Tommy is selling shell necklaces to various tourist shops at Waikiki. The graph below indicates the number of necklaces Tommy sells during his first 100 days of sales.

Determine whether each statement is true according to the graph, and select either true or false for each by placing an X in the appropriate box for each statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>The least number of necklaces Tommy sold during the first 100 days was approximately 1800.</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Tommy’s sales were increasing between days 40 and 60.</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Tommy’s sales were the greatest around day 65.</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>There are two non-overlapping intervals of days where Tommy’s sales were decreasing.</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>
2. Mark with a circle the point on the previous graph that represents Tommy’s least number of sales. See graph.

3. Mark with two stars, two points on the previous graph that represent two different days where Tommy’s sales were identical. See graph. These are not unique.

4. Mark with a square a point on the previous graph where Tommy’s sales went from decreasing to increasing. See graph. There is another point around day 25.

5. Approximate Tommy’s average increase in daily sales from day 25 to day 65. Round your answer to the nearest integer. \[ \frac{4000 - 1800}{65 - 45} = 55 \]

6. Another necklace vendor, named Tammy, is Tommy’s main competitor. Below is a table of values that indicates some of Tammy’s sales during the same time period. Assume that Tammy’s sales between each day shown followed a linear pattern.

<table>
<thead>
<tr>
<th>Day</th>
<th>0</th>
<th>10</th>
<th>25</th>
<th>30</th>
<th>40</th>
<th>55</th>
<th>75</th>
<th>85</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>1500</td>
<td>1600</td>
<td>2000</td>
<td>2100</td>
<td>2700</td>
<td>2600</td>
<td>3500</td>
<td>3000</td>
<td>4000</td>
</tr>
</tbody>
</table>

Determine whether each statement below is true according to the graph and table, and select either true or false for each by placing an X in the appropriate box for each statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tammy began with more sales per day than Tommy.</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Both Tammy’s and Tommy’s sales were increasing daily between days 25 and 65.</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Tommy sold more on day 40 than did Tammy.</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Tommy’s sales surpassed Tammy’s at some point during the first 10 days.</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Both Tommy and Tammy experienced decreasing sales between days 75 and 85.</td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>
7. Keoni walks the 4 miles from school to his home. He briskly walks the first 2 miles at a constant rate of 4 miles per hour. He takes a break during the 3rd mile and only walks at a constant rate of 2 miles per hour. Keoni then realizes his favorite TV show is coming on in exactly 10 minutes, so he runs the final mile at a constant rate to arrive just in time for his show.

a. How long does it take Keoni to get home? **One hour and ten minutes.**

b. How fast (in miles/hr) must Keoni run during the final mile? **1 mi/10 min = 6 mi/hr.**

c. Graph below Keoni’s distance $K(t)$ from home $t$ hours after he began his trip home.

d. What was Keoni’s average speed during his trip home? Round your answer to one decimal place. Note: this is the speed he could have walked at a constant rate to arrive at exactly the same time. **$4 \text{ mi}/1.167 \text{ hr} = 3.4 \text{ mi/hr.}$**
8. Pamela has a gecko infestation at her farm. When she checked at 6am on Jan. 1 the population was 20. When she checked at 6am the next day the population had increased by 50%, and when she checked at 6am on the 3\textsuperscript{rd} day the population had again increased by 50% over the previous day. Assume the population of geckos \( t \) days after she first checked can be modeled by an exponential function.

a. Plot the point on the grid below that represents the initial population of geckos that Pamela discovered at 6am on Jan 1. \((0, 20)\)

b. Plot the point on the grid below that represents the population of geckos at 6am on Jan 2. \((1, 30)\)

c. Plot the point on the grid below that represents the population of geckos at 6am on Jan 3. \((2, 45)\)

d. Find the symbolic representation for the population of geckos \( t \) days after Pamela first discovered her infestation. \( P(t) = 20 \cdot 1.5^t \)
9. Play-it-Now has a January special on video game rentals, bundles of 5 games for $20 and you can play them all month. If you rent bundles of videos they charge a monthly membership fee of $10, regardless of the number of bundles you rent. John received a Play-it-Now Christmas gift card worth $100, and he plans to rent videos on January 1 so he can get maximum enjoyment. He may choose to use none or only a portion of his gift card and wait for future promotions to use the rest.

a. What is the maximum number of video games John can rent for the month of January using his Play-it-Now gift card? 20 games.

b. The cost that John pays for his January video games can be modeled as a function of the number of video games he rents. Circle ALL statements below that correctly describe the domain or range of the cost function.

i. The domain is the set of integers greater than zero and less than or equal to 5.

ii. The domain is the set of integers greater than or equal to zero and less than or equal to 4.

iii. The domain is the set of integers \( \{0, 5, 10, 15, 20\} \).

iv. The domain is the set of integers \( \{5, 10, 15, 20\} \).

v. The range is the set of all real numbers greater than or equal to zero and less than or equal to 100.

vi. The range is the set of all multiples of 20 from 0 to 100.

vii. The range is the set \( \{0, 30, 50, 70, 90\} \)
10. Dave bought some stock, kept it for 10 days and sold it. A graph depicting the value \( V(t) \) of his stock \( t \) days after he purchased it is given below.

![Graph showing stock value over time](image)

a. On what day should Dave have sold his stock? **Day 5**

b. Did Dave lose money by buying and selling this stock? Explain.  
   **No, he bought it and sold it for the same amount.**

c. Assuming the value of the stock is computed continuously throughout the days, choose from the list below all values that are in the domain of \( V \).

   i. 3
   ii. 15
   iii. 10.5
   iv. 4.5

   **3 and 4.5**

d. What was the stock’s average daily loss or gain from day 5 to day 8? Be sure to include units.  
   **It lost $15 in 3 days, so the average loss was $5/day. This can also be written as -$5/day.**

e. What was the stock’s average daily loss or gain from day 0 to day 5? Be sure to include units.  
   **It gained $10 in 5 days, so the average gain was $2/day.**

f. What was the stock’s average daily loss or gain from day 0 to day 10? Be sure to include units.  
   **Since the selling price = purchasing price, the average daily loss or gain was $0/day. This can also be computed as ($10 - $10)/10 days = $0/day.**
11. Given the graphs of \( f \) and \( g \) below, choose which of the remaining four graphs represents the graph of the function \( h \) defined by \( h(x) = f(x) + g(x) \). Since \( f(x) > g(x) \) for large \( x \), it must be C or B. Check the value at \( x = 3 \) to see that B has a negative value there, where it should be positive.
1. Select the ordered pair that most likely lies on the graph of the function depicted below.
   a. (0, -1.6)
   b. (-.75, 0)
   c. (0, 3.2)
   d. (3.2, 0)

   ![Graph Image]

2. Select the ordered pair that most likely lies on the graph of the function depicted above.
   a. (2, -.6)
   b. (6, 15.8)
   c. (-3, -2)
   d. (-4, -6)

   This one is a bit tricky. You need to try to approximate the slope as closely as you can. It seems to be about 4.2.

Note: The SBAC asks this question in the following form. “Select the ordered pair that is most likely a solution to the equation represented by the graph.” The orientation the authors of this curriculum have taken is that a function is a “statement” and that an equation is a “question.” The SBAC authors may have taken the orientation that given a function, a natural question to ask is “which points satisfy the definition of the function, and hence lie on the graph?” Of course, they may simply use equation because of the equal sign. We include this explanation to help prepare students for the SBAC. Obviously, if an ordered pair satisfies the symbolic representation, or what SBAC refers to as the “equation,” then the ordered pair lies on the graph, and vice-versa.
3. The following is the graph of a quadratic function defined by \( y = f(x) \).

\[
\begin{array}{c}
\text{f} \\
-4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\
\end{array}
\]

a. Mark with a circle a point on the graph where \( x = 0 \).
b. Mark with a square a point on the graph where \( f(x) > 0 \).
c. Mark with a diamond a point on the graph where \( f(x) = 0 \).
d. The points \((m, 14)\) and \((n, 14)\) both lie on this graph, where \( m < n \). Select the consecutive-integer interval that contains \( m \) and mark it on the number line below. Repeat this process for \( n \). Here you need to find the equation for the parabola, \( y = 2x^2 – 2 \) and set it equal to 14 to find the plus or minus square root of 8, which is between 2 and 3.

\[
\begin{array}{c}
\text{Number Line} \\
-5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

Note: The SBAC also asks questions of the following form. “How many solutions are there for the equation, in the interval \(-3 < x < -2\)?” Here again students should interpret this as “How many points on the graph lie between -3 and -2?” which is a simple question to answer.
4. Below are the graphs of \( y = f(x) \) and \( y = g(x) \).

a. Find the solution to the equation \( f(x) - g(x) = 0 \). \( x = 2 \), where they intersect.

b. Find one solution to the inequality \( f(x) > g(x) \). Mark an x-value on the x-axis above corresponding to your solution. Mark the points on the graphs corresponding to this x-value. See graph above.

c. Find all solutions to the inequality \( f(x) > g(x) \). Represent your solution using set notation, and also by shading the correct region of the x-axis. \( \{ x : x > 2 \} \), and shade the x-axis for all values to the right of 2 with an open circle at \( x = 2 \).

d. Find the symbolic representation for \( f \) and \( g \), assuming \( g \) is linear and \( f \) is exponential. \( f(x) = 2^x, g(x) = -0.5x + 5 \)

e. Symbolically check your answer to question “a” by substituting it into the symbolic representation for the equation \( f(x) - g(x) = 0 \).

f. Symbolically check your answer to question “b” by substituting it into the symbolic representation for the inequality \( f(x) > g(x) \).

g. Explain why you think it would be difficult to solve question “a” algebraically instead of graphically. Because these belong to two different families of functions and the algebra associated with exponentials (e.g. logs) does not work with linear functions.
5. Below are the graphs of \( f(x) = 3\sqrt{x} \) and \( g(x) = x - 4 \).

![Graph of f(x) = 3\sqrt{x} and g(x) = x - 4](image)

a. Select all answer choices that best represent solutions to \( f(x) - g(x) = 0 \).
   
   i. \( x = 0 \)
   
   ii. \( x = 18 \)
   
   iii. \( x = -4 \)
   
   iv. \( x = 16 \)

b. Select all answer choices that best represent solutions to \( f(x) - g(x) < 0 \).
   
   i. \( x = 0 \)
   
   ii. \( x = 18 \)
   
   iii. \( x = 200 \)
   
   iv. \( x = 16 \)

c. Algebraically solve \( f(x) - g(x) = 0 \). Do your answer(s) match your answer(s) to question “a” above? The quadratic equation resulting from squaring both sides of \( f(x) = g(x) \) is factorable, with solutions \( x = 1 \) and 16, but 1 is not a solution. The answers DO match, if the student is careful to throw out the extraneous answer.

   d. Find all solutions to \( f(x) - g(x) < 0 \), and represent your solution symbolically and by shading the correct interval(s) on the x-axis. Explain why you think your solution is correct. \( x > 16 \), and it appears to be correct from the graph since the square root function will continue to grow at a slower and slower rate, while the linear grows at a constant rate.
6. The graph below shows a linear function $f$ and a cube root function $g$.

![Graph of f and g]

a. Select all values that are approximate solutions to $f(x) - g(x) = 0$.
   - i. $x = -.8$ (circled)
   - ii. $x = -4.2$
   - iii. $x = 4.8$
   - iv. $x = 2$
   - v. $x = -4$
   - vi. $x = 3$
   - vii. $x = 4$

b. Mark with a circle the location of a single point that is a solution to $y = f(x)$ only.

c. Mark with a square the location of a single point that is a solution to $y = g(x)$ only.

d. Mark with a star the location of a single point that is a solution to $f(x) = g(x)$.

e. Mark with a diamond the location of a single point that is a solution to $f(x) > g(x)$. 
7. The graph below represents the set of ordered pairs that are solutions to an inequality. Write down the inequality that represents the solution set shown. \( y > -0.5x + 3 \)

![Graph of the inequality \( y > -0.5x + 3 \)](image)

8. Graph the line that represents the boundary of the linear inequality \( 2x - y \geq 4 \).

![Graph of the line \( 2x - y \geq 4 \)](image)

Plot a single point representing an ordered pair that is part of the solution set of this inequality.
9. Determine if each ordered pair is a solution to the system of inequalities by placing an X in each row of the table.

\[ 2x - 4y > 4 \]
\[ 3x + 2y \leq 7 \]

<table>
<thead>
<tr>
<th>Ordered Pair</th>
<th>Solution?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, -1)</td>
<td>X</td>
</tr>
<tr>
<td>(-1, -2)</td>
<td>X</td>
</tr>
<tr>
<td>(1, 2)</td>
<td>X</td>
</tr>
<tr>
<td>(3, 0)</td>
<td>X</td>
</tr>
</tbody>
</table>

10. Graph the lines that represent the boundaries of the system of linear inequalities.

\[ 2x - y \geq -10 \]
\[ x + 3y \leq 2 \]
d. Determine if each ordered pair is a solution to the system of inequalities by placing an X in each row of the table.

<table>
<thead>
<tr>
<th>Ordered Pair</th>
<th>Solution?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>X</td>
</tr>
<tr>
<td>(-4, 2)</td>
<td>X</td>
</tr>
<tr>
<td>(-1, 1)</td>
<td>X</td>
</tr>
<tr>
<td>(0, 1)</td>
<td>X</td>
</tr>
</tbody>
</table>

Claim 2 Problems (More Difficult Perhaps)

11. The table below shows multiple inputs and outputs for a continuous function $f$. There is exactly one solution to the equation $f(x) = g(x)$, where $g(x) = 2e^x$. Identify consecutive integers on the number line below to indicate in which consecutive integer interval the solution $f(x) = g(x)$ must lie.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>20</td>
</tr>
<tr>
<td>-3</td>
<td>16</td>
</tr>
<tr>
<td>-2</td>
<td>12</td>
</tr>
<tr>
<td>-1</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-4</td>
</tr>
<tr>
<td>3</td>
<td>-8</td>
</tr>
<tr>
<td>4</td>
<td>-16</td>
</tr>
</tbody>
</table>
The Families of Functions covered throughout Algebra I and II include:

a. Linear
b. Quadratic
c. Polynomial
d. Rational
e. Exponential
f. Square Root
g. Cube Root
h. Absolute Value

1. Determine to which family each function graphed below belongs. Note: In the case of exponential functions the graph may be a translation of an exponential function.

- **A**: Quadratic (and/or polynomial)
- **B**: Polynomial
- **C**: Exponential
- **D**: Rational
2. Determine to which family each function graphed below belongs. Note: In the case of exponential functions the graph may be a translation of an exponential function.

Square root

Exponential

Absolute Value (of a linear function)

Cube Root

3. Determine to which family each function below belongs by placing an X in each row of the table below.

<table>
<thead>
<tr>
<th>Function</th>
<th>Linear</th>
<th>Quadratic</th>
<th>Polynomial</th>
<th>Rational</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = x(x-1)^2 + 1 )</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = 3x - (2x+1) - (5-x) )</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = 3 \cdot 2^x )</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>( f(x) = 2x + x(x+1) )</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = 2x + \frac{1}{x-1} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>( f(x) = \frac{x(x-1)^2}{3} )</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4. Determine whether each of the following statements must be true, and place an X in the appropriate cell for each row.

<table>
<thead>
<tr>
<th>Statement</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>If ( f ) is a cubic polynomial and ( f(100) ) is positive, then ( f(1000) ) must be positive and ( f(1000) &gt; f(100) )</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Suppose ( f ) is a polynomial with a positive leading coefficient and ( g ) is an exponential growth function. Then there exists an ( x ) such that ( g(x) &gt; f(x) ).</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Suppose ( f ) is a polynomial with a positive leading coefficient and ( g ) is an exponential growth function. Then there exists an ( x ) such that ( f(x) &gt; g(x) ).</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Suppose ( f ) is a quadratic function, ( f ) has a minimum value, ( f(100) ) is greater than 10,000, and ( g ) is a 3(^{\text{rd}}) degree polynomial with positive leading coefficient. Then there is an ( x ) such that ( f(x) &lt; g(x) ).</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Suppose ( f ) is a 4(^{\text{th}}) degree polynomial with negative leading coefficient and ( g ) is an exponential decay function. Then there is an ( x ) such that ( g(x) &lt; f(x) ).</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>If ( f ) is a 4(^{\text{th}}) degree polynomial and ( g ) is a 6(^{\text{th}}) degree polynomial then there is an ( x ) such that ( g(x) &gt; f(x) ).</td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

5. For each false statement in the table in problem 4, provide a counter example.

The point of these problems is that exponential functions eventually grow faster than polynomial functions, polynomials with larger degree eventually grow faster than those with smaller degree, and the sign of the leading coefficient matters.

**The First Statement is false.** Define \( f(x) = -(x - 100)^3 + 1 \)

This statement would be false even if we included as part of the statement that \( f \) has a positive leading coefficient. Define \( f(x) = x(x - 100.00001)(x - 1000) \). If you graph this you will see that \( f(100) \) is just a little greater than 0, and clearly \( f(1000) \) is 0, which is smaller.

**Statement 3 is false.** Define \( f(x) = x^3 \) and \( g(x) = 100 \cdot 10^x \)

**Statement 5 is false.** Define \( f(x) = -x^4 \) and \( g(x) = .5^x \). \( f(x) \) is always less than or equal to zero, and \( g(x) \) is always positive.

**Statement 6 is false.** Define \( f(x) = x^4 \) and \( g(x) = -x^6 \). In this case \( f(x) \) is always non-negative and \( g(x) \) is always non-positive. This statement would be true if we required the leading coefficients to be positive.