be built that allows for reflection on all four levels below. Of course, mathematics educators are not expected to provide introspections about every step they make. However, they should develop their knowledge to potentially report on their own thinking and actions, and on their students' assumed representations and activities. This kind of extra knowledge can be classified under the Shulmanian umbrella concept of pedagogical content knowledge. Shulman (1986, p. 9) mentions some forms of pedagogical content knowledge, e.g., the most useful forms of representations of an idea, "the most powerful analogies, illustration, examples, explanation", without which the only option left for the students is to watch and admire the master. Returning to the example of adding fractions, the teacher may use a variety of approaches based on the idea of multiple modalities, including pie charts, bar charts or special number line. One may be suitable for one student, the other for another.

Response to G.B. Shaw's Maxim

We can reformulate Shulman's (1986) witty riposte to G. B. Shaw's degrading maxim. Shulman claimed that those who can do, and those who, in addition to this, understand, can teach. In light of the four-stage model of competence development, the hypothetical fifth stage of reflective competence may be the level that indicates a high level of pedagogical competence. Good teachers are expected to possess a high level of content knowledge, reaching even the level of being unconsciously competent in the skills they want to teach. However, in order to provide effective and individual scaffolding to the students, teachers need to regain or learn some insights about how others can be helped in reaching the next level in their proximal zone of development (see Jarvis and Bayoli, 2020). Furthermore, in line with Mason's (1998, p. 249) insights, teacher educators should work with their teacher-students in order to "extend their awareness of what students might be attending to by being explicitly aware of what they are attending to." Here, both teachers and teacher educators have a tough challenge to meet, since there are a number of math skills to be used automatically. E.g., a mental calculation task such as 9 + 6 can be solved quite quickly by most people, however, math teachers need to be aware of the "making to ten" strategy by which scaffolding can be provided to their elementary students. As Jaworski (2008) emphasized using a visual model, the math teachers' knowledge is grounded in several components including the knowledge of the learner, but, of course, additional elements of knowledge are needed beyond that. Going one step further, math teacher educators need analogue equivalents of the knowledge components a math teacher must possess. Compared to the knowledge elements expected of the student, both the teachers and the teacher trainers need additional knowledge elements that are born through conscious reflection. In other words, mathematical competence at the highest possible levels (which can be indeed called unconscious level of competence) is not enough for teaching mathematics. Consequently, we consider it necessary to supplement the pyramid model with the hypothetical fifth level, i.e., reflective competence.

CONCLUSIONS

Having reviewed the historical development of the four-stage competence development model and having discussed the potential of applications in different areas, we can now formulate suggestions for both theoretical and empirical research initiatives in the field of mathematics teacher education. One pillar of our suggestions is the promotion and urging of empirical research to test how the four- or five-stage model of competence development takes place in the mathematics teaching profession. For this, we feel the need to highlight a marked difference that exists compared to the medical professions. To do so, we need to reconsider the original Shulmanian distinctions among pedagogical knowledge, content knowledge, and pedagogical content knowledge. From a theoretical point of view, there is a chance that various competence levels are

achieved and desired in different competencies of the teaching profession. In the medical professions, no such distinction is used, at least pedagogical knowledge in itself is not a central element of those professions. Content knowledge and even pedagogical content knowledge (as we saw from the examples taken from the applications of the four-stage model in the medical professions) have their analogous counterparts. We may rightly claim that the main novelty and challenge of the application of the competence development model lies in an application where the three Shulmanian clusters of competencies can be taken and investigated both separately and in an integrative way.

In some cases, "teachers" at the third stage of the competence development model (i.e., being consciously competent) can also be effective in their teaching. Just think about the possible effective use of peer-learning in order to raise awareness of mathematical ideas. What may pose a problem at this level is the perceived lack of professional commitment and the possibly excessive use of cognitive resources.

Professional commitment and responsibility for professional development occupy an important place on the list of teacher competencies judged to be important by upper-secondary students (Zagyváné, 2016). Excellence or at least a high level of knowledge in the discipline is an often-stated expectation (see e.g., Hamachek, 1969, p. 344), since "teachers rarely fail because of lack of knowledge. They fail more often because they are unable to communicate what they know".

The second issue that a person who wants to teach but is only at level three in the developmental model must deal with is the possible (and rather likely) cognitive overload. The teaching profession itself is fairly demanding in terms of making conscious decisions in planning, monitoring, and evaluation activities. Therefore, if the content knowledge itself demands significant resources, then teaching may cause cognitive overload. Is it possible to teach effectively and attentively the types of conditional sentences in English when they need conscious monitoring during formulating conditional sentences? How could one teach others to start smoothly from a parking lot if they always need to cite the MSM (mirror, signal, and maneuver) rule for themselves in such situations? Can one teach the strategy of "making to ten" when adding 9 + 6 unless they can use the strategy fluently and quickly, and can one mobilize their cognitive resources to scaffold learners effectively? Yes, one can teach, but beyond the excessive cognitive overload, they may be seen by others as a "blind leading the blind."

In addition to these model-related concerns, the pedagogical framework within which competence development is analyzed also deserves critical reflection. While this study has applied the Broadwell-Burch model within a primarily teacher-centered framework, we acknowledge that alternative pedagogical paradigms may involve different patterns of competence development. For instance, in constructivist classrooms, where learning is often co-constructed through student engagement and dialogic teaching, the progression through the stages may be less linear or more context-dependent. However, we believe the model's flexibility allows for meaningful adaptation in such settings as well. The emphasis on reflective competence, particularly at the fifth stage,

aligns with the goals of student-centered pedagogy, where teachers must be capable of critically evaluating and adjusting their instructional strategies in response to diverse learner needs. Future empirical research should explore how the model functions within constructivist environments and whether the developmental sequence retains its explanatory power across varying instructional approaches.

Teachers who reach the fourth developmental stage while also acquiring additional pedagogical content knowledge are the most likely to become effective educators. Others in the fourth stage just "can and do." The issue of peer teaching may serve as a compelling example to highlight the importance of the need for separately considering the applicability of the four-stage model for the various competencies required for effective teaching, including both content-bound knowledge and teaching skills. The hypothetical fifth stage, i.e., the "reflective competence" stage will help in describing the knowledge of those who have a high level of competence in diverse (or almost all) areas of mathematics. In line with what Krauss et al. (2008) found about the connectedness of pedagogical content knowledge and content knowledge, mathematics educators with higher mathematical expertise seem to be more capable to scaffold their learners' understanding.

The idea of the Birch- or Broadwell-model of competence development may fall on fertile ground in the field of teacher training and teachers' continuous professional development. In the spirit of "nothing is more practical than a good theory" the competence levels can be a useful starting point for empirical investigations. The complexity of such empirical research is remarkable in itself, but beyond that the complexity is further increased by an additional factor. The general pedagogical knowledge (or skills) and the primarily content-bound PCK are two constructs that may and should be measured and observed simultaneously.

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