Phase Locked Loops

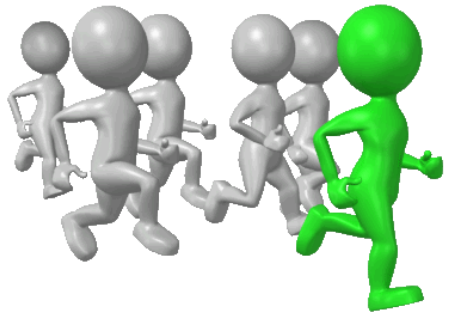
fred harris

Aligning Phase of Remote Oscillators (in the Presence of Additive Noise)

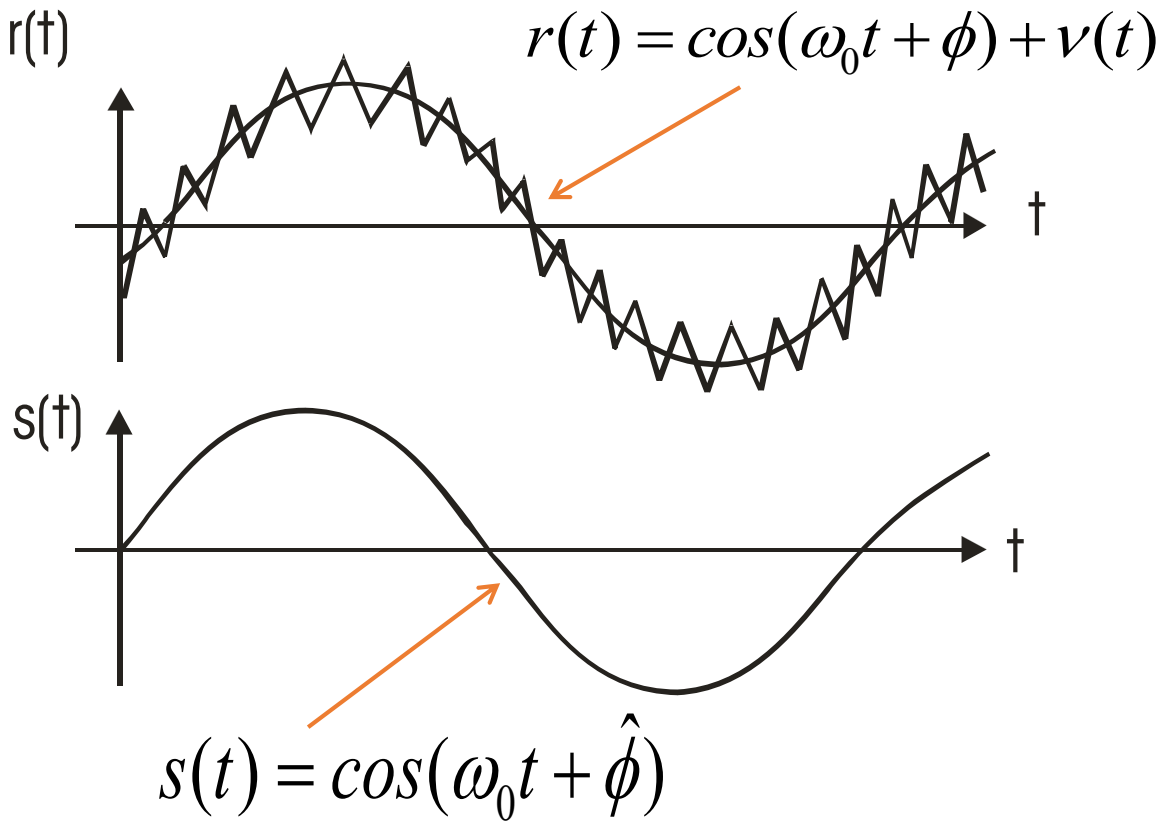
(Noise Makes the World More Interesting)

Phase Locked Loops (PLL)

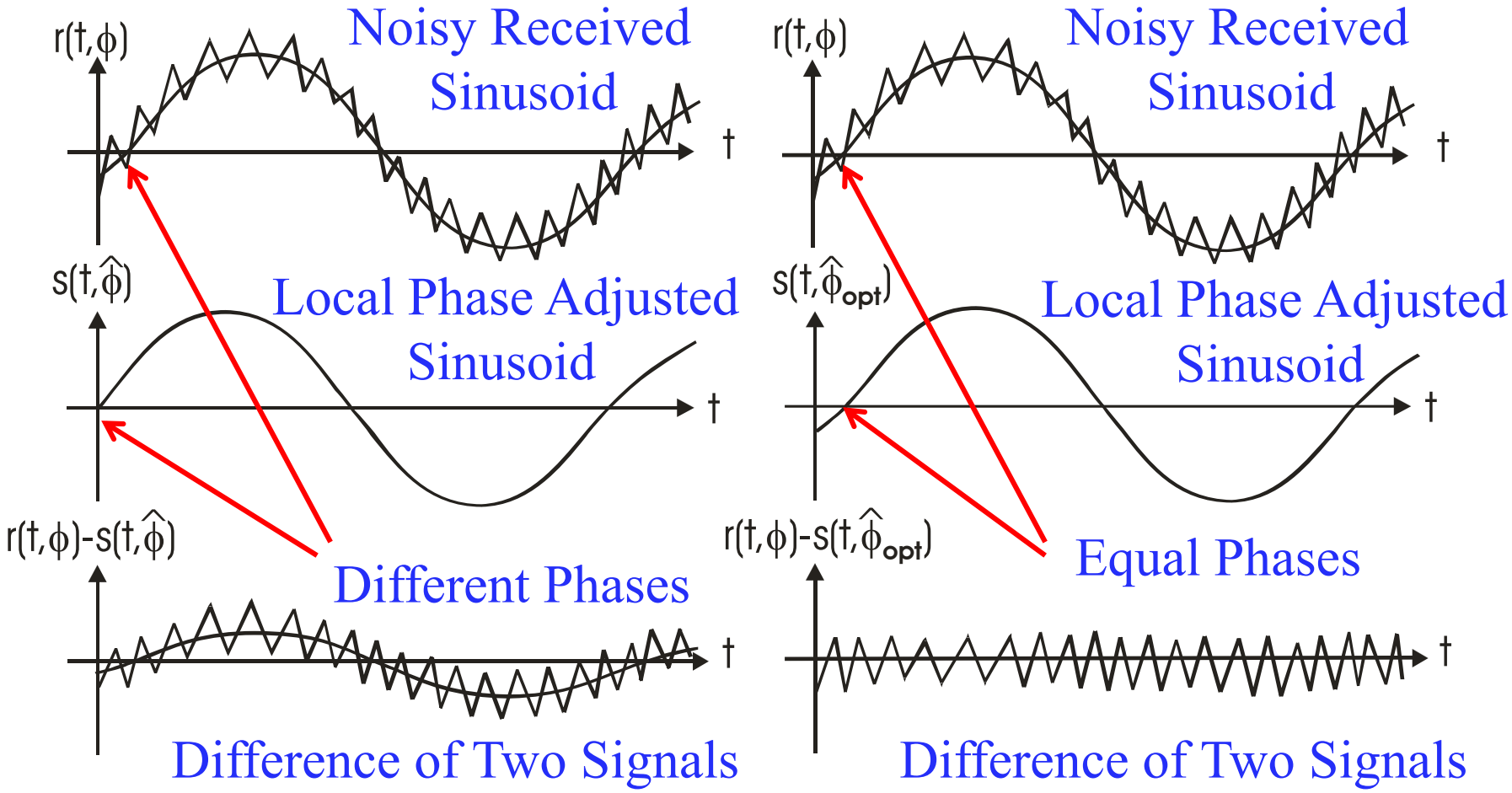
We now present the quick, back of the envelope, derivation of a phase locked loop. Hold on to your hats!



Align Phase of Local Oscillator with Phase of Received Noisy Sinusoid



Phase Aligned Sinusoids Have Smallest Difference



Minimize Energy in Difference

$$e(t) = r(t,\phi) - s(t,\hat{\phi})$$

$$e^2(t) = [r(t,\phi) - s(t,\hat{\phi})]^2$$

When phase aligned difference is noise only

$$\underset{wrt\ \hat{\phi}}{Min} \int_{T_0} [r(t,\phi) - s(t,\hat{\phi})]^2 dt$$

$$\underset{wrt\ \hat{\phi}}{Min} \left\{ \int_{T_0} r^2(t,\phi)dt + \int_{T_0} s^2(t,\hat{\phi})dt - 2\int_{T_0} r(t,\phi)\, s(t,\hat{\phi})dt \right\}$$

First two integrals are not functions of ϕ

$$\underset{wrt\ \hat{\phi}}{Max} \int_{T_0} r(t,\phi)\, s(t,\hat{\phi})dt$$

Minimizing Error is Same as
Maximizing Output of Correlator!

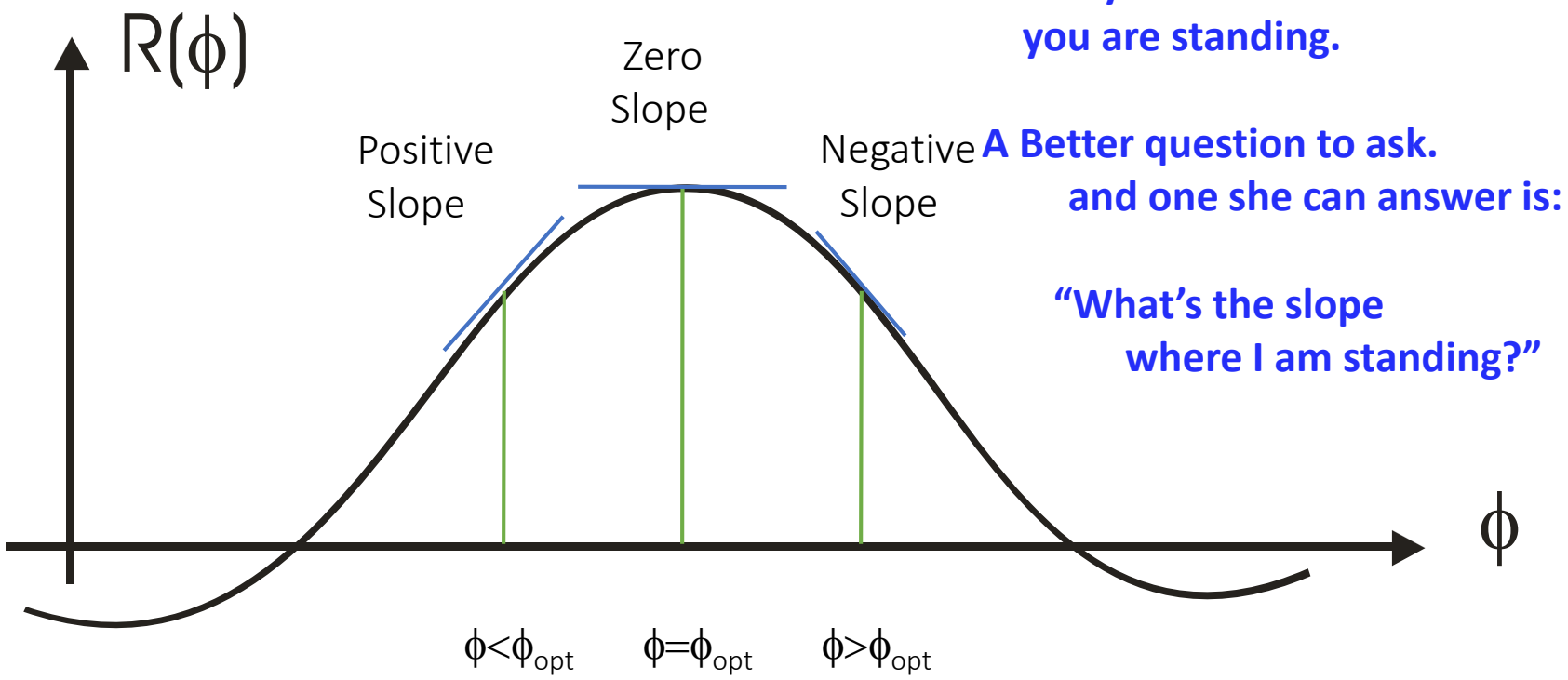
Find Peak of Correlation Function

Filter can't answer the question:
"Am I standing at the highest level?"

She can only see the level where you are standing.

A Better question to ask. and one she can answer is:

"What's the slope where I am standing?"



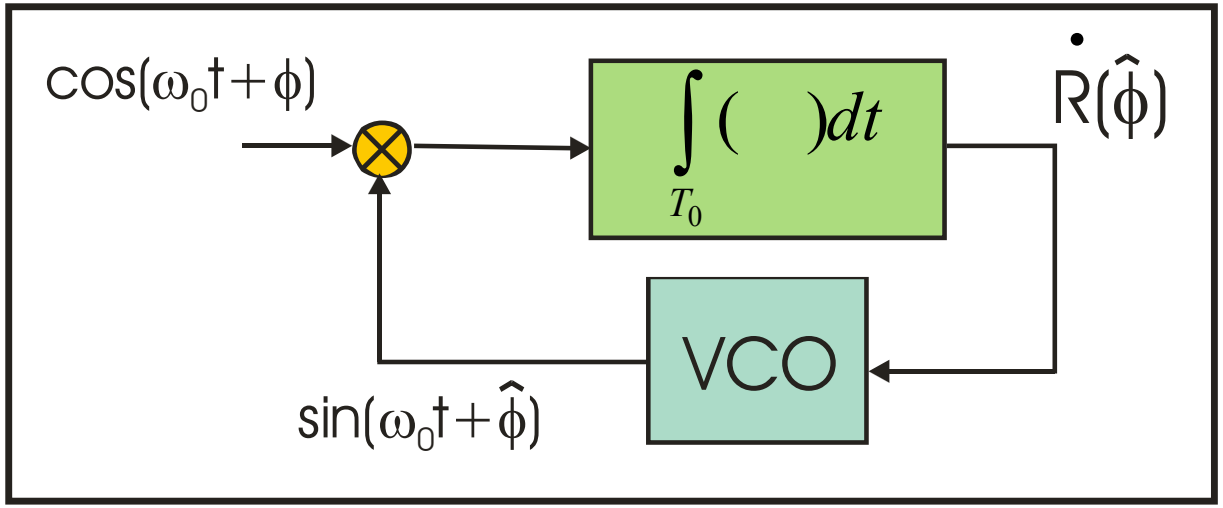
Phase Lock Loop Feedback Drives Phase Estimate to Peak of Correlation Function by Monitoring the Phase Slope.

To Locate Correlator Peak, Determine Where Derivative With Respect to Phase Is equal to Zero.

$$R(\hat{\phi}) = \int_{T_0} r(t,\phi)s(t,\hat{\phi})dt$$

$$= \int_{T_0} \cos(\omega_0 t + \phi)\cos(\omega_0 t + \hat{\phi})dt$$

$$\frac{d}{d\hat{\phi}}R(\hat{\phi}) = \int_{T_0} \cos(\omega_0 t + \phi)\sin(\omega_0 t + \hat{\phi})dt \triangleq$$

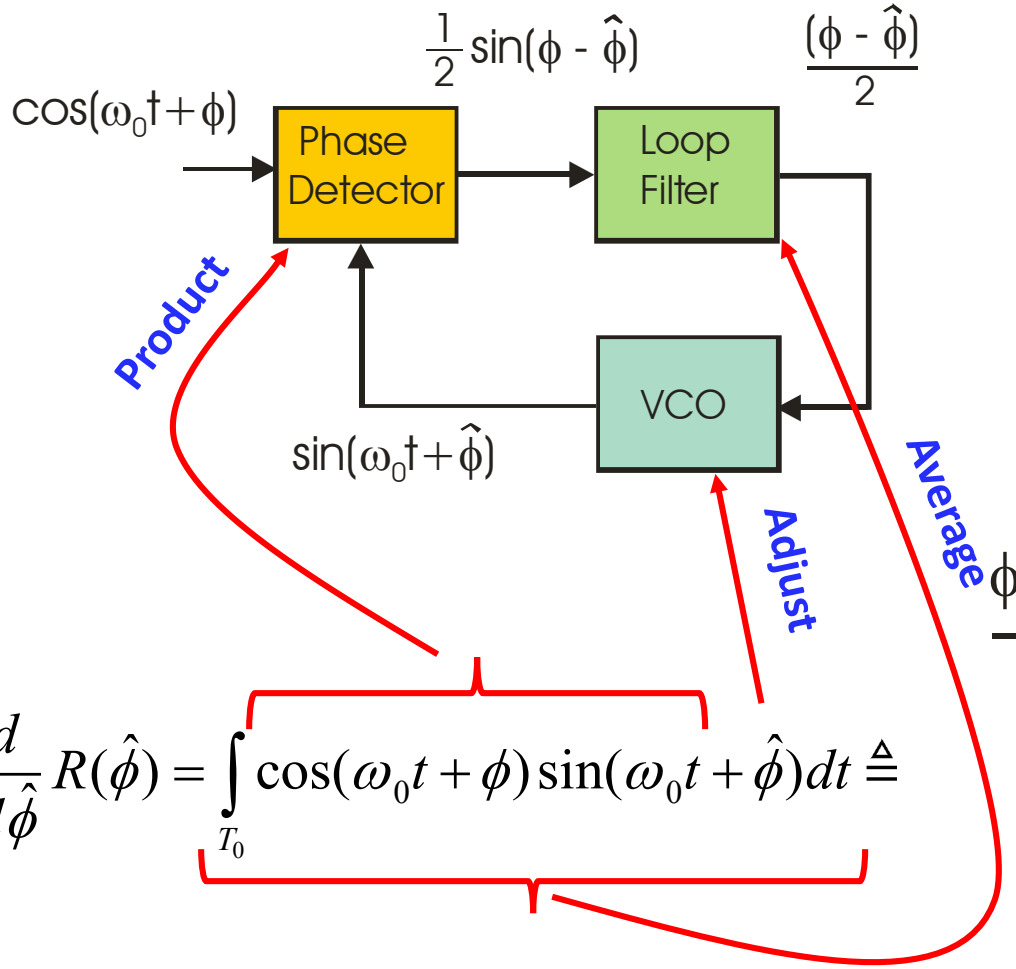


Common to All Iterative Estimators of Parameter λ. Drive to Zero The Average Value of Product

$$y(t,\lambda) \cdot \frac{d}{d\lambda} y(t,\lambda)$$

The Important Information is, not where are you, but rather where are you going!

Three Basic Components of PLL



Small Signal Model