Oliver R. Marshall

Harry Deutsch possessed a clarity of mind that demanded philosophical problems be attacked with great logical precision. As such, he was especially interested in paradoxes, the phenomena that underlie them, and in stating these in the most basic logical terms. My failures at the latter were met with, to put it mildly, emphatic demands to remove excess (elementary) technicalities from my arguments – demands made with Harry's signature flurry of capital letters and exclamation points!

His approach is clear from his three co-authored contributions to the *Stanford Encyclopedia of Philosophy* on Relative Identity, Russell's Paradox, and Alonzo Church. I was very lucky to have co-authored the latter two entries with him. Doing so allowed me to extend my philosophical education far beyond graduate school. By way of gratitude, I have chosen to focus on a few aspects of our work together that reflect his accumulated wisdom more than mine.

Harry made much of the following logical principle that underlies the two paradoxes that bear Russell's name:

Cantor's Lemma (CL)

Let f be a function with domain X and range Y. Then, on pain of contradiction, the diagonal set $D_f = \{x \in X: x \notin f(x)\} \notin Y$.

Russell's paradox of propositions (1903, Appendix B) concerns the proper background logic —or, perhaps, set theory— for semantics. The class m of propositions can be correlated with the proposition that every proposition in m is true. This, together with an (extremely) plausible fine-grained principle of individuation for propositions —implying that if the classes m and n of propositions differ, then any proposition about m will differ from any proposition about n—, leads to contradiction. The combination of the two violates (CL) because it in effect defines a function f from propositions to sets of them, and then assumes that D_f is in the range of f. Despite being very impressed with this paradox, and very unimpressed with fans of fine-grained propositions who did not attempt to solve it, Harry did not regard it as fatal to Russellian intensional logic. Together, we explored a solution in terms of Church's Russellian Ramified Type Theory – a solution also studied by Saul Kripke. Independently of me, Harry offered original proposals to formalize fine-grained propositions in untyped set theories such as GVB and Morse-Kelley. Harry's set-theoretic solutions deserve further study.

Turning to the more famous of Russell's paradoxes, Harry saw the basic question underlying it as that of what non-paradoxical objects there are (whether they be sets, or barbers). The answer is governed by the following principle of FOL, which Harry sometimes called "Russell's Law:"

$$\text{(RL)} \sim\exists y\forall x(Fxy \equiv \sim Fxx).$$

Harry showed that there is a very close purely logical relationship between Russell's paradox in the form of (RL) and Cantor's lemma, (CL). To see this, consider a formula of pure logic that distills the essence of (CL):

$$\sim\exists y\forall x(x\in f(y) \equiv x\notin f(x)).$$

Substituting an arbitrary binary relation symbol P for epsilon yields:

$$\sim\exists y \forall x(P(x,f(y)) \equiv \sim P(x,f(x)).$$

This formula is a substitution instance of (RL). That is, (CL) is a simple generalization of (RL) obtained by substituting a function letter f for the variables at certain points in (RL). Similarly, one obtains (RL) from the formula above by taking f to be the identity function. Russell was led to his set theoretical paradox by contemplating Cantor's theorem as applied to the universe consisting of everything. Harry taught me that from a purely logical point of view, the two are even more closely related than anyone might have imagined.

Turning to Kurt Grelling's Heterological paradox, after studying Church's (1976), Harry and I published a simplification of some of Church's results and observed that there is an oversight in Church's gloss of his own results. Again, we were to discover later that Kripke saw things similarly. Harry was pleased with this, but less so with Rockefeller University, who rescinded its offer to him and shut the department before he could become Kripke's colleague. Subsequently, Harry did not get the recognition he deserved and was, in my opinion, one of the most under-appreciated thinkers in analytic philosophy. The loss to the discipline and to our project on Church is enormous.

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