Peak-to-Average-Power Ratio Reduction

Average Power for an OFDM waveform

The OFDM waveform for N subcarriers is given by

$$s(t)=\sum_{i=0}^{N-1}\left\{A_i\cos(\omega_i t)+B_i\sin(\omega_i t)\right\} \tag{1}$$

The average power (or variance of the waveform) is given by

$$E[s(t)^2]=E\left(\sum_{i=0}^{N-1}\left\{A_i\cos(\omega_i t)+B_i\sin(\omega_i t)\right\}\sum_{i=0}^{N-1}\left\{A_i\cos(\omega_i t)+B_i\sin(\omega_i t)\right\}\right) \tag{2}$$

The variables A_i and B_i can be considered as random and uncorrelated defined according to the QAM constellation order. For 4QAM both A_i and B_i can be considered as equally probable of being +/-1.0. Thus

$$\begin{aligned}&E[A_iA_j]=0 \quad and \quad E[B_iB_j]=0 \quad for \quad i\neq j\\ &E[A_i^2]=1 \quad and \quad E[B_i^2]=1\\ &E[A_iB_j]=0\end{aligned} \tag{3}$$

Because $E[A_iB_j]=0$, Eq (2) becomes

$$E[s(t)^2]=E\left(\sum_{i=0}^{N-1}A_i\cos(\omega_i t)\sum_{i=0}^{N-1}A_i\cos(\omega_i t)+\sum_{i=0}^{N-1}B_i\sin(\omega_i t)\sum_{i=0}^{N-1}B_i\sin(\omega_i t)\right) \tag{4}$$

Using (3) above

$$E[s(t)^2]=E\left(\sum_{i=0}^{N-1}\cos^2(\omega_i t)+\sum_{i=0}^{N-1}\sin^2(\omega_i t)\right) \tag{5}$$

$$E[s(t)^2]=\sum_{i=0}^{N-1}E\left(\cos^2(\omega_i t)\right)+\sum_{i=0}^{N-1}E\left(\sin^2(\omega_i t)\right) \tag{6}$$

Both $\cos^2(\omega_i t)$ and $\sin^2(\omega_i t)$ vary between 0 and 1.0, the average value for both will be 0.5. Thus

$$E[s(t)^2]=\sum_{i=0}^{N-1}\frac{1}{2}+\sum_{i=0}^{N-1}\frac{1}{2} \tag{7}$$

$$E[s(t)^2]=N \tag{8}$$

Because a large number of random values are summed to get the variance of the multicarrier waveforms, the resulting waveform samples will be normally distributed – based upon the central limit theorem[1]. Eq (8) above shows that the standard deviation of the waveform will be $\sqrt{N}$. For 802.11a the number of subcarriers *N=64* so that the standard deviation of the waveform will be 8. Therefore for 802.11a, the probably of a signal's absolute value being greater than 1-, 2-, 3-sigma will be 68%, 95%, 97.2% respectively[2].

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[1] https://en.wikipedia.org/wiki/Central_limit_theorem

[2] https://en.wikipedia.org/wiki/68%E2%80%9395%E2%80%9399.7_rule