A VERSATILE PARAMETRIC FILTER USING AN IMBEDDED ALL-PASS SUB-FILTER TO INDEPENDENTLY ADJUST BANDWIDTH, CENTER FREQUENCY, AND BOOST OR CUT

fred harris
Electrical and Computer Engineering
San Diego State University
San Diego, CA 92182-0190

Eric Brooking
PS Systems
Music Technology Products
San Diego, CA 92126

1. ABSTRACT

Parametric filters use separate and non interacting controls to change bandwidth, center frequency, and cut or boost levels of an acoustic signal. These are a generalization of graphic equalizers which permits gain adjustment of a filter bank with fixed bandwidth and center frequencies. This paper describes and demonstrates the performance of a digital realization of a parametric filter.

2. INTRODUCTION

A graphic equalizer is an audio spectrum channelizer implemented as a bank of parallel second order narrowband contiguous filters. The spectral response of such a bank is shown in figure 1.

The audio band is spanned by filters with preset center frequencies and bandwidths exhibiting constant Q characteristics (equal spacing on a logarithmic frequency scale). Each filter has an adjustable gain control which can boost or cut the signal level in its band to tailor the spectral response of an audio playback system.

A generalization of the graphic equalizer is a cascade parametric filter bank. These filters have independent and non interacting controls for changing bandwidth, center frequency, and cut or boost levels over its bandwidth. The spectral response of such a filter is indicated in figure 2.
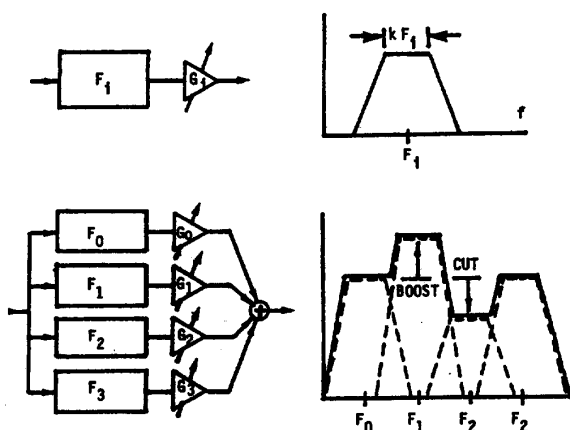

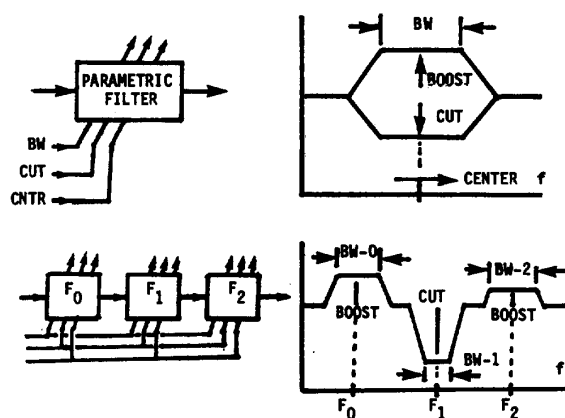FIGURE 1. SPECTRAL RESPONSE, NARROWBAND CHANNELIZER FILTER AND CHANNELIZER EQUALIZER BANK

FIGURE 2. SPECTRAL RESPONSE, ADJUSTABLE PARAMETRIC FILTER AND PARAMETRIC EQUALIZER BANK

Traditional digital recursive filter structures do not uncouple the center frequency and bandwidth of bandpass and bandstop filters nor the cut or boost levels from the selected bandedge frequencies.

The filter presented here manipulates the pole and zero of a prototype first order filter to obtain the desired cut or boost over the selected frequency bandwidth and then uses an embedded adjustable all-pass filter to effect a spectral translation to an arbitrary selected center frequeny.

The center frequency tuning is similar to digital filter structures presented in recent papers by Mitra, Neuvo, and Roivainen [1] and again by Mitra, Hirano, Nishimura, and Sugahara [2].

3. BASEBAND PROTOTYPE FILTER

The form of the baseband prototype filter is shown in figure 3 and the Z-transform of the filter is shown in (1a).

$$F(Z) = K(\beta, e_1, e_2) \frac{Z-(\beta+e_2)}{Z-(\beta+e_1)} \quad (1a)$$

The filter gain is normalized for unity gain at $Z=-1$ by (1b).

$$k(\beta, e_1, e_2) = \frac{1+\beta+e_1}{1+\beta+e_2} \quad (1b)$$
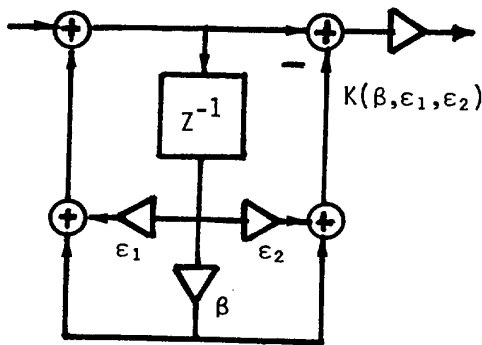

FIGURE 3. BASEBAND PROTOTYPE FILTER

The single coefficient β establishes the initial location of the filter's pole-zero pair. For stability $\beta < 1.0$. This location, termed the filter anchor point, establishes the bandwidth of the filter. Bandwidth will be discussed in more detail shortly. When the parameters $\epsilon_1 = 0$ and $\epsilon_2 = 0$, the pole and zero are both at the anchor point and cancel each other. The zero or the pole can be shifted by changing ϵ_1 or ϵ_2 from zero. The rule for the change is that only one of the pair is permitted to be non zero and the magnitude of the sum $\beta + \epsilon_1$ must be less than unity.

To better visualize the effect on spectral gain of the pole or zero shift we recognize that the filter is the image, via the bilinear Z-transform, of a polezero pair in the complex S-plane. For a specified sampled data bandwidth f_{bw} the image pole location in the S-plane is shown in (3)

$$S_\beta = -\tan(\pi \frac{f_{bw}}{f_s}) \quad (3)$$

for which the parameter β is determined in (4).

$$\beta = \frac{1+S_\beta}{1-S_\beta} = \frac{1-\tan(\pi \frac{f_{bw}}{f_s})}{1+\tan(\pi \frac{f_{bw}}{f_s})} \quad (4)$$

The pole-zero image pair in the S-plane are described by (5).

$$F(S) = \frac{S+S_z}{S+S_p} \quad (5)$$

The DC gain of this image filter is shown in (6).

$$DC\ GAIN = F(S)|_{S=0} = \frac{S_z}{S_p} \quad (6)$$

We recognize immediately that this gain is the boost (if $S_z > S_p$) or cut (if $S_p > S_z$) of the filter.

To design the lowpass prototype with a lowpass boost over a given bandwidth the design sequence proceeds as follows; we use the specified bandwidth to compute S_p from (3), the desired boost to compute S_z from (6) and then the pole and zero locations (β and $\beta + \epsilon_2$ respectively) from (4).

4. BANDWIDTH REVISITED

There is a potential source of confusion when describing the bandwidth of boost and of cut operation in a filter. At this point we resolve the confusion by explaining from where it comes. The standard definition of bandwidth only reflects the position of the pole. Thus, for a boost, bandwidth is defined by the 3-dB points relative to the boost level of the filter while, for a cut, the bandwidth is defined by the 3-dB points relative to the reference level (as opposed to the 3-dB points relative to the cut level). These relationships are shown in figure 4.
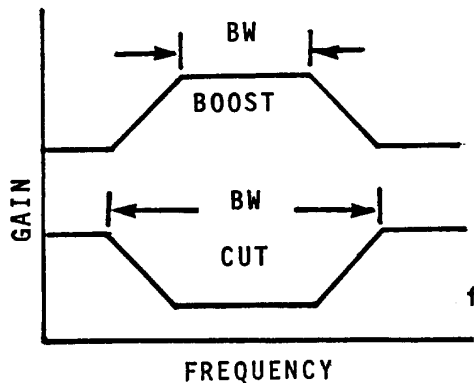
FIGURE 4. BANDWIDTH DEFINITION OF CUT AND BOOST

We have found it useful to describe cut and boost bandwidth in two different forms shown in figure 5. Figure 5a. shows the constant Q case for which both cut and boost are applied over a constant bandwidth. Note that this entails switching what we ask the pole and zero do as we move from boost to cut. We form this relationship

in this fashion; We place the pole-zero pair at the position corresponding to desired bandwidth. For boost, we anchor the pole and move the zero away from the origin ($\epsilon_2 < 0$), and for cut we anchor the zero and move the pole away from the origin ($\epsilon_1 < 0$).
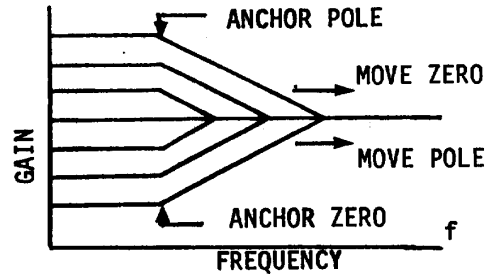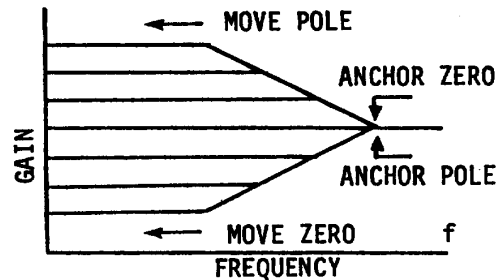
FIGURE 5a. BANDWIDTH OF CONSTANT Q CUT/BOOST

FIGURE 5b. BANDWIDTH OF VARIABLE Q CUT/BOOST

Figure 5b. shows the variable Q case for which the complementary response of the cut and boost have constant bandwidths. To emphasize the difference; here the bandwidth of the region not changing gain is held constant (while previously, the bandwidth of the part that changed gain was held constant). To accomplish this, as previously, we place the pole-zero pair at the position corresponding to the desired bandwidth. For boost, we now anchor the zero and move the pole towards the origin ($\epsilon_1 > 0$), and for cut we anchor the pole and move the zero towards the origin ($\epsilon_2 > 0$).

We use the appropriate type of boost/cut to design a baseband filter with the desired

boost/cut over the specified bandwidth. We complete the design process by translating this spectral behavior to an arbitrary specified center frequency.

5. CENTER FREQUENCY TUNING OF THE PROTOTYPE BOOST OR CUT

A standard frequency domain transformation [3] can convert a lowpass filter to a bandpass filter with the same bandwidth at an arbitrary center frequency f_c. The desired spectral transformation can be realized by the all-pass filter transformation shown in (6).

$$Z^{-1} \Rightarrow -Z^{-1}\frac{1-Zc}{Z-c}, \quad c = \cos(2\pi\frac{f_c}{f_s}) \quad (6)$$

The all-pass filter required to implement (6) can be realized a number of ways [1,2,4] but the most efficient appears to be that shown in figure 6. This form of the all-pass structure implements the extra delay and negative sign required in (6).
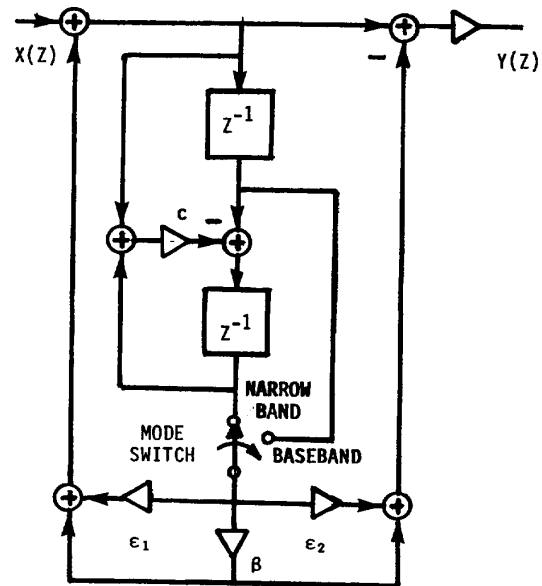
FIGURE 6. LOWPASS TO BANDPASS TRANSFORMATION

The final form of the parametric filter is shown in figure 7. Here bandwidth is set by β, cut/ boost by ϵ_1 and ϵ_2, and center frequency by c. The mode switch shown in figure 7 permits the filter to be used as a baseband or as a band centered filter.

Figure 8 shows the frequency response of the parametric filter for a range of parameters. Figures 8a and 8b demonstrate adjustable center frequency and bandwidth

respectively for the variable Q case. Figures 8c and 8d demonstrates cut and boost for variable Q and constant Q.

FIGURE 7. STRUCTURE OF PARAMETRIC FILTER

FIGURE 8a. VARIABLE CENTER FREQUENCY (VAR. Q)

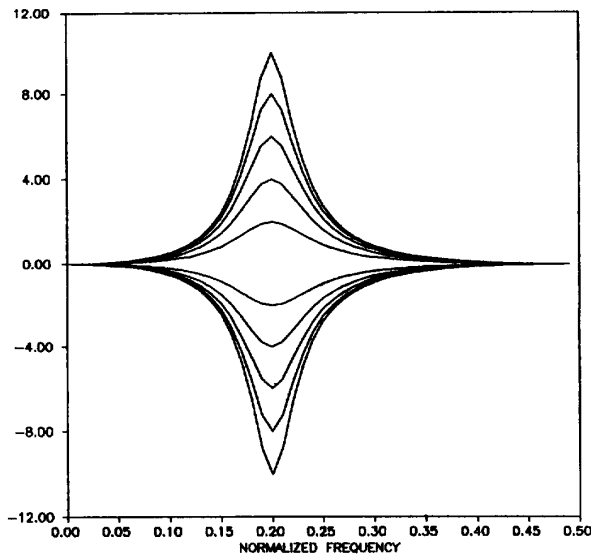FIGURE 8b. VARIABLE BANDWIDTH (VAR. Q)
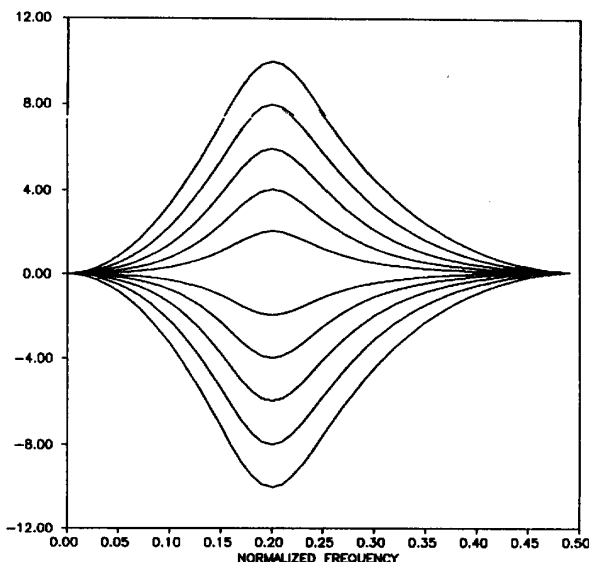
272

FIGURE 8c. VARIABLE BOOST/CUT (VAR. Q)

FIGURE 8d. VARIABLE BOOST/CUT (CONST. Q)

6. CONCLUSIONS

We have presented the design of a versatile parametric digital filter. Design equations are given to obtain specified cut/boost over a desired bandwidth. Part of this presentation addressed the definition of bandwidth for cut and boost filters and we have chosen to describe the bandwidth options in terms of constant or variable 3-dB widths relative to cut/boost level.

7. ACKNOWLEDGEMENT

5. BIBLIOGRAPHY

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