E401: Advanced Communication Theory

Professor A. Manikas
Chair of Communications and Array Processing

Imperial College London

Multi-Antenna Wireless Communications

Array Receivers for SIMO and MIMO

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General Objective

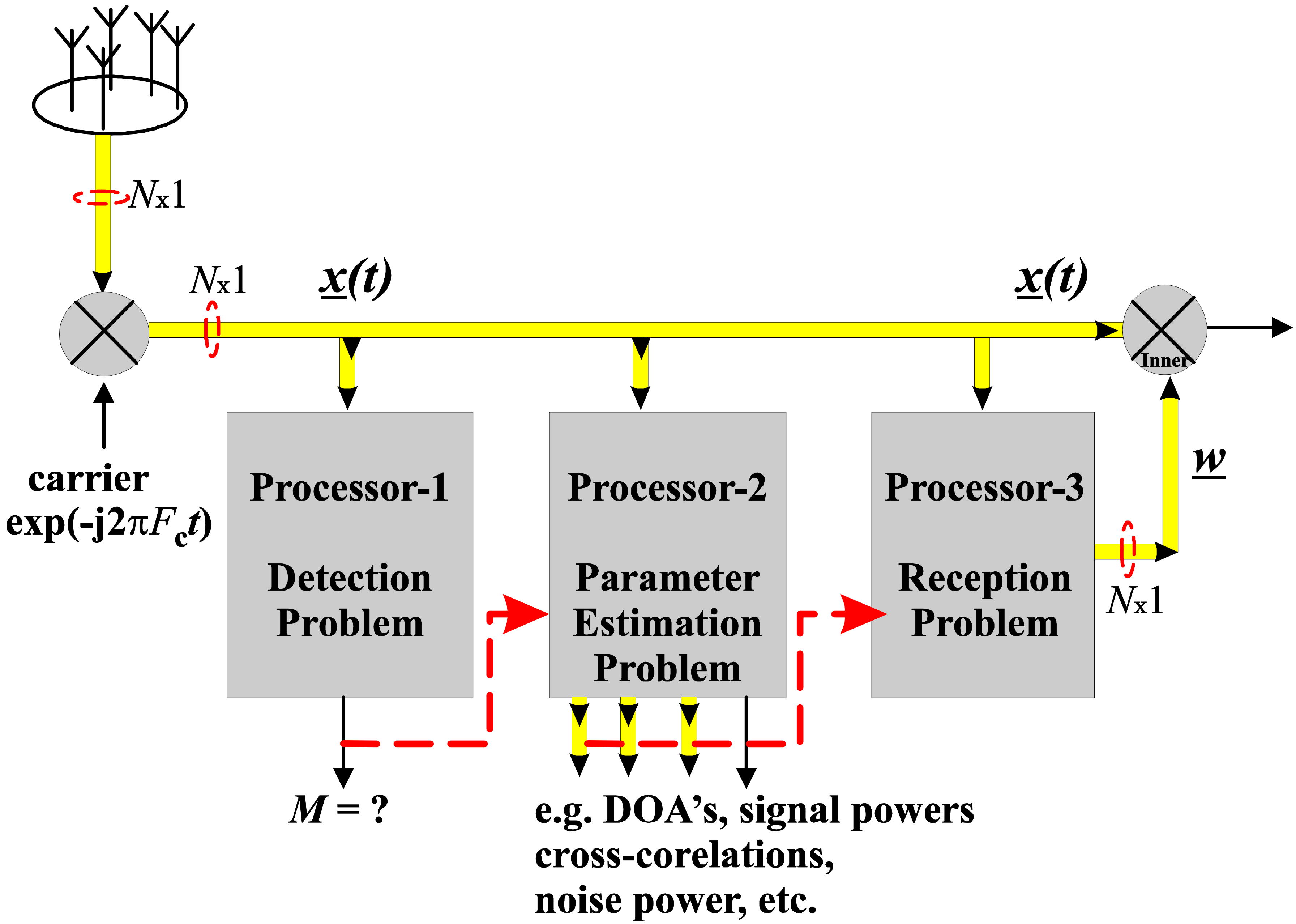
- By observing a vector-signal $\underline{x}(t) = \mathbb{S}\underline{m}(t)+\underline{n}(t)$ using an array system the aim is to obtain information about a signal environment.
- There are three general problems to solve.

1. The Detection problem: $M=?$
   (i.e. to detect the presence of M **co-channel** emitting sources)

2. The Estimation problem:
   to estimate various signal and channel parameters
   e.g. DOAs$=?\forall i$; $P_{m_i} = \mathcal{E}\{m_i^2(t)\} = ?\forall i$; $P_n = \sigma_n^2 = ?$;
   $\rho_{ij} = \mathcal{E}\{m_i(t)m_j^*(t)\} = ?\forall i,j$, with $i \neq j$
   polarization parameters, fading coefficients, signal spread.

3. The Reception problem:
   to receive one signal (desired signal) and suppress the remaining $M-1$ as unwanted cochannel interference

- These problems are highlighted in the following block structure:



General Problem Formulation M<N

- Consiner an observed $(N \times 1)$ complex signal-vector $\underline{x}(t)$ that is modelled as follows

$$\underline{x}(t) \triangleq \overbrace{\mathbb{S}(\underline{p})}^{N\times M} \cdot \overbrace{\underline{m}(t)}^{M\times 1} + \overbrace{\underline{n}(t)}^{N\times 1} \quad (1)$$

Note that by observing $\underline{x}(t)$, its 2nd order statistics become known, i.e. the covariance matrix $\mathbb{R}_{xx}$ is known, where

$$\mathbb{R}_{xx} = \mathcal{E}\{\underline{x}(t).\underline{x}(t)^H\} \quad (2)$$

- Estimate $M, p_1, p_2, ..., p_M, \mathbb{R}_{mm}, \sigma_n^2$, etc.

where
- $\mathbb{S} \triangleq \mathbb{S}(\underline{p}) = \left[S(p_1), \quad S(p_2), \quad ..., \quad S(p_M) \right]$ - (unknown)
- $\underline{m}(t)$: message signal-vector - (unknown)
- $\mathbb{R}_{mm}$: 2nd order statistics of $\underline{m}(t)$ - (unknown)
- $\underline{n}(t)$: AWGN vector - (power σ_n^2 unknown)

with
- $\underline{p} =$ the vector of generic (unknown) parameters $p_1, p_2, ..., p_M$
- $N =$ **known** (this is a system parameter)
- $M =$ **unknown** (this is a channel parameter - number of signals) with $M < N$ (later this condition will be removed)

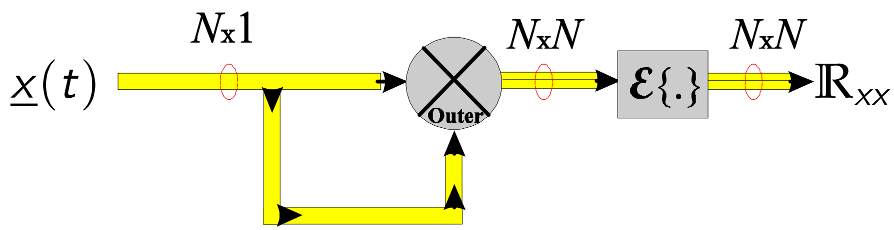
Array Covariance Matrix

Theoretical Covariance Matrix

- If the $(N \times 1)$ vector-signal $\underline{x}(t) = \mathbb{S}\underline{m}(t) + \underline{n}(t)$ is observed over infinite observation interval then its 2nd order statistics can be calculated. These are given by the theoretical covariance matrix $\mathbb{R}_{xx}$ which is an $(N \times N)$ complex matrix - always Hermitian. That is,

$$\begin{aligned}
\mathbb{R}_{xx} &\triangleq \mathcal{E}\left\{\underline{x}(t)\underline{x}(t)^H\right\} \qquad (3)\\
&= \begin{bmatrix} \mathcal{E}\{x_1(t)x_1(t)^*\}, & \mathcal{E}\{x_1(t)x_2(t)^*\}, & ..., & \mathcal{E}\{x_1(t)x_N(t)^*\} \\ \mathcal{E}\{x_2(t)x_1(t)^*\}, & \mathcal{E}\{x_2(t)x_2(t)^*\}, & ..., & \mathcal{E}\{x_2(t)x_N(t)^*\} \\ ..., & ..., & ..., & ... \\ \mathcal{E}\{x_N(t)x_1(t)^*\}, & \mathcal{E}\{x_N(t)x_2(t)^*\}, & ..., & \mathcal{E}\{x_N(t)x_N(t)^*\} \end{bmatrix}\\
&= \mathcal{E}\left\{(\mathbb{S}\underline{m}(t)+\underline{n}(t)).(\mathbb{S}\underline{m}(t)+\underline{n}(t))^H\right\}\\
&= \mathcal{E}\left\{\mathbb{S}\underline{m}(t)\underline{m}(t)^H\mathbb{S}^H + \underline{n}(t)\underline{n}(t)^H + \mathbb{S}\underline{m}(t)\underline{n}(t)^H + \underline{n}(t)\underline{m}(t)^H\mathbb{S}^H\right\}\\
&= \mathbb{S}.\underbrace{\mathcal{E}\left\{\underline{m}(t)\underline{m}(t)^H\right\}}_{\triangleq\mathbb{R}_{mm}}.\mathbb{S}^H + \underbrace{\mathcal{E}\left\{\underline{n}(t)\underline{n}(t)^H\right\}}_{\triangleq\mathbb{R}_{nn}} + \mathbb{S}\underbrace{\mathcal{E}\left\{\underline{m}(t)\underline{n}(t)^H\right\}}_{=\mathbb{O}_{M,N}} + \underbrace{\mathcal{E}\left\{\underline{n}(t)\underline{m}(t)^H\right\}}\mathbb{S}^H\\
&= \mathbb{S}.\mathbb{R}_{mm}.\mathbb{S}^H + \mathbb{R}_{nn} \qquad (4)
\end{aligned}$$

- i.e.



$$\mathbb{R}_{xx} \triangleq \mathcal{E}\left\{\underline{x}(t)\underline{x}(t)^H\right\} = \mathbb{S}.\mathbb{R}_{mm}.\mathbb{S}^H + \mathbb{R}_{nn}$$

where

$$\mathbb{R}_{mm} \triangleq \mathcal{E}\left\{\underline{m}(t).\underline{m}(t)^H\right\} = \text{2nd order statistics of } \underline{m}(t) \ \text{(unknown)}$$

$$= \begin{bmatrix} \underbrace{\mathcal{E}\{m_1(t).m_1(t)^*\}}_{\mathcal{E}\{m_1(t)^2\}=P_1}, & \mathcal{E}\{m_1(t).m_2(t)^*\}, & \ldots, & \mathcal{E}\{m_1(t).m_M(t)^*\} \\ \mathcal{E}\{m_2(t).m_1(t)^*\}, & \underbrace{\mathcal{E}\{m_2(t).m_2(t)^*\}}_{\mathcal{E}\{m_2(t)^2\}=P_2}, & \ldots, & \mathcal{E}\{m_2(t).m_M(t)^*\} \\ \vdots & \vdots & \vdots, & \vdots \\ \mathcal{E}\{m_M(t).m_1(t)^*\}, & \mathcal{E}\{m_M(t).m_2(t)^*\}, & \ldots, & \underbrace{\mathcal{E}\{m_M(t).m_M(t)^*\}}_{\mathcal{E}\{m_M(t)^2\}=P_M} \end{bmatrix}$$

$$= \text{an } (M \times M) \text{ complex matrix (always Hermitian) - unknown}$$

$$
\begin{aligned}
\mathbb{R}_{nn} &\triangleq \mathcal{E}\left\{\underline{n}(t).\underline{n}(t)^H\right\} \text{ is 2nd order statistics of } \underline{n}(t) \\
&= \begin{bmatrix}
\underbrace{\mathcal{E}\{n_1(t).n_1(t)^*\}}_{\mathcal{E}\{n_1(t)^2\}=P_{n_1}}, & \underbrace{\mathcal{E}\{n_1(t).n_2(t)^*\}}_{0}, & ..., & \underbrace{\mathcal{E}\{n_1(t).n_N(t)^*\}}_{0} \\
\underbrace{\mathcal{E}\{n_2(t).n_1(t)^*\}}_{0}, & \underbrace{\mathcal{E}\{n_2(t).n_2(t)^*\}}_{\mathcal{E}\{n_2(t)^2\}=P_{n_2}}, & ..., & \underbrace{\mathcal{E}\{n_2(t).n_N(t)^*\}}_{0} \\
..., & ..., & ..., & ... \\
\underbrace{\mathcal{E}\{n_N(t).n_1(t)^*\}}_{0}, & \underbrace{\mathcal{E}\{n_N(t).n_2(t)^*\}}_{0}, & ..., & \underbrace{\mathcal{E}\{n_N(t).n_N(t)^*\}}_{\mathcal{E}\{n_N(t)^2\}=P_{n_N}}
\end{bmatrix} \\
&= \sigma_n^2 \mathbb{I}_N \qquad (5) \\
&= \text{an } (N \times N) \text{ complex matrix (always Hermitian) - unknown}
\end{aligned}
$$

- Note that, because we have assumed isotropic AWGN noise,

$$
P_{n_1} = P_{n_2} = ... = P_{n_N} = \sigma_n^2 \qquad (6)
$$

Practical Covariance Matrix

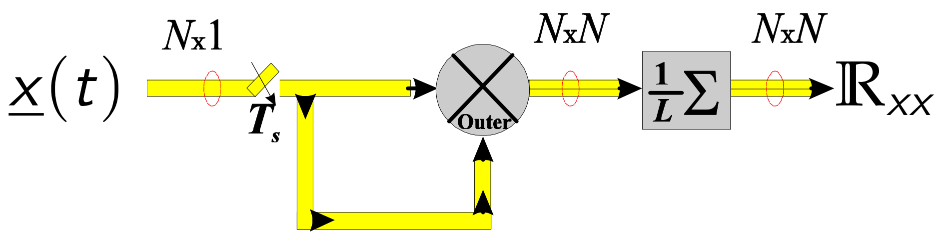
- Consider that the signal $\underline{x}(t) = \mathbb{S}\underline{m}(t) + \underline{n}(t)$ is observed over finite observation interval equivalent to L snapshots.
- These L observations (snapshots) at times $t_1, t_2, \ldots, t_L$ (i.e. finite observation interval) are denoted as $[\underline{x}(t_1), \underline{x}(t_2), \ldots, \underline{x}(t_L)]$ and represented by the $N \times L$ complex matrix $\mathbb{X}$
  i.e.

$$\begin{aligned} \mathbb{X} &\triangleq [\underline{x}(t_1), \underline{x}(t_2), \ldots, \underline{x}(t_L)] && (7a) \\ &= [\mathbb{S}.\underline{m}(t_1) + \underline{n}(t_1), \quad \mathbb{S}.\underline{m}(t_2) + \underline{n}(t_2), \quad \ldots, \quad \mathbb{S}.\underline{m}(t_L) + \underline{n}(t_L)] \\ &= \mathbb{S}.\mathbb{M} + \mathbb{N} && (7b) \end{aligned}$$

$$\textit{with} \begin{cases} \mathbb{S} = [\underline{S}_1, \underline{S}_2, \ldots, \underline{S}_M] & (N \times M) \\ \mathbb{M} = [\underline{m}(t_1), \underline{m}(t_2), \ldots, \underline{m}(t_L)] & (M \times L) \\ \mathbb{N} = [\underline{n}(t_1), \underline{n}(t_2), \ldots, \underline{n}(t_L)] & (N \times L) \end{cases} \quad (8)$$

where the matrices $\mathbb{S}$, $\mathbb{M}$ and $\mathbb{N}$ (as well as the dimension M) are unknown

- In this case the 2nd order statistics of $\underline{x}(t)$ are estimated by the practical covariance matrix $\mathbb{R}_{xx}$
- Practical Model:



$$\mathbb{R}_{xx} = \frac{1}{L}\sum_{l=1}^{L} \underline{x}(t_l)\underline{x}(t_l)^H \quad (9)$$

$$\text{i.e. } \mathbb{R}_{xx} = \frac{1}{L}\mathbb{X}\mathbb{X}^H = \mathbb{S}\underbrace{\frac{1}{L}\mathbb{M}\mathbb{M}^H}_{=\mathbb{R}_{mm}}\mathbb{S}^H + \underbrace{\frac{1}{L}\mathbb{N}\mathbb{N}^H}_{=\mathbb{R}_{nn}}$$

$$= \mathbb{S}.\mathbb{R}_{mm}.\mathbb{S}^H + \mathbb{R}_{nn} \quad (10)$$

N.B.:

- In an array system the matrix $\mathbb{R}_{xx}$ (theoretical or practical) contains all the **geometrical and other information** about the various sources relative to the array.
- Remember that, sometimes, we will use $\widehat{\mathbb{R}}_{xx}$ to denote the practical/estimated covariance matrix of the vector signal $\underline{x}(t)$.

Generating L Snapshots having a given Covariance Matrix

- To generate L snapshots of $\underline{x}(t)$ having a predefined covariance matrix $\mathbb{R}_{xx}$ the vectors $\underline{x}(t_l)$ for $l = 1, 2, .., L$ should be generated using the following expression

$$\underline{x}(t_l) = \mathbb{E}\,\mathbb{D}^{\frac{1}{2}}\,\underline{z}(t_l) \tag{11}$$

where

  - $\mathbb{E}$ and $\mathbb{D}$ are the eigenvector-matrix and eigenvalue-matrix of $\mathbb{R}_{xx}$, and
  - $\underline{z}(t_l) \in \mathcal{C}^N$ is a Gaussian random complex vector of N elements of zero mean and variance 1, i.e.

$$\mathcal{E}\{\underline{z}(t_l).\underline{z}(t_l)^H\} = \mathbb{I}_N \tag{12}$$

- That is,

$$\begin{aligned} \mathbb{X} &= \left[\underline{x}(t_1),\ \ \underline{x}(t_2),\ \ \ldots,\ \ \underline{x}(t_L)\right] \\ &= \left[\mathbb{E}\,\mathbb{D}^{\frac{1}{2}}\underline{z}(t_1),\ \ \mathbb{E}\,\mathbb{D}^{\frac{1}{2}}\underline{z}(t_2),\ \ \ldots,\ \ \mathbb{E}\,\mathbb{D}^{\frac{1}{2}}\underline{z}(t_L)\right] \end{aligned} \tag{13}$$