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This editorial introduces the main and quite diverse foci of investigations in the present issue. The mathematical thinking process has the largest number of papers (see below) followed by the investigations of stage developments of different variables, and the impact of augmented reality on learning. We have two comparison papers and three investigations looking for especially effective approaches to learning.

TR 59 has a variety of often intersecting foci—some of the authors are interested in *developmental stages* within different environments. **Biro and Csikos** explore the developmental stages of reaching competence in a given profession. The four stages are made of a 2×2 matrix with corresponding entries obtained from (competent, not competent) and (conscious, unconscious) pairs. It's interesting to see the arguments establishing the sequential developmental order within the entrees. On the other hand, **Jimenez-Valderama et al** examine reading comprehension development across four levels—literal, inferential, critical, and visual—in the context of understanding the Mean Value Theorem of Calculus (MVT). The visual level, of course, is necessary because of the conceptual understanding of MVT, so that the previous three levels seem to be the levels of critical thinking development.

Two teams embraced an Augmented Reality (AR) approach. Osman et al. are interested in the impact of AR on learning isometric geometry, or transformations that preserve the distance

between points, such as translation or rotation. Indeed, there is no doubt that AR can deepen the understanding of some mathematical concepts. **Susanto et al.** extend the use of AR into an ethno-geometric environment and are interested in the impact of that environment upon students' creative thinking, which is defined following Guilford by skills such as fluency, flexibility, elaboration and originality. This suggests that blending immersive digital experiences with culturally relevant mathematical contexts provides a promising avenue for fostering higher-order cognitive abilities in learners.

The concern for the mathematical thinking process appears in several other papers in the TR 59 collection. **Cerrado and Limjap** report on an interesting intervention in the context of Realistic Mathematics Education (RME), which uses metacognitive prompts to increase students' metacognitive activity. They find that student metacognitive activity has three components: metacognitive knowledge, metacognitive experiences, and metacognitive skills. The intervention with its specially designed performance task on quadratic functions indeed significantly increased student metacognitive awareness**. Canonigo**, on the other hand, studies reasoning approaches of students in the trigonometric domain and finds that they use several different types of reasoning: algorithmic, analytical, abductive, inductive, and deductive methods. Different teaching techniques are used in the investigation to improve student reasoning with each of the methods: scaffolding, guided inquiry, and contextualized learning experiences. He also notes that meta-cognitive reflection and discourse enhanced students' problem-solving abilities and collaborative skills. This theme of reasoning also appears in **Hassan et al**.'s paper, which investigates the relationship between working capacity of memory and the levels of mathematical anxiety as independent variables to determine their impact upon students' creative reasoning in the context of problem solving. They find out that, as one could expect, prospective mathematics teachers with high working memory capacity demonstrate flexibility and fluency (as measures of creativity) in generating new ideas. This allowed them to connect known mathematical concepts and provide logical arguments to support the truth of the ideas created. However, high and low levels of mathematics anxiety interfere with cognitive performance, causing a loss of focus in solving more complex problems and not providing arguments for the ideas created. The paper by **Fatmanissa et al.** searches for the improvement of mathematical reasoning through the development of mathematical justification of arguments. The authors used three different tasks designed for the explicit use

of technology. They showed that addressing these three situations without technological support has significantly reduced the impact of justification on reasoning.

We have two comparison papers in the issue, one by **Yi Wei et al**. and the other by **Sabri and Mainali**. The first one is concerned about the PISA levels of competence of Malaysian students. Malaysia maintains a steady average level of achievement among nations but Malaysian educators want to reach higher levels. They hope to reach them by adopting Singapore's subject-based banding system to give equal opportunities and a flexible learning environment for students; continuous improvement of professional development for teachers by adopting the Singapore SFEd roadmap and the World Bank teacher development roadmap; and incorporating PISA-released items into classroom practices and the national assessment mode. They have a good model to follow in Singapore, which continuously holds the highest levels on PISA exam. The second paper, by **Sabri and Mainali** compares the teaching approaches of Turkish and American teachers in terms of representations used in their classrooms. The main difference between them is interesting: whereas Turkish teachers emphasize symbolic and life-based representations, American teachers emphasize concrete materials, visual representations, and technology. The issue of representation is also touched upon by **Meryiati et al**. who related it to the question of self-perception vis-à-vis mathematics and technological confidence. The authors found that technological self-confidence and self-perception vis-à-vis mathematics were positively related.

The papers by Taraldsen, Doz et al., and Ngoveini present three different methods of impacting learning, correspondingly by gaming, general problem solving in a third grade, and by introducing a new framework for questioning students in the context of financial mathematics.

The issue wouldn't be complete without the Problem Corner, as always prepared by Dr. Ivan Retamoso, our Problem Corner editor……page 363