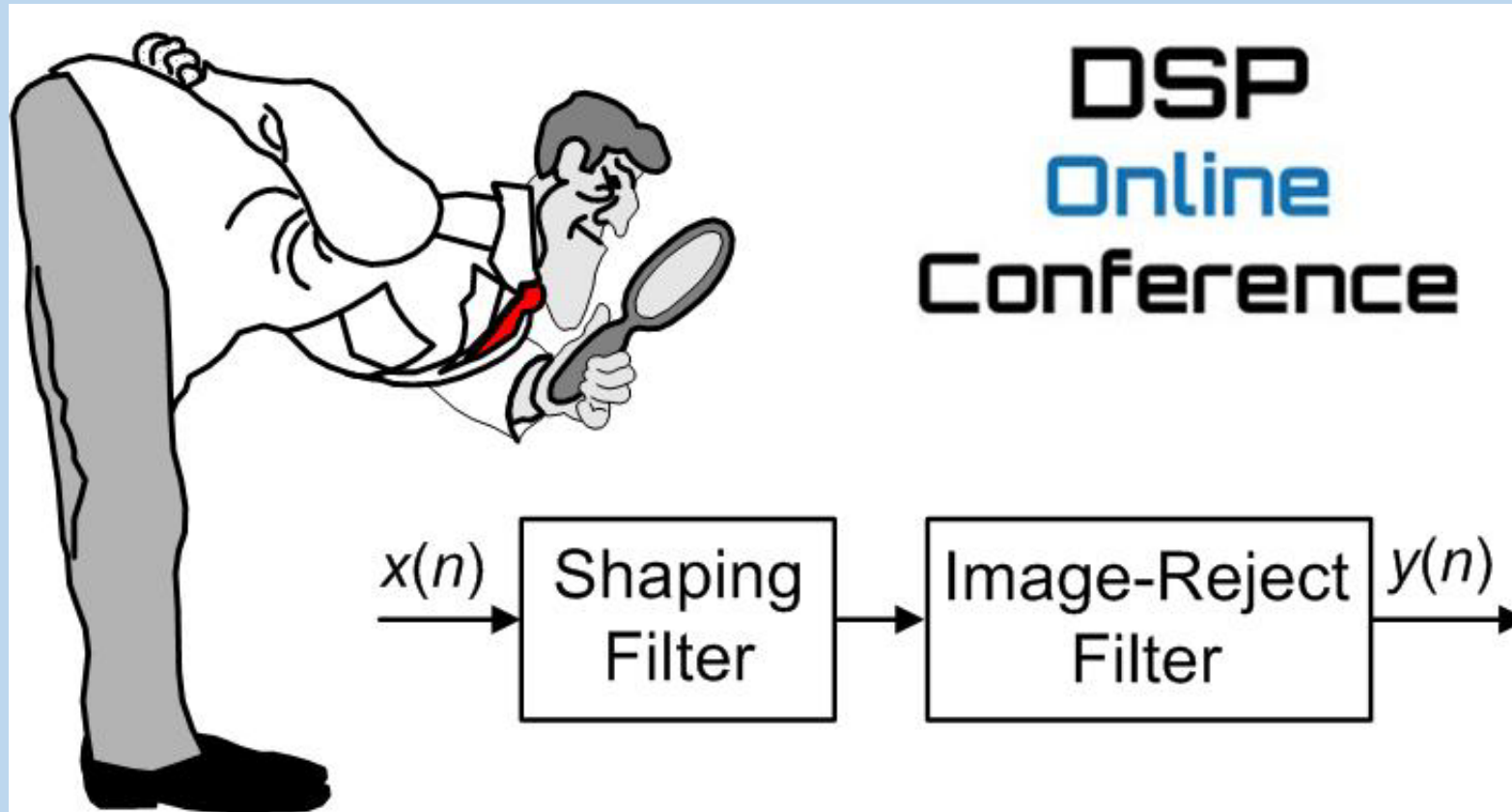


Interpolated FIR (IFIR) Filters

by Richard (Rick) Lyons [Oct., 2024]



Interpolated FIR (IFIR) Filters

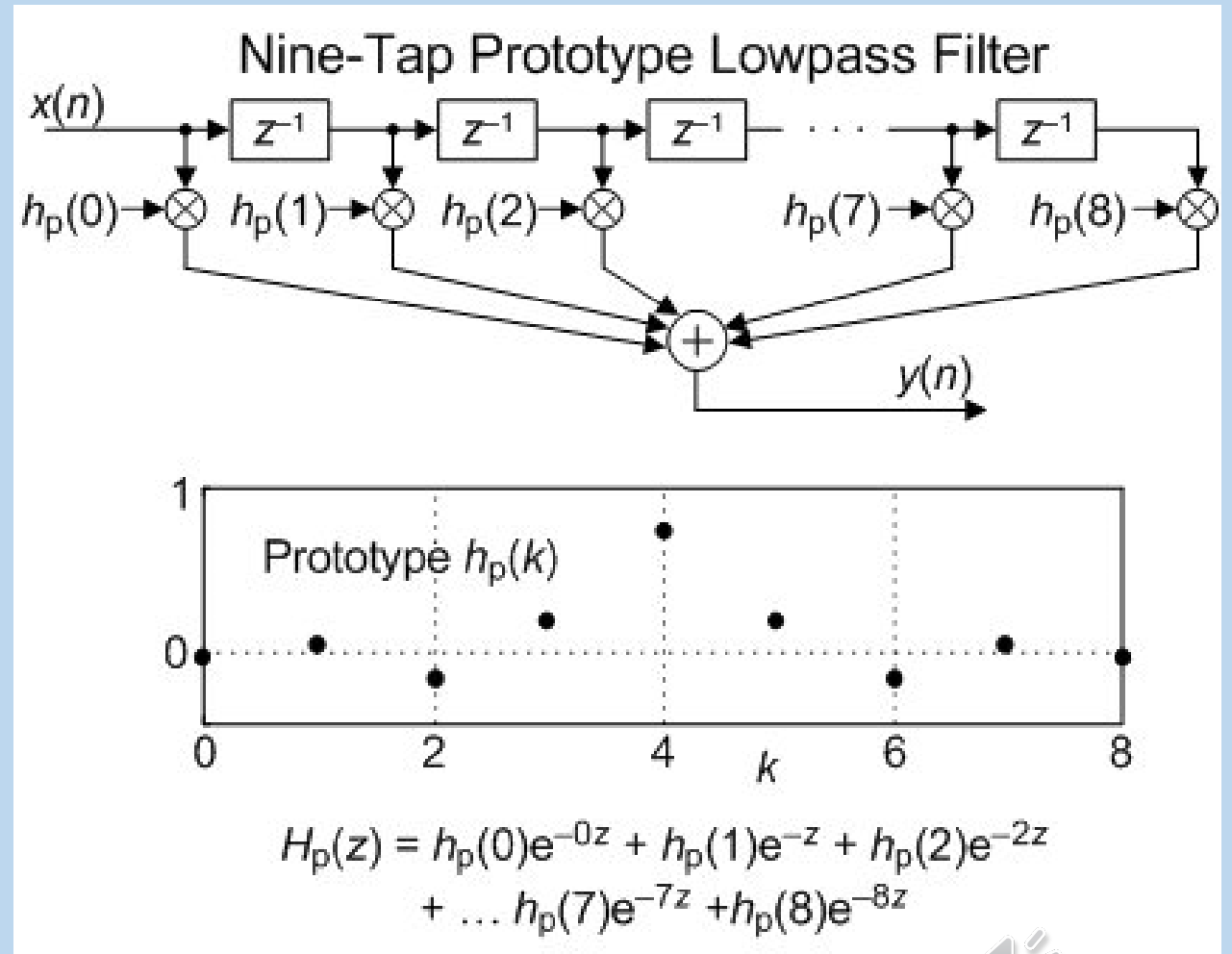
- * Invented to implement very narrowband FIR filters.
 - Computationally-efficient
 - NOT associated with the sample rate change operation of interpolation (upsampling).
 - Invented by Y. Neuvo, D. Cheng-Yu, and S. Mitra in 1984.[†]

[†] Y. Neuvo, D. Cheng-Yu, and S. Mitra, "Interpolated Finite Impulse Response Filters," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-32, June 1984, pp. 563-570.



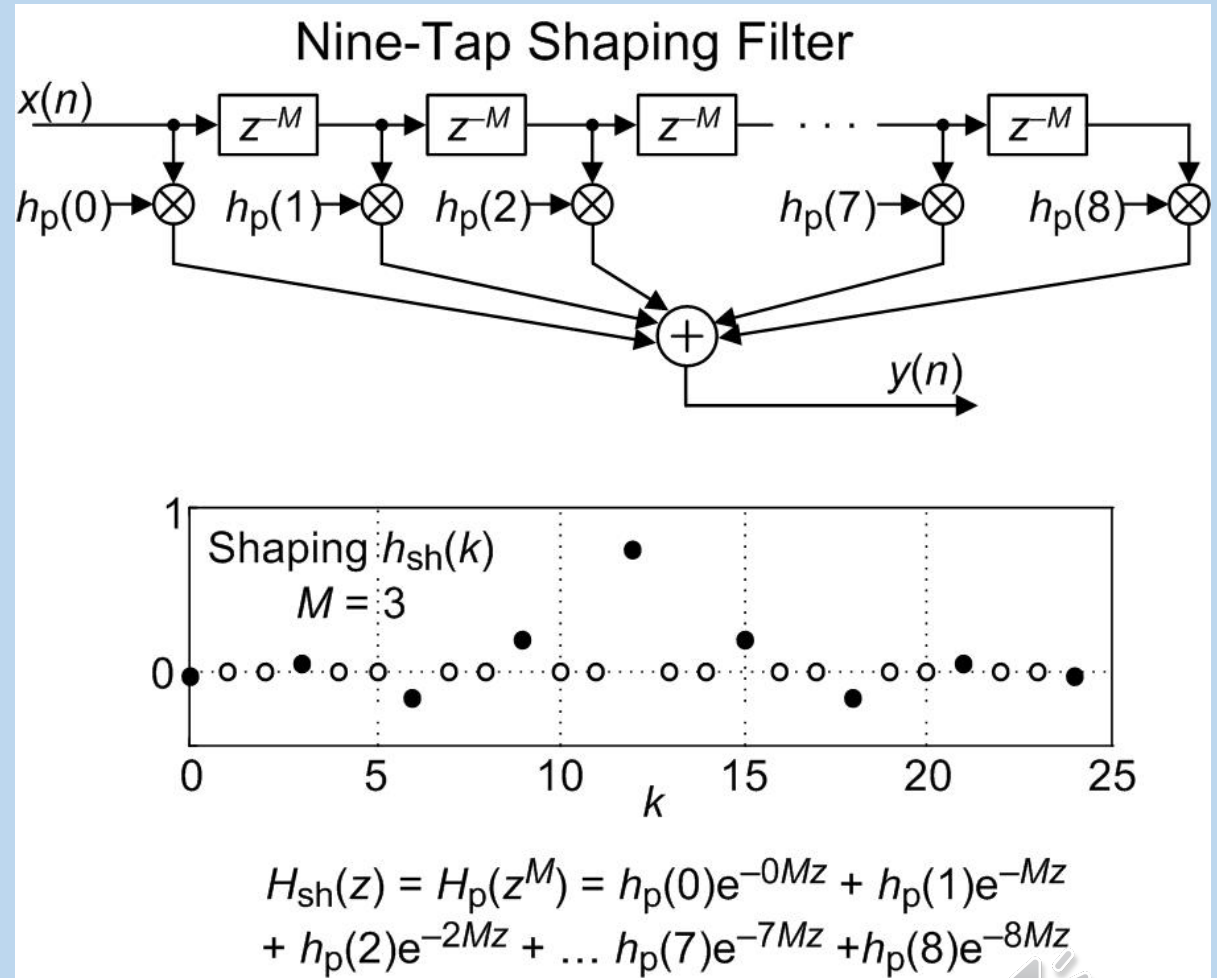
Prototype Lowpass Filter

- * Begin by considering a tapped-delay line lowpass FIR filter.
 - Has symmetrical $h_p(k)$ coefficients so it is linear phase.
- * We'll call this filter our "Prototype" filter.



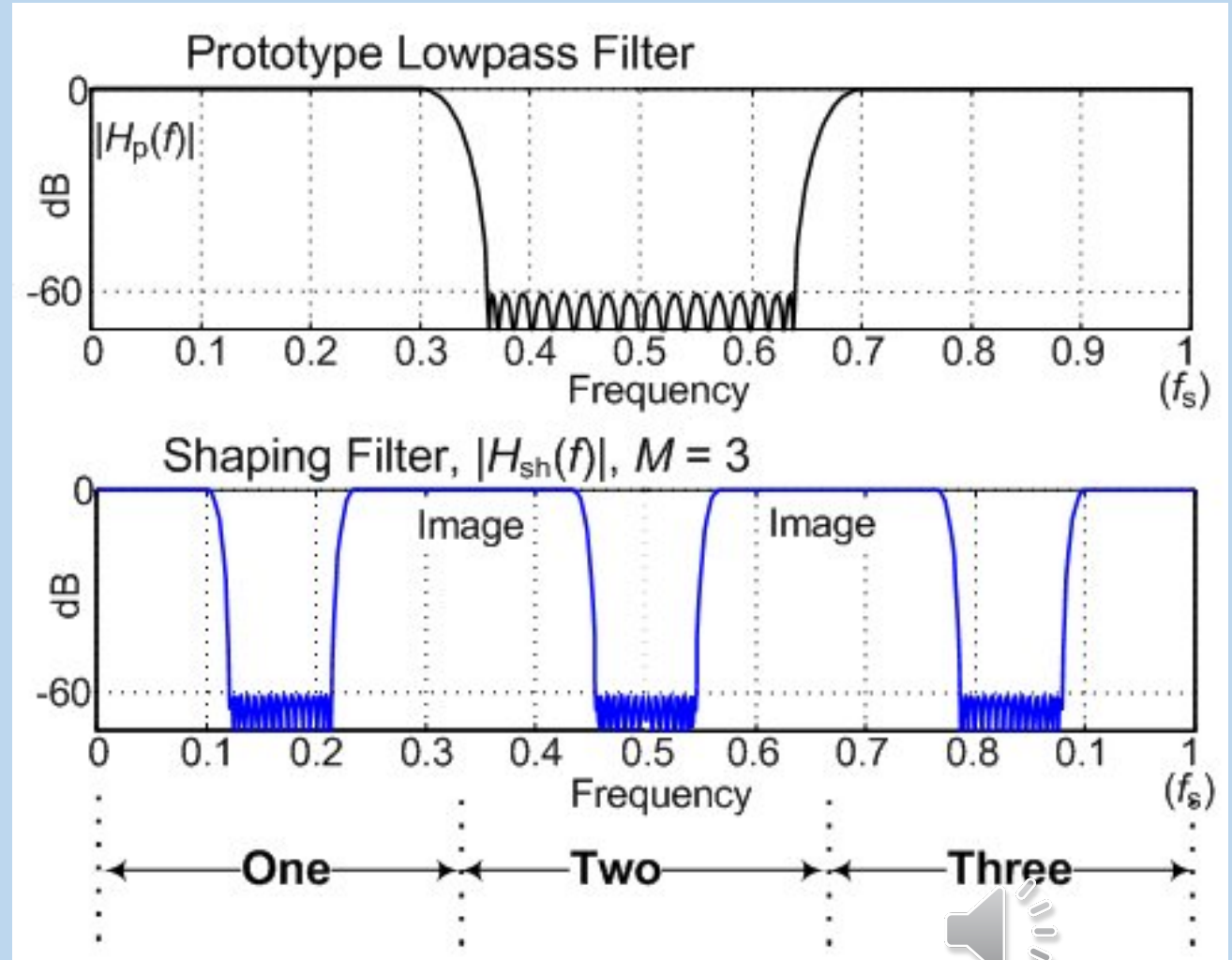
Shaping Filter

- * Replace each delay element in the Prototype filter with $M = 3$ delay elements.
 - This creates what we call the "Shaping" FIR filter.
- Variable M is called the "expansion factor."
- * The Shaping filter is also linear phase.



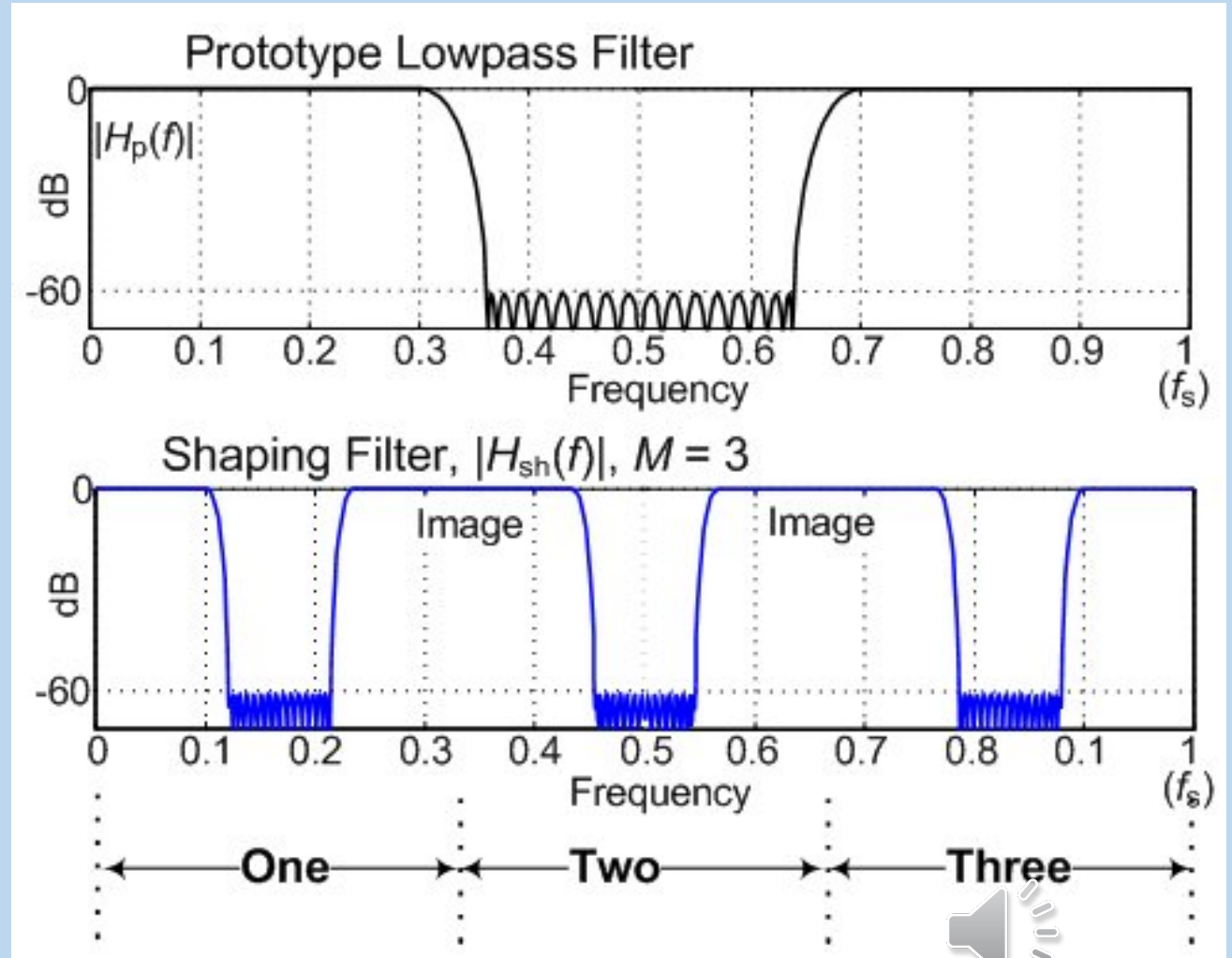
Frequency Responses

- * The M unit-delays cause an M -fold compression (and repetition) of the $H_p(f)$ frequency response.
- * While $H_p(f)$ has a single passband, $H_{sh}(f)$ has M passbands.



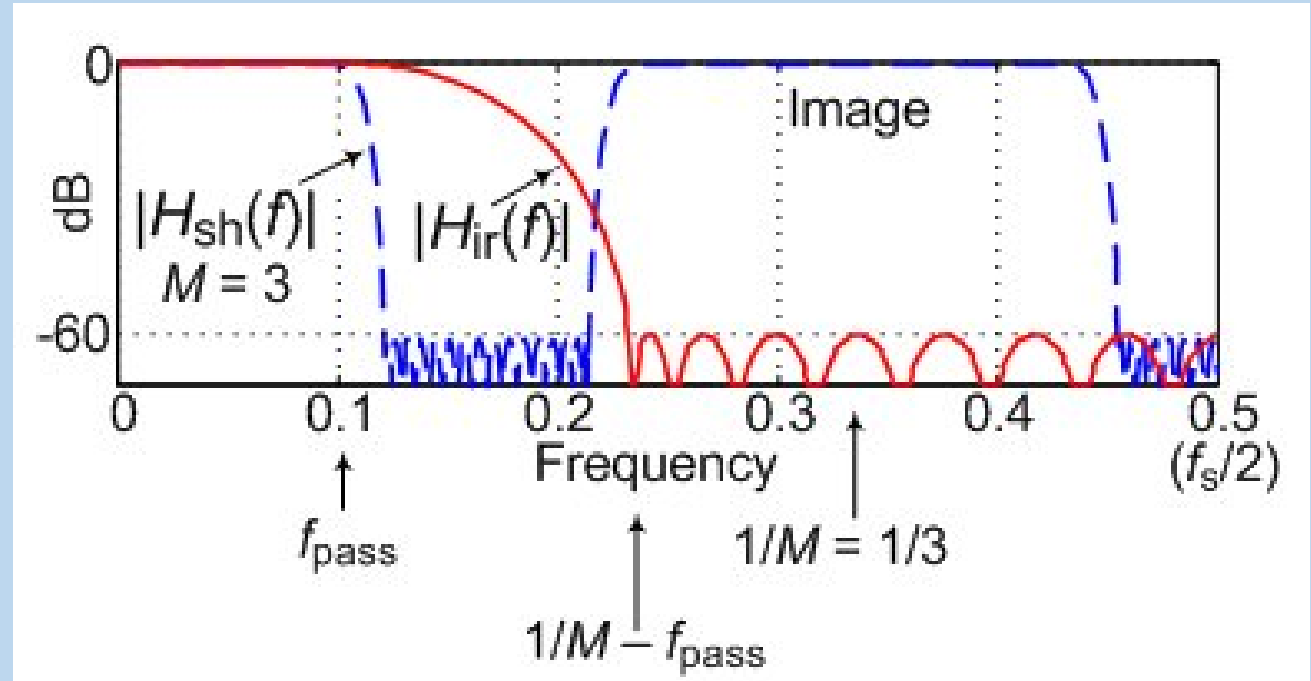
Frequency Responses

- * The frequency axis here is normalized to the f_s filter input signal sample rate.
 - For example, a frequency of 0.2 is equivalent to a frequency of $0.2f_s$ Hz.
- * The repetitive passbands in $H_{sh}(f)$, centered at integer multiples of f_s/M ($1/3$), are called *images*.



Cascaded Filters

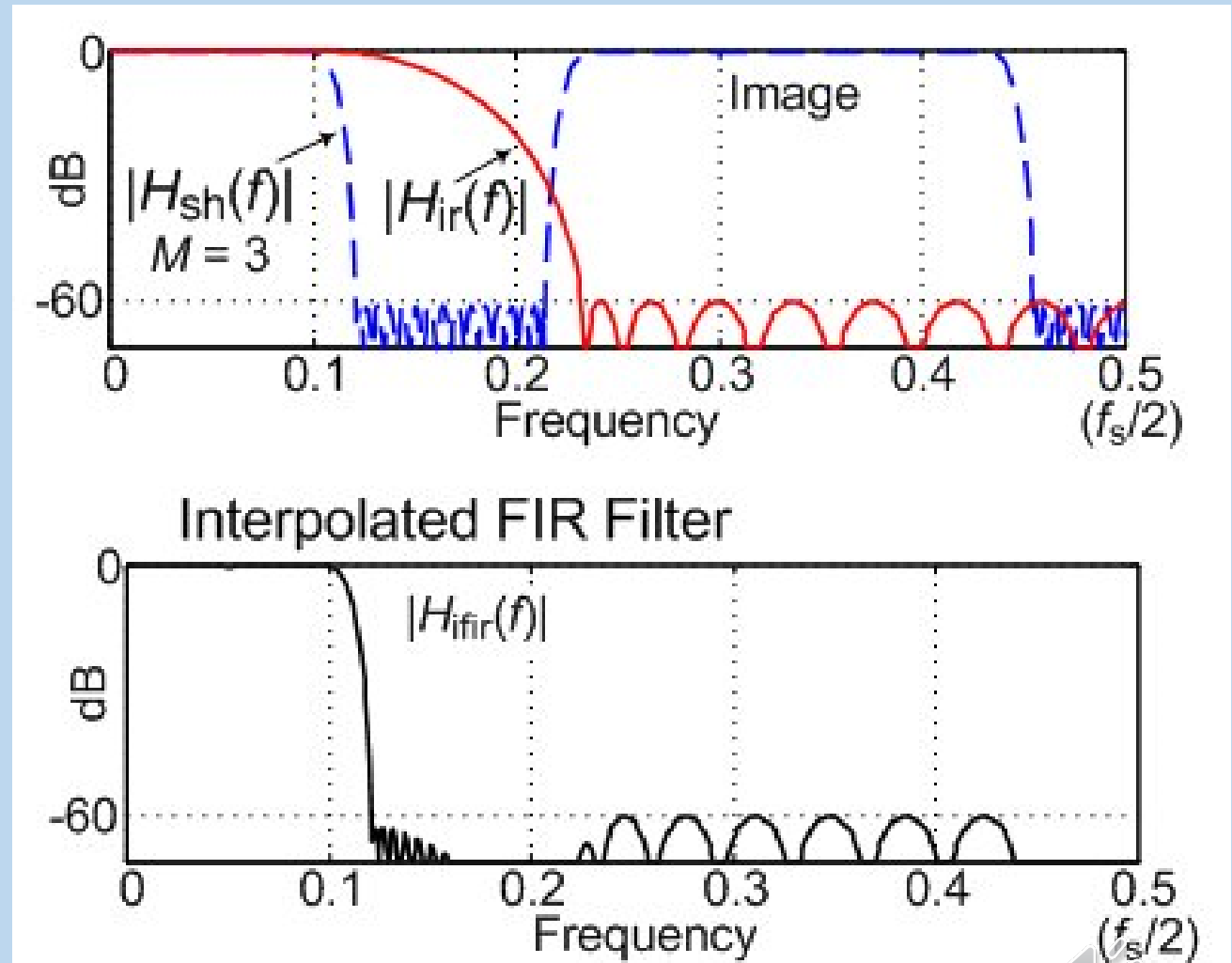
- * Next we follow the Shaping filter ($H_{sh}(f)$) with a lowpass "Image-Reject" filter ($H_{ir}(f)$).
 - $H_{ir}(f)$ passband equals the $H_{sh}(f)$ passband.
 - $H_{ir}(f)$ stopband begins at $1/M$ minus the $H_{sh}(f)$ passband frequency f_{pass} .



Cascaded Filters

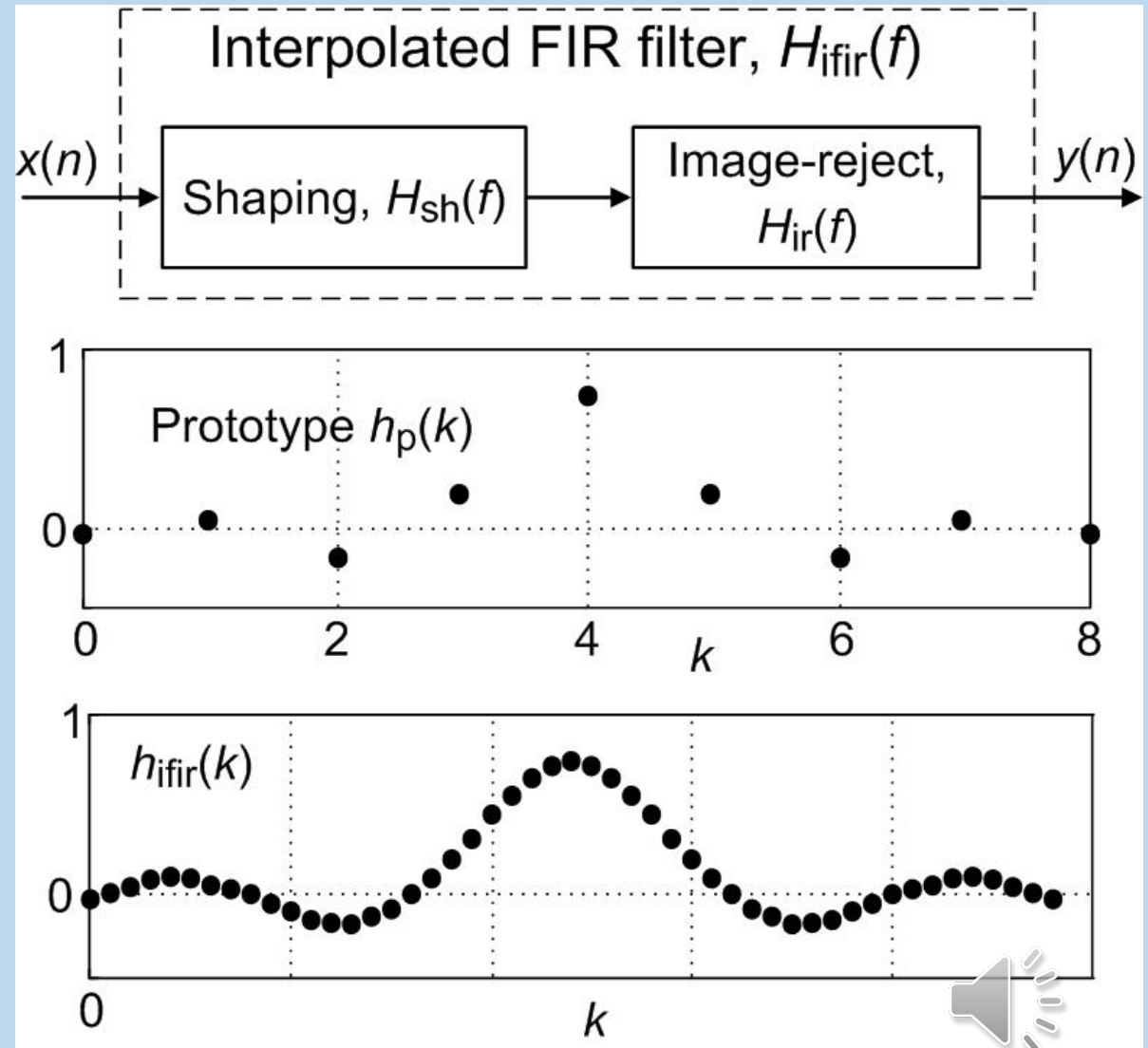
* The cascaded filters yield our desired single-passband lowpass IFIR filter.

$$\begin{aligned} - H_{\text{ifir}}(f) \\ = H_{\text{sh}}(f) H_{\text{ir}}(f) . \end{aligned}$$



Our IFIR Filter

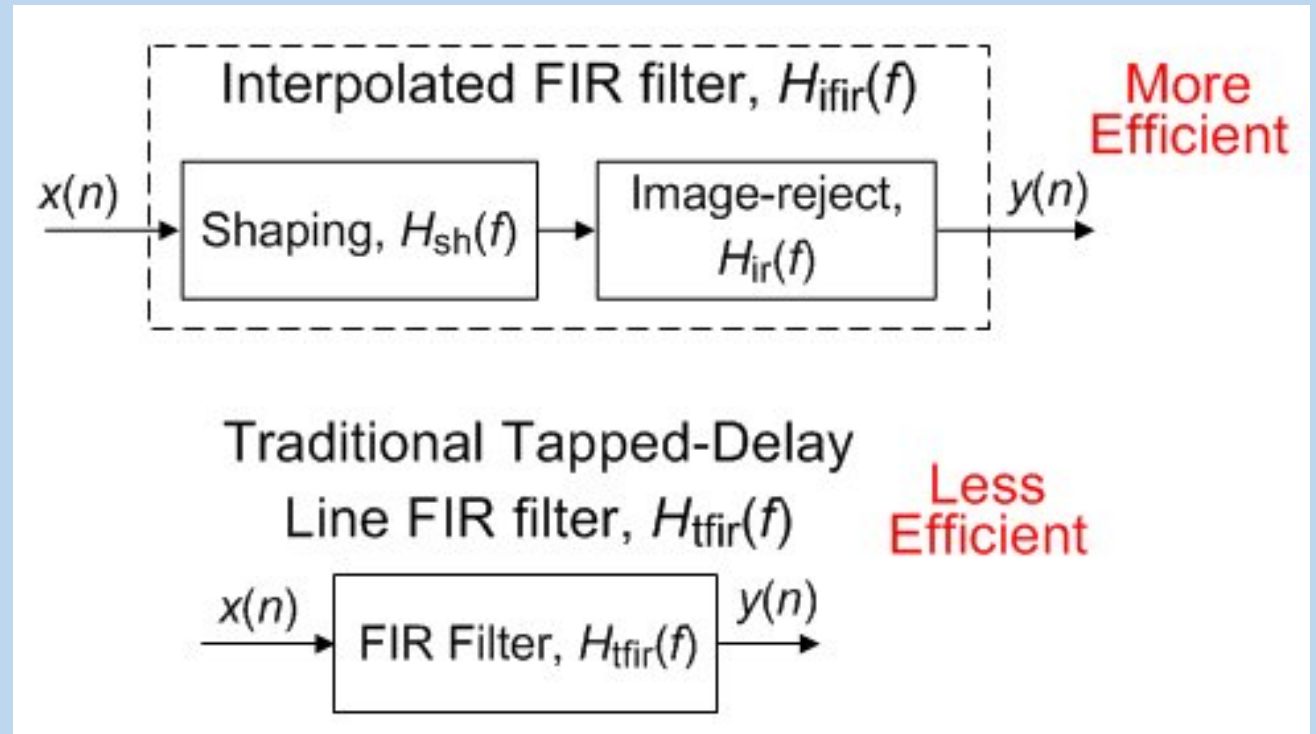
- * Two cascaded tapped-delay line linear phase FIR filters.
 - The IFIR filter's impulse response, $h_{\text{ifir}}(k)$, is an *interpolated* version of the Prototype filter's $h_{\text{ifir}}(k)$ impulse response.



Computational Efficiency

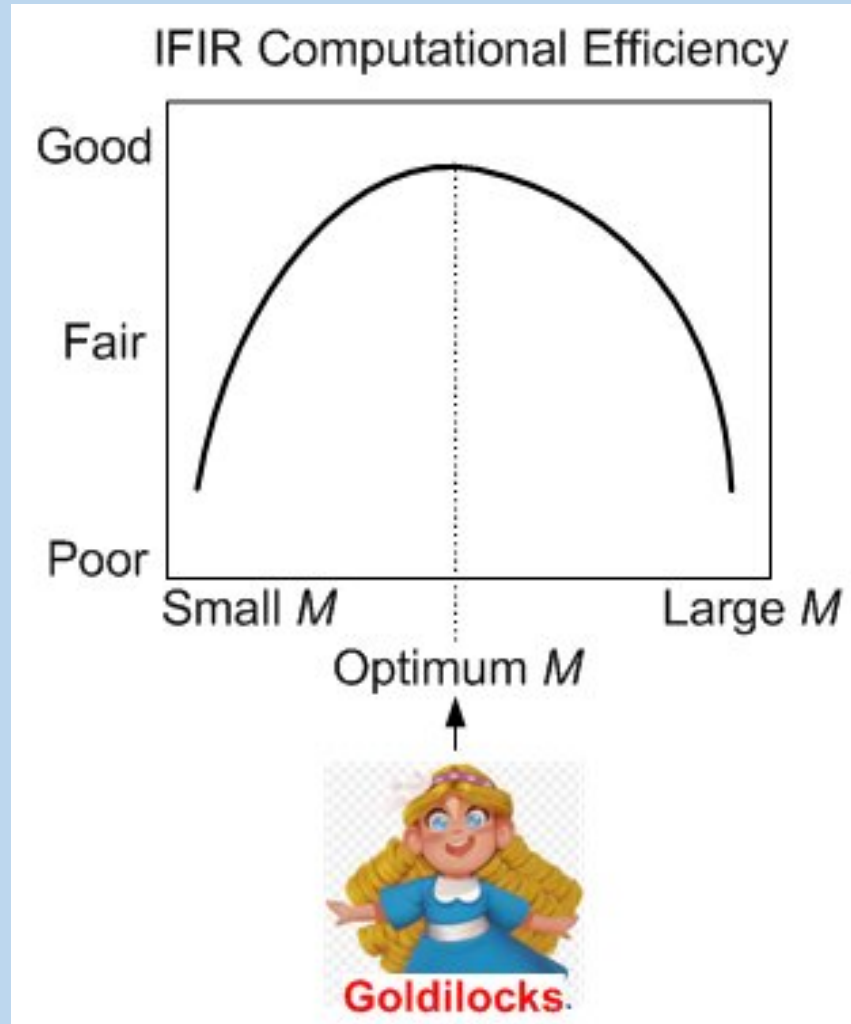
* Why care about IFIR filters?

- IFIR filters are more computationally efficient than equivalent single-stage FIR filters!



Computational Efficiency

- * IFIR computational efficiency depends on the expansion factor M .
 - There is an Optimum M value!

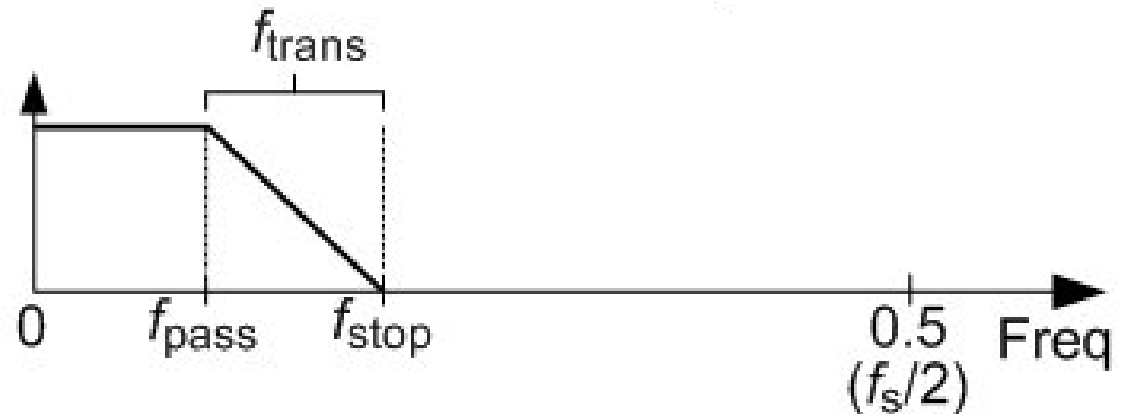


Finding the Optimum M Value

- * Here's how to find the optimum value for M :
 - Derived by Mehrnia and Willson.[†]

[†] A. Mehrnia and A. Willson Jr., "On optimal IFIR filter design", *Proc. of the 2004 International Symp. on Circuits and Systems (ISCAS)*, Volume 3, 23-26 May 2004, pp. 133-136.

Desired IFIR Lowpass Filter

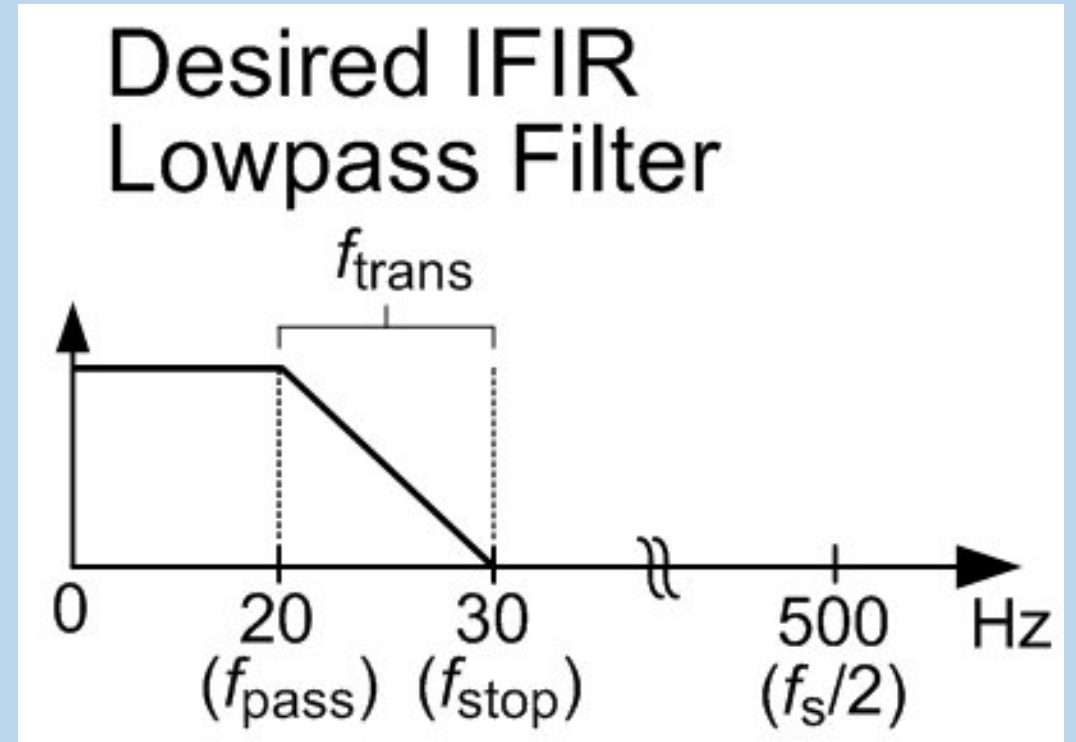


$$M_{\text{opt}} \approx \text{round} \left(\frac{1}{f_{\text{pass}} + f_{\text{stop}} + \sqrt{f_{\text{trans}}}} \right) \quad (1)$$



IFIR Design Example:

- * Assume we want a linear-phase lowpass filter where:
 - f_s sample rate = 1000 Hz.
 - Passband width $f_{\text{pass}} = 20$ Hz.
 - Stopband begins at $f_{\text{stop}} = 30$ Hz.
 - Therefore transition region width is $f_{\text{trans}} = 10$ Hz.
 - Stopband minimum attenuation = 50 dB.



IFIR Design Example:

- * Next use Eq. (1) to find the optimum value for M :
 - Doing so yields $M = 7$.

$$M_{\text{opt}} \approx \text{round} \left(\frac{1}{f_{\text{pass}} + f_{\text{stop}} + \sqrt{f_{\text{trans}}}} \right) \quad (1)$$

$$M_{\text{opt}} \approx \text{round} \left(\frac{1}{0.02 + 0.03 + \sqrt{0.01}} \right)$$

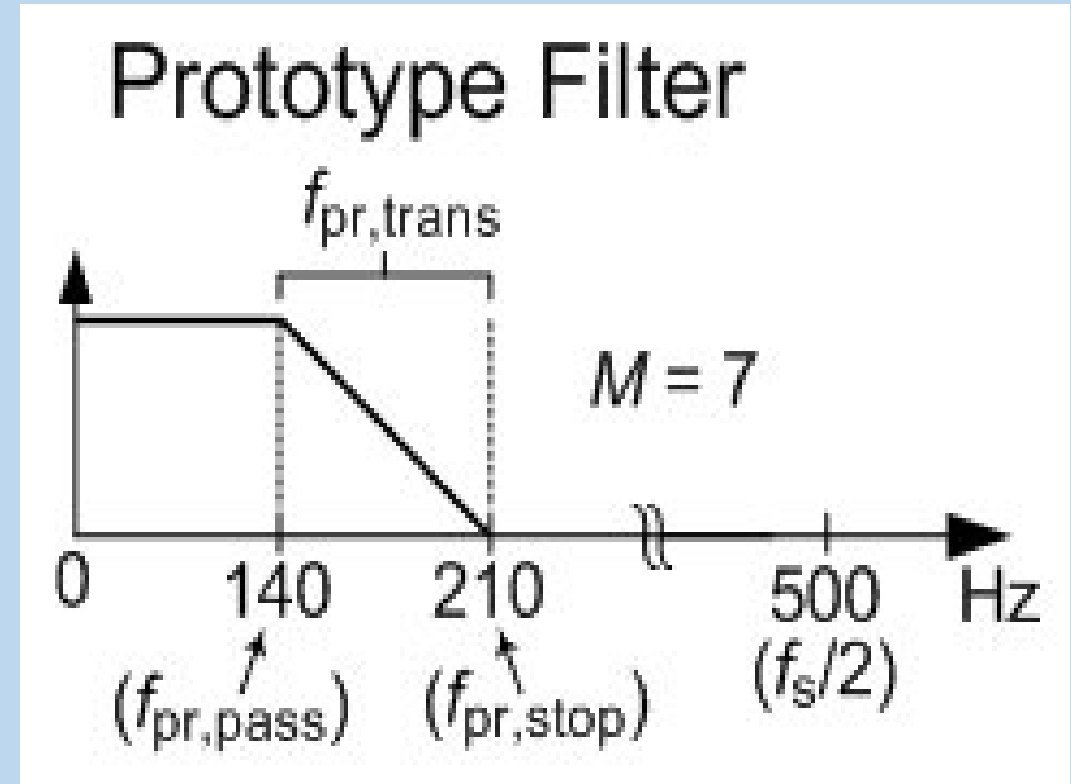
$$\approx \text{round}(6.66) = 7$$



IFIR Design Example:

* Next specify the Prototype lowpass FIR filter using:

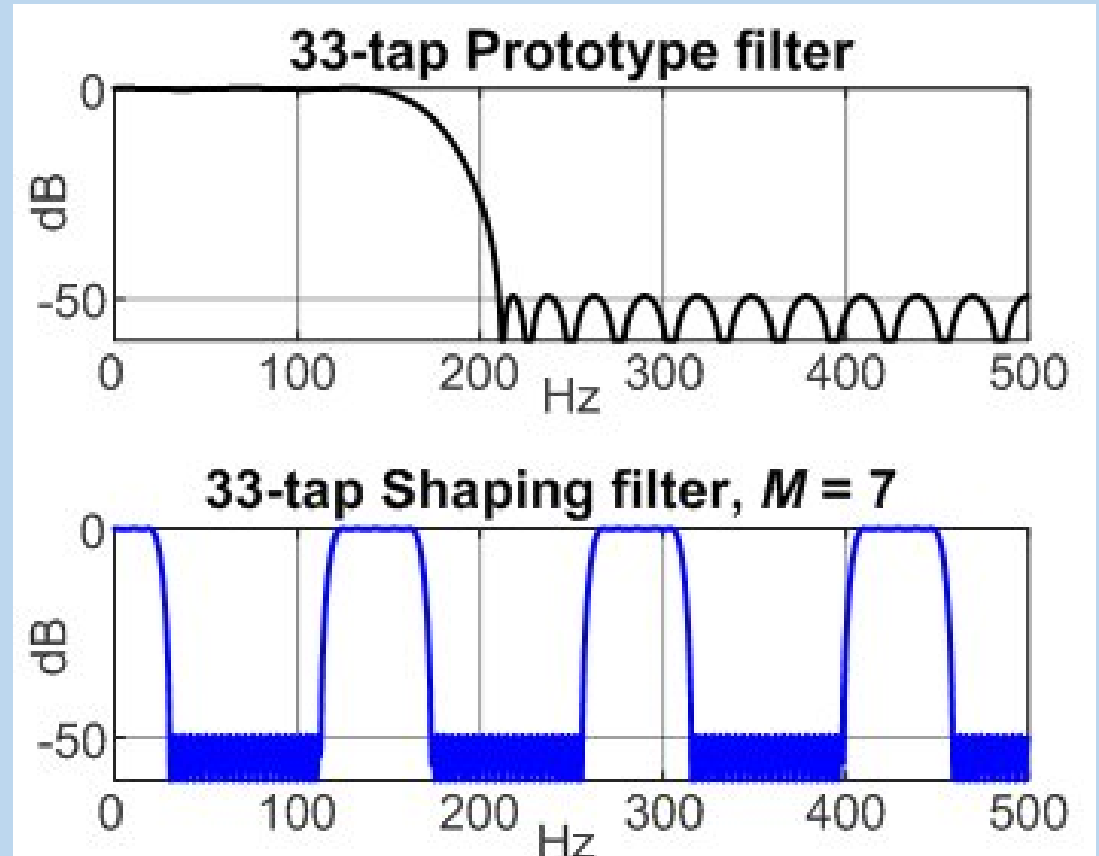
- f_s sample rate = 1000 Hz.
- Prototype passband width is $Mf_{\text{pass}} = M \cdot 20 = 7 \cdot 20 = 140$ Hz.
- Prototype stopband begins at $Mf_{\text{stop}} = M \cdot 30 = 7 \cdot 30 = 210$ Hz.
- Prototype stopband attenuation equals desired IFIR filter attenuation = 50 dB.



IFIR Design Example:

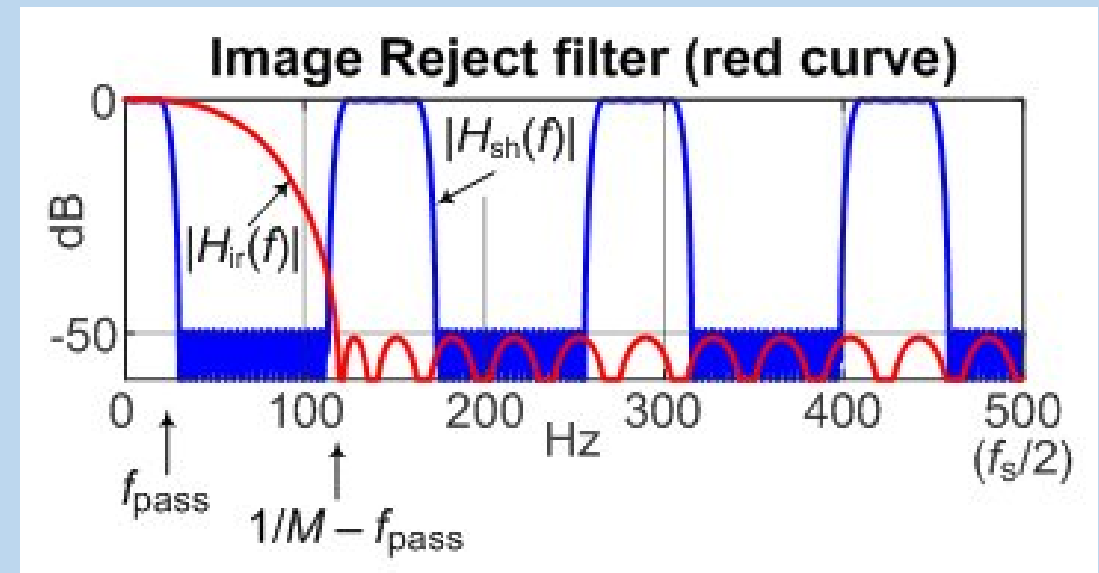
* Next design the lowpass Prototype FIR filter:

- Producing a Prototype filter having 33 taps,
- And the "compressed by $M = 7$ " 33-tap Shaping filter.



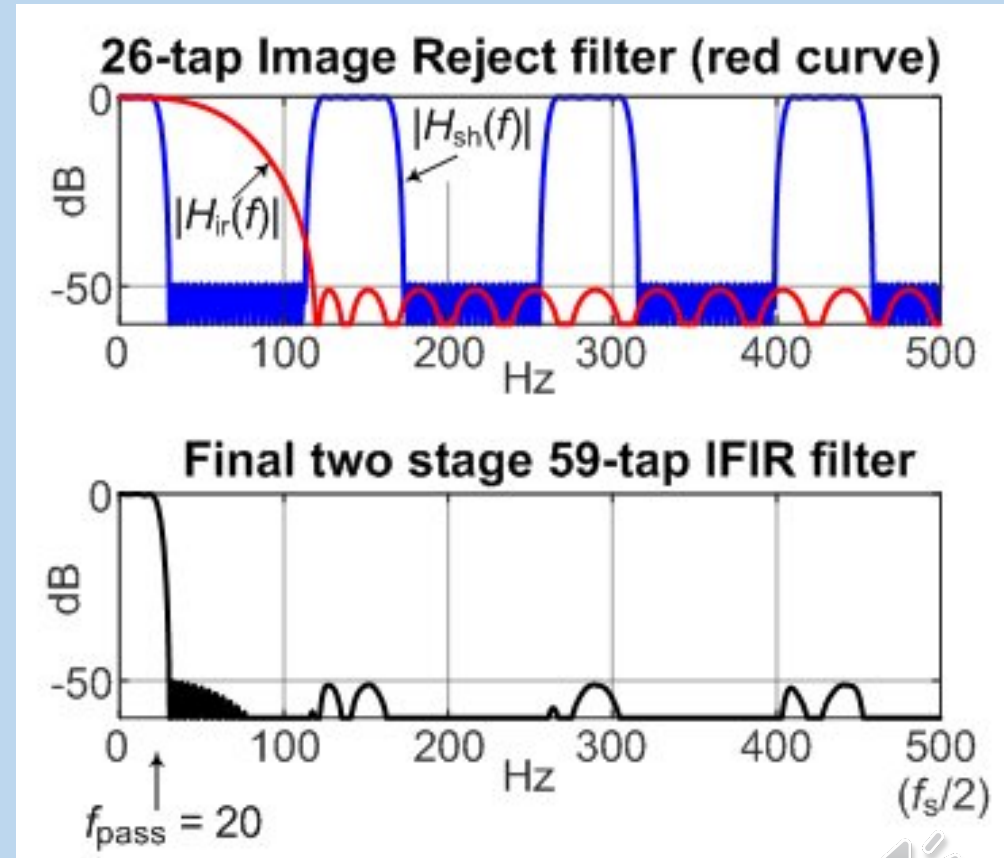
IFIR Design Example:

- * Next design the Image-Reject lowpass filter where:
 - f_s sample rate = 1000 Hz.
 - Image-Reject passband width equals $f_{\text{pass}} = 20$ Hz.
 - Image-Reject stopband begins at $(f_s/M - f_{\text{pass}}) = 1000/7 - 20 \text{ Hz} = 142.8 - 20 = 122.8 \text{ Hz}$.
 - Image-Reject stopband attenuation equals desired IFIR filter attenuation = 50 dB.



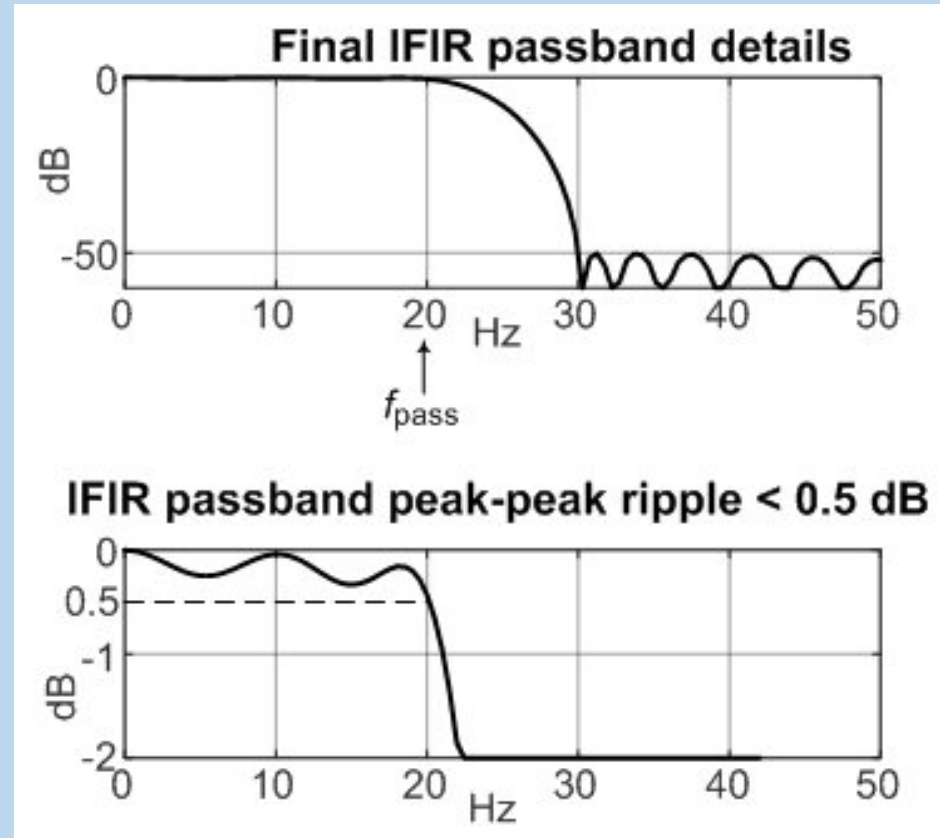
IFIR Design Example:

- * That Image-reject filter design:
 - Produces an Image-reject filter having 26 taps.
- * So our final two-stage cascaded IFIR filter has $33 + 26 = 59$ taps.



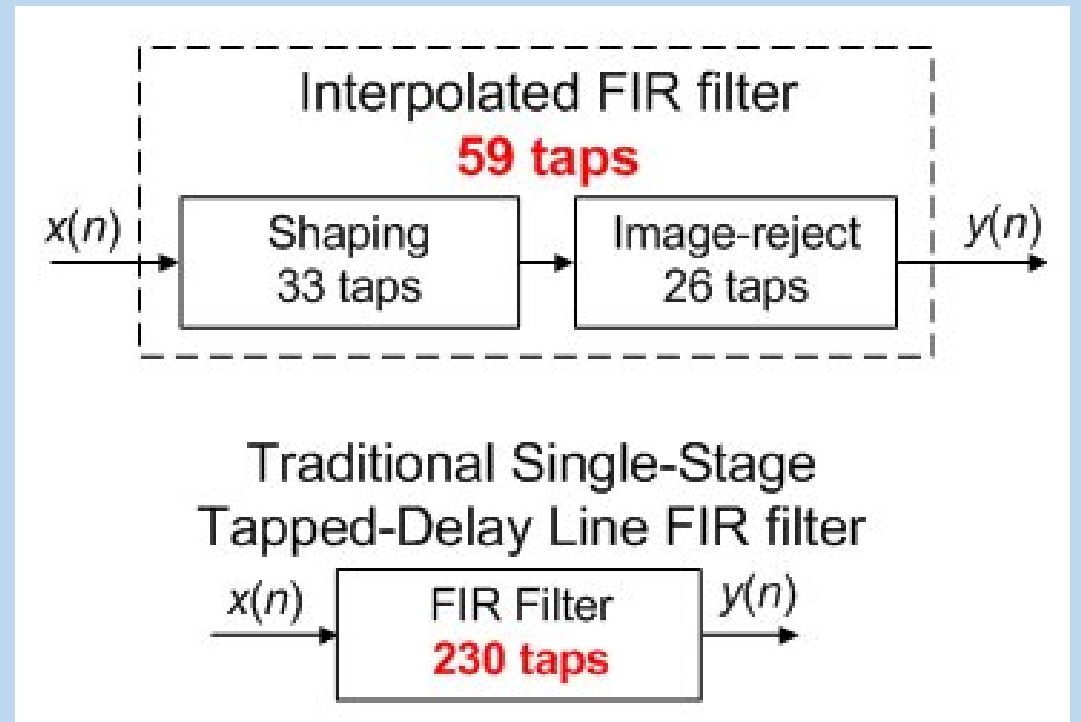
IFIR Design Example:

- * Zooming in we see that our IFIR filter satisfies:
 - The 20 Hz passband specification,
 - And the 0.5 dB peak-peak ripple specification.



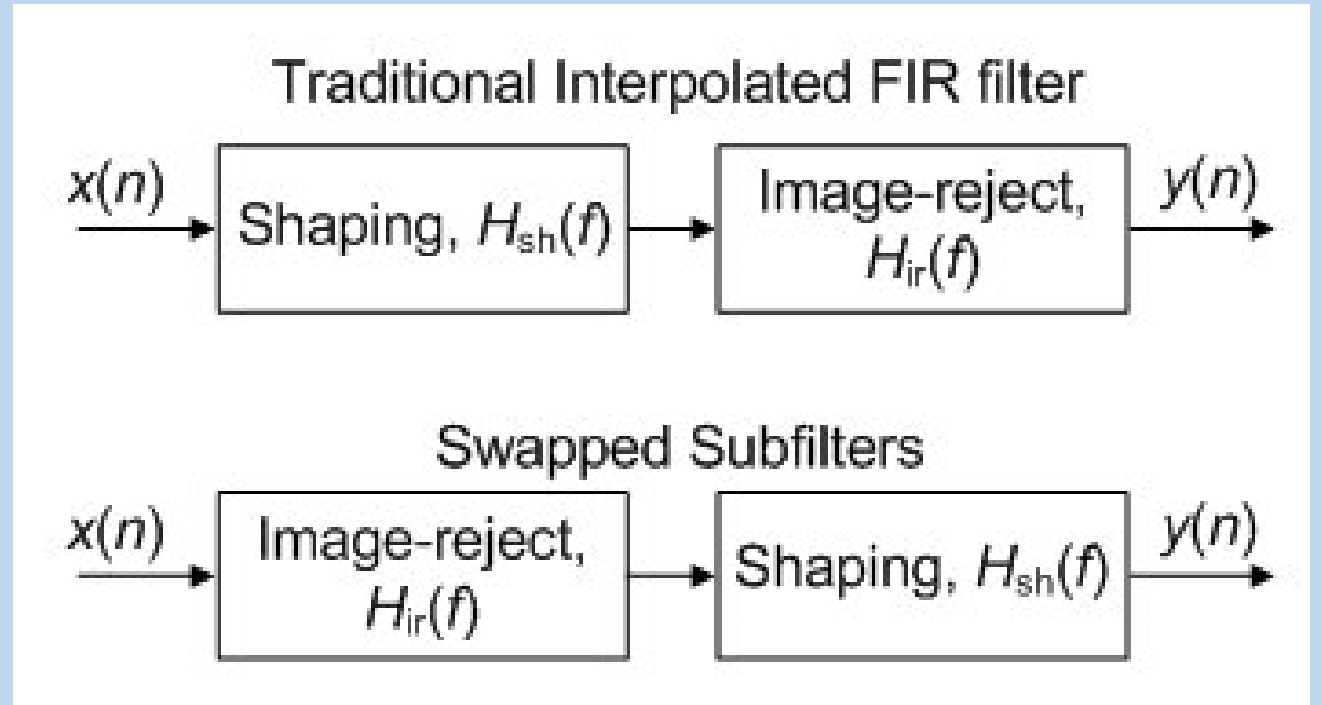
Key Point of This Presentation:

- * Our IFIR lowpass filter requires:
 - 57 adds and 59 multiplies per output sample.
- * An equivalent single-stage FIR filter requires:
 - 229 adds and 230 multiplies per output sample.
- * **Our IFIR filter reduced the computational workload by 75%**



Quiz Question:

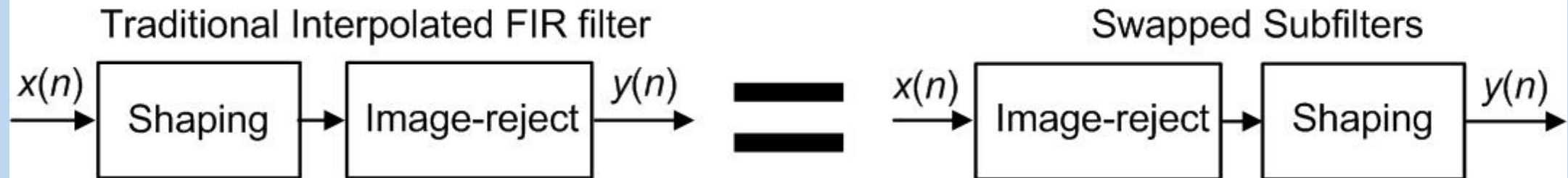
How would an IFIR filter's performance change if we swapped the order of the Shaping and Image-Reject subfilters?



Quiz Solution: Swapping subfilters has NO effect on IFIR filter performance

- Subfilters are linear time-invariant (LTI) and can be swapped.

$$H_{\text{sh}}(f) \cdot H_{\text{ir}}(f) = H_{\text{ir}}(f) \cdot H_{\text{sh}}(f)$$



Closing Comment:

* IFIR filters can significantly reduce the computational workload of linear-phase tapped-delay line narrowband FIR filters

- Lowpass,
- Bandpass, and
- Highpass.

* Thanks for watching.

