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*The Assessment/File Upload Form and many worksheets in the appendix will be used multiple times throughout this course. Please make additional copies of these pages.
Getting Started
Getting Started
Welcome to Calvert

WELCOME TO CALVERT!

We are glad you have selected our curriculum. Please take the time to read the information that follows.

Note: This lesson part, "Welcome to Calvert," is identical for all courses. Once it is finished, it will be marked complete for each course.

If you are the Learning Guide, please make sure you are logged in and have the Teaching Notes enabled. You can do this by clicking on the Teaching Notes toggle, as shown here:

CALVERT'S PLUS CURRICULUM

You will learn using Calvert's PLUS curriculum framework. Our framework is designed to motivate and engage you by using a research-based, digitally supported instructional approach.

WHY DO WE CALL THIS THE PLUS FRAMEWORK?

Our PLUS framework includes Project-Based Learning, Active Learning, Use for Mastery, and Show elements. Details on each element appear below.

Project - Projects are designed to give you fun, engaging, real-world opportunities to creatively show what you have learned. You can also collaborate with other students in the same course.
Learn - Our courses contain a variety of active learning opportunities, including interactive digital activities designed to encourage you to think independently and Quick Checks to assess your understanding.

Use - You will complete a Use for Mastery assessment at the end of each lesson to make sure you have achieved a deeper knowledge (and have "mastered" the concepts).

Show - We offer many creative and exciting opportunities for you to showcase what you have learned. You can submit audio, images, and videos from your computer or mobile device for a teacher to evaluate.

You can view the following video to learn more about the PLUS framework.

Your course is divided into units. Units are made up of lessons, and a lesson is split into lesson parts. Each lesson part is planned to be a day's work.

Please go online to view this video ▶

WHAT YOU WILL FIND IN YOUR COURSE

PROJECT OPENER

Some units in your course are built around a project. When there is a project in your unit, you will see an introduction and description in the beginning of the unit that will tell you:

• What the project will be about
• What you will be doing as part of the project
• How the project will be graded
• Any work that needs to be created or submitted as part of the project

Projects often encourage you to be creative by adding audio, video, or images to make your presentation more interesting and informative. For hints and tips on creating and uploading your projects, click here.

LESSON PARTS

Each unit is made up of lessons. Each lesson helps you learn a new idea in the unit. The lessons are divided into parts. Each part makes up one day's work.

SHOW

“Show” lessons are places in the unit that focus on your project. They give you a chance to show what you have
UNIT QUIZ
At the end of every unit, a unit quiz checks your understanding of all the concepts from the unit. Some questions will be scored by the computer, and some will be marked by your teacher.

In lower grades, the Learning Guide will need to help Grade K and Grade 1 students by reading assessments aloud in cases where Text-to-Speech is not available and taking dictation to submit students’ answers online or helping them to upload responses completed using paper and pencil.

You can view the following video to learn more about what you will find in a course.

Please go online to view this video ▶

WHAT YOU WILL FIND IN A LESSON
At the beginning of each lesson, you will see a lesson title and part number at the top of the screen. You will also see resource buttons to the right of the screen. These resource buttons will identify what you will be working on for your project (if applicable) and will also include lesson objectives, books and materials, assignments, as well as the ability to use Text-to-Speech and print the lesson.
RESOURCE BUTTONS
Here's what each resource button will include:

- **Project** – The Project button provides a short description of the project you are doing as part of the lesson.

- **Objectives** – Objectives are statements that describe what you will be learning. The objective will be your goal for the lesson across all lesson parts.

- **Assignments** – The Assignments list highlights the lesson's work at a glance. This list includes reading assignments, labs, activities, and exercises.

- **Books & Materials** – All books and materials needed for the day's lesson are listed here. You may find it helpful to review this list before each day's lesson part.

- **Standards** show how each lesson is aligned with national or state standards.

- **Text-to-Speech** will read the page text aloud or allow you to look up the definition of a word that appears in the lesson.

- **Print** allows you to print the lesson, unit, or course you are currently viewing.

You can view the following video to learn more about what your course and lessons will look like.

Please go online to view this video ▶

COLORS AND CARD TYPES

COLORS

Each lesson card is color-coded.

- **Green** refers to Learn sections.

- **Purple** refers to Use sections.

- **Orange** refers to Project/Show sections.
CARD TYPES

All content in a lesson part is laid out as a series of cards. Each card indicates a distinct activity that you will do as part of your daily work. Here are the different types of cards:

- **Collaboration** is a way you can share information, data, or projects with other Calvert students in your school. Calvert uses an online collaborative tool to allow you to chat with other students in the classes in specifically designed lessons.

- **Final Project** cards will be a place to showcase what you have learned at the end of your project. You can be creative and submit audio, images, or video from your computer or from your mobile device.

- **Interactive Activities** are fun digital tools that will help you learn more about a topic. Interactive Activities are digital activities that may include virtual labs, simulations, videos, and more.

- **More to Explore** is additional content that can help you either learn more about a concept or help you understand a new concept. More to Explores can include videos, additional readings, or digital activities that help you apply knowledge of a concept a different way.

- **Project Progress** cards provide the opportunity to share pieces of project work for feedback in advance of pulling all the pieces together for the final Show.

- **Quick Checks** are short assessments that will help you clarify what topics you have mastered and what concepts you may need to review. After you complete a Quick Check, you will be given the correct answer and a resource to help you review the concept in a new way.

- **Rate Your Enthusiasm** will appear periodically after your lessons, so you can give us real-time feedback during your course.

- **Rate Your Excitement** will appear periodically after your lessons so you can give us real-time feedback while you complete each course.
We want to check in with you to see how you are progressing through your project. **Rate Your Progress** will appear on some of the days you are working on a project so you can let us know where you are in the project and how things are going.

We want to check in with you to see how ready you feel for the course. **Rate Your Readiness** will appear in lessons in the Getting Started unit.

We want to check in with you to see how you are understanding each lesson part. **Rate Your Understanding** will appear periodically after your lessons so you can give us real-time feedback while you complete each course.

At the end of every unit, we provide a **Unit Quiz** where you will be assessed on your understanding of all the key concepts learned in that unit. The concepts that are tested are based on the key standards identified by your state.

Each lesson has a **Use for Mastery** assessment. These open-ended response questions help assess how well you understood the lesson concepts. The 'Use For Mastery Guidelines & Rubric' below each question will provide helpful information on how and what to submit for your response. You may be asked to type into a text box or upload a document.

---

**ONLINE PLATFORM ACCESS**

You can complete our course using a fully online approach with access to a computer or with a hybrid approach, with the help of printed materials. When online, you can use our content in one of two ways:

1. Our online platform called Calvert Teaching Navigator (CTN). You can access CTN online at [http://login.calvertlearning.com](http://login.calvertlearning.com). Your school's Learning Management System (LMS).

2. If you are viewing the Calvert product through your school's LMS, please contact your school for how to get access.

Please review our [Technology Requirements](#) to make sure your computer is set up to allow full access to our courses.
SUGGESTED DAILY SCHEDULE

The following is a suggested daily schedule as it displays in CTN. Although each subject can be studied in a designated order, know that you can adapt the schedule and pace to meet your individual educational needs.

A complete course is planned for an average school year of about nine months. There are 160–180 daily lesson parts in a course. The number of lesson parts and tests for individual subjects will vary based on the amount of material that must be covered in the course during the school year.

Each day, we recommend that you spend approximately 120-150 minutes in grades K-2 and 100-120 minutes in grades 3-8 on English Language Arts, 45 minutes on Math, 45 minutes on Science, 45 minutes on Social Studies, and 30 minutes reading independently.

You can view the following video to learn more about the Suggested Daily Schedule.

Please go online to view this video ►

KNOW YOUR ROLE

ROLE OF THE LEARNING GUIDE

The Learning Guide is a responsible adult (usually a parent) who guides the student through his or her academic journey.
Your certified school teacher directs the instruction, determines the pacing, and makes decisions for intervention and enrichment. However, the Learning Guide has an essential role in helping you on the road to academic success.

The Learning Guide has access to the all course materials. Additionally, teacher-specific instructions (Teaching Notes) written specifically to the Learning Guide or instructor give information, directions, and suggestions for leading you through a lesson.

When Teaching Notes are enabled, teacher-specific instructions for a card will appear just below that card.

You can view the following video to learn more about the role of Teaching Notes and the Learning Guide.

Please go online to view this video ▶

ROLE OF THE STUDENT

While the lessons in this curriculum are written to you, the student, that does not mean you are expected to work completely on your own. Keep in mind that your Learning Guide is here to support and help you. You and your Learning Guide will work as partners. Together you will decide which assignments you will work on independently and which you will do jointly. During the course, there will be times when you will be directed to read a selection aloud for your Learning Guide, share information you have learned, or take part in a discussion.

When working on your own, ask for your Learning Guide's assistance if you have any questions or if directions do not seem clear. You should also check with your Learning Guide before linking to any of the websites listed in the lessons or activities.

ROLE OF THE CALVERT SUPPORT STAFF

At Calvert, we understand the importance of having support when you need it. We offer many resources to help you along the way. If you have a question about our curriculum, our Education Counselors are available to help you Monday through Friday, 9:00 a.m. to 5:00 p.m. Eastern time, by phone at 1-888-487-4652, or email at support@calvertservices.org.

RATE YOUR READINESS

Please go online to view and submit this assessment.
PRINT VS. DIGITAL EXPERIENCE

If you plan to do this course exclusively online, you will have access to all the course material digitally.

If you are going to complete some of this course offline, you might have already received a printed version of the lesson manual. If not, you can print at any time using our Print-On-Demand functionality. Using this functionality, you can print a single lesson, an entire unit, or the entire course.

Print-On-Demand does not print the textbooks that you will need as part of your course. Please contact your school directly to have the textbooks shipped to you.

As part of your project work or assessment, you may be required to submit a file, image, or video to your teacher. To do this, you will need access to a computer and a camera-equipped mobile phone.

WORKSHEETS

If you are working in the print version of our lessons, all the worksheets that are needed to complete the course are provided in the Appendix as part of the printed packet. Otherwise, PDFs of all worksheets will be linked to the individual lessons. You will need Adobe Reader® to use these worksheets. Most of these worksheets are fillable, and you can use your computer keyboard to type directly in them and save them on your computer.

NOTEBOOKS AND JOURNALS

You may be directed to use a notebook throughout this course. The Math Notebook should be used to reflect on your learning and can serve as a single place to record information as you move through the course. You can take notes in your physical notebook or even digitally by using an application such as Evernote®.

ONLINE ACTIVITIES

Your course may include interactive digital activities, videos from publishers such as YouTube®, virtual simulations, and digital assessments that cannot be completed without going online.
BOOKS AND MATERIALS

MATH IN FOCUS TEXTBOOK

You will find textbook page numbers in the lesson that are underlined. We refer to this as hyperlinking. Clicking directly on the link opens the corresponding page of the textbook. You can then scroll through the pages of your textbook.

The e-text will not allow you to directly type into any blanks.

INSTRUCTIONAL VIDEOS

The Math in Focus course is based on the Singapore Math method, which may be new to some Learning Guides. For this reason, Calvert Learning has produced a series of instructional videos to provide training in the basics of this method. These videos will be linked directly in the appropriate lessons, but for your convenience they are listed and linked here as well:

How to Teach Problem Solving
How to Teach Bar Models, Part 1
How to Teach Bar Models, Part 2
How to Teach Bar Models, Part 3

Calvert Learning instructional videos for students are also available to provide review of important skills in preparation for learning new material. These videos are directly linked to the lessons where appropriate.
BRAINPOP®

Calvert Learning is pleased to offer BrainPOP®, an engaging web-based interactive program that supports the core curriculum. BrainPOP® activities include animated video tutorials, interactive activities, and assessments that provide a rich, multisensory experience designed to improve learning. These research-based activities were developed in accordance with national and state academic standards. These engaging activities are accessed through the online course. When a BrainPOP® activity is appropriate for a lesson, the link is located with the online lesson for that day. Click on the link, and you will be directed to the instructional activities.

DISCOVERY EDUCATION™ VIDEOS

Your course may include videos from Discovery Education™, which provides thousands of subject and grade specific videos to enrich your learning experience. Discovery EducationTM videos have been aligned to lessons throughout the Calvert curriculum to reinforce lesson objectives. These videos can be accessed through the online lessons in Grades K–8. If a video has been aligned to a lesson, you will find a link to that video in the online lesson.

ADDITIONAL MATERIALS

We have included many resources designed to provide additional help and support as you complete your course. These supplementary resources are provided to you in the appropriate lessons as downloadable PDFs that you can print as needed.

Your course may also use these materials that are commonly found throughout your home.

Please go online to view this video ▶

RATE YOUR READINESS

Please go online to view and submit this assessment.
Unit 1 - Real Number System
Numbers can be placed in many different categories. One type of number that may be new to you is irrational numbers. Remember that rational numbers can be written in the form \( \frac{m}{n} \), where \( m \) and \( n \) are integers and \( n \) does not equal zero. A number like \( \sqrt{2} \) can be written as \( \frac{\sqrt{2}}{1} \), but it is not a rational number. It cannot be expressed as a ratio of two integers.

How can you tell if a number is irrational? The use of a radical \( \sqrt{ } \) is a clue, but it is not totally reliable. Consider, for example, \( \sqrt{25} \). You know that \( 5 \times 5 = 25 \) and \( -5 \times -5 = 25 \), so \( \sqrt{25} \) could be 5 or \( -5 \), both of which are integers. However, \( \sqrt{5} \) and \( \sqrt{12} \) are irrational because they do not have roots that are integers. The best way to tell if a number is irrational is to examine it in decimal form. Irrational numbers can be rewritten as nonrepeating, nonterminating decimals.
Study these examples:

These are rational numbers.

0.75 This is a nonrepeating decimal.

You can easily find its equivalent in fractional form.

\[
\frac{75}{100} = \frac{75 \div 25}{100 \div 25} = \frac{3}{4}
\]

0.757575... This is a repeating decimal.

This is equivalent to \( \frac{25}{33} \). (You will learn how to convert repeating decimals to fractions in a later lesson.)

These are irrational numbers.

1.414213562373... This decimal does not repeat or terminate.

It is equivalent to \( \sqrt{2} \).

3.14159265358... This decimal does not repeat or terminate.

It is equivalent to \( \pi \).

Even though you do not know the exact value of an irrational number, you can still find its approximate location on the number line.

Read p. 2–3 in *Math in Focus 3A*. Examine the number line in *Recall Prior Knowledge*. Notice the locations of \( \sqrt{2} \), \( \pi \) and \( -0.04004004... \), which are irrational numbers. These locations were found by rounding the decimal equivalents. For example, to place \( \sqrt{2} \) on the number line, you can round its decimal equivalent to 1.4, which is almost halfway between 1 and 2. If you round out farther (to 1.4142, for example), you can place \( \sqrt{2} \) much closer to its exact location. How could you find the approximate location of \( \sqrt{5} \)? You know that \( 2^2 \) is 4 and \( 3^2 \) is 9, so \( \sqrt{5} \) must lie somewhere between 2 and 3. You can use a calculator to find the decimal equivalent, 2.23606..., which will help you place \( \sqrt{5} \) more accurately.

You can also use approximations to compare irrational numbers. You know that \( \sqrt{2} \) is almost halfway between 1 and 2 on the number line and \( \sqrt{5} \) is somewhere between 2 and 3. Therefore, \( \sqrt{5} \) must be greater than \( \sqrt{2} \). Comparing their decimal equivalents (1.414... and 2.236...) confirms this idea.
HELPFUL ONLINE RESOURCES

Instructional Video: BrainPop: *Rational and Irrational Numbers* (02:33)

PRACTICE

Complete the *Rational and Irrational Numbers Worksheet*.

### TEACHING NOTES

Worksheet Answers:

A. 1. rational (equals 11) 2. irrational 3. irrational 4. rational (terminates) 5. irrational 6. rational (repeats)

B. 1. 0.416 2. –0.5625 3. 8.3 4. 14 5. –0.35 6. –7.83

C. 1. 4, 5 2. –7, –6 3. 7, 8 4. –10, –9 5. 11, 12 6. 13, 14

D.  

E. 1. > 2. > 3. < 4. > 5. < 6. <

F. 1. 3.66, √14 2. –4.671, √21 3. π, 1.809, 6

WRAP-UP

Today you learned about irrational numbers. Irrational numbers cannot be written in the form $\frac{m}{n}$, but they can be rewritten as nonrepeating, nonterminating decimals. You can place irrational numbers on the number line by rounding their decimal equivalents. The farther you round the decimal form of an irrational number, the closer you can place it to its exact location. You can also use approximations to compare irrational numbers.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Today’s lesson covers important ideas you need to understand before beginning the lessons in this chapter. Review the categories of numbers on pp. 2–3 in *Math in Focus 3A*. Also examine the number line. Be sure you understand why the numbers indicated by green arrows are placed where they are. Also, look carefully at the diagram showing the relationships between the different types of real numbers.

You will also be reviewing operations with integers. Review the rules for these operations on p. 4 in *Math in Focus 3A*.
Today you learned about different kinds of numbers.

- counting numbers beginning at 1 → positive integers
- 0 + positive integers → whole numbers
- whole numbers + negative integers → integers
- integers + fractions → rational numbers
- rational numbers + irrational numbers → real numbers

You also reviewed how to add, subtract, multiply, and divide integers.

Please go online to view and submit this assessment.
Real Numbers - Part 3

LEARN

WARM-UP
Evaluate each expression.

1. \((-8) + 13\)
2. \((-16) ÷ (-8)\)
3. \((-6) \cdot 7\)

WARM-UP ANSWERS
1. 5 2. 2 3. -42

TEACHING NOTES

INSTRUCTION
Read Understand Exponential Notation on p. 5 in Math in Focus 3A. Copy the repeated multiplication for \(10^9\). Multiply pairs of 10s together and write the product above each pair. Keep multiplying pairs of numbers until there are no more pairs left. Since there are an odd number of 10s, you will need to multiply the leftover 10 by the product of the pairs of numbers to determine the final product.

Review Example 1 on p. 5. Then complete the first Guided Practice section on p. 6.

Review Example 2 on p. 6. Then complete the second Guided Practice section on p. 6.

Review Example 3 on p. 6. Notice that the base can be positive or negative and may contain a variable. Complete Guided Practice on p. 7. Remember to use parentheses in your answer if the base includes them.
Review Example 4 on p. 7. Multiplying a number between 0 and 1 by itself will result in a number less than what you started with. For example:

\[ 0.04 \times 0.04 = 0.0016 \]

Complete Guided Practice on p. 8. In the second row of each practice problem, the number of lines drawn is the same as the exponent.

WATCH FOR THESE COMMON ERRORS

Your student might ignore the negative sign on the base and then re-insert it in the answer. He must understand that the negative sign is part of the base and is included in each multiplication. Sometimes a negative base results in a positive answer (for an even exponent) and sometimes a negative base results in a negative answer (for an odd exponent).

Complete problems 1–18 of Practice 1.1 on p. 12 in Math in Focus 3A.

Textbook Answer Key
WRAP-UP

Today you learned how to write expressions in exponential notation. You also identified the base and exponent in an exponential expression.

You learned to take an expression that is in exponential notation (e.g. one in which the base is negative two-fifths and the exponent is three) and expand it into a repeated multiplication. Then you learned to do the repeated multiplication to evaluate the expression and find the final answer for an exponential notation expression. Check to see if you got the right answer.

\[
\left( -\frac{2}{5} \right)^3 = \left( -\frac{2}{5} \right) \cdot \left( -\frac{2}{5} \right) \cdot \left( -\frac{2}{5} \right)
\]

\[
= -\frac{8}{125}
\]

✓ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Real Numbers - Part 4

Books & Materials
- Math in Focus - Teacher Edition

Assignments
- Complete Interactive Activity.
- Complete Rate Your Enthusiasm.

LEARN

INTERACTIVE ACTIVITY
Click on this activity to practice your skills with exponents.

RATE YOUR ENTHUSIASM
Please go online to view and submit this assessment.
LEARN

WARM-UP
Expand and evaluate each expression in exponential notation.

1. \(3.4^3\)
2. \((2.5)^4\)
3. \((-6)^2\)

WARM-UP ANSWERS
1. 39.304  
2. 16/625  
3. 36

TEACHING NOTES

INSTRUCTION
Read Use Exponents to Write the Prime Factorization of a Number on p. 8 in Math in Focus 3A. Review Example 5 on p. 9. For each part, find the product of the prime factors to check that the factorizations are correct. Sometimes when you are factoring numbers, the same factor appears more than once. In this case, you can use exponential notation as a sort of shorthand. For instance, the factors of 1,500 are 2, 2, 3, 5, 5, and 5. This is written in the following way:

\[1,500 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 5\]

Instead of writing all of the factors out, you can use exponential notation to write it this way:

\[1,500 = 2^2 \cdot 3 \cdot 5^3\]
Complete **Guided Practice** on p. 10. Find the product of the factors to check your answers.

Review **Example 6** on p. 10. The drawing in the solution for part a shows only one purple blob for the first sample. This blob represents all 20 bacteria. After one hour, there are two purple blobs or $2 \cdot 20$ bacteria.

In part b, Shana deposits $100. Use this value to complete the problem. Estimate the answer mentally before you solve the problem to help you catch errors.

...she will earn $5 in the first year. If she earns $5 interest per year for five years, Shana will earn $25 in five years. However, her interest *increases* each year because this is an exponential relationship, so Shana’s account value at the end of five years is probably between $125 and $130. Now complete the problem and compare your answer to the estimate.

Complete **Guided Practice** on p. 11. Make an estimate for each answer before completing the problem.

**HELPFUL ONLINE RESOURCE**

**Instructional Video:** [Prime Factorization](#)

---

**TEACHING NOTES**

Often students anticipate linear results, even for exponential problems. As long as the base is greater than one and the exponent is greater than one, exponential expressions grow at an *increasing* (vs. a constant) rate. In part b of **Example 6**, Shana earns $5 interest the first year, but more than $5 ($5.25) in the second year because she is now earning interest on the first year’s interest as well as on the principal. In the third year, she earns still more ($5.51), and so on. Your student can calculate the interest and new account amount at the end of each year to arrive at $127.63 at the end of the fifth year. Point out that using the formula with the exponent gives this result much faster.

---

**PRACTICE**

Complete problems 19–26 of **Practice 1.1** on p. 12 in *Math in Focus 3A*. 
Textbook Answer Key

If your student struggles with problem 26, ask how much paper Jen has remaining after she throws away half. Your student can draw the first three situations to determine the base is \( \frac{1}{2} \) for this problem.

Today you learned how to write the prime factors for a number in exponential notation. You factored numbers, then recognized multiple factors and wrote them in exponential notation. You also used your knowledge of exponential expressions to solve real-world problems.

If you are able, use the text box to show your work and enter your final answer to each question. If not, complete your work on paper and upload it below.

10,000; \( \sqrt[3]{1,000,000} \); \( \sqrt{1,000,000} \)

1. Write each number as 10 raised to a power.
2. Order the expressions from greatest to least.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Write each number as 10 raised to a power?
- Order the expressions from greatest to least?
- Show all your work either directly in the text box or on a piece of paper uploaded to the site?
**Properties of Exponents - Part 1**

**Objectives**
- Identify the Product of Powers Property.
- Use the Product of Powers Property to simplify expressions.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*
- online graphing calculator (Optional)

**Assignments**
- Complete Warm-up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete Practice Questions.

---

**LEARN**

---

**WARM-UP**
Use mental math to simplify each expression.

1. $100 \cdot 100$
2. $8 \cdot 8$

---

**TEACHING NOTES**

---

**WARM-UP ANSWERS**
1. 10,000  
2. 64

---

**INSTRUCTION**

Read **Understand the Product of Powers Property** on p. 13 in *Math in Focus 3A*.

You can apply the Product of Powers Property to the problems in the Warm-up. The left side of each equation can also be written in scientific notation.

\[
100 \cdot 100 = 10^4 \quad 8 \cdot 8 = 2^6 \\
(10 \cdot 10) \cdot (10 \cdot 10) = 10^4 \quad (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = 2^6 \\
10^2 \cdot 10^2 = 10^4 \quad 2^3 \cdot 2^3 = 2^6 \\
\]

These equations validate the Product of Powers Property. In both cases, the exponential expressions on each side of the equal sign have the same base. The Product of Powers Property only works when the exponential expressions have the same base.

Review **Example 7** on p. 14. Notice that raising a number to the first power results in the same number. Complete **Guided Practice** on p. 14. Use a calculator to check your answers.
Review **Example 8** on p. 15. Use the Product of Powers Property to simplify the expressions. Complete **Guided Practice** on p. 15.

Read the instructional section (in the purple box) on the bottom of p. 15 in *Math in Focus 3A*.

Then review **Example 9** on p. 16. This example also uses the *Commutative Property of Multiplication*, which states that factors can be multiplied in any order without changing the value of the product. In this example, you will use this property to rearrange the factors before multiplying.

Complete **Guided Practice** on p. 16. Remember to group factors with common bases and apply the Product of Powers Property only to exponential expressions with the same base.

### TEACHING NOTES

Make sure your student spends some time on **Example 7**. Expanding each exponential expression into factors will help him understand the Product of Powers Property. A thorough understanding of this property is essential for comprehending **Example 9**. Be sure to point out to your student that the property only applies to expressions with the same base.

*Note:* You can find a list of suggested online graphing calculators in the Online Resources section in the Appendix of this manual.

### WATCH FOR THESE COMMON ERRORS

Your student may try to combine exponents even when the bases are not the same. For instance, your student may try to say $x^2y^2=(xy)^4$. To help him see this is not correct, substitute different numbers for $x$ and $y$ and simplify the expression, similar to the steps in **Example 7**.

### PRACTICE

Complete problems 1–5 and 7–9 of **Practice 1.2** on p. 24 in *Math in Focus 3A*.

### TEACHING NOTES

[Textbook Answer Key]
**WRAP-UP**

Today you learned how to multiply exponential expressions with the same base. To find the product of two exponential expressions with the same base, add the exponents.

\[ 3a^2b^3c \cdot 2ac^2 = (3 \cdot 2)a^{2+1}b^3c^{1+2} \]
\[ = 6a^3b^3c^3 \]

You can check your answer by expanding the exponential expressions into their factors.

\[ 3a^2b^3c \cdot 2ac^2 = (3 \cdot a \cdot a \cdot b \cdot b \cdot c) \cdot (2 \cdot a \cdot c \cdot c) \]
\[ = (3 \cdot 2)(a \cdot a \cdot c)(b \cdot b \cdot c \cdot c) \]
\[ = 6a^3b^3c^3 \]

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
# Properties of Exponents - Part 2

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Books &amp; Materials</th>
<th>Assignments</th>
</tr>
</thead>
</table>
| - Identify the Quotient of Powers Property.  
- Use the Quotient of Powers Property to simplify expressions. | - *Math in Focus 3A*  
- *Math in Focus - Teacher Edition* | - Complete Warm-up.  
- Read and complete assigned pages in *Math in Focus 3A*.  
- Complete Practice Questions. |

## LEARN

### WARM-UP

Solve each problem.

1. What is $2^{10}$?
2. What is $2^5$?

### TEACHING NOTES

#### WARM-UP ANSWERS

1. 1,024  
2. 32

### INSTRUCTION

Read **Understand the Quotient of Powers Property** on p. 17 in *Math in Focus 3A*. When dividing two expressions with the same base, the exponent of the quotient is the difference of the exponents in the dividend and divisor.

The Quotient of Powers Property can be used to simplify $3^7 ÷ 3^3$ because the bases are the same. It cannot be used in $3^7 ÷ 2^3$ because the bases are not the same.

Review **Example 10** on p. 18. Then complete **Guided Practice** on pp. 18–19. Remember, when you divide numerical expressions with the same base, subtract the exponents.

Review **Example 11** on p. 19. You can use the Quotient of Powers Property with variable bases in the same way you do when the base is a number. Complete **Guided Practice** on p. 19.
Read the bottom instructional section on p. 19. The Quotient of Powers Property applies to algebraic expressions that have the same base. Subtract the exponent of the divisor from the exponent of the dividend. The base (the variable) stays the same.

Review Example 12 on p. 20. Rewrite the expressions as a product of fractions. Finally, complete Guided Practice on p. 20. Remember, to divide algebraic expressions that have the same base, subtract the exponents.

\[ d^6 ÷ d^4 = d^{6-4} = d^2 \]

**TEACHING NOTES**

Have your student simplify a division expression by writing it in expanded form. For example,

\[
\frac{2^5 ÷ 2^3}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{32}{8} = 4
\]

Then, have him use the Quotient of Powers Property to subtract exponents.

\[
2^5 ÷ 2^3 = 2^{5-3} = 2^2 = 4
\]

He should notice he finds the same answer.

**WATCH FOR THESE COMMON ERRORS**

Make sure your student understands that when dividing powers, he should not divide the exponents.

\[ 4^6 ÷ 4^2 = 4^4, \text{ not } 4^3. \]
INTERACTIVE ACTIVITY

Begin by watching a BrainPOP video, *Multiplying and Dividing Exponents*. Then complete a quiz and/or an activity.

PRACTICE

Complete problems 6 and 10–15 of **Practice 1.2** on p. 24 in *Math in Focus 3A*.

TEACHING NOTES

Textbook Answer Key

WRAP-UP

Today you learned how to use the Quotient of Powers Property to simplify numeric and algebraic expressions. The Quotient of Powers Property states that you can divide expressions with the same base by subtracting the exponent of the divisor from the exponent of the dividend.

\[
a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}, a \neq 0
\]

\[
x^9 \div x^3 = x^{9-3} \quad \text{Use the Quotient of Powers Property.}
\]

\[
= x^6 \quad \text{Simplify.}
\]

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Properties of Exponents - Part 3

Objectives

- Multiply and divide expressions in exponential notation.

Books & Materials

- Math in Focus 3A

Assignments

- Complete Warm-up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete Quick Check.

LEARN

WARM-UP

Write each of the following in standard form.

1. \(10^1\)
2. \(10^2\)
3. \(10^3\)
4. \(10^4\)

TEACHING NOTES

WARM-UP ANSWERS

1. 10  2. 100  3. 1,000  4. 10,000

INSTRUCTION

Read Multiply and Divide Expressions in Exponential Notation on p. 21 in Math in Focus 3A. In this lesson, you will multiply and divide exponents within the same expression. Remember:

- To multiply expressions with like bases, add the exponents.
- To divide expressions with like bases, subtract the exponents.

You may need to use the Commutative Property to regroup like bases together.

\[x^6 \cdot 2y^5 \cdot 2y \cdot x^4\] is equivalent to \[x^6 \cdot 2y^5 \cdot 2y \cdot x^4\] .

Review Example 13 on pp. 21–22. Notice these problems have the same base in the numerator and denominator. Complete Guided Practice on p. 22. Remember to rewrite the problem to group like bases if needed.
Review Example 14 on p. 23. In this example, 100 could have been rewritten as $10^2$. The problem is still solved the same way. Complete Guided Practice on p. 23.

### TEACHING NOTES

Your student can break down expressions into simpler problems. For example, $4^4 \cdot 4^3 \cdot 4^3 \cdot 4^2 \cdot 4^2$ can be broken down by simplifying the numerator and denominator separately. By using the Product of Powers Property, the numerator, $4^4 \cdot 4^3 \cdot 4^3$, simplifies to $4^{10}$. By using the Product of Powers Property, the denominator, $4 \cdot 4^2 \cdot 4^2$, simplifies to $4^5$. By using the Quotient of Powers Property, $4^{10} / 4^5$ simplifies to $4^5$.

### PRACTICE

Complete problems 16–23 of Practice 1.2 on p. 24 in Math in Focus 3A.

### TEACHING NOTES

Textbook Answer Key

If your student struggles with problem 23 on Math in Focus 8, Volume A, Chapter 1, p. 24, remind him that the formula for volume is length times width times height. Therefore, in part a he should initially multiply $3x \cdot 2x \cdot x$. He can complete part b similarly; each dimension is just doubled. Finally in part c, he can use the Quotient of Powers Property to find how much greater the volume of the larger prism is.

### WRAP-UP

Today you learned to simplify expressions using the properties of exponents.

\[
\left( \frac{1}{5} \right)^4 \cdot \left( \frac{1}{5} \right)^2 \cdot \left( \frac{1}{5} \right)^4 = \left( \frac{1}{5} \right)^{12}
\]

Apply the Product of Powers Property: The bases are the same, so add the exponents.

\[
\left( \frac{1}{5} \right)^2 \cdot \left( \frac{1}{5} \right)^2 \cdot \left( \frac{1}{5} \right)^2 = \left( \frac{1}{5} \right)^{12}
\]

Apply the Quotient of Powers Property: The bases are the same, so subtract the exponents.

### QUICK CHECK

Please go online to view and submit this assessment.
MORE TO EXPLORE

If you chose the wrong answer, practice multiplying and dividing exponents by writing out the factors and simplifying. Revisit the material from this lesson.
LEARN

WARM-UP
Solve each problem.

1. What is the product when 5 is used as a factor 3 times?

2. What is the product when 2 is used as a factor 9 times?

3. What is the product when –3 is used as a factor 3 times?

4. What is the product when –3 is used as a factor 4 times?

WARM-UP ANSWERS
1. 125  2. 512  3. –27  4. 81

INSTRUCTION
Read p. 25 in *Math in Focus 3A*. When you have an exponential expression that is raised to a power, you can multiply the exponents in the form of \((a^m)^n = a^{mn}\).

The parentheses indicate that the entire expression contained within them is the base. For example, \((3^2)^4\) is not \(3^6\). You should recognize that \((3^2)^4\) means that \(3^2\) (or \(3 \times 3\)) is used as a factor 4 times.

\[(3 \times 3) \times (3 \times 3) \times (3 \times 3) = 3^8\]

Complete **Hands-On Activity** on p. 26. Draw 3 cards and choose which card to use for \(a\), \(m\), or \(n\) in the form of \((a^m)^n\). Try to make the greatest number possible and the least number possible using the cards.
Notice which value \((a, m, \text{ or } n)\) should be greatest to find the greatest number possible and which value should be least to find the least number possible.

Review Example 15 on p. 27. Then complete Guided Practice on p. 28.

### TEACHING NOTES

Your student can evaluate an expression in the form of \((a^m)^n\) alternatively by using the order of operations to evaluate an expression. Evaluate the expression inside the parentheses, \(a^m\). Then use the product inside the parentheses as a factor of the exponent. For example, \((5^3)^2 = (5 \cdot 5 \cdot 5)^2 = 125^2 = 125 \cdot 125 = 15,625\).

By using the Power of a Power Property, \((5^3)^2 = (5 \cdot 5 \cdot 5)^2 = (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) = 5^6 = 15,625\)

### PRACTICE

Complete problems 1–13 of Practice 1.3 on p. 31 in Math in Focus 3A.

### TEACHING NOTES

Textbook Answer Key

In problem 13, your student is asked to explain if the student work \((a^3)^2 = a^5\) is correct. Remind him when you raise a power to a power, you multiply the exponents. Therefore, \((a^3)^2 = a^{3 \cdot 2}\).

### WRAP-UP

Today you learned that when you raise a power to a power, you use the base and multiply the exponents. In the numeric expression, \((2^3)^5\), you use the base 2 as a factor \(3 \cdot 5\) times, so, \(2^3 \cdot 5\) simplifies to \(2^{15}\). The expression \(2^{15}\) means using the expression 2 as a factor 15 times. Then \(2^{15}\) simplifies to 32,768.

In the algebraic expression \([(4b)^2]^3\), you use the base, \(4b\), as a factor \(2 \cdot 3\) times. \((4b)^2 \cdot 3\) simplifies to \((4b)^6\). The expression \((4b)^6\) means use the expression \(4b\) as a factor 6 times, so, \(4^6 \cdot b^6\) simplifies to \(4,096b^6\).
Please go online to view and submit this assessment.
Properties of Exponents - Part 5

Objectives
- Use properties of exponents to simplify expressions.

Books & Materials
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*

Assignments
- Complete Warm-up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete Quick Check.

LEARN

WARM-UP

Simplify.

1. \((-3)^2\)  
2. \((-3)^2\)  
3. \((-2)^3\)  
4. \((-2)^3\)

WARM-UP ANSWERS

1. 9  
2. -9  
3. -8  
4. -8

TEACHING NOTES

WARM-UP ANSWERS

1. 9  
2. -9  
3. -8  
4. -8

INSTRUCTION

Read *Use Properties of Exponents to Simplify Expressions* on p. 28 in *Math in Focus 3A*. Recall the properties of exponents that you have learned so far:

- The *Product of Powers Property* allows you to multiply two expressions with the same base by adding the exponents and keeping the common base.

- The *Quotient of Powers Property* allows you to divide two expressions with the same base by subtracting the exponents and keeping the common base.

- The *Power of a Power Property* allows you to raise a power to a power by keeping the base and multiplying the exponents.
Review Example 16 on pp. 28–29. Remember that a variable without an exponent such as $x$ can be written as $x^1$.

Complete Guided Practice on p. 30. Remember that when working with expressions with brackets, first simplify inside the brackets.

Your student may notice a pattern of when solutions are positive or negative when simplifying expressions. Even powers of negative numbers allow for the negative values to be arranged in pairs. This pairing guarantees that the answer will always be positive, since $(-1)(-1) = 1$. A negative number raised to an even power will be positive. For example, $(-3)^2 = 9$.

Odd powers of negative numbers, however, always leave one factor of the negative number not paired. This one lone negative term guarantees that the answer will always be negative. A negative number raised to an odd power will be negative. For example, $(-3)^3 = -27$.

In this game, Exponent Pirates, you will practice using properties of exponents to earn gear for your pirate.

Complete problems 14–29 of Practice 1.3 on p. 31 in Math in Focus 3A.
Problems 28 and 29 provide an additional challenge, as they require multiple steps. Remind your student to break down each problem into simpler problems. The dividend and the divisor of the expression should be simplified separately.

In the problem \((12)^2 \cdot (13)^4 (1232)^2\), simplify \((13)^2 \cdot (13)^4\) first. Next, simplify \((1332)^2\).

Then the quotient can be determined.

Today you learned how to use properties of exponents to simplify expressions. Remember to simplify within grouping symbols first, and then apply the Power of a Power Property.

\[
(s^3 \cdot s^3)^4 = (s^{3+3})^4 \\
= (s^6)^4 \\
= s^{6\cdot4} \\
= s^{24}
\]

Use the Product of Powers Property.

Simplify.

Use the Power of a Power Property.

Simplify.

Please go online to view and submit this assessment.

If you struggled with this question, write out each step and make one simplification in each step, naming the property you used for that step, such as Product of Powers Property, Quotient of Powers Property, etc. Revisit the material in this lesson.
Properties of Exponents - Part 6

Objective
- Identify the Power of a Product Property.
- Use the Power of a Product Property to simplify expressions.

LEARN

WARM-UP
Write each product in expanded form.

1. Use 3 as a factor 3 times.
2. Use 2 as a factor 4 times.
3. Use 5 as a factor 2 times.
4. Use 6 as a factor 5 times.

WARM-UP ANSWERS

1. $3 \cdot 3 \cdot 3$
2. $2 \cdot 2 \cdot 2 \cdot 2$
3. $5 \cdot 5$
4. $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

TEACHING NOTES

INSTRUCTION

Read Understand the Power of a Product Property on p. 32 in Math in Focus 3A. Remember, when multiplying numbers, you can group factors together in different ways; this is the Associative Property of Multiplication. For example:

$$(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5)$$

Both sides of the equation give a value of 60.

You may have noticed a common theme in this chapter. Most of the properties are presented using numbers and then generalized to variables. This lesson follows that same pattern. To find the product of
You may have noticed a common theme in this chapter. Most of the properties are presented using numbers and then generalized to variables. This lesson follows that same pattern. To find the product of two expressions with the same exponent, multiply their bases and raise that product to the common exponent.

Review Example 17 on p. 33. Remember, a negative number raised to an odd power gives a negative result.

Complete Guided Practice on p. 33. To simplify a numeric expression using the Power of a Product Property, keep the exponent the same and multiply the bases.

Review Example 18 on p. 34. In part c, you could choose to use the Power of a Power Property first:

\[
\left(\frac{1}{4x}\right)^7(-20x^2)^{14} = \left(\frac{1}{4x}\right)^7(-20)^{14}x^{14} \\
= \left(\frac{1}{4}\right)^7(-20)^{14}\left(\frac{1}{x}\right)^{14}x^{14} \\
= \left(-\frac{20}{4}\right)^{14}\left(\frac{1}{x}\right)^{14}x^{14} \\
= (-5)^{14}x^{14} \\
= (-5x)^{14} \\
\]

Use the Power of a Power Property.
Regroup numerals and unknowns.
Multiply \(x^{14}\) by \(\left(\frac{1}{x}\right)^{14}\) to get \(\left(\frac{x^{14}}{x^{14}}\right)\).
Simplify and use the Quotient of Powers Property.
Simplify.

Complete Guided Practice on p. 34.

### TEACHING NOTES

As problems become more complex, such as part c of Example 18, there is often more than one way to solve them. Encourage your student to solve the problem in the way that makes the most sense to him, as long as he can support each step. As an added challenge, you can ask him to begin with a different method and work through the problem again. If all work is done correctly, he should find the same answer. See the previous instructional section regarding part c of Example 18 for an example.

### PRACTICE

Complete problems 1–7 of Practice 1.4 on p. 39 in Math in Focus 3A.

### TEACHING NOTES

Textbook Answer Key
WRAP-UP

Today you learned the Power of a Product Property. For two expressions with the same exponent, keep the exponent the same and multiply the bases.

\[ a^n \cdot b^n = (a \cdot b)^n \]

A numeric example:

\[ 7^4 \cdot 3^4 = (7 \cdot 3)^4 \]

Use the Power of a Product Property.

\[ = 21^4 \]

Simplify.

An algebraic example:

\[ f^n \cdot g^n = (f \cdot g)^n \]

Use the Power of a Product Property.

\[ = (fg)^n \]

Simplify.

✓ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Properties of Exponents - Part 7

**Objectives**
- Identify the Power of a Quotient Property.
- Use the Power of a Quotient Property to simplify expressions.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*

**Assignments**
- Complete Warm-up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete Practice Questions.

---

**LEARN**

---

**WARM-UP**

Write each product in exponential notation.

1. 13 as a factor 3 times
2. 25 as a factor 2 times
3. 37 as a factor 4 times
4. 38 as a factor 5 times

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. $(1/3)^3$  
2. $(2/5)^2$  
3. $(3/7)^4$  
4. $(3/8)^5$

---

**INSTRUCTION**

Read *Understand the Power of a Quotient Property* on p. 35 in *Math in Focus 3A*. You could also simplify the algebraic expression at the bottom of p. 35 by completing the steps in a different order.

\[
\left(\frac{2m}{6n}\right)^3 = \left(\frac{m}{3n}\right)^3
\]

Simplify the fraction.

\[
= \frac{m^3}{27n^3}
\]

Use the Power of a Quotient Property.

Review *Example 19* on p. 36. Remember, when the negative number is inside the parentheses, it is part of the base, so the exponent applies to it.

Complete *Guided Practice* on p. 36. To divide two numeric expressions with the same exponent, keep the exponent and divide the bases. Also, reduce fractions to simplest form.
Review Example 20 on p. 36. An expression may already be in simplest form when written as a power of a quotient.

Finally, complete Guided Practice on p. 37. Again, remember to reduce fractions to simplest form.

TEACHING NOTES

Emphasize to your student that he might choose to start a problem in a different way than the book. As long as his steps can be supported by mathematical principles and he does them correctly, the solution approach is valid.

WATCH FOR THESE COMMON ERRORS

Your student may confuse the Power of a Quotient Property with the Quotient of Powers Property. For the Quotient of Powers Property, you have two powers that are being divided. This property states that to divide powers having the same base, subtract the exponents. That is, for a nonzero base \( a \) and two exponents \( m \) and \( n \), \( \frac{a^m}{a^n} = a^{m-n} \). For the Power of a Quotient Property, you already have a quotient, \((a \div b)\), and that quotient is being raised to a power: \((a \div b)^m\).

PRACTICE

Complete problems 8–16 of Practice 1.4 on p. 39 in Math in Focus 3A.

TEACHING NOTES

Textbook Answer Key

WRAP-UP

Today you learned how to find the quotient of two expressions with the same exponent: divide their bases and keep the same exponent.

\[
\frac{a^m}{b^n} = \left(\frac{a}{b}\right)^n, \quad b \neq 0
\]

A numeric example:

\[
3^5 \div 6^5 = \left(\frac{3}{6}\right)^5
\]

Use the Power of a Quotient Property.

\[
= \left(\frac{1}{2}\right)^5
\]

Simplify.

An algebraic example:

\[
r^5 \div s^5 = \left(\frac{r}{s}\right)^5
\]

Use the Power of a Quotient Property.
PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Properties of Exponents - Part 8

Books & Materials
- Math in Focus - Teacher's Edition
- Math Notebook

Assignments
- Complete Interactive Activity.
- Complete Use For Mastery.

LEARN

INTERACTIVE ACTIVITY

Follow these instructions for the activity shown below.

Click here to view the activity in a new window.

This Gizmo will help you practice simplifying exponential expressions.

When you begin, you should see the expression \((3^4)^5\) at the top of the Gizmo. Follow these steps, writing your answers in your Math Notebook:

1. Write \(x^5\) as the product of repeated factors. \(x^5 =\)

2. Now write \((3^4)^5\) as the product of repeated factors. \((3^4)^5 =\)

3. Simplify the product so it has a single exponent.

In the Gizmo, choose the correct step. If your choice is incorrect, read the given feedback and try again. What is the simplified final answer? Share your response with your Learning Guide.

Click New. You should now see the expression \((4b^{-3})^{-2}\) in the Gizmo. Each factor in parentheses \((4\text{ and }b^{-3})\) is raised to the \(-2\) power. How can you rewrite this expression to show that? Select that tile in the Gizmo. You should now have \(4^{-2}(b^{-3})^{-2}\). How does each factor of this product simplify? Choose the last correct step. What is the simplified final answer? Share your response with your Learning Guide.
Keep practicing in the Gizmo. Then simplify these expressions and write your answers in your Math Notebook.

1. \((5x^4)^2\)
2. \((3a^3b^5)^4\)
3. \((2m^6)^{-4}\)

**Answers:**

1. \(x^5 = x \cdot x \cdot x \cdot x \cdot x\)
2. \((3^4)^5 = 3^4 \cdot 3^4 \cdot 3^4 \cdot 3^4 \cdot 3^4\)
3. \(3^{20}\)

\((4b^{-3})^{-2} = 4^{-2}(b^{-3})^{-2} = 1\ 16\ (b^6) = 1\ 16\ b^6\)

1. \((5x^4)^2 = (5^2)x^6 = 25x^6\)
2. \((3a^3b^5)^4 = (3^4)a^7b^9 = 81a^7b^9\)
3. \((2m^6)^{-4} = (2^{-4})m^2 = 1\ 16\ m^2\)

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.
Did you:

- Simplify the expression shown and enter your answer in the box provided?
- Use the text box to show your work OR use a piece of paper to show your work and upload when finished?
- Solve the problem using a logical, ordered approach?

Simplify.

\[
\frac{7^7 \cdot (3^4)^3}{21^9}
\]

a. Type the correct answer in the box.

b. If you are able, use the text box to show your work. If not, complete your work on paper and upload it below.
Exponents and Roots - Part 1

Objectives
- Use properties of exponents to simplify expressions.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete Practice Questions.

LEARN

WARM-UP
Simplify each expression.

1. $5^7 \cdot 7^7$
2. $x^8 \cdot y^8$
3. $14^3 \div 7^3$
4. $p^5 \div q^3$

TEACHING NOTES

WARM-UP ANSWERS
1. $35^7$ 2. $(xy)^8$ 3. $2^3$ 4. $(p/q)^5$

INSTRUCTION

Read Use Properties of Exponents to Simplify Expressions on p. 37 in Math in Focus 3A.

Review Example 21 on pp. 37–38. Be sure you understand the properties used to simplify the expressions in each step.
Complete Guided Practice on p. 38. Remember you can use a combination of the Product of Powers, Quotient of Powers, Power of a Power, Power of a Product, and Power of a Quotient Properties. The properties are summarized in the following table.

<table>
<thead>
<tr>
<th>Property</th>
<th>Base</th>
<th>Exponent</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product of Powers</td>
<td>Same</td>
<td>Add</td>
<td>(a^n \cdot a^m = a^{n+m})</td>
</tr>
<tr>
<td>Quotient of Powers</td>
<td>Same</td>
<td>Subtract</td>
<td>(\frac{a^n}{a^m} = a^{n-m})</td>
</tr>
<tr>
<td>Power of a Power</td>
<td>Same</td>
<td>Multiply</td>
<td>((a^n)^m = a^{nm})</td>
</tr>
<tr>
<td>Power of a Product</td>
<td>Different (\rightarrow) Multiply</td>
<td>Same</td>
<td>(a^n \cdot b^n = (ab)^n)</td>
</tr>
<tr>
<td>Power of a Quotient</td>
<td>Different ((b \neq 0) \rightarrow) Divide</td>
<td>Same</td>
<td>(\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n)</td>
</tr>
</tbody>
</table>

**TEACHING NOTES**

In this part, your student is simplifying expressions using both the Power of a Product Property and the Quotient of a Product Property. He can use the table as a review of the chapter (thus far) by looking through the chapter to find on which page each property is explained.

**PRACTICE**

Complete problems 17–26 of Practice 1.4 on p. 39 in Math in Focus 3A.

**TEACHING NOTES**

Textbook Answer Key

In problem 25, your student is asked to explain if \(a^3 \cdot b^3 = ab^3\) is correct. Remind him the Power of a Product Property states that when you have two bases with the same exponent, keep the common exponent and multiply the bases.

**WRAP-UP**

Today you learned to combine the properties of exponents to simplify expressions. These problems require you to first simplify the power of a product and then the power of a quotient.

\[
\frac{3^8 \cdot 6^8}{3^8 \cdot 3^8} = \frac{(3 \cdot 6)^8}{(3 \cdot 3)^8} \quad \text{Use the Power of a Product Property.}
\]

\[
= \frac{18^8}{9^8} \quad \text{Simplify.}
\]

\[
= \left(\frac{18}{9}\right)^8 \quad \text{Use the Power of a Quotient Property.}
\]

\[
= 2^8 \quad \text{Simplify.}
\]
Please go online to view and submit this assessment.
Exponents and Roots - Part 2

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Books &amp; Materials</th>
<th>Assignments</th>
</tr>
</thead>
</table>
| ● Identify the value of an exponent of zero as 1.  
● Simplify expressions with an exponent of zero. | ● Math in Focus 3A  
● Math in Focus - Teacher Edition  
● online graphing calculator (Optional) | ● Complete Warm-up.  
● Read and complete assigned pages in Math in Focus 3A.  
● Complete Practice Questions. |

WARM-UP

Answer the following questions.

1. What happens when you add or subtract 0 from a number?

2. What happens when you multiply a number by 0?

3. What happens when you divide a number by 0?

LEARN

TEACHING NOTES

WARM-UP ANSWERS

1. The number stays the same.  
2. The result is 0.  
3. You cannot divide by zero.

INSTRUCTION

Complete Hands-On Activity on p. 40 in Math in Focus 3A. In Step 1, use the Quotient of Powers Property to simplify the expressions. (Note: The instructions in the book direct you to use the Power of a Quotient Property, but this is an error.) Remember the Quotient of Powers Property states that to find the quotient of two expressions with the same base, you subtract their exponents. Pay close attention when simplifying $3^5 / 3^3$.

Steps 2 and 3 are shown as follows.

\[
\frac{3^5}{3^3} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3} \quad \text{Write in expanded form.}
\]

\[
= \frac{243}{243} \quad \text{Simplify.}
\]

\[
= 1 \quad \text{Evaluate.}
\]
This can be rewritten in exponential form.

\[ 3 \times 3 \times 3 = 3^{-5} = 3^0 = 1 \]

Read **Understand the Zero Exponent** on p. 41.

Review **Example 22** on pp. 41–42. Pay careful attention to the **Math Note**. This will be very useful when you study scientific notation.

Finally, complete **Guided Practice** on p. 42. Remember, any number (except zero) raised to the power of 0 is equal to 1. For example, \(-12^0 = 1\).

---

**TEACHING NOTES**

Your student can add a row to the table he constructed in the previous lesson with the Zero Exponent Property. He should fill in all the information for each column. Recommend to your student to use summarizing activities such as tables to keep information handy and organized.

---

**PRACTICE**

Complete problems 1–8 of **Practice 1.5** on p. 46 in *Math in Focus 3A*.

---

**TEACHING NOTES**

**Textbook Answer Key**

---

**WRAP-UP**

Today you learned that any nonzero number raised to the power of 0 equals 1. Sometimes it is not obvious that a problem involves a zero exponent.

\[ \frac{2 \cdot 2^7}{2^8} \]

For example, in simplifying the expression the Product of Powers Property is used first.
\[
\frac{2 \cdot 2^7}{2^8} = \frac{2^{1+7}}{2^8} \quad \text{Use the Product of Powers Property.}
\]
\[
= \frac{2^8}{2^8} \quad \text{Simplify.}
\]
\[
= 2^{8-8} \quad \text{Use the Quotient of Powers Property.}
\]
\[
= 2^0 \quad \text{Simplify.}
\]
\[
= 1 \quad \text{Evaluate.}
\]

This expression involved a power of 0 as it was being simplified.

**TEACHING NOTES**

Remind your student to use the previous properties: Product of Powers, Quotient of Powers, Power of a Power, Power of a Product, and Power of a Quotient. The expressions in this section may look similar to problems solved in previous lessons; however, at some point in the problem, your student may have an exponent of 0.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Exponents and Roots - Part 3

**Objectives**
- Identify the value of a negative integer exponent as a fraction.
- Simplify exponents with negative integer exponents.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*

**Assignments**
- Complete Warm-up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete Quick Check.

---

**LEARN**

**WARM-UP**

Write the reciprocal of each number or expression.

1. 6
2. \(\frac{1}{5}\)
3. \(\frac{1}{y}\)
4. \(x\)

**WARM-UP ANSWERS**

1. \(\frac{1}{6}\)  2. \(\frac{1}{5}\)  3. \(\frac{1}{y}\)  4. \(\frac{1}{x}\)

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. \(\frac{1}{6}\)  2. \(\frac{1}{5}\)  3. \(\frac{1}{y}\)  4. \(\frac{1}{x}\)

---

**INSTRUCTION**

Complete *Hands-On Activity* on p. 43 in *Math in Focus 3A*. In Step 3, you are simplifying \(\frac{4^5}{4^7}\) as shown.

\[
\frac{4^5}{4^7} = \frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} \\
= \frac{1}{4 \cdot 4} \\
= \frac{1}{4^2}
\]

Write in expanded form.

Simplify.

Write using exponents.
You can also use the Quotient of Powers Property to simplify \( \frac{4^5}{4^7} \). The Quotient of Powers Property states that to find the quotient of two expressions with the same base, subtract their exponents.

\[
\frac{4^5}{4^7} = 4^{5-7} \quad \text{Use the Quotient of Powers Property.}
\]

\[
= 4^{-2} \quad \text{Simplify.}
\]

\[
= \frac{1}{4^2} \quad \text{Write using a positive exponent.}
\]

A negative exponent is used to represent a fraction that is in the form of 1 over the base. For example, \( x^{-3} \) is equivalent to \( \frac{1}{x^3} \).

Review Example 23 on p. 44. Then complete Guided Practice on p. 45. Remember to rewrite an expression with a negative exponent as a positive exponent by writing the reciprocal of the base.

### TEACHING NOTES

If your student is having difficulty deciding what part of the term is raised to a power, have him look at the following examples.

\( 5a^{-3} \) Only the term immediately to the left of the exponent is affected, so only \( a \) is raised to the power of \(-3\).

\( (5a)^{-3} \) The parentheses group \( 5a \) together, so both 5 and \( a \) are raised to the power of \(-3\).

### PRACTICE

Complete problems 9–26 of Practice 1.5 on p. 46 in Math in Focus 3A.

### TEACHING NOTES

Textbook Answer Key

### WRAP-UP

Today you learned to identify the value of an expression with a negative integer exponent and to simplify exponential expressions with negative integer exponents.
QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

If you struggled with this question, try writing out the factors. You might also wish to revisit the material from this lesson.
# Exponents and Roots - Part 4

## Objectives
- Find the square root of a positive real number.
- Find the cube root of a positive real number.

## Books & Materials
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*
- online graphing calculator (Optional)

## Assignments
- Complete Warm-up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete Practice Questions.

## LEARN

### WARM-UP

Simplify.

1. $4^2$
2. $(–5)^2$
3. $(-2)^3$
4. $3^3$

### WARM-UP ANSWERS

1. 16  
2. 25  
3. –8  
4. 27

## TEACHING NOTES

### WARM-UP ANSWERS

1. 16  
2. 25  
3. –8  
4. 27

## INSTRUCTION

Read **Evaluate Square Roots of Positive Real Numbers** on p. 47 in *Math in Focus 3A*.

Squaring a number, or raising it to the second power, is multiplying a number by itself. For example, $2 \times 2 = 4$ and $-2 \times -2 = 4$. The positive square root of 4 is shown by $\sqrt{4} = 2$. The negative square root of 4 is shown by $-\sqrt{4} = -2$.

Review **Example 24** on p. 47. Notice, there are two square roots: a positive square root and a negative square root.

Complete **Guided Practice** at the top of p. 48.

To find the square root without a calculator, start by guessing a solution and squaring it, such as $11^2 = 121$. 
Read **Evaluate Cube Roots of Positive Real Numbers** on p. 48. Be sure you understand that cubing a number, or raising it to the third power, is using a number as a factor three times. For example, \(2^3\) means \((2 \cdot 2 \cdot 2) = 8\). The positive cube root of 8 is shown by \(8^{1/3} = 2\).

Review **Example 25** on p. 48. Notice there is only one cube root. Think back to when you learned to use exponents. When the base is negative, the solution has a pattern, based on the exponent. A negative base raised to an even exponent gives a positive answer: \(-4^2 = 16\). A negative base raised to an odd exponent gives a negative answer: \(-4^3 = -64\). So there is only one cube root of \(-64\), which is \(-4\), and only one cube root of +64, which is 4. Every number has exactly one cube root.

Complete **Guided Practice** on the bottom of p. 48.

Read **Solve Equations Involving Squares and Cubes of Variables** at the top of p. 49. It is a good idea to look at the equation first and determine how many answers you will have. For a squared equation, such as \(x^2 = 25\), you will have two solutions, and for a cubed equation, such as \(y^3 = 125\), you will have a single solution.

Review **Example 26** on p. 49. Notice you can use *perfect squares* (numbers for which the square root is an integer) to help you estimate the square root of nonperfect squares. For example, 20 is not a perfect square, but 16 is a perfect square:

\[
\sqrt{16} = 4 \quad \text{and} \quad -\sqrt{16} = -4
\]

The next perfect square is 25:

\[
\sqrt{25} = 5 \quad \text{and} \quad -\sqrt{25} = -5
\]

You can estimate that 20 is between 4 and 5.

Finally, complete **Guided Practice** on pp. 49–50. Remember, you will have two solutions to a square root problem and only one to a cube root problem.

**HELPFUL ONLINE RESOURCE**

**Instructional Video:** [Square Roots](#)

---

### TEACHING NOTES

It is helpful if your student knows the square of integers from 1 to 15. He can make flash cards with the numbers 1 to 15 on one side and the square of those numbers on the reverse side to practice.

### PRACTICE

Complete problems 1–16 of **Practice 1.6** on p. 53 in *Math in Focus 3A.*
Today you learned to find the square root and cube root of a number. The better you know your perfect squares and cubes, the easier they are to work with. Practice memorizing as many perfect squares and cubes as you can.

The first twelve squares are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, and 144.

The first nine cubes are 1, 8, 27, 64, 125, 216, 343, 512, and 729.

Please go online to view and submit this assessment.
Exponents and Roots - Part 5

Books & Materials
- Math in Focus - Teacher Edition
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

Assignments
- Complete Interactive Activity.
- Complete Rate Your Understanding.

LEARN

INTERACTIVE ACTIVITY

In this activity Ordering Rational and Irrational Numbers, you will use this GeoGebra applet to order rational and irrational numbers on a number line.

RATE YOUR UNDERSTANDING

Please go online to view and submit this assessment.
Exponents and Roots - Part 6

Objectives
- Solve real-world problems involving square roots and cube roots.

Books & Materials
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*
- Online graphing calculator (Optional)

Assignments
- Complete Warm-up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete Quick Check.

LEARN

WARM-UP

Answer the following questions.

1. What are the square roots of 49?
2. What are the square roots of 144?
3. What is the cube root of 216?
4. What is the cube root of -1,331?

WARM-UP ANSWERS

1. 7 and -7  2. 12 and -12  3. 6  4. -11

INSTRUCTION

Read Solve Real-World Problems Involving Squares and Cubes on p. 50 in *Math in Focus 3A*. Here is a quick review of areas and volumes: To find the area of a rectangle, multiply the length by the width. A square is a special rectangle, one for which the length and width are the same. If the side of a square is $x$ units long, the area of the square is $x^2$. To find the volume of a rectangular prism, multiply the length, width, and height. A cube is a special rectangular prism, one for which all three sides are the same. If the side of a cube is $y$, its volume is $y^3$.

Review Example 27 on p. 50. Remember that just because a square root has two solutions does not mean both the positive and negative solutions make sense for real-world situations. If the real-world
problem involves lengths or distances, it is likely that only the positive solution to the square root makes sense.

Complete Guided Practice on p. 51. Use a calculator to find square roots of unknowns. Remember the negative square root may not always make sense in a real-world situation.

Review Example 28 on p. 51. Then complete Guided Practice on p. 52. The volume is given in terms of π. Do not multiply the expression for volume and then approximate it with a decimal. Leave it in terms of π.

If your student struggles with when to use square roots and cube roots in the real-world problems, remind him that area involves square units and volume involves cubic units. Area involves two dimensions: length and width. Two dimensions are represented by square units. Volume involves three dimensions: length, width, and height. Three dimensions are represented by cubic units.

Textbook Answer Key

To solve problem 18, your student will first need to find the total area of land needed. There are 3,136 apple trees, and each tree needs 4 square meters to grow. He will need to multiply the number of trees by the amount of space needed for each tree to find the area of the field. The area of the field is 12,544 square meters.

Today you learned how to solve real-world problems involving square roots and cube roots. To solve a squared or cubed root equation, you need to find the value(s) of the variable that makes the equation true by finding the square or cube root of both sides of the equation. Real-world problems involving length can only have positive solutions.
Please go online to view and submit this assessment.

Remember to carefully read word problems to make sure you understand what is being asked. You might also want to revisit the material in this lesson.
Exponents and Roots - Part 7

Objectives
- Apply properties of exponents, square roots, and cube roots to solve problems.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete Brain @ Work in Math in Focus A.
- Complete the Practice activity.
- Complete Math Practice Questions.

LEARN

WARM-UP

Solve.

1. Anna has a jewelry box that has dimensions of 10 inches by 12 inches by 8 inches. Find its volume.
2. Emma has a ball that has a radius of 9 inches. Find its volume \((V = \frac{4}{3}\pi r^3)\).

WARM-UP ANSWERS

1. 960 cubic inches 2. \(0.972\pi\) cubic inches

TEACHING NOTES

INSTRUCTION

Read and complete Brain @ Work on p. 54 in Math in Focus 3A.

When working on problem 1, you are using mental math. Remember to evaluate the numerator and then the denominator. Then simplify. You may check your answer with a calculator.

To solve problem 2, simplify the left side of the equation using the properties of exponents. Begin with the Power of a Power Property. Look for matching factors in the numerator and the denominator. Hint: When you have an
expression of the form \( x^4 = 16 \), take the square root of both sides. In this case, taking the square root of both sides gives \( x^2 = 4 \). (Only use the positive root of 16 for now. You have not learned how to solve \( x^2 = -4 \).) From here, you know how to solve the problem.

Another method for solving problem 2 is to guess a solution and check it. Read on for more information about the guess-and-check method.

Problem 3 involves using the properties of exponents presented in this chapter. You can solve this problem by using the guess-and-check strategy. When you do not know how to solve a problem, making an educated guess is a good strategy to try. You may want to use a table or an organized list to keep track of your guesses.

To use the guess-and-check strategy, look at the problem and make a reasonable guess. Your first guess will probably be wrong, but it will give you information to help you make a better guess next time. Check your guess. If you are not correct, use what you found out in checking your guess to make a better guess. Continue to guess, check, and revise until you find the solution.

When working on problem 4, break the problem into simpler problems. You may not be able to find the radius of one marble in one step from the information given, but you are given enough information to find the volume of one marble. Then you can find the radius of the marble from there.

### TEACHING NOTES

**Textbook Answer Key**

For problem 2, if your student finds only a positive solution for \( x \), ask him if there could be a negative solution.

For problem 3, your student will need to keep track of which digit he tried in each position of the problem. Be sure he writes down the problem completely as he attempts to solve it each time.

For problem 4, your student first needs to find the volume of one marble by dividing 9,720 (the volume of the tank) by 360 marbles.

### PRACTICE

Throughout the course you will be asked to write constructed responses. When you answer a constructed response question, you not only solve the problem, but you also explain how you found the answer. The explanation of a constructed response can use words, a drawing, a chart or table, an equation, or any combination of these. The explanation of your constructed response answers the question *How do you know?*
You may want to write your constructed responses in a separate notebook or Math Journal so that you can evaluate your progress as you continue through the course.

Write a constructed response for each of the following problems to practice.

1. Jacinda wants to build a cover for the sandbox in her backyard. The sandbox is a square with an area of 56.25 square feet. How long should the cover be on each side?
2. Tell whether the following statement is true or false and explain your answer: If \( x^y = 25 \), there are only two possible values for \( x \).

### TEACHING NOTES

**Practice Answers:**

1. The area of a square is the length multiplied by the width. Since the sides of a square are equal, I was able to find the square root of the area to figure out the length of each side. The square root of 56.25 is 7.5, so each side of the sandbox is 7.5 feet long.

2. False; \( 5^2, (-5)^2, (1/5)^{-2}, \) and \( (-1/5)^{-2} \) all satisfy the given equation.

### WRAP-UP

Today you worked with different problem-solving strategies. You used the **guess-and-check** strategy, which requires you to take a guess, test its correctness, and refine the guess using rational thinking. You can make a table or keep an organized list to keep track of your guesses and their accurateness.

When a problem seems too difficult, solve a simpler problem. Instead of solving the given problem, concentrate on only solving part of the problem at a time.

There are many ways to solve a problem. There is no one correct strategy as long as you arrive at the correct solution.

### PRACTICE QUESTIONS

Please go online to view and submit this assessment.
## Exponents and Roots - Part 8

### Books & Materials
- Math in Focus - Teacher Edition
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

### Assignments
- Complete Use for Mastery.

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### USE

---

### USE FOR MASTERY

Simplify the expression. Write your answer in exponential notation.

\[
\frac{(7^5 \cdot 7^3)^4}{7^{-7} \cdot 7^{20}}
\]

If you are able, use the text box to show your work and enter your final answer to each question. If not, complete your work on paper and upload it below.

---

Supported file formats: PDF, JPG, GIF, PNG

---

0 / 12 File Limit
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Simplify the expression?
- Write your answer in exponential notation?
- Use the text box to show your work OR complete your work on paper and upload it to the site?
- Enter your final answer to each question?
- Show your work in a logical, ordered way?

Solve the problem using a logical, ordered approach?

Simplify.

\[
\frac{7^7 \cdot (3^4)^3}{21^9}
\]

a. Type the correct answer in the box.

b. If you are able, use the text box to show your work.

If not, complete your work on paper and upload it below.
Scientific Notation - Part 1

**Objectives**
- Multiply and divide decimals by powers of 10.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*

**Assignments**
- Complete Warm-Up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete Practice Questions.

---

LEARN

**WARM-UP**

Use mental math to multiply or divide.

1. $34 \cdot 10$
2. $179 \cdot 100$
3. $760 \div 10$
4. $22,800 \div 100$

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. 340
2. 17,900
3. 76
4. 228

---

**INSTRUCTION**

Today's lesson covers important ideas you need to understand before beginning the lessons in this chapter.

Read p. 58 in *Math in Focus 3A*. Look at the number given in the paragraph. Imagine having to tell someone else about that number. Scientific notation provides a shorter way to express that number.

Read Recall Prior Knowledge on p. 59. Notice that the exponent in the power of 10 tells you how many places to move the decimal point.
As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the Quick Check.

Complete Quick Check on p. 59 in Math in Focus 3A. Remember to compute using only scientific notation and the rules of exponents. Do not rename the numbers in standard form.

Textbook Answer Key

Notice that in the Quick Check, three problems involve multiplication and three problems involve division. Also note that of the six powers of 10, three are written in standard form and three are written in exponential form. Take some time to relate one form to another.

RETEACH

After your student completes the Quick Check in the Recall Prior Knowledge lesson of this chapter, review the questions that were answered incorrectly. If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him or her complete the activity.

QUICK CHECK

<table>
<thead>
<tr>
<th>Question(s)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–6</td>
<td>Multiply and Divide by Powers of Ten</td>
</tr>
</tbody>
</table>

WRAP-UP

Today you reviewed how to multiply and divide decimals by a power of 10. When you multiply by a positive power of 10, the decimal point moves to the right. It moves the number of places indicated by the exponent in the power of 10 (or by the number of zeros if the number is in standard form). When you divide by a positive power of 10, the decimal point moves to the left. It moves the number of places indicated by the exponent in the power of 10 (or by the number of zeros if the number is in standard form).

\[
4.61 \times 10^3 = 4,610
\]

\[
98.3 \div 10^5 = 0.000983
\]
Today you reviewed how to multiply and divide decimals by a power of 10. When you multiply by a positive power of 10, the decimal point moves to the right. It moves the number of places indicated by the exponent in the power of 10 (or by the number of zeros if the number is in standard form). When you divide by a positive power of 10, the decimal point moves to the left. It moves the number of places indicated by the exponent in the power of 10 (or by the number of zeros if the number is in standard form).

\[ 4.61 \cdot 10^3 = 4610 \]

\[ 98.3 \div 10^5 = 0.000983 \]

Please go online to view and submit this assessment.
LEARN

WARM-UP
Evaluate each power of 10.

1. \(10^5\)
2. \(10^8\)
3. \(10^{-2}\)
4. \(10^{-6}\)

WARM-UP ANSWERS
1. 100,000 2. 100,000,000 3. 1/100 4. 1/1,000,000

TEACHING NOTES

INSTRUCTION
Read p. 60 in Math in Focus 3A. Brainstorm measurements that have very large or very small values. One example of a measurement with very large value is the number of blood cells in a human body. One example of a measurement with a very small value is the mass of an electron.

Then read Write Numbers in Scientific Notation on p. 61. Notice that when a very large number is written in scientific notation, the exponent in the power of 10 is positive. However when a very small number is written in scientific notation, the exponent in the power of 10 is negative. A positive exponent means to multiply, and a negative exponent means to divide.
Review Example 1 on p. 61. Then complete the first Guided Practice on p. 62. Remember that for a number to be written correctly in scientific notation, the coefficient must be at least 1 but less than 10.

Review Example 2 on p. 62. Then complete the second Guided Practice on p. 62. For each number, decide whether the exponent in the power of 10 will be positive or negative.

Read Write Numbers in Standard Form on p. 62. Then review Example 3 and complete Guided Practice on p. 63. You can use multiplication to check your work. For example, for problem 6, you can multiply $9 \times 10 \times 10 \times 10 \times 10$.

HELPFUL ONLINE RESOURCE

Instructional Video: Scientific Notation

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**TEACHING NOTES**

Your student can underline digits to help her write numbers given in standard form as numbers written in scientific notation. For example, given the number 45,000,000, she could underline the 5 and the six zeros: $45,000,000$. The underlining starts at the second digit and the number of digits underlined is the exponent in the power of 10. $45,000,000$ written in scientific notation is $4.5 \times 10^7$.

When given a small number, your student can use the same process in reverse. The underlining starts at the first nonzero digit and ends at the decimal point. The opposite of the number of digits underlined is the exponent in the power of 10. The number $0.0000072$ would look like $0.0000072$. Written in scientific notation, $0.0000072$ is $7.2 \times 10^{-6}$.

---

**PRACTICE**

Complete problems 1–12 and 17–19 of Practice 2.1 on pp. 66–67 in Math in Focus 3A. Remember to compute using only scientific notation and the rules of exponents. Do not rename the numbers in standard form.

---

**TEACHING NOTES**

Textbook Answer Key

---

**WRAP-UP**

Today you learned how to write numbers in scientific notation. A number in scientific notation is a number that is at least 1 but less than 10 multiplied by a power of 10. The purpose of scientific notation
is to prevent having to count zeros to understand the value of a number. A number with a positive exponent in the power of 10 is a large number. A number with a negative exponent in the power of 10 is a small number.

384,000,000,000 in scientific notation is $3.84 \cdot 10^{11}$

0.00006392 in scientific notation is $6.392 \cdot 10^{-5}$

---

**SUPPLEMENTAL**

- BrainPOP: Standard and Scientific Notation

---

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Scientific Notation - Part 3

**Books & Materials**
- Math in Focus - Teacher Edition
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

**Assignments**
- Complete the Interactive Activity.
- Complete Rate Your Enthusiasm.

### LEARN

**INTERACTIVE ACTIVITY**

Try this activity to practice scientific notation.

**RATE YOUR ENTHUSIASM**

Please go online to view and submit this assessment.
Scientific Notation - Part 4

Objectives
- Compare numbers written in scientific notation.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-Up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete Quick Check.

WARM-UP

Compare the numbers using <, >, or =.

1. 5.6 〇 3
2. 6.1 〇 0.61
3. 18.2 〇 18.2
4. -17 〇 -12
5. 0.291 〇 0.2761

TEACHING NOTES

WARM-UP ANSWERS
1. > 2. > 3. = 4. < 5. >

INSTRUCTION

Read Compare Numbers in Scientific Notation on p. 63 in Math in Focus 3A. Be sure you understand that a number written in scientific notation has two parts—a coefficient and a power of 10.

Review Example 4 on p. 64. Then complete Guided Practice on p. 64. Remember to first compare the exponent in the power of 10. Only when the exponents are equal should you compare the coefficients.

Review Example 5 on pp. 64–65. Notice there are two methods presented for solving. The first method shows the numbers given in the problem in standard form and then compares them. The second method shows the numbers in scientific notation and then compares them. The key is that in each method, both numbers need to be written in the same form before comparing.
Complete **Guided Practice** on p. 65. For problems 11–12, be sure to write the numbers in the same form before making a comparison.

### TEACHING NOTES

When given a pair or set of numbers to compare in scientific notation, encourage your student to circle the exponent in the power of 10 to aid in the comparison. For example, if asked to identify the lesser number in the pair $4.3 \cdot 10^3$ and $3.6 \cdot 10^3$, she should circle the exponent 3 in each number. Since $3 = 3$, she would need to compare the coefficients. Since $4.3 > 3.6$, $3.6 \cdot 10^3$ is the lesser number in the pair.

**WATCH FOR THESE COMMON ERRORS**

A student may think that a greater negative exponent results in a greater number, when in fact it is a smaller number. For example, $1.5 \cdot 10^{-4}$ is less than $1.5 \cdot 10^{-2}$ because $-4$ is less than $-2$. You can demonstrate this by comparing the standard forms of the numbers: $0.00015 < 0.15$.

### PRACTICE

Complete problems 13–16 and 20–22 of **Practice 2.1** on pp. 66–67 in **Math in Focus 3A**. Remember to compute using only scientific notation and the rules of exponents. Do not rename the numbers in standard form.

### TEACHING NOTES

Textbook Answer Key

### WRAP-UP

Today you learned how to compare numbers in scientific notation. You can follow these steps to compare numbers.

**Step 1:** Given numbers in different forms, convert all numbers to either scientific notation or standard form.

**Step 2:** Compare the exponents.

**Step 3:** If the exponents are the same, compare the coefficients.
Today you learned how to compare numbers in scientific notation. You can follow these steps to compare numbers.

Step 1: Given numbers in different forms, convert all numbers to either scientific notation or standard form.

Step 2: Compare the exponents.

Step 3: If the exponents are the same, compare the coefficients.

Please go online to view and submit this assessment.

View the video Comparing Numbers in Scientific Notation (06:57) to see how to compare numbers in scientific notation.

Please go online to view this video ►
**Scientific Notation - Part 5**

**Objectives**
- Add and subtract numbers in scientific notation with the same power of 10.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*

**Assignments**
- Complete Warm-up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete Practice Questions.

---

**LEARN**

**WARM-UP**
Rewrite each expression by applying the Distributive Property.

1. \(12 \cdot 6 + 8 \cdot 6\)
2. \(31 \cdot 5 - 19 \cdot 5\)
3. \(2 \cdot 3^2 + 5 \cdot 3^2\)
4. \(10 \cdot 12^{-3} - 2 \cdot 12^{-3}\)

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. \((12 + 8) \cdot 6\)
2. \((31 - 19) \cdot 5\)
3. \((2 + 5) \cdot 3^2\)
4. \((10 - 2) \cdot 12^{-3}\)

**INSTRUCTION**

Read *Add and Subtract Numbers in Scientific Notation with the Same Power of 10* on p. 68 in *Math in Focus 3A*. The first step is to make sure the exponents are equal. Then add or subtract the coefficients. If the coefficient in the sum or difference is not at least 1 and less than 10, use the rules of exponents to change it to a number written in correct scientific notation.

Review *Example 6* on p. 69. In part a, note that the sum of the coefficients, 11.88, cannot be the coefficient of the answer written in scientific notation. Rewrite it as 1.188, and increase the exponent in the power of 10 by 1.
Complete **Guided Practice** on p. 70. Fill in the blanks as you go. Refer to **Example 6** if you need guidance on what type of number goes on each line.

Review **Example 7** on p. 71. Then complete **Guided Practice** on p. 72. Fill in the blanks as you go. Refer to **Example 7** if you need guidance on what type of number goes on each line. Note that in problem 2b, the difference is a number that is not correctly written in scientific notation, so you will need to use the rules of exponents to rewrite it.

Since problems with adding and subtracting scientific numbers involve factoring out the power of 10, have your student begin a problem by writing the power of 10 with an empty set of parentheses for the coefficient. Your student can then cross out the powers of 10 from each scientific number and fill in the parentheses with the coefficients.

For example, begin the problem 3.5 \( \cdot \) 10\(^8\) − 2.1 \( \cdot \) 10\(^8\) by writing (______) \( \cdot \) 10\(^8\). She should then cross out the powers of 10 in the original problem. Then she should fill in the parentheses with the coefficients as shown: (3.5 − 2.1) \( \cdot \) 10\(^8\). Your student can then simplify the problem. Once that step is complete, she should make sure the answer is in scientific notation. If the coefficient is not at least 1 and less than 10, the answer will need to be adjusted.

**WATCH FOR THESE COMMON ERRORS**

Remind your student to check that each final answer is correctly written in scientific notation. For example, using the Distributive Property, 7.8 \( \cdot \) 10\(^5\) + 6.3 \( \cdot \) 10\(^5\) = (7.8 + 6.3) \( \cdot \) 10\(^5\) = 14.1 \( \cdot \) 10\(^5\). The answer in scientific notation is 1.41 \( \cdot \) 10\(^6\).

**PRACTICE**

Complete problems 1, 2, 6, and 8 of **Practice 2.2** on p. 79 in **Math in Focus 3A**. Remember to compute using only scientific notation and the rules of exponents. Do not rename the numbers in standard form.

**TEACHING NOTES**

**Textbook Answer Key**

Note that the table used for problem 6 is also used for problems 5 and 7. Only problem 6 includes numbers in scientific notation with the same power of 10. The other two problems associated with this table will be assigned in the next lesson.
WRAP-UP

Today you learned how to add and subtract numbers in scientific notation with the same powers of 10. Follow the steps in this example to solve this type of problem.

\[9.33 \times 10^7 + 5.41 \times 10^7\]

1. Factor the common power of 10 from each term.
2. Simplify within the parentheses.
3. Write in scientific notation.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
LEARN

**WARM-UP**

Evaluate each expression and express the answer in scientific notation.

1. \(8 \cdot 10^{-3} + 3 \cdot 10^{-3}\)
2. \(6.6 \cdot 10^{11} + 3.2 \cdot 10^{11}\)
3. \(9.83 \cdot 10^4 - 4.2 \cdot 10^4\)

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. \(1.1 \cdot 10^{-2}\)
2. \(9.8 \cdot 10^{11}\)
3. \(5.63 \cdot 10^4\)

**INSTRUCTION**

Read p. 73 in *Math in Focus 3A*. In the previous lesson, you learned how to add and subtract numbers in scientific notation with the same power of 10. In this lesson, you will add and subtract numbers that do not have the same power of 10. When you do this, one of the numbers (or both) will no longer be in scientific notation, but this is a necessary step so that you can add or subtract the coefficients.
Review Example 8 on p. 74. In part a, the speech bubble in the middle of the example shows an alternate method for solving the problem. Here the power $10^7$ is used, whereas in the main example the power used is $10^6$.

Complete Guided Practice on p. 75. Fill in the blanks as you go. Refer to Example 8 if you need guidance on what type of number goes on each line.

Review Example 9 on p. 76. In parts a and b, the power $10^{-3}$ was used for the computation.

Complete Guided Practice on p. 76. Be sure the final answers are written in correct scientific notation.

Read Introduce the Prefix System on p. 77. Then review Example 10 on pp. 77–78. In part a, one of the numbers is given in scientific notation and one is given in standard form.

Complete Guided Practice on p. 78. You can use the chart on p. 77 to help you convert Pluto’s distance to kilometers in scientific notation.

---

**TEACHING NOTES**

When your student is asked to add or subtract numbers with different powers of 10, you might want to have her solve the problem once using the greater exponent in the power of 10 and then a second time using the lesser exponent in the power of 10. This will show her that regardless of which power of 10 she chooses to use as a common factor, she will obtain the same answer when the problem is completed.

To extend learning, have your student do some Internet research and learn how these prefixes in the chart on p. 77 are used in the fields of computer technology and medical science.

---

**PRACTICE**

Complete problems 3–5, 7, and 9–15 of Practice 2.2 on pp. 79–80 in Math in Focus 3A. Remember to compute using only scientific notation and the rules of exponents. Do not rename the numbers in standard form.

---

**TEACHING NOTES**

Textbook Answer Key
WRAP-UP

Today you learned how to add and subtract numbers in scientific notation with different powers of 10. Follow the steps in this example to solve this type of problem.

\[ 5.4 \times 10^5 + 7.7 \times 10^7 \]

\[ 5.4 \times 10^5 + 770 \times 10^5 \]
Rewrite \( 7.7 \times 10^7 \) as \( 770 \times 10^5 \).

\[ (5.4 + 770) \times 10^5 \]
Factor \( 10^5 \) from each term.

\[ 775.4 \times 10^5 \]
Simplify within the parentheses.

\[ 7.754 \times 10^7 \]
Rewrite in scientific notation.

Quick Check

Please go online to view and submit this assessment.

More to Explore

View the video, *Scientific Notation: Adding and Subtracting* (07:07) to learn how to add and subtract expressions that include scientific notation.

Please go online to view this video ➤
Scientific Notation - Part 7

Objectives
- Multiply and divide numbers in scientific notation.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition
- online graphing calculator (Optional)

Assignments
- Complete Warm-up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete Practice Questions.

LEARN

WARM-UP
1. $8^3 \cdot 8^5$
2. $36^{-4} \cdot 36^7$
3. $12^{12} + 12^9$
4. $7^{18} + 7^5$

TEACHING NOTES

WARM-UP ANSWERS
1. $8^8$  2. $36^3$  3. $12^3$  4. $7^{13}$

INSTRUCTION

Read Multiply and Divide Numbers in Scientific Notation on p. 81 in Math in Focus 3A.

Review Example 11 on pp. 81–82. Remember that the Commutative Property states that the order you multiply numbers does not affect the product. Think of a number written in scientific notation as the product of two numbers. So when you multiply two numbers in scientific notation, the product is really the product of four numbers. You can rearrange these numbers so that the coefficients are together and the powers of 10 are together.

Complete Guided Practice on p. 82. Fill in the blanks as you go. Refer to Example 11 if you need guidance on what type of number goes on each line.
Review Example 12 on p. 83. On some graphing calculators, you enter numbers in scientific notation different ways than others. Refer to the calculator’s manual or online instructions to understand how to enter these numbers.

Complete Guided Practice on p. 84. Fill in the blanks as you go.

### TEACHING NOTES

When given a problem to solve, have your student underline the coefficients and circle the powers of 10. For example, in the division expression \((3.6 \cdot 10^5) \div (1.8 \cdot 10^3)\), your student would underline 3.6 and 1.8 and circle \(10^5\) and \(10^3\). She can then more easily identify that she needs to solve two division problems, \(3.6 \div 1.8\) and \(10^5 \div 10^3\). The answer is the product of these quotients. \(3.6 \div 1.8 = 2\), and \(10^5 \div 10^3 = 10^{5-3} = 10^2\). The product of these quotients is \(2 \cdot 10^2\), or 200 when written in standard form.

### PRACTICE

Complete Practice 2.3 on pp. 85–86 in Math in Focus 3A. Remember to compute using only scientific notation and the rules of exponents. Do not rename the numbers in standard form.

### TEACHING NOTES

Textbook Answer Key

Encourage your student to show her work on these problems, so that you can review possible errors. The work that should be shown is similar to the work shown in Examples 11 and 12.

Problems 3 and 4 on p. 85 are shown incorrectly in the book. If the division of two numbers in scientific notation is written horizontally using the division symbol (instead of using a fraction bar), the dividend and divisor should each be enclosed in parentheses. Otherwise, calculating according to the order of operations would result in an incorrect answer. For example, the expression for problem 3 should be written: \((5.75 \cdot 10^{-5}) \div (7.15 \cdot 10^7)\). Encourage your student to observe and explain this error, and to be on the lookout for it in later exercise sets.
WRAP-UP

Today you learned how to multiply and divide numbers in scientific notation. The steps in the following example will help you solve problems of this type.

\[ 5.12 \cdot 10^{11} \cdot 1.8 \cdot 10^{15} \]

\[ 5.12 \cdot 1.8 \cdot 10^{11} \cdot 10^{15} \quad \text{Rewrite using the Commutative Property.} \]

\[ 9.216 \cdot 10^{11} \cdot 10^{15} \quad \text{Multiply the coefficients.} \]

\[ 9.216 \cdot 10^{11 + 15} \quad \text{Use the Product of Powers Property.} \]

\[ 9.216 \cdot 10^{26} \quad \text{Write in scientific notation.} \]

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
LEARN

WARM-UP
Simplify each expression. Write each answer in scientific notation.

1. $6.15 \cdot 10^{12} + 5.3 \cdot 10^{12}$
2. $9.4 \cdot 10^{7} - 7.8 \cdot 10^{6}$
3. $4.12 \cdot 10^{4} \cdot 2.3 \cdot 10^{9}$
4. $(21.6 \cdot 10^{32}) ÷ (5.4 \cdot 10^{29})$

WARM-UP ANSWERS

1. $1.145 \cdot 10^{13}$  
2. $8.62 \cdot 10^{7}$  
3. $9.476 \cdot 10^{13}$  
4. $4 \cdot 10^{3}$

TEACHING NOTES

INSTRUCTION
Read and complete Brain @ Work on p. 86 in Math in Focus 3A. Be sure to show your work. For problem 1, remember that the cube root of a number is a number that when multiplied by itself and by itself again equals the given number.

For problem 2, make sure you follow the order of operations.

For problem 3, use the chart of prefixes on p. 77 in Math in Focus 3A to rewrite each measure in scientific notation. Then add or subtract the numbers.
For many problems involving scientific notation, your student will need to make a substitution before attempting to solve, whether it is for a variable or a prefix unit. For example, consider the problem 8 gigameters + 6 megameters. First, your student will need to replace the terms with the corresponding exponents.

\[ 8 \times 10^9 + 6 \times 10^6 \]

Next, she must rewrite one of the numbers so that they have the same exponent.

\[ 8,000 \times 10^6 + 6 \times 10^6 \]

She can then proceed to solve this problem by adding numbers in scientific notation.

Solve the following problem. Remember to compute using only scientific notation and the rules of exponents. Do not rename the numbers in standard form.

If the sun is \( 1.5 \times 10^{11} \) meters from Earth, and the speed of light is \( 3 \times 10^8 \) meters/second, how many seconds does it take sunlight to reach the Earth?

Practice Answer:

\[ 1.5 \times 10^{11} \text{ meters} = \frac{1 \text{ second}}{3 \times 10^8 \text{ meters}} \]

\[ 1,500 \times 10^8 \text{ meters} \times \frac{1 \text{ second}}{3 \times 10^8 \text{ meters}} = 500 \text{ seconds} \]

Today you learned how to apply problem-solving skills to problems involving numbers in scientific notation. When you are asked to problem solve, remember the following:

- Make sure the powers of 10 are the same before adding or subtracting.
When multiplying numbers in scientific notation, you can multiply the powers of 10 using the Product of Powers Property. This means you will add the exponents.

When dividing numbers in scientific notation, you can divide the powers of 10 using the Quotient of Powers Property. This means you will subtract the exponents.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Scientific Notation - Part 9

Objectives
- Review previously learned concepts.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete assignments in Math in Focus 3A.
- Complete Practice Questions.

LEARN

WARM-UP
Use mental math to simplify each expression.

1. 2.6 + 3.5
2. 8.4 − 3.6
3. 3.1 · 2.4
4. 4.2 ÷ 1.4

WARM-UP ANSWERS
1. 6.1  2. 4.8  3. 7.44  4. 3

TEACHING NOTES

INSTRUCTION
Read Chapter Wrap Up on p. 87 in Math in Focus 3A. The concept map reviews that the coefficient of a number written in scientific notation must be at least 1 and less than 10. It also shows that when the exponent in the power of 10 is negative, the number represents a very small value and when the exponent in the power of 10 is positive, the number represents a very large value.

Read over the key concepts to review more information about scientific notation.
Scientific Notation - Part 9

LEARN

Use mental math to simplify each expression.

1. 2.6 + 3.5
2. 8.4 − 3.6
3. 3.1 \cdot 2.4
4. 4.2 ÷ 1.4

WARM-UP ANSWERS

1. 6.1
2. 4.8
3. 7.44
4. 3

Read Chapter Wrap Up on p.87 in Math in Focus 3A. The concept map reviews that the coefficient of a number written in scientific notation must be at least 1 and less than 10. It also shows that when the exponent in the power of 10 is negative, then the number represents a very small value and when the exponent in the power of 10 is positive, then the number represents a very large value.

Read over the key concepts to review more information about scientific notation.

PRACTICE

Complete Chapter Review/Test on pp. 88–89 in Math in Focus 3A. Remember to compute using only scientific notation and the rules of exponents. Do not rename the numbers in standard form.

WRAP-UP

Today you reviewed concepts associated with scientific notation, including how to write numbers in scientific notation, how to compare numbers in scientific notation, and how to operate with numbers in scientific notation.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Scientific Notation - Part 10

Objectives
- Review previously learned concepts.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete Use for Mastery.

LEARN

WARM-UP
Use mental math to simplify each expression.

1. $15 \cdot 15$
2. $21 \cdot 21$
3. $6^4$
4. $7^3$

TEACHING NOTES

WARM-UP ANSWERS

1. 225
2. 441
3. 1,296
4. 343

INSTRUCTION
Today you will complete Cumulative Review for Chapters 1 and 2. If you have difficulty with any of the problems, make sure you go back and look at the corresponding lesson. Review as necessary to complete the problems.

PRACTICE
Complete Cumulative Review on pp. 90–91 in Math in Focus 3A. Remember to compute using only scientific notation and the rules of exponents. Do not rename the numbers in standard form.
TEACHING NOTES

Textbook Answer Key

Your student has learned many new skills, vocabulary terms, and strategies in the last two chapters. Once your student has completed the review, look over it with her to determine areas of strength and areas that may need more development and practice.

WRAP-UP

Today you reviewed concepts from Chapters 1 and 2.

USE FOR MASTERY

The United States has a labor force of approximately \((1.4\cdot10^8)\) people. The table shows the distribution of the labor force among five categories of occupations. Distribution represents a percentage of the total population.

<table>
<thead>
<tr>
<th>Categories</th>
<th>Distribution of U. S. Labor Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Farming, forestry, and fishing</td>
<td>0.01</td>
</tr>
<tr>
<td>Manufacturing, extraction, transportation, and crafts</td>
<td>0.20</td>
</tr>
<tr>
<td>Managerial, professional, and technical</td>
<td>0.37</td>
</tr>
<tr>
<td>Sales and office</td>
<td>0.24</td>
</tr>
<tr>
<td>Other services</td>
<td>0.18</td>
</tr>
</tbody>
</table>

a. Approximately how many people work in the "Sales and office" category? Show the answer in scientific notation.

b. Find the difference between the number of people working in the "Farming, forestry, and fishing" category and the number of people working in the "Managerial, professional, and technical" category. Show the answer in scientific notation.
If you are able, use the text box to show your work and enter your final answer to each question. If not, complete your work on paper and upload it below.

**USE FOR MASTERY GUIDELINES & RUBRIC**

Did you:

- Use the information given in the chart to answer the questions?
- Approximate how many people work in the "Sales and office" category and show the answer in scientific notation?
- Find the difference between the number of people working in the "Farming, forestry, and fishing" category and the number of people working in the "Managerial, professional, and technical" category and show the answer in scientific notation?
- Use the text box to show your work and enter your final answer to each question OR complete your work on paper and upload it to the site?
- Show your work in an organized, thoughtful way to explain how you found your answers?

The United States has a labor force of approximately \(1.4 \times 10^8\) people. The table shows the distribution of the labor force among five categories of occupations. Distribution represents a percentage of the total population.

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</table>

**a.** Approximate how many people work in the "Sales and office" category? Show the answer in scientific notation.

**b.** Find the difference between the number of people working in the "Farming, forestry, and fishing" category and the number of people working in the "Managerial, professional, and technical" category. Show the answer in scientific notation.

If you are able, use the text box to show your work and enter your final answer to each question. If not, complete your work on paper and upload it below.
Unit Quiz - Real Number System

Books & Materials
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

UNIT QUIZ

Please go online to view and submit this assessment.
Unit 2 - Algebraic Equations
Today’s lesson covers important ideas you need to understand before beginning the lessons in this chapter.

Read **p. 92** in *Math in Focus 3A*. Explain in words or a number statement how much it would cost for you and your friends to play 1 game. Then show how much it would cost if you all played 4 games.

Read **Understanding equivalent equations** on **p. 93**. You should be able to use mental math to find each unknown value of \( x \).

Read **Expressing the relationship between two quantities with a linear equation** on **p. 93**. Remember that the value of the dependent variable is calculated using the value of the independent variable.
Today you reviewed how to identify whether two equations are equivalent. Two equations are equivalent when they have the same solution.

\[ x + 8 = 15 \] and \[ x - 1 = 6 \] are equivalent equations because you can subtract 9 from both sides of \( x + 8 = 15 \) to obtain \( x - 1 = 6 \). The solution to both equations is \( x = 7 \).

You also learned how to write a linear equation for a given situation. To do this, define independent and dependent variables and write an equation that expresses the relationship between the quantities.

Suppose each light bulb costs $3.50. The cost, \( C \), of \( n \) light bulbs is \( C = 3.50n \).
Today you reviewed how to identify whether two equations are equivalent. Two equations are equivalent when they have the same solution.

\[ x + 8 = 15 \text{ and } x - 1 = 6 \] are equivalent equations because you can subtract 9 from both sides of \(x + 8 = 15\) to obtain \(x - 1 = 6\). The solution to both equations is \(x = 7\).

You also learned how to write a linear equation for a given situation. To do this, define independent and dependent variables and write an equation that expresses the relationship between the quantities.

Suppose each light bulb costs $3.50. The cost, \(C\), of \(n\) light bulbs is \(C = 3.50n\).

Please go online to view and submit this assessment.
LEARN

WARM-UP
Perform long division to simplify each expression.

1. \(1,143 \div 3\)
2. \(4,575 \div 5\)
3. \(5,292 \div 14\)
4. \(87,494 \div 82\)

WARM-UP ANSWERS
1. 381  2. 915  3. 378  4. 1,067

TEACHING NOTES

INSTRUCTION
Today’s lesson covers important ideas you need to understand before beginning this lesson. Read Solving algebraic expressions on p. 94 in Math in Focus 3A. Remember that the goal in solving an equation is isolating the variable. In the first example, you need to get \(4x\) by itself before you can find \(x\). Likewise, in the second example, you need to get \(8x\) by itself before you can find \(x\). Read Representing fractions as repeating decimals on p. 95. You can stop the long division process once you are sure of the pattern in repeating digits.
Today you reviewed how to solve algebraic equations. If possible, simplify both sides of an equation. When choosing which operation to do to both sides, you ensure the pattern in repeating digits.

Representing fractions as repeating decimals

The equation is isolating the variable. In the first example, you need to get $4x$. Likewise, in the second example, you need to get $8x$.

WRAP-UP

SKILLS CHECK

Quick Check sections on pp. 94–95 in *Math in Focus 3A*.

TEACHING NOTES

Textbook Answer Key

Be sure your student shows all work in Quick Check on pp. 94. It is tempting to combine multiple steps and use mental math skills, but it is best for your student to show work with one step completed on each line to help as he progresses through more advanced math topics. It is also important as you will be able to see if any errors are because of a miscalculation or because he does not fully understand the process.

Review your student’s answers to the Quick Check sections, noting the problems that your student answered incorrectly. Click on the link to access the appropriate Reteach activity that your student should complete for the remainder of this lesson.

RETEACH

After your student completes the Quick Check in the Recall Prior Knowledge section of this lesson, review the questions that were answered incorrectly. If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.

Note this chapter opener spans two lessons.

Quick Check

<table>
<thead>
<tr>
<th>Question(s)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>9–12</td>
<td>Two-Step Equations</td>
</tr>
<tr>
<td>13–16</td>
<td>Converting Fractions and Decimals</td>
</tr>
</tbody>
</table>
WRAP-UP

Today you reviewed how to solve algebraic equations. If possible, simplify both sides of an equation before performing an operation on both sides. When choosing which operation to do to both sides, choose one that will help isolate the variable term.

Solve $4(x + 3) = 32 + 2x$.

\[
\begin{align*}
4(x + 3) &= 32 + 2x \\
4x + 12 &= 32 + 2x & \text{Use the Distributive Property.} \\
4x - 2x + 12 &= 32 + 2x - 2x & \text{Subtract } 2x \text{ from both sides.} \\
2x + 12 &= 32 & \text{Simplify.} \\
2x + 12 - 12 &= 32 - 12 & \text{Subtract } 12 \text{ from both sides.} \\
2x &= 20 & \text{Simplify.} \\
\frac{2x}{2} &= \frac{20}{2} & \text{Divide both sides by } 2. \\
x &= 10 & \text{Simplify.}
\end{align*}
\]

You also reviewed how to represent fractions as repeating decimals. Use long division to divide the numerator of a fraction by the denominator. Use bar notation to indicate which digits repeat.

To write $\frac{16}{3}$ as a repeating decimal, divide 16 by 3.

\[
\begin{array}{c|c}
5 & 33 \\
\hline
1 & 6.00 \\
\hline
15 & 10 \\
\hline
9 & 10 \\
\hline
9 & 1 \\
\hline
1 & 0 \\
\hline
\end{array}
\]

So, $\frac{16}{3} = 5.33\ldots = 5\ \overline{3}.$

✅ QUICK CHECK

Please go online to view and submit this assessment.

📚 MORE TO EXPLORE

If you struggled with this question, try to write out each step as you simplify and solve equations. Remind yourself you can check your answer by substituting the value into the original equation and check to see if it makes a true statement when simplified. You might also want to revisit the material in this lesson.
Equations in One Variable - Part 3

Objectives
- Solve linear equations in one variable.

Books & Materials
- **Math in Focus 3A**
- **Math in Focus - Teacher Edition**

Assignments
- Complete Warm-Up.
- Complete **Math in Focus 3A**.
- Complete Practice Questions.

LEARN

WARM-UP

Simplify each expression.

1. \(4x + 2(2x - 5)\)
2. \(-6(9 - 3x)\)
3. \(\frac{3}{4} + 5(x + 2)\)
4. \(0.4(1.2x + 10)\)

TEACHING NOTES

WARM-UP ANSWERS

1. \(8x - 10\)  
2. \(18x - 54\)  
3. \(5x + 10 - 3/4\)  
4. \(0.48x + 4\)

INSTRUCTION

Read p. 96 in **Math in Focus 3A**. Remember that the goal when solving equations is to isolate the variable on one side of the equation by itself. Notice the first step is to combine both terms that include the variable \(x\). The next step is to multiply both sides by the same number to eliminate the coefficient of the variable.

Review Example 1 on p. 97. Even though many steps are shown, the basic concept involves two steps:

1. Combine like terms.
2. Isolate the variable

Notice how only one step is completed from one line to the next. It is good practice to show this detail of work when solving equations.
Complete **Guided Practice** on p. 97.

Review **Example 2** on p. 98. Notice that 10 was chosen to be the number by which to multiply both sides because there is one digit that repeats. For a number that has two digits that repeat, multiply both sides by 100. For a number that has three digits that repeat, multiply both sides by 1,000, and so on. This way, when you subtract the infinite string of digits, the difference is 0.

Complete **Guided Practice** on p. 98. Be sure to multiply both sides by a power of 10 that will allow you to eliminate the infinite string of repeating digits.

---

**TEACHING NOTES**

There are often multiple ways to solve a linear equation involving fractions. Your student may choose to rewrite two fractions as a single fraction, or he or she may choose to use multiplication to eliminate the fractions altogether.

For example, the problem \( \frac{5x}{3} + \frac{x + 2}{6} = 4 \) can be approached in various ways. The expressions on the left side of the equation can be written as a single fraction with a common denominator. The equation becomes \( \frac{10x + x + 2}{6} = 4 \), which can be simplified further to \( \frac{11x + 2}{6} = 4 \). Alternately, your student may prefer to immediately eliminate the fractions by multiplying both sides of the equation by 6. This results in the equation \( 10x + x + 2 = 24 \), which can be simplified to \( 11x + 2 = 24 \). Both methods lead to the same solution.

**WATCH FOR THESE COMMON ERRORS**

Remind your student that the fraction bar acts like a grouping symbol, so care must be taken when working with fractions preceded by a negative sign. For example, in the problem \( \frac{2x}{3} - \frac{4x + 2}{3} = 9 \), your student might make the mistake of rewriting the problem as \( \frac{2x - 4x + 2}{3} = 9 \). To avoid this error, instruct him or her to group the expression \( 4x + 2 \) in parentheses when rewriting as a single fraction. This would look like: \( \frac{2x - (4x + 2)}{3} = 9 \).

---

**PRACTICE**

Complete problems 1–18 of **Practice 3.1** on p. 102 in *Math in Focus 3A*.

---

**TEACHING NOTES**

Textbook Answer Key
If your student struggles with determining what number to multiply the variable by to eliminate the repeating digits, point out that she can multiply by $10^n$, with $n$ being the number of repeated digits. If 2 digits repeat, for example, she will multiply the variable by $10^2$, or 100.

Follow these instructions for the activity shown below.

Click [here](#) to view the activity in a new window.

You will see the equation $2x + 2 = 6$ at the top of Gizmo. To model this equation, click on the $x$ cup on the left two times. You will see that 2 cups have appeared in the blue section on the left, showing $2x$. Now click on the red counter on the left two times. You now have 2 cups and 2 counters, representing $2x + 2$. To show 6 on the other side of the equation, click the red counter on the right six times. Your screen should now look like this:

Think about how you would solve for $x$. You can click on 2 counters on each side and remove them from the screen. This leaves $2x = 4$.

Look at the remaining counters and cups. You only want to find the number of counters in 1 cup, so remove half the cups from the left and half the counters from the right. This shows that there would be 2 counters in each cup, which models the solution, $x = 2$. Click on Check to verify your answer.

Now click New. Model this equation and find the solution. Continue with 5 more equations. Be sure to check your work for each one.
Today you learned how to solve linear equations. You can follow these simplified steps as a guide to solving linear equations.

**Step 1**: Simplify both sides of the equation to combine like terms.

**Step 2**: Perform inverse operations to both sides in the reverse order of operations until the variable is isolated.

Please go online to view and submit this assessment.
Equations in One Variable - Part 4

Objectives
- Solve real-world problems by using linear equations in one variable.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete Math in Focus 3A.
- Complete Practice Questions.

LEARN

WARM-UP
Solve each equation.

1. 3x + 3 = 2x + 5
2. 4x + 6 + x3 = 12
3. 8x 3 - 12x - 186 = 17

WARM-UP ANSWERS
1. x = 2  2. x = 1/2  3. x = 21

TEACHING NOTES

INSTRUCTION
Read p. 99 in Math in Focus 3A. The bar model first shows the ratio of the costs of the shirt and jeans, or how the costs relate to one another. The bar model then shows that adding 30 to the cost of the belt equals the cost of the jeans. The total adds the three costs together (1 block for the shirt, 2 blocks for the jeans, and the part representing the belt).

If you have difficulty understanding where the equation comes from, draw lines from the Total row in the bar model to the parts of the equation. Each orange block represents an x in the equation.
Review Example 3 on p. 100. The equation has 3 parts on the left side: the distance from the bottom of the wall to the mirror (x), the height of the mirror (2814), and the distance from the top of the mirror to the top of the wall (12x). Follow the steps for solving the equation. Like terms are combined and then the variable is isolated.


TEACHING NOTES
Once your student is able to translate a real-world problem into an equation involving a variable, he or she can solve the linear equation he or she wrote. Your student may struggle in generating an equation to solve. You can help him or her develop this skill by writing a list of phrases commonly used in translating words into variable expressions. Some examples you might have on the list are: $15 more than twice the cost of jeans (15 + 2c) and 5 minutes less than half his time (t + 2 − 5).

WATCH FOR THESE COMMON ERRORS
Your student may solve an equation he wrote for the variable, but this may not answer the question. Have him check to see what the variable represents. Then check to see what the question is asking. These may not always be the same.

PRACTICE
Complete problems 19–28 of Practice 3.1 on pp. 102–103 in Math in Focus 3A.

TEACHING NOTES
Textbook Answer Key

WRAP-UP
Today you learned how to solve real-world problems involving linear equations. This guided example shows the basic steps that can be taken to solve this type of problem.

Tony has $2.65 in quarters and nickels. The number of nickels is 4 more than twice the number of quarters. How many quarters and nickels does Tony have?
Let $q$ be the number of quarters and $n$ be the number of nickels.

\[ n = 4 + 2q \]

\[ 0.25q + 0.05(4 + 2q) = 2.65 \]
\[ 0.25q + 0.2 + 0.1q = 2.65 \]
\[ 0.35q + 0.2 = 2.65 \]
\[ 0.35q + 0.2 - 0.2 = 2.65 - 0.2 \]
\[ 0.35q = 2.45 \]
\[ \frac{0.35q}{0.35} = \frac{2.45}{0.35} \]
\[ q = 7 \]

Assign a variable.  
Relate the variables.  
Write an equation for the problem.  
Use the Distributive Property.  
Combine like terms.  
Subtract 0.2 from both sides.  
Simplify.  
Divide both sides by 0.35.  
Simplify.

Find the number of nickels.

\[ n = 4 + 2q = 4 + 2(7) = 18 \]

Tony has 7 quarters and 18 nickels.

✔️ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
WARM-UP

Solve each equation.

1. \(6x - (3 - 2x) = 25\)
2. \(0.15(4x + 2) + 0.4(x - 6) = 9.9\)
3. \(5(x+10) 4 - x+12 2 = 9.5\)

TEACHING NOTES

WARM-UP ANSWERS
1. \(x = 3.5\) 2. \(x = 12\) 3. \(x = 4\)

INSTRUCTION

Read Understand and Identify Linear Equations with No Solution on p. 104 in Math in Focus 3A. Try using mental math to find a value of \(x\) that makes the equation true. This equation is an inconsistent equation because it has no solution.

Review Example 4 on pp. 104–105. In part a, 15 is not equal to 3, and the statement is false. There are no values of \(x\) that make the equation true. In part b, check the answer.

\[
3(10 - 4) = 2(10 - 1)
\]

\[
3(6) = 2(9)
\]

It is important to note that just because there is an \(x\) on both sides of the equation does not mean the equation has no solution.

Complete Guided Practice on p. 105. Check your solutions.
Read the instructional section on p. 105. Be sure to read the Caution box. The solution is not 5. The statement $5 = 5$ means that no matter what value you substitute in for $x$, the equation is true. Try a few values to verify this.

Review Example 5 on p. 106. In part a, $-10$ is equal to $-10$, and the statement is true. There are an infinite number of values for $x$ that make the equation true. In part b, check the answer in the original equation.

Complete Guided Practice on p. 107.

Encourage your student to check his solution to a linear equation. Review the vocabulary terms in the lesson. For an inconsistent equation (equation with no solution), the result after simplifying both sides of the equation will be a false statement.

For a consistent equation (equation with one solution), only the value of the variable that your student found will make the equation true. For an identity (equation with infinitely many solutions), let your student substitute two or more values for the variable. Choosing multiple values will help demonstrate to him that the equation will be true for any value of the variable he chooses.

Complete Practice 3.1 on p. 108 in Math in Focus 3A.

Textbook Answer Key

Today you learned how to identify the number of solutions to a linear equation.

An inconsistent equation is an equation with no solution.

\[ 2x - 12 = 2x \]

When you subtract $2x$ from both sides of the equation, you get the false statement $-12 = 0$. This means no value of $x$ will make the equation true.
A *consistent equation* has one solution.

$$3x + 1 = 7$$

The value $x = 2$ is the only value for $x$ that will make the equation true.

An *identity* is an equation that is true for all values of the variable, resulting in infinitely many solutions.

$$6x + 3 = 3(2x + 1)$$

After you use the Distributive Property and then subtract $6x$ from both sides, you get the true statement $3 = 3$. This means any value of $x$ will make the equation true.

---

**USE**

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**USE FOR MASTERY**

1. The ratio of the perimeter of triangle $PQR$ to the perimeter of rectangle $ABCD$ is $5 : 9$.

   a. Write algebraic expressions for the perimeters of triangle $PQR$ and rectangle $ABCD$.

   b. Write a linear equation using the algebraic expressions written in part a. Then solve for $x$.

   c. Find the area of rectangle $ABCD$.

   If you are able, use the text box to show your work and enter your final answer to each question. If not, complete your work on paper and upload it below.
Did you:

- Use the information given to solve the problems?
- Write algebraic expressions for the perimeters of triangle $PQR$ and rectangle $ABCD$?
- Write a linear equation using the algebraic expressions from part A and then solve for $x$?
- Find the area of rectangle $ABCD$?
- Use the text box to show your work and enter your final answer OR complete your work on paper and upload it to the site?
- Show your work in an organized, logical manner?
- Use correct labels to give meaning to the numeric values?
Objective: Write a linear equation to express a relationship between two variables.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition
- Online graphing calculator (Optional)

Assignments
- Complete Warm-up.
- Read and complete Math in Focus 3A.
- Complete the Practice Questions.

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**LEARN**

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**WARM-UP**

Find the next three values in each pattern.

1. 0, 3, 6, 9, 12, ...
2. −15, −13, −11, −9, ...
3. 0, 1 4 , 1 2 , 3 4 , 1,...

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. 15, 18, 21  2. −7, −5, −3  3. 1−1/4, 1−1/2, 1−3/4

---

**INSTRUCTION**

Read p. 109 in Math in Focus 3A. Look at the patterns in the table. Each year, Dorothy’s age is three more than Benjamin’s age. Notice how the variables are defined, with x representing Benjamin’s age and y representing Dorothy’s age.

Review Example 6 on p. 110. In part b, make sure you know that t and T, even though they are the same letter, represent two different quantities. In this case, capitalization matters. If this is confusing to you, change one of the variables to a different letter.

---
Complete **Guided Practice** on pp. 110–111. Fill in the blanks in problems 1 and 2. For problem 1, try to write the linear equation for \( w \) in terms of \( d \).

If you have access to an online graphing calculator, you may complete the optional **Technology Activity** on p. 111.

---

**TEACHING NOTES**

To determine an equation that shows a relationship between two variables, your student can observe patterns from a table or can draw on prior knowledge to generate the relationship. For example, to write a relationship between yards and feet, your student should recall that 1 yard is equivalent to 3 feet. He can write a linear equation for \( f \) in terms of \( y \) as \( f = 3y \).

*Note*: A list of suggested online graphing calculators is listed in the Online Resources section in the Appendix of this manual.

**WATCH FOR THESE COMMON ERRORS**

When your student is told that 1 yard is 3 feet, your student may incorrectly translate this to \( y = 3f \). Encourage her to check the equation by substituting in values for \( f \) and seeing if the answer is reasonable. When \( f = 2 \), \( y = 6 \). Your student should ask herself if 2 feet is the same as 6 yards. Since it is not, she should know that her linear equation is not correct.

---

**PRACTICE**

Complete problems 1–4 and 21 of **Practice 3.3** on pp. 116–117 in *Math in Focus 3A*.

---

**TEACHING NOTES**

*Textbook Answer Key*

Problem 21 shows a proportional relationship. As \( g \) increases by 1, \( d \) increases by 40.5. As your student progresses through topics associated with proportional relationships, make sure he or she is aware that in a proportional relationship, when the independent variable is 0, the dependent variable is also 0. In this case, when the number of gallons is 0, the distance traveled is 0.
WRAP-UP
Today you learned how to write a linear equation to show the relationship between two variables.

A car gets 22 miles for each gallon of gas. Write a linear equation to express $d$, the distance traveled in miles, in terms of $g$, the gallons of gas used.

The relationship is $d = 22g$.

✔️ PRACTICE QUESTIONS
Please go online to view and submit this assessment.
Linear Equations in Two Variables - Part 2

Objectives
- Make a table of values to represent a linear relationship between two variables.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-Up.
- Read and complete Math in Focus 3A.
- Complete Quick Check.

LEARN

WARM-UP
Find the value of the expression when \( x = 5 \).

1. \( 3x + 4 \)
2. \( -0.5x + 7 \)
3. \( \frac{4x + 2}{9} \)

TEACHING NOTES

WARM-UP ANSWERS
1. 19 2. 4.5 3. 2–4/9

INSTRUCTION

Read p. 112 in Math in Focus 3A. When making a table of values presented horizontally, the independent variable is usually the top row. When the table is presented vertically, the independent variable is usually in the first column.

Review Example 7 on p. 113. Once the value of the variable is substituted, simplify. Then use inverse operations to isolate the variable.

Complete Guided Practice on p. 113. Show your work.

Review Example 8 on pp. 114–115. In part a, first set up a table with 2 rows. The top row is for values of \( x \). The problem states to use the values from \(-1\) to \(1\), which are \(-1\), \(0\), and \(1\). Each value is substituted into the original equation and the corresponding value of \( y \) is determined.

Notice in part b, sometimes the \( x \) value is given and sometimes the \( y \) value is given. When asked to solve problems like this, be sure to substitute the known quantity for the correct variable.

Complete Guided Practice on p. 115. Show your work.
When your student is making a table of values for a linear equation and input values are not provided, encourage him or her to substitute values that are convenient. Substituting the value 0 for a variable is always convenient, unless it causes division by 0. In some problems, especially those involving fractions, your student will need to make the decision of what values will work best for substitution. Although substituting 1 for a variable often results in a simple calculation, it may not always be the most convenient choice. For example, in the problem \( \frac{x-5}{2} = \frac{3y}{2} + 4 \), your student will find it easier to substitute a value of \( x \) or \( y \) that will make the numerator divisible by 2 to avoid fractions. Similarly, in the example \( \frac{4y}{3} = x + 7 \), your student might want to choose values of \( y \) that are divisible by 3.

**PRACTICE**
Complete problems 5–20 and 22–24 of Practice 3.3 on p. 116–117 in Math in Focus 3A.

**WRAP-UP**
Today you learned how to represent linear equations using tables. Follow these steps as a guideline to making a table for a linear equation.

**Step 1:** Pick a convenient value to substitute for one of the variables in the equation.

**Step 2:** Isolate the remaining variable on one side of the equation to determine its value. This will usually involve performing inverse operations.

**Step 3:** Repeat these steps several times using additional values to substitute for one of the variables.

**Step 4:** Make a table using each pair of corresponding values that you calculated.

**QUICK CHECK**
Please go online to view and submit this assessment.
MORE TO EXPLORE

If you struggled with this question, make sure that you substitute each given value for \( y \) into the equation and then solve for \( x \). Remember to write out each step so you can catch your errors. You might also want to revisit the material in this lesson.
Linear Equations in Two Variables - Part 3

Objectives
- Solve for a variable in an equation with two variables.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-Up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete Practice Questions.

LEARN

WARM-UP
Solve each equation.

1. \(5x - 3(x + 4) = -12 + 6x\)

2. \(4x + 15 = 9x\)

3. \(18 - 2x = -5x\)

4. \(x - 8 = 2 = x 4\)

TEACHING NOTES

WARM-UP ANSWERS
1. \(x = 0\)  
2. \(x = 3\)  
3. \(x = -6\)  
4. \(x = 16\)

INSTRUCTION

Read p. 118 in Math in Focus 3A. You will use this process of solving an equation for one variable in terms of another often as you continue your study of mathematics. It has applications in graphing and geometry.

Review Example 9 on p. 119. In part a, you may recognize the final equation as one you would use to convert a given Celsius temperature to a Fahrenheit temperature. The steps show you how one formula relates to the other. Notice that in part b, it takes fewer steps to substitute the four Celsius temperatures when the equation is solved for \(F\) in terms of \(C\). If it is not in this form, you would have to replace \(C\) in the original equation four times and solve four equations.

Complete Guided Practice on p. 120.
Review Example 10 on pp. 120–121. A calculator is used in part b to simplify each expression when the values of \( P \) are substituted into the equation solved for \( s \) in terms of \( P \).

Complete Guided Practice on p. 121. To write the initial equation, think about how you would find the average of three numbers. Use a calculator for part b.

---

### TEACHING NOTES

Let your student discover that when completing a table of values, sometimes it is quicker to solve for one of the variables before substituting any values into the equation. For example, if he is given the equation 3(2x + 5) – y = 2y and asked to complete a table of values for \( x = 2, 4, \) and 6, have him first complete the table by substituting each value in for \( x \) and solving for the corresponding value of \( y \). Set a timer and see how long it takes him to complete. Then have your student complete the table using the method shown in the lesson.

Your student should first solve the original equation for \( y \). Next, he can substitute each value into this equation and find the corresponding value of \( y \). Both methods result in the same answers, but ask your student if he felt one way was quicker or easier. (The corresponding values of \( y \) are 9, 13, 17.)

---

### PRACTICE

Complete Practice 3.4 on pp. 122–123 in Math in Focus 3A.

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### TEACHING NOTES

[Textbook Answer Key](#)

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### INTERACTIVE ACTIVITY

Try this activity to practice your skills with equations.

Click [here](#) to view the activity in a new window.

In the Solving Formulas for any Variable Gizmo, you will be given a formula (a rule or relationship between variables) expressed as an equation. You will solve each formula for a variable, by isolating that variable, step by step. Even though there are no numbers in this formula, you will follow the same procedures as if there were numbers.
The pink tiles under the screen give you four choices for the next step. Choose the one you think is correct and drag it into the gray area above. You may use just one tile, or you may use more than one. If you choose an incorrect step, you will receive feedback to help you make the correct choice. If you choose correctly, you will see a green check mark when you are finished. Practice with at least 5 different formulas.

Today you learned how to solve for a variable in an equation with two variables. Follow the steps in this example to solve for \( y \) in terms of \( x \) in the equation \( 1.5(10x - 2y) = 6x + 15 \).

\[
\begin{align*}
1.5(10x - 2y) &= 6x + 15 \\
15x - 3y &= 6x + 15 \\
15x - 3y - 15x &= 6x + 15 - 15x \\
-3y &= -9x + 15 \\
\frac{-3y}{-3} &= \frac{-9x + 15}{-3} \\
y &= 3x - 5
\end{align*}
\]

Use the Distributive Property.

Subtract 15x from both sides.

Simplify.

Divide both sides by -3.

Simplify.

Please go online to view and submit this assessment.
Linear Equations in Two Variables - Part 4

**Objectives**
- Apply knowledge of linear equations to solve problems.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*

**Assignments**
- Complete Warm-up.
- Read and complete Brain @ Work in *Math in Focus 3A*.
- Complete the Practice activity.
- Complete the Practice Questions.

---

**LEARN**

---

**WARM-UP**
Find the value of y when x = 4.

1. $0.2(3x + y) = x$
2. $\frac{30 - 4y}{x} = y + 7$
3. $16y + 10x = 18y$

---

**TEACHING NOTES**

**WARM-UP ANSWERS**
1. $y = 8$  
2. $y = 0.25$  
3. $y = 20$

---

**INSTRUCTION**

Read and complete Brain @ Work on p. 124 in *Math in Focus 3A*. Be sure to show your work. Underline key information when problem solving, and then look at the information that you underlined to create a table or drawing to assist you in solving the problem.

For problem 1, determine how much money Lynnette makes from each student per month.

For problem 2, consider the relationship between distance, speed, and time. Notice that the time Stefanie has until the train leaves is given in minutes, while the rates are given in miles per hour. For problem 3, look for a pattern in the $n$ values and a pattern in the $r$ values. Then relate the patterns to one another.
If your student needs additional help with problem 2, you can have him create and complete the following table.

<table>
<thead>
<tr>
<th></th>
<th>Walk</th>
<th>Run</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (mi)</td>
<td>$w$</td>
<td></td>
<td>$y$</td>
</tr>
<tr>
<td>Speed (mi/h)</td>
<td>4</td>
<td>8</td>
<td>N/A</td>
</tr>
<tr>
<td>Time (h)</td>
<td>4</td>
<td></td>
<td>$\frac{24}{60}$</td>
</tr>
</tbody>
</table>

Instruction Answers:

Write constructed responses to explain your answers to problems 1c and 2c in Brain @ Work in Math in Focus 3A.

Then solve the following problem:

Kelly read for a total of 7 hours over 3 days. On Monday, she read half as long as she did on Sunday. On Tuesday, she read 2 hours less than she did on Sunday and Monday combined. For how many hours did she read on Sunday?

Practice Answer:

3 hours
WRAP-UP

Today you learned how to solve problems involving linear equations with two variables. Given a problem situation, you can write a two-variable linear equation to solve for any value of one of the variables.

Step 1: First write an equation relating the two variables.

Step 2: Solve for one variable in terms of the other.

Step 3: Substitute in a known value and solve the equation.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
**Linear Equations in Two Variables - Part 5**

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Books &amp; Materials</th>
<th>Assignments</th>
</tr>
</thead>
</table>
| - Review previously learned concepts. | - *Math in Focus 3A*  
- *Math in Focus - Teacher Edition* | - Complete Warm-up.  
- Read and complete assigned pages in *Math in Focus 3A*.  
- Complete the Use For Mastery. |

**LEARN**

**WARM-UP**

Find the value of $y$ when $x = -2$.

1. $\frac{3x+8}{y} = 5$
2. $7(2y - x) = 21$
3. $y + 2.2x = 3y - x$

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. $y = -\frac{1}{5}$  
2. $y = 1/2$  
3. $y = -3.2$

**INSTRUCTION**

Read Chapter Wrap Up on p. 125 in *Math in Focus 3A*. The concept map shows information about algebraic linear equations. Review the vocabulary terms to the right of the number of possible solutions to linear equations. Then read over the key concepts to review more information about linear equations.

**TEACHING NOTES**

Review with your student some basic information he used throughout the chapter to solve problems. Solving one-variable linear equations uses similar methods as expressing a two-variable linear equation in terms of one of the variables. Both types of problems involve isolating the variable on one side of the equation using inverse operations, which are performed in the reverse order of operations.
Tables and drawings are useful aids in both expressing a relationship between two variables and in problem solving. They will also help your student determine the reasonableness of an answer.

**PRACTICE**

Complete **Chapter Review/Test** on pp. 126–127 in **Math in Focus 3A**.

**TEACHING NOTES**

**Textbook Answer Key**

**WRAP-UP**

Today you reviewed concepts associated with linear equations, including how to solve a one-variable linear equation, how to find the number of solutions to a one-variable linear equation, and how express a two-variable linear equation as an equation for one variable in terms of another.

**USE**

**USE FOR MASTERY**

1. Melody is paid $M$ per week. She earns a weekly flat wage of $80 plus an additional $0.15 for each of the $n$ customers she advises. Her salary, in terms of the number of customers she advises, $n$, is represented by the equation $M = 80 + \frac{15n}{100}$.

   a. Find the salary she receives for advising 360 customers per week for 4 consecutive weeks.

   b. Melody received $140 at the end of a certain week. How many customers did she advise that week?

   c. Melody's employer decides to decrease her basic wage to $65 and increase her salary per customer advised to $0.21. Write a linear equation to represent her new salary, $M$, in $n$ terms of the number of customers she advises, $n$.

   d. Find the number of customers she would have to advise in one week to receive the same amount of money before the decrease in basic wage.
If you are able, use the text box to show your work and enter your final answer to each question. If not, complete your work on paper and upload it below.

---

**USE FOR MASTERY GUIDELINES & RUBRIC**

Did you:

- Use the information given to help you answer parts A–D?
- Find the salary Melody receives for advising 360 customers per week for 4 consecutive weeks?
- Tell how many customers Melody advised if she received $140 at the end of a certain week?
- Write a linear equation to represent her new salary, $M$, in terms of the number of customers she advises, after her employer decides to decrease her basic wage to $65 and increase her salary per customer advised to $0.21?
- Find the number of customers she would have to advise in one week to receive the same amount of money before the decrease in basic wage?
- Use the text box to show your work and enter your final answer OR complete your work on paper and upload it to the site?
- Show your work in an organized, logical manner?
Slope - Part 1

Objectives
- Identify a direct proportion and the constant of proportionality.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-Up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete Practice Questions.

LEARN

WARM-UP
Express \( y \) in terms of \( x \).

1. \( 5x + y = 3 + 2x \)
2. \( \frac{3(x+7)}{y} - 3 = 4 \)
3. \( \frac{y}{3} + 4x = 12 \)

TEACHING NOTES

WARM-UP ANSWERS
1. \( y = -3x + 3 \)  2. \( y = (3/7)x + 3 \)  3. \( y = -12x + 36 \)

INSTRUCTION

This part of the lesson covers important ideas you need to understand before beginning the lessons in this chapter. Read p. 128 in Math in Focus 3A.

Think about the steepest mountain you have ever seen. Another word for steepness is slope. One way to describe the slope of a mountain is how many feet you climb up for every foot you walk forward. The steeper the mountain, the higher you climb up with every step forward.
Read p. 129 and look at the graph. You can tell that the graph shows a direct proportion because it is a straight line that passes through the origin, (0, 0).

Another way to find the constant of proportionality is to substitute the values for the x- and y-coordinates of a point on the line in the direct proportion equation, \( y = kx \). For example, for the point \((1, 2)\), \( y = 2 \) and \( x = 1 \).

\[
2 = k(1)
\]

\[
\frac{2}{1} = k
\]

\( k = 2 \)

The constant of proportionality for the graph on p. 129 is 2. The constant of proportionality tells you that for every point \((x, y)\) on the line, \( y \) is 2 times \( x \).

---

**TEACHING NOTES**

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the Quick Check.

**SKILLS CHECK**

Complete the Quick Check on p. 129 in *Math in Focus 3A*.

---

**TEACHING NOTES**

[Textbook Answer Key](#)
In the direct proportion equation, $k$ is the constant of proportionality, so every $y$ is equal to $k$ times the corresponding $x$.

Please go online to view and submit this assessment.

**Quick Check**

<table>
<thead>
<tr>
<th>Question(s)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td>Constant of Proportionality Made Easy</td>
</tr>
</tbody>
</table>

**WRAP-UP**

In this part, you reviewed important concepts that will help you to be successful in this chapter.

- The graph of a direct proportion always passes through the origin $(0, 0)$.
- The equation of a direct proportion is $y = kx$.
- In the direct proportion equation, $k$ is the constant of proportionality, so every $y$ is equal to $k$ times the corresponding $x$.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
LEARN

WARM-UP

Each point is on the graph of a direct proportion. Write the equation of the direct proportion for each.

1. (1, 3)  
2. (1, 2 3 )  
3. (1, 5)  
4. (1, 2 3 )

WARM-UP ANSWERS

1. \(y = 3x\)  
2. \(y = (2/3)x\)  
3. \(y = 5x\)  
4. \(y = (1–1/5)x\)

TEACHING NOTES

INSTRUCTION

Read Define the Slope of a Line and Relate Unit Rate to Slope on pp. 130–131 in Math in Focus 3A. Both graphs on p. 130 show linear relationships that increase at a steady rate, but the graph in the Math Note does not represent a direct proportion because the line does not pass through the origin.

To find the slope of a line, choose two points on the line. To find the \(\text{rise}\), subtract the two \(y\)-coordinates of the points. To find the \(\text{run}\), subtract the \(x\)-coordinates. Then divide the change in \(y\) (vertical change) by the change in \(x\) (horizontal change). The slope can be thought of as the ratio of rise to run, or rise over run: \(\frac{\text{rise}}{\text{run}}\).

Review Example 1 on p. 133. You can use the slope to find unit rate. Choose points on the graph where two grid lines intersect so you can easily read the \(x\)- and \(y\)-values. Then complete Guided Practice on p. 134.

Read Find Slopes of Slanted Lines on p. 135. Notice that the slope of a line can be positive or negative. Then review Example 2 on pp. 136-137. When you subtract to find slope, you may obtain...
negative numbers in the slope ratio. Follow the rules for simplifying fractions and dividing integers to determine whether the slope is negative or positive.

Complete Guided Practice on p. 137.

Review Example 3 on p. 138. You can also solve this problem by graphing. The graph of the speed of the faster car will have the steeper line and the greater slope. The graph shows the red car’s trip. You can show the blue car’s trip by plotting a point at (4, 140) and drawing a line from the origin to that point. Because the lines are graphed on the same grid, compare the lines to see which car traveled faster.

Complete Guided Practice on p. 139.

The slope formula can be used to find the slope of any line. Your student will learn more about the slope formula in Lesson 41. Any two points on the line can be used to calculate the rise and the run, but it is important to subtract the coordinates in the same order.

Complete problems 1–2 and 5 of Practice 4.1 on p. 145 in Math in Focus 3A.

If your student needs help with problem 5, prompt her to notice the intervals on the y-axes of each graph.
WRAP-UP

Today you learned how to find the slope of a line by dividing the vertical change, rise, by the horizontal change, run. You also learned that slope can be negative or positive.

\[
\begin{align*}
\text{rise} &= \text{vertical change: } 6 - 2 = 4 \\
\text{run} &= \text{horizontal change: } 3 - 1 = 2 \\
\text{slope} &= \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2 \\
\text{rise} &= \text{vertical change: } -12 - (-6) = 6 \\
\text{run} &= \text{horizontal change: } 4 - 2 = 2 \\
\text{slope} &= \frac{\text{rise}}{\text{run}} = \frac{-6}{2} = -3
\end{align*}
\]

QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

If you struggled with this question, practice finding the slopes of lines by marking the grid to count the change in y and the change in x. Remember that the slope is the vertical change divided by the horizontal change. Also remember that in order to determine if the slope is positive or negative, examine the line going left to right. If the slope is going up, then it is positive. If the slope is going down, then it is negative.
On grid paper, plot the points (0, 2) and (−3, 8). Then draw a line through the points. What is the slope of the line?

slope: –2

Since horizontal lines go straight across, the rise is zero. Since zero divided by a number is always zero, it does not matter what the run is. The slope of any horizontal line is 0.
Look at the vertical line graphed on p. 140. Since vertical lines go up and down, the run is zero. No matter what the rise is, any number divided by zero is undefined. Since division by 0 is undefined, the slope of any vertical line is undefined.

Review Example 4 on the top of p. 141. Then complete Guided Practice on p. 141.

**TEACHING NOTES**

The slopes of horizontal and vertical lines may be confusing. For horizontal lines, the vertical change (rise) is always 0, so the slope is always 0 divided by a number, which equals 0.

For vertical lines, the horizontal change (run) is always 0, so the slope is always a number divided by 0, which is undefined.

**WATCH FOR THESE COMMON ERRORS**

Your student might think the slope of the vertical line is 0 and the horizontal line is undefined. Have her visualize a flat surface and discuss reasons why the slope would be 0. Then have her visualize something being dropped from a tall height. Finally, have her discuss reasons why that slope would not be 0 but would be undefined.

**PRACTICE**

Complete problems 3–4 and 6 of Practice 4.1 on pp. 145–146 in Math in Focus 3A.

**TEACHING NOTES**

Textbook Answer Key

For problem 6, your student may sketch the graph described in the problem and use the graph to find the solution.
WRAP-UP

Today you learned that the slope of a horizontal line is 0 and the slope of a vertical line is undefined.

\[
\text{slope} = \frac{-1 - (-1)}{3 - (-2)} = \frac{0}{5} = 0
\]

\[
\text{slope} = \frac{3 - (-1)}{-1 - (-1)} = \frac{4}{0} \quad \text{undefined}
\]

✔ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
LEARN

WARM-UP

Answer the following questions.

1. What is the slope of any horizontal line?
2. What is the slope of any vertical line

WARM-UP ANSWERS

1. 0  2. undefined

INSTRUCTION

Read Understand The Slope Formula on p. 142 in Math in Focus 3A. To keep track of the x- and y-coordinates of the two points you use to find the slope, label them as \((x_1, y_1)\) and \((x_2, y_2)\). The subscript numbers identify which coordinate goes with which point. Read \(x_1\) as \(x\ sub 1\) and \(x_2\) as \(x\ sub 2\).

See the following example:

\[
\begin{align*}
(x_1, y_1) &= (6, 5) \\
(x_2, y_2) &= (-3, -1) \\
\text{rise} &= \text{vertical change} = \text{difference between y-coordinates} \\
y_1 - y_2 &= 5 - (-1) = 6 \\
\text{run} &= \text{horizontal change} = \text{difference between x-coordinates} \\
x_1 - x_2 &= 6 - (-3) = 9 \\
\text{slope} &= \frac{\text{rise}}{\text{run}} = \frac{6}{9} = \frac{2}{3} \\
\text{slope} &= \frac{y_1 - y_2}{x_1 - x_2}
\end{align*}
\]
This formula is a formal way of writing what you have already learned in this chapter. Because you can subtract the coordinates of the points in either order, you can also write the formula as slope = $\frac{y_2 - y_1}{x_2 - x_1}$.

Review Example 5 on p. 143. You can find the slope of a line without a graph by using the slope formula. Substitute the values of two given points into the slope formula and simplify to find the slope.

Complete Guided Practice on p. 144.

**TEACHING NOTES**

**WATCH FOR THESE COMMON ERRORS**

Your student might mix up the coordinates that she subtracts when finding slope. Have her circle the $y$-coordinates and put a square around the $x$-coordinates. Then have her draw an arrow from the $y$-coordinate in the first point to the $y$-coordinate in the second point. Repeat with the $x$-coordinates. Allow your student to refer to this sketch as she proceeds through the rest of the chapter.

![Sketch](image)

**PRACTICE**

Complete problems 7–14 of Practice 4.1 on p. 146 in *Math in Focus 3A*.

**TEACHING NOTES**

Textbook Answer Key

For problem 14, be sure your student draws the graphs of the two stations on the same coordinate grid with two different colored pencils. Have her compare the slope of each line with the steepness of each line and explain to you the relationship. (The steeper line has the greater slope.)
WRAP-UP

Today you learned how to use two points on a line and the slope formula to find the slope of the line.

Points: \((1, 0)\) and \((-9, 5)\)

\[
\text{Slope formula: } \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
\frac{5 - 0}{-9 - 1} = \frac{5}{-10} = -\frac{1}{2}
\]

✔️ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Slope - Part 5

**Books & Materials**
- Math in Focus - Teacher Edition
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

**Assignments**
- Complete Interactive Activity.
- Complete Use For Mastery.

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**LEARN**

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**INTERACTIVE ACTIVITY**

Try this activity to explore slope.

Click [here](#) to view the activity in a new window.

In this activity, you will practice finding rise, run, and slope. Start by dragging the red point to (2, 1). Drag the blue point to (5, 7). Turn on Show rise and run. You can see that the rise was found by subtracting the \(y\) value of the red point from the \(y\) value of the blue point (\(y_2 - y_1\)). It is shown by the green arrow on the graph. Similarly, the run was found by subtracting the \(x\) value of the red point from the \(x\) value of the blue point (\(x_2 - x_1\)) and is shown with the purple arrow. Notice that the values were subtracted in the same order: the value of the blue point minus the value of the red point. Practice moving the points to different locations to see how the rise and run are affected. Can you find two points that give you a negative rise? A negative run? Share them with your Learning Guide.

Remember that the ratio of rise and run (\(\frac{\text{rise}}{\text{run}}\)) gives you the slope of the line. Drag the red point back to (2, 1) and the blue point to (5, 7). Find the ratio of the rise (6) over the run (3). Now click on “Show slope computation.” You should see that the slope of this line is \(\frac{6}{3}\), or 2.

Experiment with a variety of lines in the Gizmo. Which is steeper, a line with a slope of 2 or a line with a slope of 3? Which is steeper, a line with a slope of 1 2 or a line with a slope of 1 3? Use the terms rise and run as you explain your answers to your Learning Guide.

Continue to experiment with other lines and slopes to find the answers to these questions:

1. How does the graph of a line with a positive slope differ from a graph with a negative slope?
2. What is true about a graph with a slope of 1?
3. What is the slope of a perfectly vertical line? Why do you think this is so?
Sample answers: A negative rise will occur when the red point is above the blue point; a negative run will occur when the red point is to the right of the blue point. A line with a slope of 3 is steeper than a line with a slope of 2; a line with a slope of 1 \(\frac{1}{2}\) is steeper than a line with a slope of 1 \(\frac{1}{3}\).

Answers to questions:

1. A graph with a positive slope runs from lower left to upper right; a line with negative slope runs from the lower right to the upper left.

2. It is perfectly horizontal. All the points on the line have the same y value.

3. It is undefined. This is because all the x values are the same, so when you subtract them, the difference is zero. You cannot subtract by zero.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

USE FOR MASTERY

1. A line with slope \(-\frac{1}{2}\) passes through point Q \((6, 7)\). Does point S \((2, 5)\) lie on the same line? Without graphing, explain how you found your answer.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information given to tell if the point S (2, 5) lies on the same line?
- Explain how you came to your answer?
- Show your work?
LEARN

WARM-UP

Find the slope of the line passing through the following pairs of points.

1. (3, 2) and (1, −3)
2. (1, 4) (−2, 5)

WARM-UP ANSWERS
1. 5/2  2. −1/3

TEACHING NOTES

INSTRUCTION

Read pp. 147–148 in Math in Focus 3A. You can use the equation of a direct proportion, \( y = mx \), to represent a line that goes through \((0, 0)\). You can also use equations to represent lines that do not go through the origin.

One type of equation for representing lines is called slope-intercept form. It is written as \( y = mx + b \), where \( m \) is the slope and \( b \) is the y-intercept.

You can find the y-intercept by looking at the graph. The y-coordinate of the point where the graph of the equation and the y-axis cross is called the y-intercept.

Read Technology Activity on p. 149. If you do not have a graphing calculator, you can complete the activity by graphing the lines on grid paper as follows.
1. Set up a table for each equation like the one shown.

<table>
<thead>
<tr>
<th>x</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
</tr>
</tbody>
</table>

2. Choose three $x$-values and substitute those $x$-values into the equation to find the corresponding $y$-value to form ordered pairs in the form ($x$-value, $y$-value).

3. Use those points to graph the line.

Review Example 6 on pp. 149–150. Remember, if the graph is a direct proportion and passes through the origin, then $b = 0$ and you only need to find the value for $m$.

Complete Guided Practice on p. 151. For problem 2, the $y$-intercept is negative. When you substitute for $b$ in the equation and simplify, there will be a subtraction sign. Note that although there is no addition sign in the final equation, it is still in slope-intercept form. Be sure to simplify your equations in this way when possible.

TEACHING NOTES

In this lesson, your student will use the general form of the slope-intercept equation, $y = mx + b$, to write specific equations for lines. To do this, she needs the slope ($m$) and the $y$-intercept ($b$). If she is given two points on a graph, she can find the slope. The $y$-intercept is the $y$-coordinate of the point at which the line crosses the $y$-axis. The slope-intercept form for a direct proportion is $y = mx$.

Even if your student has difficulty remembering this, she will be able to write a correct equation as long as she correctly substitutes the values in $y = mx + b$. In a direct proportion, the line passes through the origin, so $b$ always equals 0.

PRACTICE

Complete Practice 4.2 on pp. 152–153 in Math in Focus 3A.

TEACHING NOTES

Textbook Answer Key
WRAP-UP
Today you learned how to identify $y$- and $x$-intercepts. You also learned how to write an equation for a line in the form $y = mx + b$, where $m$ is the slope and $b$ is the $y$-intercept.

![Graph of a line with points (-2, 5) and (0, -2)]

$y$-intercept: $b = -2$

slope: $m = \frac{-2 - 5}{0 - (-2)} = \frac{-7}{2} = -\frac{7}{2}$

slope-intercept form: $y = -\frac{7}{2}x - 2$

SUPPLEMENTAL
- [BrainPop: Slope and Intercept](#)

PRACTICE QUESTIONS
Please go online to view and submit this assessment.
Slope-Intercept Form - Part 2

Objectives
- Rewrite a linear equation in slope-intercept form.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition
- index cards (Optional)

Assignments
- Complete Warm-up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete the Quick Check.

LEARN

WARM-UP
Write the slope-intercept form for the line passing through each pair of points.

1. (3, 4) and (0, −2)
2. (−4, 2) and (0, −5)

TEACHING NOTES

WARM-UP ANSWERS
1. \( y = 2x - 2 \)  2. \( y = -(7/4)x - 5 \)

INSTRUCTION
Read Write an Equation of a Line in Slope-Intercept Form on p. 154 in Math in Focus 3A. Then review Example 7. Equations written in slope-intercept form may look different from each other if some of the values are 0.

- If every variable has a nonzero value, the equation is in the form \( y = mx + b \).
- If \( b = 0 \) (as in a direct proportion), the equation is in the form \( y = mx \).
- If \( x = 0 \) or \( m = 0 \), or both equal 0, the equation is in the form \( y = b \).
- If \( y = 0 \), you can write the equation in terms of \( x \), for example, \( x = b \).

Complete Guided Practice on p. 155. Be sure that the equation you write and the given equation are equivalent. The resulting \( y \)-values in both equations should be the same.

Review Example 8 on p. 156. Then complete Guided Practice on p. 156. Remember to include the sign when substituting the values for slope and \( y \)-intercept.
Today you learned more about the slope-intercept form equation. You learned the following skills:

- rewriting an equation in slope-intercept form
- using the slope-intercept form to identify the slope and y-intercept of a line
- writing an equation of a line given its slope and y-intercept

---

**WATCH FOR THESE COMMON ERRORS**

If the values are negative, your student may forget to include the sign when identifying the slope and y-intercept or when substituting \( m \) and \( b \) in the slope-intercept form equation. Your student should circle the values for \( b \) and \( m \), being sure to include the sign when they are negative.

---

**PRACTICE**

Complete problems 1–6 of Practice 4.3 on p. 164 in *Math in Focus 3A*.

---

**WRAP-UP**

Today you learned more about the slope-intercept form equation. You learned the following skills:

- rewriting an equation in slope-intercept form
- using the slope-intercept form to identify the slope and y-intercept of a line
- writing an equation of a line given its slope and y-intercept
Please go online to view and submit this assessment.

If you struggle with this question, write the slope-intercept form of the equation $y = mx + b$ and then rewrite the equation, substituting the values for slope and $y$-intercept. Watch the signs of the values. You might also wish to revisit the material from this lesson.

In the equation $y = -3x - 5$, the slope is $-3$ and the $y$-intercept is $-5$. 

Complete problems 1–6 of Practice 4.3 on p.164 in Math in Focus 3A.

Today you learned more about the slope-intercept form equation. You learned the following skills:

- Rewriting an equation in slope-intercept form
- Using the slope-intercept form to identify the slope and $y$-intercept of a line
- Writing an equation of a line given its slope and $y$-intercept

In the equation $y = -3x - 5$, the slope is $-3$ and the $y$-intercept is $-5$. 

Complete problems 1–6 of Practice 4.3 on p.164 in Math in Focus 3A.

Textbook Answer Key
LEARN

**WARM-UP**

Identify the slope and $y$-intercept of the graph represented by each equation. *Hint:* First write the equation in slope-intercept form.

1. $y - 4x + 7 = 10$
2. $2y - 6x = -5$

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. slope = 4; $y$-intercept = 3  
2. slope = 3; $y$-intercept = $-5/2$

**INSTRUCTION**

Read *Identify and Write Equations of Parallel Lines* on p. 157 in *Math in Focus 3A*. To help you understand the relationship between parallel lines and their equations, sketch the following lines on grid paper. You may draw each line in a different color.

Then write an equation in slope-intercept form for each line.

If two lines are parallel, the slope-intercept form of their equations will be the same, except for the value of $b$. Parallel lines must have different $y$-intercepts or they would be identical lines. As the $y$-intercept changes, the lines move up or down the $y$-axis, but the slope of the lines stays the same.

Review Example 9 on p. 157. Then complete Guided Practice on p. 158. Write the equation in slope-intercept form before identifying the slope.
Read **Write an Equation of a Line Given its Slope and a Point on the Line** on pp. 158–159. If you know any three of the values in the slope-intercept form equation, you can find the missing value. Remember that the values for $x$ and $y$ are the $x$- and $y$-coordinates of any point on the line.

Review **Example 10** on p. 159. Then complete **Guided Practice** on p. 160. Remember to include the signs when the values are negative.

**TEACHING NOTES**

Lines with the same rate of change have the same slope and are parallel. To use an equation to identify parallel lines, your student should first write it in slope-intercept form. If two lines are parallel, the coefficient of $x$ (the slope value, $m$) in the equations will be the same, but the $b$ value (the constant value) will be different because it represents where the line passes through the $y$-axis.

If your student knows the $x$- and $y$-values of a point on a line and the slope of the line, she can substitute the known values and solve for $b$ to find the $y$-intercept. Then she can use the $y$-intercept and given slope to write an equation for the line in slope-intercept form.

**PRACTICE**

Complete problems 7–11 of **Practice 4.3** on p. 164 in *Math in Focus 3A*.

**TEACHING NOTES**

*Textbook Answer Key*

**WRAP-UP**

Today you learned how to write an equation of a line given its $y$-intercept and the equation of another line parallel to it. You also learned how to write an equation of a line given its slope and a point on the line.
The slope of a line is \(-3\). The line passes through \((4, -1)\).

1. Find the \(y\)-intercept.

\[
\begin{align*}
    y &= mx + b \quad \text{← Use the slope-intercept form equation.} \\
    -1 &= (-3)(4) + b \quad \text{← Substitute the values for } y, m, \text{ and } x. \\
    -1 &= -12 + b \quad \text{← Simplify.} \\
    11 &= b \quad \text{← Solve for } b.
\end{align*}
\]

2. Substitute the values for \(m\) and \(b\) to write an equation.

\[
\begin{align*}
    y &= mx + b \quad \text{← } m = -3, b = 11 \\
    y &= -3x + 11
\end{align*}
\]

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
LEARN

WARM-UP

Find the slope of the line that passes through each pair of points.

1. A (0, 3) and B (2, −3)
2. C (−4, 4) and D (−3, 2)

TEACHING NOTES

WARM-UP ANSWERS

1. −3  2. −2

INSTRUCTION

Read part A on the Slope-Intercept Form Worksheet. In the slope formula, substitute the slope, \( \frac{1}{2} \), for \( m \) and the coordinates of the \( y \)-intercept, (0, 2), for \( x_1 \) and \( y_1 \). Use inverse operations to write the slope formula in slope-intercept form.

Read part B. When a line crosses the \( y \)-axis, \( x = 0 \), so the point at which the line crosses the \( y \)-axis is always (0, \( b \)). Note that a vertical line never intersects the \( y \)-axis at exactly one point, so for nonvertical lines, \( m = \frac{y - b}{x} \).

Substitute \( x_1 = 0 \) and \( y_1 = b \) in the slope equation. Then write the equation in terms of \( y \) with the constant term, \( b \), to the right of the \( mx \) term. The result is the general form of the slope-intercept equation.

Complete the Your Turn section. Fill in the missing values. In a direct proportion, the line intersects both axes at (0, 0), so \( b = 0 \).
In this part, your student uses the slope formula to write the general form of the slope-intercept equation, \( y = mx + b \), as well as the specific slope-intercept equation for a given line. It is important that your student understands that for an equation to be in slope-intercept form, it must be written in terms of \( y \), and that \( b \) (the constant term) needs to be to the right of the \( mx \) term. Your student also will need to understand that \( y = mx \) is the slope-intercept form for a direct proportion. Since the line passes through the origin, \( b = 0 \).

**Your Turn Answers:**

1 0 2 0; 0 3 \( m = \frac{y - 0}{x - 0} \) 4 \( m = \frac{y}{x} \) 5 \( m = \frac{y}{x} \) 6 \( mx = y \)

**PRACTICE**

Complete the **Practice** section of the **Slope-Intercept Form Worksheet**.

**TEACHING NOTES**

For problem 5, your student should start by finding the slope of the line. Then she should replace \( m, y_1 \) and \( x_1 \) and use inverse operations to write the equation in terms of \( y \). This will produce a specific equation in slope-intercept form.

**Practice Answers:**

1a no b \( y = \frac{3}{4} x + \frac{5}{4} \) c \( \frac{3}{4} \) d \( \frac{5}{4} \)

2a no b \( y = x + \frac{-1}{3} \) c 1 d \( \frac{-1}{3} \)

3a yes b N/A c \( \frac{5}{2} \) d \( \frac{-4}{2} \)

4a no b \( y = \frac{-1}{7} x + 5 \) c \( \frac{-1}{7} \) d 5

5a \( m = \frac{4 - 0}{0 - (-2)} = 4/2 = 2 \)

b \( m = \frac{y - y_1}{x - x_1} \)

\( 2 = \frac{y - 4}{x - 0} \)

\( 2 = \frac{y - 4}{x} \)

\( 2x = y - 4 \)

\( 2x + 4 = y \)
Sample answer: The slope and y-intercept are not correct because the equation is not in slope-intercept form. To use the equation to find the slope and y-intercept, Gerrard should have first written it in slope-intercept form:

\[-3y = (3/4)x + (-2)\]
\[-1/3 \cdot (-3y) = -1/3 \cdot (3/4)x + (-1/3) \cdot (-2)\]
\[y = (-1/4)x + 2/3\]

The slope is \(-1/4\), and the y-intercept is \(2/3\).

WRAP-UP

Today you learned how to use the slope formula to derive the slope-intercept form equation.

\[
m = \frac{y - y_1}{x - x_1}
\]
\[
m = \frac{y - b}{x - 0}
\]
\[
m = \frac{y - b}{x}
\]
\[
mx = y - b
\]
\[
mx + b = y
\]
\[
y = mx + b
\]

QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

If you struggled with this question, write the equation \(y = mx + b\) and then rewrite it with words to tell the values \(m\) and \(b\) represent; \(y = \text{(slope)}x + \text{(y-intercept)}\). You might also wish to revisit the material from this lesson.
**Slope-Intercept Form - Part 5**

**Objectives**
- Write an equation of a line given a point and the equation of a parallel line.
- Write the equation of a line given two points.

**Books & Materials**
- Math in Focus 3A
- Math in Focus - Teacher Edition

**Assignments**
- Complete Warm-up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete the Use For Mastery.

**LEARN**

**WARM-UP**
Find an equation of a line parallel to the line represented by the given equation with the given y-intercept.

1. equation: \(-3y = 3x + 7\)
y-intercept: -13

2. equation: \(y + 5x = 2\)
y-intercept: 4

**WARM-UP ANSWERS**
1. \(y = -x - 1/3\)  
2. \(y = -5x + 4\)

**TEACHING NOTES**

**INSTRUCTION**
Read the instructional section on p. 160 in Math in Focus 3A. Then review Example 11 on p. 161. Notice that in the equation \(y = 2 - 4x\), the constant term, 2, is written to the left of the \(mx\) term, \(-4x\), so it is not in slope-intercept form.

\[
y = 2 - 4x \\
  = -4x + 2
\]

The slope is negative.

Read Think Math on p. 161. To answer the question, think about what parallel lines have in common. Since the slope is the same for both lines, you can look at the \(y\)-intercept and determine which number
Read Think Math on p. 161. To answer the question, think about what parallel lines have in common. Since the slope is the same for both lines, you can look at the y-intercept and determine which number is greater. The line with the greater y-intercept would be found above the other line.

Complete Guided Practice on p. 162.

Read Write an Equation of a Line Given Two Points on p. 162. Then review Example 12 on p. 163. Notice that both of the points (1, 3) and (2, −4) are on the line, so either one can be used to find the y-intercept. Always be sure you use the x- and y-values from one point rather than one value from one point and one value from the other.

Complete Guided Practice on p. 163.

The purpose of this lesson and the previous lessons is to teach your student how to write an equation to represent a line. The slope-intercept form equation is one equation that she can write. To write an equation in slope-intercept form, she needs the slope and y-intercept of the line. She can use the slope formula or information about a parallel line to find the slope of a line. She can substitute the slope and the values for a point on the line into the slope-intercept form equation to find the y-intercept.

As a matter of habit, your student should always write any given equation for a line in slope-intercept form. This will enable her to more easily identify the slope and y-intercept and will make equations of different lines easier to compare.

Practice

Complete problems 12–16 of Practice 4.3 on p. 164 in Math in Focus 3A.

Textbook Answer Key
WRAP-UP

Today you learned how to write an equation of a line given a point on the line and the equation of a parallel line. You also learned how to write an equation of a line given two points on the line.

Example 1: Find the equation for a line parallel to \( y = -3x + 2 \) that passes through (3, 2).
1. Parallel lines have the same slope. The slope of the line is -3.
2. Substitute the values for \( m, x, \) and \( y \) and solve for \( b \).
   \[ y = mx + b \]
   \[ 2 = -3(3) + b \]
   \[ 2 = -9 + b \]
   \[ 11 = b \]
3. Substitute \( m \) and \( b \).
   \[ y = -3x + 11 \]

Example 2: A line passes through (1, 7) and (4, 1).
1. Find the slope.
   \[ m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{7 - 1}{1 - 4} = \frac{6}{-3} = -2 \]
2. Substitute the values for \( m, x, \) and \( y \) and solve for \( b \).
   \[ y = mx + b \]
   \[ 7 = -2(1) + b \]
   \[ 7 = -2 + b \]
   \[ 9 = b \]
3. Substitute \( m \) and \( b \).
   \[ y = -2x + 9 \]

USE

USE FOR MASTERY

1. There were 900 students enrolled in a high school in 2009 and 1,500 students enrolled in the same high school in 2012. The student enrollment of the high school, \( P \), has increased at a constant rate each year, \( t \), since 2009.

A) Write the given enrollment numbers as a pair of points in the form \((t, P)\).

B) Find the slope of the line passing through the pair of points from part A and explain the information the slope gives about the situation.

C) Write an equation that relates the high school's student enrollment, \( P \), to the number of years since 2009, \( t \).

D) Predict the high school's enrollment in 2017.
If you are able, use the text box to show your work and enter your final answer to each question. If not, complete your work on paper and upload it below.

**USE FOR MASTERY GUIDELINES & RUBRIC**

Did you:

- Use the information given to answer parts A - D?
- Write the given enrollment numbers as a pair of points in the form \((t, P)\)?
- Find the slope of the line passing through the pair of points from part A and explain the information the slope gives about the situation?
- Write an equation that relates the high school's student enrollment, \(P\), to the number of years since 2009, \(t\)?
- Predict the high school's enrollment in 2017?
- Use the text box to show your work and enter your final answer to each question OR complete your work on paper and upload it to the site?
- Add labels to give meaning to the numeric values?
- Show your work in an organized, logical manner?
Graphing Linear Equations - Part 1

Objectives
- Graph a linear equation when given two points.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition
- grid paper
- straightedge
- online graphing calculator (Optional)

Assignments
- Complete Warm-up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete the Practice Questions.

LEARN

WARM-UP
Find the slope-intercept form equation for each line.

1. A line passes through point (1, 2) and is parallel to the line represented by the equation \( y = -x - 5 \).
2. A line passes through points (−1, 3) and (−2, 1).

WARM-UP ANSWERS
1. \( y = -x + 3 \)  2. \( y = 2x + 5 \)

TEACHING NOTES

INSTRUCTION
Read p.165 in Math in Focus 3A. In previous lessons you learned that in the slope-intercept form equation, \( x \) and \( y \) values are the coordinates of a point on the line. Additionally, if you know three of the values in the slope-intercept form equation, you can find the fourth value.

If you substitute values for \( x \), \( m \), and \( b \) in the slope-intercept form equation, you can find the value for \( y \). This \( xy \) pair is a point on the line. You can write the point in the form \( (x-value, y-value) \). You only need two points to graph a line, but graphing a third point will help you make the graph more accurate.

Review Example 13 on p.166. To find the \( y \)-values using this equation, you need to multiply the \( x \)-values by 34. If you choose multiples of 4 for your \( x \)-values, the products will be integers. Choosing at least one negative \( x \)-value, 0, and one positive \( x \)-value can make your graph more accurate because the points will be spread out rather than clustered together.
Complete **Guided Practice** on p. 166. If the three points are not in a line, you need to check your work. Another way to check your work is to determine the slope of the line you graphed and see if it is the same as the slope in the equation.

1. Make sure that the given equation is in slope-intercept form so you can easily identify the \(m\)-value.
2. Find the slope of the line you graphed using the slope formula or another method you have learned in this chapter.
3. Compare the slope with the \(m\)-value in the equation.

---

**TEACHING NOTES**

The slope-intercept form equation is useful because it can help your student graph a line. The \(x\) and \(y\) in the equation represent the \(x\)- and \(y\)-values of a point on the line, and there are infinitely many points on the line. If your student knows \(m\) and \(b\), and then chooses any value for \(x\), she can find the corresponding \(y\)-value. In this way, she can make ordered pairs in the form of \((x, y)\) that are on the line. With two or more points, your student can draw the graph.

When making tables with \(x\)- and \(y\)-values, your student can make a horizontal table as shown on p. 166 in *Math in Focus 3A*, or she can make a vertical table as shown here:

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

**PRACTICE**

Complete problems 1–4 and 9–10 of **Practice 4.4** on p. 170 in *Math in Focus 3A*. Make \(xy\) tables to create the graph for each equation.

---

**TEACHING NOTES**

[Textbook Answer Key]
WRAP-UP

Today you learned how to graph a linear equation using two or more points.

Graph the equation \(y = -2x + 1\).

1. Make a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>-3</th>
<th>0</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>7</td>
<td>1</td>
<td>-5</td>
</tr>
</tbody>
</table>

2. Graph the ordered pairs.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Graphing Linear Equations - Part 2

**Objectives**
- Graph a linear equation using the slope and y-intercept.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*
- grid paper
- straightedge
- online graphing calculator (Optional)

**Assignments**
- Complete Warm-Up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete Quick Check.

---

**LEARN**

**WARM-UP**

Use grid paper. Use 1 square to represent 1 unit for the x-interval from −4 to 4 and the y-interval from −4 to 4. Graph the equation $y = \frac{1}{2}x - 2$.

---

**TEACHING NOTES**

**WARM-UP ANSWER**

[Graph image]

---

**INSTRUCTION**

Read Sketch a Linear Graph by Using $m$ and $b$ on p. 166 in *Math in Focus 3A*. The y-intercept is where the line crosses the y-axis, so if the y-intercept is $b$, the point where the line crosses the y-axis is $(0, b)$. 
Review Example 14 on pp. 167–168. When you are given a point and the slope, you can plot the point on grid paper and then use the slope to find other points. Once you have found 3 points, use a ruler or a straightedge to connect the points to make the line. This line is the graph of the equation \( y = -12x - 3 \).

Complete Guided Practice on p. 168. Check that you have moved in the correct direction by being sure that lines with positive slope rise and lines with negative slope fall.

Read Sketch a Linear Graph by Using \( m \) and a Point and review Example 15 on p. 169. Then complete Guided Practice on p. 169.

### TEACHING NOTES

In this lesson, your student will learn that she can use the slope and a point on the line to draw a graph. The point can be the \( y \)-intercept or any other point on the line. One difficult concept in this lesson is how to use the slope to determine how to move from one point to a second point on the line. The signs of the numerator and denominator of the slope ratio indicate which direction she should move. Your student should move up and left or down and right if the slope is negative. If the slope is positive, she should move up and right or down and left.

**WATCH FOR THESE COMMON ERRORS**

Your student may use the vertical change to move right or left and the horizontal change to move up or down. Point out that \( m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} \) and that rise is up and down and run is left and right. Have her circle the numbers in the ratio and label them up/down or right/left before she uses them to locate a second point on the graph.

### PRACTICE

Complete problems 5–8 and 11–15 of Practice 4.5 on p. 170 in *Math in Focus 3A*. For problems 5–8, you may want to use the slope and \( y \)-intercept to graph each equation. To help you find Emily’s mistake in problem 15, you can graph the given equation and then compare your graph to Emily’s.

### TEACHING NOTES

Textbook Answer Key
WLAP-UP

Today you learned how to graph a linear equation using the slope and the coordinates of any point on the line.

Graph a line with slope $-\frac{3}{2}$ and point $(-2, 3)$.

1. Plot point $A$ at $(-2, 3)$.
2. Use the slope to find a second point, point $B$. $-\frac{3}{2} = -\frac{3}{2}$, so move 3 units down and 2 units right.

![Graph of the line with slope $-\frac{3}{2}$ and point $(-2, 3)$]

**QUICK CHECK**

Please go online to view and submit this assessment.

**MORE TO EXPLORE**

If you struggled with this question, practice describing graphing an equation in slope-intercept form by plotting the $y$-intercept and using the slope to find the next point. You might also want to revisit the material from this lesson.
Graphing Linear Equations - Part 3

Books & Materials
- Math in Focus - Teacher Edition

Assignments
- Complete Interactive Activity.
- Complete Rate Your Enthusiasm.

LEARN

INTERACTIVE ACTIVITY

Use the GeoGebra applet Graph the Line to adjust the line to match the equation.

RATE YOUR ENTHUSIASM

Please go online to view and submit this assessment.
LEARN

WARM-UP

Use grid paper. Use 1 grid square to represent 1 unit for the x-interval from −5 to 5 and the y-interval from −5 to 5. Graph the line that has a slope of \( \frac{3}{4} \) and passes through (−2, 1).

TEACHING NOTES

WARM-UP ANSWER

![Graph of a line with slope \( \frac{3}{4} \) passing through (−2, 1)](image)

INSTRUCTION

Read the instructional section on pp. 171–172 in *Math in Focus 3A*. The *vertical intercept* is another name for the *y*-intercept.
Then review **Example 16** on pp. 172–173. In this lesson, you have used $x$ to represent the independent variable and $y$ to represent the dependent variable. However, you can use any letters for the variable pair. The independent variable is always on the $x$-axis, and the dependent variable is always on the $y$-axis.

Look at the graph on p. 172. When a graph represents a real-life situation, the points on the graph have a particular meaning in that context. For example, the point (200, 248,000) means that after 200 hours, there are 248,000 gallons of water in the pool.

The slope of a line also has a specific meaning in context.

\[
\frac{\text{vertical change}}{\text{horizontal change}} = \frac{400,000}{200} = -2,000 \quad 1
\]

The slope means that the amount of water decreases by 2,000 gallons for every one hour that passes.

Complete **Guided Practice** on p. 174. To understand the meaning of slope in a real-world context, think about its numerical value and its sign. Look at the intervals on the scales of the graph. The intervals are varied, and using the wrong interval will result in an incorrect slope.

Then review **Example 17** on pp. 175–176. Even though Joanne’s salary is greater than Chris’s, that does not guarantee she will make more than Chris each month. Since Chris’s commission rate is so much more than Joanne’s, he could earn more than her in a month.

Complete **Guided Practice** on p. 177.

Review **Example 18** on pp. 178–179. Although Scarlet’s account is changing at a faster rate than Britney’s, the accounts are changing in different ways. The negative slope indicates the money in Scarlet’s account is decreasing, and the positive slope indicates the money in Britney’s account is increasing. The equation given for Scarlet’s bank account, $y = -24x + 120$, shows that she started with $120 in her bank account and withdrew $24 every week.

Complete **Guided Practice** on p. 179.

---

**TEACHING NOTES**

One of the most difficult concepts in this lesson is how to interpret the slope, $y$-intercept, and point on a line in the context of a real-life situation. Your student should look at these aspects for each graph given in the lesson and determine what each means in context.

A graph and an equation are two different ways to represent the same relationship and so they can be compared. The equation written in slope-intercept form tells your student the slope and the $y$-intercept. These features can also be found on the graph. The numerical value of a slope indicates the rate of change. The sign indicates whether the change is decreasing or increasing.
The independent variable is always on the x-axis, and the dependent variable is always on the y-axis.

Look at the graph on p. 172. When a graph represents a real-life situation, the points on the graph have a particular meaning in that context. For example, the point (200, 248,000) means that after 200 hours, there are 248,000 gallons of water in the pool.

The slope of a line also has a specific meaning in context. The slope means that the amount of water decreases by 2,000 gallons for every one hour that passes.

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Complete Guided Practice on p. 179.

One of the most difficult concepts in this lesson is how to interpret the slope, y-intercept, and point on a line in the context of a real-life situation. Your students should look at these aspects for each graph given in the lesson and determine what each means in context.

A graph and an equation are two different ways to represent the same relationship and so they can be compared. The equation written in slope-intercept form tells your students the slope and the y-intercept. These features can also be found on the graph. The numerical value of a slope indicates the rate of change. The sign indicates whether the change is decreasing or increasing.
Graphing Linear Equations - Part 5

**Objectives**
- Apply knowledge of linear equations to solve problems.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*
- grid paper
- straightedge
- colored pencils

**Assignments**
- Complete Warm-up.
- Read and complete Brain @ Work in *Math in Focus 3A*.
- Complete the Practice Questions.

---

**LEARN**

**WARM-UP**

Mary Jo is traveling from a gas station near her home to her grandmother’s house. The graph shows her distance \( d \) miles from home as she travels \( h \) hours. What are the slope and \( y \)-intercept of the line? What does each value mean in the context of the graph?

![Graph of Mary Jo's Trip](image)

**WARM-UP ANSWERS**

Slope is 50, \( y \)-intercept is 20. The slope means Mary Jo traveled 50 miles an hour; the \( y \)-intercept means the gas station was 20 miles from her house.

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

Slope is 50, \( y \)-intercept is 20. The slope means Mary Jo traveled 50 miles an hour; the \( y \)-intercept means the gas station was 20 miles from her house.
**INSTRUCTION**

In this lesson, you will continue solving real-life problems involving linear relationships. If you need to review these concepts, look back at Lesson 49.

Read and complete **Brain @ Work** on p. 182 in *Math in Focus 3A*. When you graph two relationships on one line, you can compare them visually. Use different colored pencils to draw Conrad and Angeline’s graphs in problem 1. When graphing Angeline’s line, remember that the two students have the same amount of money after 4 days. The slope of Angeline’s line tells you how much allowance she saves each day.

For problem 2, the time that Gordon left Townsville is represented by a point at the origin. Since Jonathan started at Kingston 1.5 hours past 12:00 P.M. and 50 miles from Townsville, his graph starts at (1.5, 50).

Remember to look carefully at the intervals on the scales of the graphs. Using the wrong interval will result in finding an incorrect slope.

**TEACHING NOTES**

In this chapter, your student has learned that linear relationships can be represented by graphs and linear equations. In the last lesson, she learned that linear equations and graphs can represent real-world situations that show a constant rate of change.

When your student graphs an equation, the line is a visual way to represent the same relationship. If your student knows the graph of one relationship and the equation of another, she can compare those relationships by comparing their slopes and y-intercepts. By interpreting the equations and graphs in the context of the real-life situations, your student can compare quantities and solve problems.

**PRACTICE**

Complete problems 2–5 of **Practice 4.5** on pp. 180–181 in *Math in Focus 8A*.

**TEACHING NOTES**

[Textbook Answer Key](#)

If your student has difficulty with problem 4, she may graph the line represented by the given equation to represent the amount of Winnie’s gift card. It may be easier for her to compare two lines than the equation and the line.
WRAP-UP

Today you continued to solve real-world problems involving equations and graphs of linear relationships.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Graphing Linear Equations - Part 6

**Objectives**
- Review previously learned concepts.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*

**Assignments**
- Complete Warm-up.
- Read and complete assigned pages in *Math in Focus 3A.*
- Complete Practice Questions.

**LEARN**

**WARM-UP**
Write a slope-intercept form equation for the line that passes through each pair of points.

1. (0, 0) and (2, −3)
2. (1, 3) and (2, 4)
3. (−1, 2) and (−3, 2)

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. \(y = \frac{-3}{2}x\)  
2. \(y = x + 2\)  
3. \(y = 2\)

**INSTRUCTION**

Read Chapter Wrap Up on p. 183 in *Math in Focus 3A.* Study the concept map about linear equations. The slope is the ratio of the vertical change to the horizontal change.

The points \((x_1, y_1)\) and \((x_2, y_2)\) are two points on the line.

The \(y\)-intercept, or the vertical intercept, is the \(y\)-coordinate of the point at which the line crosses the \(y\)-axis. The coordinate of this point is \((0, b)\). The equation of a line can be written in slope-intercept form, \(y = mx + b\).

Read over the key concepts to review additional information about equations of lines.
In this chapter, your student learned about linear relationships. The concept map on p. 183 shows how the ideas in this chapter are related.

Complete Chapter Review/Test on pp. 184–185 in Math in Focus 3A.

Textbook Answer Key

In this lesson, you reviewed important concepts from the chapter about linear relationships.

Please go online to view and submit this assessment.
Graphing Linear Equations - Part 7

LEARN

WARM-UP
For each line, state its slope and its y-intercept.

1. \( y = -\frac{3}{2}x - 6 \)
2. \(-3y = 6x - 1\)
3. \(2y - 5x = 2\)

WARM-UP ANSWERS

1. slope: \(-\frac{3}{2}\); y-intercept: \(-6\)
2. slope: \(-2\); y-intercept: \(1/3\)
3. slope: \(5/2\); y-intercept: \(1\)

PRACTICE

Complete Cumulative Review on pp. 186–189 in Math in Focus 3A. If you have trouble with any of the problems, make sure you go back and look at the corresponding lesson.

TEACHING NOTES

Textbook Answer Key

Your student has learned many new skills, vocabulary terms, and strategies in the last two chapters. Once your student has completed the review, look over it with her to determine areas of strength and areas that may need more development and practice.
WRAP-UP

Today you reviewed concepts from Chapters 3 and 4.

USE

USE FOR MASTERY

1. The table shows the cost of strawberries at two supermarkets.

<table>
<thead>
<tr>
<th>Weight of Strawberries (x pounds)</th>
<th>Cost at Supermarket A (y dollars)</th>
<th>Cost at Supermarket B (y dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4.00</td>
<td>3.00</td>
</tr>
<tr>
<td>3</td>
<td>6.00</td>
<td>4.50</td>
</tr>
</tbody>
</table>

A. Click here to download the Cost of Strawberries coordinate grid. On the grid, graph the relationship between cost and weight of strawberries for each supermarket. Use 2 units on the horizontal axis to represent 1 pound for the x interval from 0 to 3. Use 1 unit on the vertical axis to represent $0.50 for the y interval from 0 to 6.

Upload your completed graphs here.
B. Find the slope of each line. Then tell what the slopes represent and how they help you determine which supermarket has less expensive strawberries.

C. Which supermarket has less expensive strawberries per pound? Explain how you determined your answer.

USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information given to help you answer the questions?
- Download the Cost of Strawberries coordinate grid?
- Graph the relationship between cost and weight of strawberries for each supermarket?
- Use 2 units on the horizontal axis to represent 1 pound for the x interval from 0 to 3?
- Use 1 unit on the vertical axis to represent $0.50 for the y interval from 0 to 6?
- Tell which supermarket has less expensive strawberries per pound AND explain how you determined your answer?
- Upload your completed graphs?
- Label your graph correctly?
- Show your work in an organized, logical manner?
Systems of Linear Equations - Part 1

Objectives
- Graph a linear equation from a table of values.
- Write a linear equation to represent a real-world situation.

Books & Materials
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*

Assignments
- Complete Warm-up.
- Read and complete assigned pages in Math in Focus A.
- Complete the Practice Questions.

LEARN

WARM-UP

Find the quantities described in each problem.

1. If $2x - y = 13$, what is the value of $x$ when $y = 5$?

2. Chandler is 3 years younger than Elizabeth. If $x$ represents Elizabeth’s age, what expression represents Chandler’s age?

3. Pierre has $n$ quarters. What algebraic expression represents the amount of money Pierre has in quarters?

WARM-UP ANSWERS

1. $x = 9$  
2. $x - 3$  
3. $0.25n$

TEACHING NOTES

INSTRUCTION

Today’s lesson covers important ideas you need to understand before beginning the lessons in this chapter. Read pp. 190–192 in *Math in Focus 3A*. Recall that the solutions of linear equations in two variables are ordered pairs, which, when graphed, form a line. A table of values shows several solutions, but there are infinitely many solutions to these types of equations.
In the example on p. 192, you use a linear equation with one variable to solve a real-world problem. One of the unknown quantities is represented by a variable. The second unknown quantity is written in terms of the variable (the first unknown) and a number. The equation shows how the quantities in the situation are related. You solve the equation using Properties of Equality to isolate the variable. Remember to interpret the solution of the equation in the context of the problem.

### SKILLS CHECK

Complete the Quick Check sections on pp. 191–192 in Math in Focus 3A.

### TEACHING NOTES

#### Textbook Answer Key

Review your student’s answers to the Quick Check sections, noting the problems that he answered incorrectly. Click on the link to access the appropriate Reteach activity that your student should complete for the remainder of this lesson.

#### RETEACH

After your student completes the Quick Check in the Recall Prior Knowledge lesson of this chapter, review the questions that were answered incorrectly.

If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.

<table>
<thead>
<tr>
<th>Quick Check Question(s)</th>
<th>Activity</th>
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</thead>
<tbody>
<tr>
<td>1–2</td>
<td>Refer to material from previous lessons.</td>
</tr>
<tr>
<td>3</td>
<td>Writing Equations for Word Problems</td>
</tr>
</tbody>
</table>
**WRAP-UP**

Today you reviewed how to use a table of values to graph a linear equation in two variables.

**Step 1:** Substitute values of \( x \) into the equation and solve the equation to find the corresponding values of \( y \). Record your results in a table of values.

**Step 2:** Plot each ordered pair from the table on a coordinate grid.

**Step 3:** Draw the line that passes through the points you plotted.

You also reviewed how to use a linear equation in one variable to solve a real-world problem.

There are 29 students in Esteban’s math class. There are 5 more girls than boys. How many boys are there in the class?

Let \( x \) represent the number of boys. Then \( x + 5 \) represents the number of girls. The number of boys plus the number of girls is equal to the total number of students in the class.

\[
\begin{align*}
x + x + 5 &= 29 \\
2x + 5 &= 29 \\
2x + 5 - 5 &= 29 - 5 \\
2x &= 24 \\
\frac{2x}{2} &= \frac{24}{2} \\
x &= 12
\end{align*}
\]

There are 12 boys in the class.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Systems of Linear Equations - Part 2

Objectives
- Identify a system of linear equations.
- Solve a system of linear equations by making tables.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition
- online graphing calculator (Optional)

Assignments
- Complete Warm-up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete the Practice Questions.

LEARN

WARM-UP
Find a solution to each equation.

1. \( y = 6x - 1 \)
2. \( x - y = 4 \)
3. \( 2x + 3y = 12 \)

TEACHING NOTES

WARM-UP ANSWERS
Answers will vary. 1. Sample answer: (1, 5) 2. Sample answer: (6, 2) 3. Sample answer: (6, 0)

INSTRUCTION
Read p. 193 in Math in Focus 3A. When a system of linear equations with two variables has a unique solution, that solution is the only ordered pair that satisfies both equations in the system.

Review Example 1 on p. 194. When making a table of values for a real-world problem, be sure to use values that make sense in the situation. If you do not see the same pair of values for both equations, examine the trends in both tables. For instance, if you start with \( x = 4 \) for your first values and increase Lionel’s age by 1 for each successive pair of numbers, you will not find the ordered pair \((3, 2)\). However, notice that the \( y \)-values in the two tables become farther apart as \( x \) increases. This indicates that you should choose a smaller value for \( x \) to find the ordered pair that satisfies both equations.

Complete Guided Practice on pp. 194–195. The directions for problems 2 and 3 state that the values of \( x \) and \( y \) for these systems of equations are positive integers. Note, however, that the solutions of systems of equations are not always ordered pairs of positive integers, as you will see in the following lessons.
If you have access to a graphing calculator, complete Technology Activity on p. 195. When you use the table function, your calculator is creating a table just as you have done by hand.

**TEACHING NOTES**

Help your student understand the meaning of the solution of a system of equations by encouraging him to substitute the solution for the system in Example 1 into both of the equations in the system and observe that the solution satisfies both equations. Emphasize that whereas there are many ordered pairs that satisfy each equation, the solution is the only ordered pair that satisfies both equations.

**PRACTICE**

Complete Practice 5.1 on p. 196 in Math in Focus 3A.

**TEACHING NOTES**

Textbook Answer Key

You may want to help your student determine reasonable values to use for the tables of values in problems 10–12. Guide him to consider what numbers make sense in the context of the situation. Also, in problem 11, the second equation has already been simplified. You may need to lead your student to this equation. The conditions give the equation \((x + 3) + (y + 3) = 27\). Rearranging gives \((x + y) + (3 + 3) = 27\). Subtracting 6 from both sides gives \(x + y = 21\).

While Math in Focus presents the equations of a system simply as two stacked equations, some texts use a brace to group the equations that form a system together. Use the example shown in the Wrap-up to ensure your student is exposed to this notation.

**WRAP-UP**

Today you learned how to use tables of values to find the solution of a system of linear equations. You learned that the solution is the ordered pair that satisfies both equations in the system.
The following is the solution to the system of equations \( y = 3x - 1 \) and \( x + 2y = 5 \).

\[
\begin{array}{c|cccc}
\hline
x & 0 & 1 & 2 & 3 \\
y & -1 & 2 & 5 & 8 \\
\hline
\end{array}
\]

\[
\begin{array}{c|cc}
\hline
x & 0 & 1 \\
y & 2.5 & 2 \\
\hline
\end{array}
\]

The solution of the system \( \begin{cases} 
y = 3x - 1 \\
x + 2y = 5 \end{cases} \) is \( x = 1, y = 2 \).

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Systems of Linear Equations - Part 3

Objectives
- Use elimination to solve systems of linear equations.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete the Quick Check.

LEARN

WARM-UP

Find the value of the variable for each equation.

1. If $3x - 2y = 15$, what is the value of $y$ when $x = 3$?

2. If $-x + 4y = -1$, what is the value of $x$ when $y = -2$?

WARM-UP ANSWERS
1. $y = -3$  
2. $x = -7$

INSTRUCTION

Read the instructional section on pp. 197–198 in Math in Focus 3A. Notice that the equations in this system are numbered. This system of equations is solved by subtracting.

Review Example 2 on pp. 198–199. In these equations, the coefficients of the $y$-terms are opposites, so you need to add the equations to eliminate the $y$-terms. Remember to find values for both variables to solve the problem completely.

Complete Guided Practice on p. 199. Use substitution to check your answers.

Read p. 200. The bar models show that neither adding nor subtracting these equations will eliminate a variable. To find the solution to equations like this, you need to find equivalent expressions for one or both of them. You can accomplish this by multiplying one or both of the equations by the same number. When you do this, be sure to use the Distributive Property correctly on both sides of the equation.
Note that Equation 3 is equivalent to Equation 2. If you want to verify this, make a table of ordered pairs for each equation.


Notice that in this example, you need to multiply both equations by numbers to eliminate a variable when you add (or subtract) the equations.

This procedure is commonly used to convert repeating decimals into fractions. Consider the repeating decimal 0.333… It is not possible to rewrite this as a fraction because there is never a final place to determine the value of the denominator. However, you can multiply it by a power of ten to create a second equation and solve using the elimination method.

EXAMPLE 1

Let \( x = 0.\overline{3} \). Then \( 10x = 3.\overline{3} \).

Solve the system of equations by subtraction.

\[
\begin{align*}
10x &= 3.\overline{3} \\
x &= 0.\overline{3} \\
9x &= 3 \\
x &= \frac{3}{9} = \frac{1}{3}
\end{align*}
\]

EXAMPLE 2

Let \( x = 0.\overline{45} \). Then \( 10x = 4.\overline{54} \). This is not helpful because it does not give us the same number after the decimal point and will not eliminate the repeating decimal. Try \( 100x = 45.\overline{45} \).

Solve the system of equations by subtraction.

\[
\begin{align*}
100x &= 45.\overline{45} \\
x &= 0.\overline{45} \\
99x &= 45 \\
x &= \frac{45}{99} = \frac{5}{11}
\end{align*}
\]

With your Learning Guide, try converting the following repeating decimals to fractions:

\( 0.\overline{7}, \ 2.\overline{8}, \ 0.\overline{19}, \ \text{and} \ 6.\overline{95} \).

The fractions for the repeating decimals are $\frac{7}{9}, \frac{2}{9}, \frac{19}{99}, \frac{6}{9599}$. If your student struggles with these problems, try to determine whether the difficulty lies in finding the multipliers or in performing the computations.

If it is the former, present several examples of systems of equations and have him brainstorm different combinations of multipliers that would work. If it is the latter, check to see whether he is making errors when subtracting or distributing.

**WATCH FOR THESE COMMON ERRORS**

Some students forget to multiply the entire equation by the number. Encourage your student to write the multiplier on both sides of the equation and to watch the signs when multiplying by a negative integer.

**PRACTICE**

Complete problems 1–9 and 25 of Practice 5.2 on p. 209 in Math in Focus 3A.

**WRAP-UP**

Today you learned how to solve a system of equations using elimination.

**Step 1:** If necessary, multiply one or both of the equations by a number or numbers that will make the coefficients of either the x- or y-terms the same or opposites.

**Step 2:** Add or subtract the equations, as necessary, to eliminate either the x- or y-terms.

**Step 3:** Solve the resulting equation for the remaining variable.

**Step 4:** Substitute the known value into one of the original equations to find the value of the other variable.

**Step 5:** Check your solution by substituting both values into the original equations.

With your Learning Guide, try converting the following repeating decimals to fractions:

\[ x = 0.9999999 \]

Complete problems 1–9 and 25 of Practice 5.2 on p. 209 in Math in Focus 3A.
WATCH FOR THESE COMMON ERRORS

Somestudents forget to multiply the entire equation by the number. Encourage your student to write the multiplier on both sides of the equation and to watch the signs when multiplying by a negative integer.

Complete problems 1–9 and 25 of Practice 5.2 on p. 209 in Math in Focus 3A.

Textbook Answer Key

Today you learned how to solve a system of equations using elimination.

Please go online to view and submit this assessment.

If you answered incorrectly, you can check your answers by substituting the values of the variables into the equations to see if they make true statements. You might also want to revisit the material in this lesson.
In this activity, Solving a System of Equations by Elimination, you will practice solving systems of linear equations by using the elimination method.

Please go online to view and submit this assessment.
LEARN

WARM-UP

Solve each equation for x.

1. $2x + 8y = 5$
2. $3y - x = 1$

WARM-UP ANSWERS

1. $x = -4y + 2.5$  
2. $x = 3y - 1$

TEACHING NOTES

INSTRUCTION

Read *Solve Systems of Linear Equations Using the Substitution Method* on pp. 203–204 in *Math in Focus 3A*. You can use either equation and solve for either of the variables in the first step of the solution. However, it is usually easiest to solve for a variable that has a coefficient of 1. Be sure to substitute for the correct variable in the other equation.

Review Example 4 on pp. 204–205. In part a, one of the equations is already solved for a variable, so Step 1 is already complete. In this example, parentheses are used when $x - 4$ is substituted for $y$ in the first equation. This will help you keep track of negative signs and ensure they are distributed. (*Note:* In part b, the second equation, $3p - 5q = \frac{1}{2}$, is labeled *Equation 1* in the textbook, but it should be labeled *Equation 2*.)

Neither equation contains a variable term that has a coefficient of 1. However, both equations contain $3p$. Solving one equation for $3p$ allows you to substitute for $3p$ in the other equation.
Recall that the solution of a system of equations is the pair of values that satisfies both equations. Thus, you can check the solution by substituting the values into the equations. You can also use the elimination method to check your solution.

Complete **Guided Practice** on p. 206. Use the elimination method to check your answers.

**TEACHING NOTES**

Help your student to understand that when he solves an equation for a variable, the resulting equation provides an algebraic expression that is equivalent to the variable. The resulting expression can be substituted into the other equation without changing the value of the expressions in that equation.

**WATCH FOR THESE COMMON ERRORS**

Some students make sign errors when substituting an expression for a variable that is being subtracted. (See Example 4, part a.) Encourage your student to use parentheses when making the substitution, and point out the sign error that occurs if parentheses are not used.

**PRACTICE**

Complete problems 10–18 of **Practice 5.2** on p. 209 in **Math in Focus 3A**.

**TEACHING NOTES**

**Textbook Answer Key**

You may want to read through problems 10–18 with your student and brainstorm what the first step in each solution will be. Recognize that there is more than one correct way to solve each system of equations.

**WRAP-UP**

Today you learned how to solve a system of equations using the substitution method.

\[
\begin{cases}
2x + y = 1 \\
4x + 3y = -3
\end{cases}
\]

Solve the system:

Solve Equation 1 for \(y\). Substitute the expression for \(y\) into Equation 2 and solve for \(x\).
Recall that the solution of a system of equations is the pair of values that satisfies both equations. Thus, you can check the solution by substituting the values into the equations. You can also use the elimination method to check your solution.

Complete Guided Practice on p. 206. Use the elimination method to check your answers.

Help your student to understand that when he solves an equation for a variable, the resulting equation provides an algebraic expression that is equivalent to the variable. The resulting expression can be substituted into the other equation without changing the value of the expressions in that equation.

WATCH FOR THESE COMMON ERRORS

Some students make sign errors when substituting an expression for a variable that is being subtracted. (See Example 4, part a.) Encourage your student to use parentheses when making the substitution, and point out the sign error that occurs if parentheses are not used.

Complete problems 10–18 of Practice 5.2 on p. 209 in Math in Focus 3A.

Textbook Answer Key

You may want to read through problems 10–18 with your student and brainstorm what the first step in each solution will be. Recognize that there is more than one correct way to solve each system of equations.

Today you learned how to solve a system of equations using the substitution method.

Solve the system:

Substitute \( x = 3 \) into Equation 1 to solve for \( y \).

\[
2x + y = 1 \\
y = 1 - 2x
\]

\[
4x + 3(1 - 2x) = -3 \\
4x + 3 - 6x = -3 \\
2x + 3 = -3 \\
-2x = -6 \\
-x = -3 \\
x = 3
\]

Substitute \( x = 3 \) into Equation 1 to solve for \( y \).

\[
y = 1 - 2 \cdot 3 \\
= 1 - 6 \\
= -5
\]

The solution is \( x = 3, y = -5 \).

✔️ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Systems of Linear Equations - Part 6

Objectives
- Choose an appropriate method to solve systems of linear equations.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete the Quick Check.

LEARN

WARM-UP
Solve each system of equations using the given method.

1. Use the elimination method to solve the following system of equations.

\[
\begin{align*}
3x + 4y &= -8 \\
5x - 2y &= -22
\end{align*}
\]

2. Use the substitution method to solve the following system of equations.

\[
\begin{align*}
m &= 2m - 8 \\
5m - 3n &= 21
\end{align*}
\]

 TEACHING NOTES

WARM-UP ANSWERS
1. \(x = -4, \ y = 1\)  2. \(m = 3, \ n = -2\)

INSTRUCTION
Review Example 5 on pp. 207–208 in Math in Focus 3A. Although it is possible to solve any system of linear equations using either the elimination method or the substitution method, some systems are more easily solved using one method over the other. For example, if one of the equations is already solved for one variable in terms of the other, the substitution method will be easier to apply. If none of the variable terms have a coefficient of 1, the elimination method will often be a better choice. For other problems, use the method you like best.

Complete Guided Practice on p. 208.
Select a few systems of linear equations for your student and have him use both methods to find the solution. Then he should review his work and determine why one method would be preferable over the other in each instance.

Complete problems 19–24 of Practice 5.2 on p. 209 in Math in Focus 3A.

Textbook Answer Key

Today you learned how to choose an appropriate method to solve a system of linear equations. For example, in the following system, the second equation is already solved for $a$ in terms of $b$, so the substitution method is a good choice. You could also solve it by elimination, but it might require a few extra steps.

\[
\begin{align*}
2a - 5b &= -8 \\
a &= 3b - 6
\end{align*}
\]

In the following system, neither of the equations contains a variable that has a coefficient of 1, so the elimination method is a good choice.

\[
\begin{align*}
3x + 7y &= -4 \\
2x - 3y &= 5
\end{align*}
\]

Please go online to view and submit this assessment.
MORE TO EXPLORE

If you answered incorrectly, look carefully at the systems to decide the best way to solve it. Make sure to show each step as you solve. Remember that you can check your answer by substituting the values into the original equations to make sure you make a true statement. You might also wish to revisit the material in this lesson.
Systems of Linear Equations - Part 7

Objectives

- Use knowledge of systems of linear equations to solve real-world problems.

Books & Materials

- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*
- online graphing calculator (Optional)

Assignments

- Complete Warm-up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete the Practice Questions.

**LEARN**

**WARM-UP**

Write expressions for the following situations.

1. Connor buys frozen pizzas for $6.99 each and containers of ice cream for $4.99 each for his basketball team. Write an expression for the amount of money Connor spends.

2. Shannon cuts a board into 4 pieces, 2 of one length and 2 of another. Write an expression for the total length of the board Shannon started with.

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. $6.99p + 4.99i$ (Variable letters may vary.)  
2. $2x + 2y$ (Variable letters may vary.)

**INSTRUCTION**

Read pp. 210–211 in *Math in Focus 3A*. The problems in this lesson involve two unknown quantities, each represented by a different variable. Since there are two variables, two equations are needed to solve each problem. You can solve the resulting system using either the elimination method or the substitution method. Be sure to interpret your solution in terms of the original situation.

Review Example 6 on p. 212. When one of the equations shows how one variable is related to the other (as is the case in this example), the substitution method is a good choice.

Complete Guided Practice on pp. 212–213.
TEACHING NOTES

If your student has difficulty with these problems, help him to see that each of the two equations represents one of the two relationships described in the problem. It may be helpful to have him write two sentences describing the relationships in his own words before attempting to write the equations or to create a chart similar to the one on p. 210.

PRACTICE

Complete problems 1–7 of Practice 5.3 on pp. 215–216 in Math in Focus 3A.

TEACHING NOTES

Textbook Answer Key

It may be helpful to read through the problems in this assignment with your student before having him solve. Identify the two relationships described in each problem and discuss how those relationships might be represented algebraically. Have your student write these relationships as number statements and apply the method that he chooses to solve the systems.

WRAP-UP

Today you learned how to solve real-world problems using systems of linear equations. You learned how to write two equations with two unknowns to represent the relationships described in the situation. You then solved the resulting system of equations and interpreted the solution of the system in terms of the situation.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Systems of Linear Equations - Part 8

**Objectives**
- Use knowledge of systems of linear equations to solve real-world problems.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*

**Assignments**
- Complete Warm-up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete the Use For Mastery.

---

**LEARN**

---

**WARM-UP**

Solve the following systems of equations using your choice of method.

1. \[
\begin{align*}
2j + 3k &= -1 \\
3j - 2k &= 18
\end{align*}
\]

2. \[
\begin{align*}
4c + d &= 2 \\
3c - \frac{1}{2}d &= -6
\end{align*}
\]

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. \(j = 4, \ k = -3\)  \(2. \ c = -1, \ d = 6\)

---

**INSTRUCTION**

Review Example 7 on p. 213 in *Math in Focus 3A*. Then complete Guided Practice on p. 214. In problem 2, be sure you note the distinction between the digits of the number (which could be any of the numbers from 0 to 9) and the number itself (which could be any two-digit number). As indicated in the speech bubble, a two-digit number with tens digit \(c\) and ones digit \(d\) is the number \(10c + d\). In other words, if the tens digit is 4 and the ones digit is 6, the number is \(10 \cdot 4 + 6\), or 46.

---

**TEACHING NOTES**

If your student is having difficulty solving problem 2 in Guided Practice, have him choose several two-digit numbers. Then have him write the relationship between the digits for each number in words. For example, if he chooses 24, the difference between the tens digit and the ones digit is 2. He could also say the ones digit is twice the tens digit.
PRACTICE
Complete problems 8–15 of Practice 5.3 on pp. 216–217 in Math in Focus 3A.

TEACHING NOTES
Textbook Answer Key
It may be helpful to read through the problems in this assignment with your student before having him work on the solutions. Identify the two relationships described in each problem and discuss how those relationships might be represented algebraically.

WRAP-UP
Today you continued to learn how to solve real-word problems using systems of linear equations. You learned how to write two equations with two unknowns to represent the relationships described in the situation. You then solved the resulting system of equations and interpreted the solution of the system in terms of the situation.

USE

USE FOR MASTERY
1. Two cars leave town at the same time and travel in opposite directions. The average rate of speed of one car is 15 miles per hour faster than the average rate of speed of the other car. The cars are 315 miles apart after three hours. Find the average rate of speed of the two cars.

Type your work in the box to show how you found the answer.
Two cars leave town at the same time and travel in opposite directions. The average rate of speed of one car is 15 miles per hour faster than the average rate of speed of the other car. The cars are 315 miles apart after three hours. Find the average rate of speed of the two cars.

Did you:

- Use the information given to find the average rate of speed of the two cars?
- Type your work in the box to show how you found the answer?
- Show your work in an organized, logical manner?
Graphing Systems of Linear Equations - Part 1

Objectives
- Graph systems of linear equations to find the unique solution.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition
- Online graphing calculator (Optional)

Assignments
- Complete Warm-up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete the Practice Questions.

LEARN

WARM-UP
For each problem, write the system of linear equations in slope-intercept form. Then solve.

1. \[\begin{align*}
2x + 3y &= 14 \\
4x - 2y &= 12
\end{align*}\]

2. \[\begin{align*}
x + 2y &= 9 \\
x + y &= 6
\end{align*}\]

TEACHING NOTES

WARM-UP ANSWERS

1. \[\begin{align*}
y &= (-2/3)x + 14/3 \\
y &= 2x - 6 \\
x &= 4 \text{ and } y = 2
\end{align*}\]

2. \[\begin{align*}
y &= (-1/2)x + 9/2 \\
y &= -x + 6 \\
x &= 3 \text{ and } y = 3
\end{align*}\]

INSTRUCTION
Complete Technology Activity on p. 218 in Math in Focus 3A. This activity shows how to use a graphing calculator to solve a system of linear equations. Remember, the unique solution is the ordered
Today you learned how to solve a system of linear equations using the graphical method. Graphically, the solution is the intersection point of the two lines.

If you do not have a graphing calculator, make a table of values for each equation. Plot the points and graph each line to find the solution.

Read **Solve Systems of Linear Equations Using the Graphical Method** on pp. 219–220. Remember, the slope-intercept form of the equation for a line is \( y = mx + b \) where \( m \) is the slope of the line and \( b \) is the \( y \)-intercept.

Review **Example 8** on pp. 220–221. Then complete **Guided Practice** on p. 221.

---

**TEACHING NOTES**

There are many ways to solve a system of linear equations. The graphical method allows you to work on one equation at a time, but it sometimes lacks precision. There will be times when the graphical method will only give an approximation of the point of intersection. Encourage your student to check the solution by substituting values into both original equations. This will indicate when the answer found graphically is an estimate or an exact solution.

**PRACTICE**

Complete problems 1–10 of **Practice 5.4** on pp. 223–224 in **Math in Focus 3A**.

**TEACHING NOTES**

**Textbook Answer Key**

**WRAP-UP**

Today you learned how to solve a system of linear equations using the graphical method.

- Equation 1: \( 3x + y = 1 \)
- Equation 2: \( 2x - y = 4 \)
Step 1: Make a table of values for each equation.

\[
\begin{array}{|c|c|c|}
\hline
x & 0 & 1 & 2 \\
\hline
y & 1 & -2 & -5 \\
\hline
\end{array}
\quad
\begin{array}{|c|c|c|}
\hline
x & 0 & 1 & 2 \\
\hline
y & -4 & -2 & 0 \\
\hline
\end{array}
\]

Step 2: Plot the points represented by the tables of values. Then draw the graph of each equation.

Step 3: Locate the point of intersection.

The coordinates of the point of intersection are (1, −2).

✔️ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Graphing Systems of Linear Equations - Part 2

LEARN

Books & Materials
- Math in Focus - Teacher Edition

Assignments
- Complete Interactive Activity.
- Complete Rate Your Understanding.

INTERACTIVE ACTIVITY

Play the game X-Wars to explore solving linear systems by graphing.

TEACHING NOTES

In this activity, your student will experiment with the slopes and intercepts of linear equations to shoot a given target. Allow your student to play the game freely, but encourage him to observe how the changes he makes affects the graphs of the equations. After the game is finished, engage your student in a discussion around these questions:

- How did you solve the equations so your shot would be at the intersection of the two lines?
- What does the solution look like when there is an infinite number of solutions?
- What does a system of equations look like when there is no solution?
- What strategies did you use to figure out which slope to use?
- What strategies did you use to figure out which y-intercept to use?

Playing this game will set the stage for further learning in this lesson.
RATE YOUR UNDERSTANDING

Please go online to view and submit this assessment.
Graphing Systems of Linear Equations - Part 3

Objectives
- Graph systems of linear equations to find the unique solution.

Books & Materials
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*
- online graphing calculator (Optional)

Assignments
- Complete Warm-up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete the Quick Check.

LEARN

WARM-UP

Write the point of intersection for the system of linear equations.

1. \[
\begin{align*}
  y &= 4 \\
  y &= 3x + 1
\end{align*}
\]

2. \[
\begin{align*}
  y &= 9 \\
  3x - y &= 6
\end{align*}
\]

TEACHING NOTES

WARM-UP ANSWERS
- 1. (1, 4)  
- 2. (5, 9)

INSTRUCTION

Review Example 9 on p. 222 in *Math in Focus 3A*. When solving a real-world system of linear equations, it sometimes helps to think about what the equations mean. One car is traveling 60 miles per hour, and the other car is traveling 50 miles per hour with a 20-mile head start on the first car. Without doing any calculations, you can conclude that the car going 60 miles per hour will eventually meet the car going 50 miles per hour.

The point where the two cars are at the same place on the highway is part of the solution to the problem. The other part is the time it takes for the cars to get to that point.

Complete Guided Practice on p. 222. It is important to use the information that is given to you in a problem-solving situation. By following the instructions, you can graph the lines for each equation to find the point of intersection. Both equations are written in slope-intercept form, so they can be graphed quickly. If it helps, make a table before graphing.
When solving a real-world system of linear equations, your student should make sure that his answer makes sense. For example, the solution to a problem concerning temperatures of an oven where a variable represents the temperature cannot be a negative number because an oven, at its coldest, is at room temperature, and room temperature is well above zero.

### PRACTICE

Complete problems 11–15 of Practice 5.4 on p. 224 in Math in Focus 3A.

### TEACHING NOTES

Textbook Answer Key

### WRAP-UP

Today you learned how to solve real-world problems involving a system of linear equations using the graphical method. First write the equations describing the situation. Then rearrange the equations so they are in slope-intercept form. Next, graph both lines. Finally, find the intersection point of the lines; this is the solution that satisfies both equations.

Check your answer first to be sure it makes sense for the real-world problem. If it does, further check it by substituting values into each equation.

### QUICK CHECK

Please go online to view and submit this assessment.

### MORE TO EXPLORE

If you answered incorrectly, think about common x- and y-values before deciding on a window or scale. For example, if $t = 2$, then $d$ would equal 100 or 120. So the x-scale will definitely need to be different than the y-scale. Check your solution algebraically before making a final decision. You might also want to revisit the material from this lesson.
Graphing Systems of Linear Equations - Part 4

**LEARN**

**WARM-UP**

Write each equation in slope-intercept form.

1. $3x + 4y = 20$
2. $x - 2y = 8$

**WARM-UP ANSWERS**

1. $y = (-3/4)x + 5$
2. $y = (1/2)x - 4$

**TEACHING NOTES**

**INSTRUCTION**

Read pp. 225–227 in *Math in Focus 3A*. When a system of linear equations has a unique solution, that solution is the ordered pair of numbers that satisfies both equations. This ordered pair is the point where the graphs of the equations in the system intersect. If the graphs do not intersect, the lines have no point in common, and the system is, therefore, *inconsistent*. This means that there is no ordered pair of numbers that satisfies both equations in the system.

If you try to solve an inconsistent system of equations algebraically, both variables will be eliminated, and the resulting equation will be false. If a real-world problem yields an inconsistent solution, there is no answer that satisfies both equations. In other words, the real-world problem has no real answer.

Review Example 10 on pp. 228–229. The methods of solution demonstrated here involve writing the equations in slope-intercept form to compare the slopes and $y$-intercepts of their graphs, and using the equations as written in standard form to compare the coefficients of the variables. Note that these
problems can also be solved by using the elimination method, the substitution method, or the graphing method and interpreting the results. No matter how you solve the system, you should find the same result.

Complete Guided Practice on p. 230. Try using two or more of the methods just described to solve and check each problem.

If your student is having difficulty understanding this concept, it may be helpful to make a comparison chart using a system of equations that has a unique solution and a system of equations that is inconsistent. Have your student solve the two systems both algebraically and graphically. Also have him write the equations in each system in both slope-intercept form (to compare the slopes and y-intercepts) and standard form (to compare the coefficients). Use the chart to help him recognize the differences that exist between the two different types of systems.

Complete problems 1, 4–5, 7–8, and 13 of Practice 5.5 on p. 234 in Math in Focus 3A.

Today you learned how to determine whether a system of linear equations is inconsistent. Inconsistent systems of equations have no solution, meaning that there is no ordered pair of numbers that satisfies both equations in the system. The following system is inconsistent.

\[
\begin{align*}
9x + 6y &= 5 \\
3x + 2y &= 4
\end{align*}
\]

You can tell that the system is inconsistent by comparing the ratios of the coefficients and constants. The following ratios are for the coefficients of \(x\), the coefficients of \(y\), and the constants, in that order:

\[
\frac{93}{362} = \frac{354}{3}
\]

Since both coefficients are in the same ratio but the constants are in a different ratio, the system is inconsistent.
You can also tell that the system is inconsistent by comparing the slope-intercept forms of the equations:

\[ 9x + 6y = 5 \rightarrow y = -\frac{3}{2}x + \frac{5}{6} \]

\[ 3x + 2y = 4 \rightarrow y = -\frac{3}{2}x + 2 \]

Since the lines have the same slopes and different \( y \)-intercepts, the graphs of the equations in this system are parallel lines and thus have no points in common.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Graphing Systems of Linear Equations - Part 5

LEARN

WARM-UP

Determine whether or not each pair of equations is equivalent. Explain your reasoning.

1. $3x - 2y = -5$ and $9x - 6y = -15$
2. $2x + y = 1$ and $-2x - y = 1$

WARM-UP ANSWERS

1. equivalent; the second equation is the first equation multiplied by 3 on both sides.
2. not equivalent; the second equation is the first equation multiplied by $-1$ on the left side only.

TEACHING NOTES

INSTRUCTION

Read Understand and Identify Dependent Systems of Linear Equations on pp. 230–232 in Math in Focus 3A. A dependent system of equations has infinitely many solutions, or an infinite number of ordered pairs of numbers that satisfy both equations. When the graphs of two equations have the same slopes and $y$-intercepts, or one equation is a multiple of the other, you can determine that it is a dependent system of linear equations.

Review Example 11 on p. 233. These methods of solution involve checking to see if the equations are equivalent, writing the equations in slope-intercept form, and comparing the slopes and $y$-intercepts of their graphs. Note that these problems can also be solved using the elimination method, the substitution method, or the graphing method and then interpreting the results.

Complete Guided Practice on p. 233. Try using two or more of the methods just described to solve and check each problem.
It is important that your student be able to distinguish among systems that have a unique solution, systems that are inconsistent (those having no solutions), and systems that are dependent (those having an infinite number of solutions). Have him summarize how these systems compare, both algebraically and graphically. Have him also recap the various methods that can be used to determine the nature of a given system.

**WATCH FOR THESE COMMON ERRORS**

Some students may observe that the ratios of the coefficients of the x- and y-terms are equal and conclude that the system is dependent. Point out that this conclusion is only correct if the ratio of the constant terms is equal to the ratio of the coefficients of the variable terms.

**PRACTICE**

Complete problems 2–3, 6, 9–12, and 14–15 of *Practice 5.5* on p. 234 in *Math in Focus 3A*.

**TEACHING NOTES**

Encourage your student to use different methods for solving these problems. You may also want to encourage him to check his answer to any problem using a second method.

**WRAP-UP**

Today you learned how to determine whether a system of linear equations is dependent. Dependent systems of equations have infinitely many solutions. The following system is dependent.

\[
\begin{align*}
2x - 5y &= 3 \\
6x - 15y &= 9
\end{align*}
\]

The second equation is the first equation multiplied by 3.

\[
\begin{align*}
3(2x - 5y) &= 3(3) \\
6x - 15y &= 9
\end{align*}
\]
It is also possible to tell that the system is dependent by comparing the slope-intercept forms of the equations.

\[ 2x - 5y = 3 \rightarrow y = \frac{25}{2}x - \frac{3}{2} \]

\[ 6x - 15y = 9 \rightarrow y = \frac{25}{3}x - 3 \]

Since the lines have the same slopes and the same \( y \)-intercepts, the graphs of the equations in this system are the same line, and the system is dependent.

**QUICK CHECK**

Please go online to view and submit this assessment.

**MORE TO EXPLORE**

If you answered incorrectly, review how to classify a system based on the number of solutions. You might also want to view the video, *Solving Linear Systems by Graphing* (08:29), to review how to solve linear systems by graphing.
Graphing Systems of Linear Equations - Part 6

**Objectives**
- Use knowledge of systems of linear equations to solve real-world problems.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*

**Assignments**
- Complete Warm-up.
- Read and complete Brain @ Work in *Math in Focus 3A*.
- Complete the Practice activity.
- Complete the Practice Questions.

---

**LEARN**

---

**WARM-UP**

Write a system of equations and solve each problem.

1. Mrs. Walker purchases 3 adult tickets and 5 student tickets at the Science Center. Mrs. Devlin purchases 2 adult tickets and 6 student tickets. If Mrs. Walker pays $44 and Mrs. Devlin pays $40, what is the price of each type of ticket?

2. The width of a rectangular desktop is 3 inches shorter than the length. The perimeter of the desktop is 62 inches. What are the dimensions of the desktop?

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. $3a + 5s = 44$
   
   $2a + 6s = 40$
   
   adult ticket: $8, student ticket: $4

2. $w = l - 3$
   
   $2l + 2w = 62$
   
   17 in. by 14 in.
### INSTRUCTION

Read and complete problem 1 of Brain @ Work on p. 235 in Math in Focus 3A. Use the information in this problem to write a system of two equations, each solved for $C$. Once you have written these equations, think about which method would be easiest to use to solve the system.

Read and complete problem 2. Consider how you might use the information to find the mass of one block of metal A and the mass of one block of metal B. Then find the total mass and the total volume of the alloy in order to find its density.

Read and complete problem 3. Think about how you might write a system of equations that you could use to find the cost per minute for local calls and the cost per minute for out-of-state calls. Recognize that if the charges are correct, these rates should be the same for every month. If your system has a unique solution, this solution should satisfy all three equations. If your system has no solution, then there is an error in the charges.

### TEACHING NOTES

**Textbook Answer Key**

Prompt your student to read each problem thoroughly to identify two relationships that can be used to make a system of equations. Note that in problem 3, there is more than one pair of relationships that could be used.

Remind your student that when using a system of equations to solve a problem, the solution to the system is not necessarily the solution to the problem. He will need to reread the problem to determine how the solution to the system enables him to find the answer to the problem.

### PRACTICE

Write a constructed response to explain your answer to problem 3 in Brain @ Work in Math in Focus 3A.

Then solve the following problems.

1. Given the following system of equations, what is the value of $x + y$?

   \[
   \begin{align*}
   x + y + 4z &= 95 \\
   x + y + z &= 35
   \end{align*}
   \]

2. Nick is laying his kitchen floor with 1 ft × 1 ft white, blue, and green tiles. The floor is 12 ft × 15 ft. He is using twice as many white tiles as green tiles. The number of blue tiles and green tiles add up to 70. How many of each color is he using?
In Practice problem 1, your student can solve each equation for \((x + y)\), and then use substitution to find the value of \(z\). He can then substitute into one of the original equations to find the value of \(x + y\).

In problem 2, your student will need to determine the area of the floor and write an equation that shows that the total number of tiles is equal to the area (since the area of each tile is 1 square foot). He can then use the other equations he writes to substitute into this equation to solve for each quantity.

**Practice Answers:**

Possible answer: The charges are not correct. The system of equations that represents the charges for January and March is:

\[
60x + 30y = 45 \\
40x + 20y = 34
\]

This system is inconsistent, which means that it has no solution. Therefore, the charges are not correct.

1 15 2 110 white, 55 green, 15 blue

**WRAP-UP**

Today you learned how to apply what you have learned about solving systems of linear equations in two variables to solve multistep problems.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Graphing Systems of Linear Equations - Part 7

LEARN

WARM-UP

Solve each system of equations.

\[
\begin{align*}
5h + k &= 1 \\
-2h - 3k &= 10 \\
3x - 4y &= -2 \\
5x + 2y &= -12
\end{align*}
\]

WARM-UP ANSWERS

1. \( h = 1, \ k = -4 \)  
2. \( x = -2, \ y = -1 \)

INSTRUCTION

Read Chapter Wrap Up on pp. 236–237 in Math in Focus 3A. The concept map shows what you learned in this chapter about systems of equations. You learned three methods for solving systems of equations: the graphical method, the substitution method, and the elimination method. You learned that systems may have unique solutions, or they may be inconsistent or dependent systems, in which case they have either no solution or infinitely many solutions. You also learned how to use systems of equations to solve real-world problems.

Study the key concepts summarized in the list on p. 237.
You may want to encourage your student to review the steps for the different methods. Discuss the factors that might make one method preferable over another. Review the terms *inconsistent* and *dependent* and the various methods that can be used to determine whether a system is inconsistent, dependent, or has a unique solution.

**PRACTICE**

Complete Chapter Review/Test on pp. 238–239 in *Math in Focus 3A*.

**WRAP-UP**

Today you practiced solving systems of linear equations using different methods. You practiced classifying systems as inconsistent, dependent, or having a unique solution. You also practiced using systems of equations to solve real-world problems.

**USE**

**USE FOR MASTERY**

1. A fitness club has two options, one for members and one for nonmembers. Members pay a one-time registration fee of $12 plus $8 per gym visit. Nonmembers pay $10 per gym visit.

   Click [here](#) to download a coordinate grid. On the grid, solve the system of linear equations graphically.
1. A fitness club has two options, one for members and one for nonmembers. Members pay a one-time registration fee of $12 plus $8 per gym visit. Nonmembers pay $10 per gym visit. Click here to download a coordinate grid. On the grid, solve the system of linear equations graphically.

When you are finished, upload your work below.

a. Upload your finished graphs here.

b. Type the system of equations you used to graph the solutions. Use $C$ for the cost of the two payment options and $n$ for the number of visits.

c. After how many gym visits is the payment for the member option more beneficial than the payment for the nonmember option? Explain how you determined your answer.

---

**USE FOR MASTERY GUIDELINES & RUBRIC**

Did you:

- Use the information given to help you answer ALL of the questions?
- Download a coordinate grid to show the scenario graphically?
- Upload your finished graphs?
- Type the system of equations you used to graph the solutions?
- Use $C$ for the cost of the two payment options and $n$ for the number of visits?
- The number of gym visits after which the payment for the member option is more beneficial than the payment for the nonmember option?
- Explain how you determined your answer?
- Show your work in an organized, logical manner?
- Check to make sure your graph has two lines in it?
Unit Quiz: Algebraic Equations

Books & Materials
- Math in Focus - Teacher Edition
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

UNIT QUIZ

Please go online to view and submit this assessment.
Unit 3 - Functions
Our bodies were designed to move! Our heads are constantly buried in computers, tablets, and smartphones. We need to set time aside for exercise so we can be healthy. If you have ever researched the cost of gyms, you will see that there are a variety of costs, and they often involve a joining fee and then a monthly cost after you start. In this project, you have decided that you want to join a gym to motivate you to spend more time exercising. You have done some research and decided on three gyms. These gym prices are provided, and you will use them to try to find the best price. Just looking at the initial price will not be enough, though. You have to consider how much the memberships will cost over several months to see which will be the best deal in the long run.

PROJECT DETAILS

In this project, you will:

- Create tables to represent the cost of belonging to each of three gyms by month.
- Create linear equations representing the cost of belonging to each gym as a function of the number of months.
- Graph the functions representing the cost of belonging to each gym, based on the number of months.
- Use the graphs to determine the month in which the cost of different gyms will be the same.

PROJECT RUBRIC

The Project Rubric will help you understand how your project will be scored. Your goal should be to earn all possible points for each part.
COLLABORATION

What kinds of exercise do you like to do? Do you run? Do you play outside a lot? Do you play any sports? Swim? How many hours per day do you think you do this exercise? What do you think your weekly average is? Respond to two classmates.

RATE YOUR EXCITEMENT

Please go online to view and submit this assessment.
Today’s lesson covers important ideas you need to understand before beginning the lessons in this chapter.

Read p. 240 in *Math in Focus 3A*. The relationship where there is exactly one output for one input is called a *function*.

Read the first instructional section on p. 241. Draw a bar model if you have trouble figuring out which operation to use to write an expression.

Read the second instructional section on p. 241. Be sure to follow the order of operations when you substitute a quantity for the variable.
Today you reviewed how to write an algebraic expression for a word problem. Many word problems have multiple parts. Translate each part of the situation.

Alice sold $x$ bouquets of flowers for $6 each. She spent $15 on supplies for the bouquets. An expression representing the amount she had after the sales is $6x - 15$.

You also reviewed how to evaluate algebraic expressions for a given value of a variable. Substitute the value of the variable and then simplify the expression using the order of operations.

The value of $4x + 1$ when $x = -2$ is $4(-2) + 1 = -8 + 1 = -7$. 
The value of $4x + 1$ when $x = -2$ is $4(-2) + 1 = -8 + 1 = -7$.

Please go online to view and submit this assessment.
Functions - Part 2

**Objectives**
- Identify a mathematical relation.
- Identify input and output values.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teachers Edition*

**Assignments**
- Complete Warm-Up.
- Read and complete assigned pages in Math in Focus 8A.
- Complete problems 1–8 in Math in Focus 8A.
- Complete Practice Questions.

---

**LEARN**

**WARM-UP**
Tickets to a movie cost $8 each. Find the cost for each amount of tickets.

1. 6 tickets
2. 10 tickets
3. 25 tickets

**TEACHING NOTES**

**WARM-UP ANSWERS**
1. $48  2. $80  3. $200

---

**INSTRUCTION**

Read **Understand Relations** on pp. 242–243 in *Math in Focus 3A*. Pay close attention to the definitions in this section.

Read **Represent Relations Using Mapping Diagrams** on pp. 243–244. Study the four types of relations. The red arrows in type 2 show why this is a one-to-many relation. The red arrows in type 3 show why this is a many-to-one relation, and the red and blue arrows in type 4 show why this is a many-to-many relation.

Review **Example 1** on p. 244. Then complete **Guided Practice** on p. 245.
Have your student make her own relation. Instruct her to write a relation of her family members and the ages of each. She may use ordered pairs or a table. Ask her to identify the input and the output values. (The input values are the family members’ names, and the output values are the ages.) Have her make a mapping diagram of the relation. Next, ask your student to identify the type of relation. If her parents’ ages are the same, or if siblings have the same age, then the relation is a many-to-one relation. Otherwise, it is a one-to-one relation.

Complete problems 1–8 of Practice 6.1 on pp. 255–256 in Math in Focus 3A.

Textbook Answer Key

Today you learned how to identify a mathematical relation and its input and output values. You can use a mapping diagram to show how each input is related to each output and identify the type of relation.

The following relation is a many-to-one relation because the input values Blake, Charlie, and Dante all map to the same output value, 6.

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adam</td>
<td>2</td>
</tr>
<tr>
<td>Blake</td>
<td>6</td>
</tr>
<tr>
<td>Charlie</td>
<td>13</td>
</tr>
</tbody>
</table>

Please go online to view and submit this assessment.
Functions - Part 3

Objectives
- Identify a function.
- Determine whether a given relation is a function.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-Up.
- Read and complete assigned pages in Math in Focus 3A.
- Complete Quick Check.

LEARN

WARM-UP

Identify the type of relation.

1. Input | Output
   x → 8
   y → 15
   z → 21

2. Input | Output
   1 → 0
   2 → 5
   3 → 9

TEACHING NOTES

WARM-UP ANSWERS
1. many-to-many relation  2. one-to-one relation
INSTRUCTION

Read **Understand Functions** on pp. 246–248 in *Math in Focus 3A*. A *function* is a relation in which each input has only one output. Different inputs can share the same output, as long as each input has only one output. One-to-one relations and many-to-one relations are functions. The other relations, one-to-many and many-to-many, are not functions because their inputs have more than one output.

Review **Example 2** on pp. 249–250. You may be able to tell from the table or list of ordered pairs that the relation is a function. The table, the list of ordered pairs, and the mapping diagram all show the same information, but in different formats.

Complete **Guided Practice** on p. 250. For any relations that are not functions, try to tell what input value causes the relation to not be a function.

---

**TEACHING NOTES**

When helping your student identify whether or not a relation is a function from a list, if there is one x-coordinate repeated more than once, and that x-coordinate has different y-coordinates, then the relation is not a function. Likewise in a table, if the column or row showing the independent variable has a number (or word) repeated, look at the corresponding output values of those terms. In a mapping diagram, regardless of what is occurring with the output values, if each input value has only one arrow coming from it, the relation is a function.

---

**PRACTICE**

Complete problems 9–15 and 23 of **Practice 6.1** on p. 256 and p. 258 in *Math in Focus 3A*.

---

**TEACHING NOTES**

**Textbook Answer Key**

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**WRAP-UP**

Today you learned how to identify a function and determine whether a relation is a function. Use this graphic organizer to determine whether a relation is a function.
Complete Guided Practice on p.250. For any relations that are not functions, try to tell what input value causes the relation not to be a function.

When helping your student identify whether or not a relation is a function, from a list, if there is one x-coordinate repeated more than once, and that x-coordinate has different y-coordinates, then the relation is not a function. Likewise in a table, if the column/row showing the independent variable has a number (or word) repeated, look at the corresponding output values of those terms. In a mapping diagram, regardless of what is occurring with the output values, if each input value has only one arrow coming from it, the relation is a function.

Complete problems 9–15 and 23 of Practice 6.1 on p.256 and p.258 in Math in Focus 3A.

Textbook Answer Key

Today you learned how to identify a function and determine whether a relation is a function. Use this graphic organizer to determine whether a relation is a function.

Please go online to view and submit this assessment.

MORE TO EXPLORE

If you struggled with this question, view the Instructional Video, Determining the Function Rule.
Functions - Part 4

Learn

Interactive Activity

Follow these instructions for the activity shown below.

Click here to view the activity in a new window.

In a function, a rule is applied to an input value to give an output value. In this game, Machines A–F have rules that you will use to generate outputs. Start by dragging Machine A to the blue stand. Look at the row of numbers above the machine. Click on the number 1 and drop it into Machine A. Make a note of what happens in your Math Notebook.

Now drop in the numbers 2, 3, and 4. Write the results in your Math Notebook. What do you think will happen if you drop the number 10 into Machine A? Write down your prediction and then test it in the Gizmo. What is the function rule for Machine A?

Now find the function rules for Machines B–F. When you have finished, choose one of the other machines and drag it to the gray stand. Select the operation and the number you wish to use in your function. Move the machine to the red stand and try it with different numbers from the list.

Teaching Notes

Answers for Machine A: The output values are 4, 5, 6, 7, and 13. The function rule is \( f(x) = x + 3 \).

Answers for Machines B–F:  
B \( f(x) = x + 8 \)  
C \( f(x) = x - 2 \)  
D \( f(x) = 10x \)  
E \( f(x) = 4x \)  
F \( f(x) = x^2 \)

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

Rate Your Enthusiasm

Please go online to view and submit this assessment.
LEARN

**WARM-UP**

Tell whether each relation is a function.

1. (2, 15), (4, 5), (6, 2), (3, 10)

2.

![Input vs Output Table]

3.

<table>
<thead>
<tr>
<th>x</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. function 2. not a function 3. function
INSTRUCTION

Read Identify Functions Graphically on pp. 251–252 in Math in Focus 3A. Notice the relationship between the mapping diagram and the graph. Each ordered pair on the graph corresponds to an ordered pair from the mapping diagram. Discuss the Think Math question with your Learning Guide.

Review Example 3 on pp. 252–253. To determine if a relation on a graph is a function, draw vertical lines through each point. If one or more of the vertical lines intersects more than one point, the relation is not a function.

Complete Guided Practice on p. 254.

Review Example 4 on p. 254. Even if the graph of a relation is curved, the vertical line test provides a method to determine whether the relation is a function.

TEACHING NOTES

Emphasize to your student that the vertical line test has to work at every point on the graph. To demonstrate the vertical line test, have your student place her pencil vertically on the edge of a given graph. Instruct her to roll the pencil across the graph, keeping it vertical. If the pencil touches more than one point at once on any part of the graph, the graph is not a function.

PRACTICE

Complete problems 16–22 and 24–27 of Practice 6.1 on pp. 257–258 in Math in Focus 3A.

TEACHING NOTES

Textbook Answer Key

WRAP-UP

Today you learned how to identify whether a graph represents a function. The vertical line test will help you do this.

• If every vertical line intersects the graph in at most one point, then the graph represents a function.

• If any vertical line intersects the graph at more than one point, then the graph does not represent a function.
Emphasize to your student that the vertical line test has to work at every point on the graph. To demonstrate the vertical line test, have your student place her pencil vertically on the edge of a given graph. Instruct her to roll the pencil across the graph, keeping it vertical. If the pencil touches more than one point at once on any part of the graph, the graph is not a function.

Complete problems 16–22 and 24–27 of Practice 6.1 on pp. 257–258 in Math in Focus 3A.

Today you learned how to identify whether a graph represents a function. The vertical line test will help you do this.

• If every vertical line intersects the graph in at most one point, then the graph represents a function.
• If any vertical line intersects the graph at more than one point, then the graph does not represent a function.

Please go online to view and submit this assessment.
Functions - Part 6

**Objectives**
- Represent a function as an equation, a table of values, and a graph.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*

**Assignments**
- Complete Warm-Up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete Quick Check.

---

**LEARN**

**WARM-UP**

Write the ordered pair \((x, y)\) for each equation and given value of \(x\).

1. \(y = -2x\) for \(x = 4\)
2. \(10x = 3y + 24\) for \(x = 3\)
3. \(x = y + 8\) for \(x = 5\)
4. \(2y = 4x + 6\) for \(x = -1\)

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. \((4, -8)\)  
2. \((3, 2)\)  
3. \((5, -3)\)  
4. \((-1, 1)\)

**INSTRUCTION**

Read the instructional section on pp. 259–260 in *Math in Focus 3A*. The color coding will help you relate the independent variable (input) and dependent variable (output). The algebraic description assigns the variable \(x\) to the input values and the variable \(y\) to the output values and links the quantities to one another using the verbal description. The graph shows the relation of the ordered pairs generated from the table. You can see from the graph that the relation is a function. Select different amounts of time (hours) that Janice could spend on lessons and determine the input and output values.

Review Example 5 on p. 261. For part a, \(x\) is the input value, or number of minutes. The variable \(y\) is the output value, or total amount of water.
For part b, the values 1, 2, and 3 are substituted in for x. There is no steadfast rule on how many values to choose, or which values to choose. However, you need to consider that you will be graphing these values, so you want to choose convenient values that will not result in decimals and fractions if possible.

For part c, a scale of 2 was chosen (with y starting at 8), so that all the values display without the graph becoming too large. Discuss the Math Note with your Learning Guide.

Complete Guided Practice on p. 262.

### TEACHING NOTES

Help your student understand whether a graph should contain just plotted points or a continuous line through the points. Give her a list of scenarios to practice and ask if each graph would be represented by plotted points or a continuous line.

- the graph of the distance a car travels over a given time (continuous line)
- the graph of the total amount paid for a given number of pizzas (plotted points only)
- the amount of water lost from a leak in a swimming pool over time (continuous line)
- the number of students absent from school on given days (plotted points only)

### PRACTICE

Complete problems 1–5 of Practice 6.2 on p. 265 in Math in Focus 3A.

### TEACHING NOTES

Textbook Answer Key
WRAP-UP

Today you learned how to represent a given function as an equation, a table of values, and a graph.

Julie sells handmade hats for $7 each. The total amount of money Julie collects, y dollars, is a function of the number of hats, x, that she sells.

The total amount of money Julie collects equals the product of the cost of each hat and the number of hats sold.

Equation: $y = 7x$

Table of values:

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>7</td>
<td>14</td>
<td>21</td>
<td>28</td>
</tr>
</tbody>
</table>

In this example, it only makes sense for the inputs to be whole numbers. Therefore, the graph should only be the plotted points, not a continuous line.

![Graph of Julie's Hat Sales]

In this example, it only makes sense for the inputs to be whole numbers. Therefore, the graph should only be the plotted points, not a continuous line.

✅ QUICK CHECK

Please go online to view and submit this assessment.

MOVED TO EXPLORE

If you struggled with this question, write the equation in words and then substitute the known values. For example, total paycheck = percent commission times sales, plus salary. You might also want to revisit the material in this lesson.
Functions - Part 7

**Objectives**
- Write an equation and draw a graph for a given set of values in a table.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*
- online graphing calculator (Optional)

**Assignments**
- Complete Warm-Up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete Use For Mastery.

**LEARN**

**WARM-UP**
Determine the $y$-values of each function for $x = 0, 1,$ and $2$.

1. $y = 7x - 3$
2. $y = 8x + 1$
3. $y = \frac{3x + 1}{2}$

**TEACHING NOTES**

**WARM-UP ANSWERS**
1. $y = -3, 4, 11$  
2. $y = 1, 9, 17$  
3. $y = 1/2, 2, 7/2$

**INSTRUCTION**
Review Example 6 on pp. 263–264 in *Math in Focus 3A*. Time is the input value, and distance is the output value because the distance depends on the time Rachel cycles. (Note that in part a, the vertical axis uses 1 unit to represent 2 meters, not 4 meters as the text indicates.) For part b, be sure to read the Caution box. If you substitute incorrectly into the slope formula, the line representing the function will be incorrect.

Complete Guided Practice on p. 264. Refer to Example 6 and apply the values given here to the solution process.
Your student should test the equation she develops for a table of values to make sure it is correct. To do this, she should pick a nonzero value of $x$ from the table to substitute into the equation. Then she should solve for $y$. This value should match the corresponding value for $y$ in the table.

Use the following table as an example:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

Your student may calculate the slope using the points (0, 4) and (1, 6). The slope formula (shown on p. 264) elicits a slope of 2. The $y$-intercept, $b$, is the value of $y$ when $x = 0$. In this case, $b = 4$.

These values can be substituted into the equation $y = mx + b$ to get $y = 2x + 4$. To check this equation, use a nonzero value of $x$ from the table. Substitute $x = 2$ into the equation: $y = 2 \cdot 2 + 4 = 4 + 4 = 8$. This is the correct value of $y$ in the table.

**PRACTICE**

Complete problems 6–11 of Practice 6.2 on pp. 265–266 in Math in Focus 3A.

**TEACHING NOTES**

**Textbook Answer Key**

For problems 6 and 7, your student should identify points where the line intersects the grid lines on the graph. Using these points, she can find the slope and the $y$-intercept to generate the algebraic equation that represents the function.

**WRAP-UP**

Today you learned how to draw a graph and write an equation for a given set of values in a table. Use the following steps.

**Step 1**: Write the values in the table as a set of ordered pairs.

**Step 2**: Graph the ordered pairs on a coordinate plane.
**Step 3:** Determine if the points on the graph need to be connected with a continuous line.

**Step 4:** Write an algebraic equation to represent the function by determining the slope \( m \) and the y-intercept \( b \). Substitute these values into the equation \( y = mx + b \).

### USE FOR MASTERY

1. A sequence of dots is shown in the diagram.

![Figure Diagram](image)

   a. Fill in the input-output table for the sequence. Use the figure number, \( n \), as the input and the number of dots, \( D \), as the output.

<table>
<thead>
<tr>
<th>Figure Number (( n ) input)</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Dots (( D ) output)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Identify the type of relation between the figure number and the number of dots in a figure.

   0 / 50 Word Limit

### TEST TEACHING NOTES

**PRACTICE**

To check this equation, use a nonzero value of \( x \) from the table. Substitute \( x = 2 \) into the equation:

\[
y = 2 \cdot 2 + 4 = 4 + 4 = 8.
\]

This is the correct value of \( y \) in the table.

Complete problems 6–11 of Practice 6.2 on pp. 265–266 in Math in Focus 3A.

**Textbook Answer Key**

For problems 6 and 7, your student should identify points where the line intersects the grid lines on the graph. Using these points, she can find the slope and the y-intercept to generate the algebraic equation that represents the function.

Today you learned how to draw a graph and write an equation for a given set of values in a table. Use the following steps.

**Step 1:** Write the values in the table as a set of ordered pairs.

**Step 2:** Graph the ordered pairs on a coordinate plane.

**Step 3:** Determine if the points on the graph need to be connected with a continuous line.

**Step 4:** Write an algebraic equation to represent the function by determining the slope \( m \) and the y-intercept \( b \). Substitute these values into the equation \( y = mx + b \).
c. Tell whether this relation is a function.

- yes
- no

d. Explain your answer to part c and represent the relation with an algebraic expression.

---

### USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Fill in the input-output table for the sequence shown?
- Identify the type of relation between the figure number and the number of dots in the figure?
- Tell whether this relation is a function?
- Explain your answer to part C and represent the relation with an algebraic expression?
- Show your work in an organized, logical manner?
Functions - Part 8

SHOW

You have done some research and found two gyms in your area. The pricing plans are as follows:

Fitness/Activity Center Price Plans

<table>
<thead>
<tr>
<th></th>
<th>Gym A</th>
<th>Gym B</th>
<th>Gym C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$160 enrollment fee</td>
<td>$100 enrollment fee</td>
<td>$50 enrollment fee</td>
</tr>
<tr>
<td></td>
<td>$40 per month ($10 per week)</td>
<td>$60 per month ($15 per week)</td>
<td>$100 per month ($25 per week)</td>
</tr>
</tbody>
</table>

For each plan, create a table in your Math Notebook, showing the cost of belonging to each gym for the first 12 months. (You may want to create these tables in a word processing program to submit as part of your final project.) Remember to include the enrollment fee. The input value \((x)\) should be the number of months, and the output value \((y)\) should be the cost based on the number of months.

RATE YOUR PROGRESS

Please go online to view and submit this assessment.
**Objectives**
- Identify linear and nonlinear functions from a table of values.

**Books & Materials**
- *Math in Focus 3A*
- *Math in Focus - Teacher Edition*

**Assignments**
- Complete Warm-Up.
- Read and complete assigned pages in *Math in Focus 3A*.
- Complete Practice Questions.

---

**LEARN**

**WARM-UP**

Find the next two terms in each pattern.

1. 4, 8, 12, 16, ...

2. 0.3, 0.6, 0.9, 1.2, ...

3. 9, 18, 27, 36, ...

4. 50, 45, 40, 35, ...

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. 20, 24  
2. 1.5, 1.8  
3. 45, 54  
4. 30, 25

---

**INSTRUCTION**

Read *Identify a Linear Function from a Table* on pp. 267–268 in *Math in Focus 3A*. In the table in part a, the x-values all increase by the same amount, 2, and the y-values all increase by the same amount, 6. Tables may not always be presented that way. As long as the rate of change is constant, though, the function is linear.

In part b, the x- and y-values do not increase by the same amount each time, but the function is linear because it has a constant rate of change. To help see the difference between linear and nonlinear functions (like the
function in part c), plot the values from the table on a graph. You will see the points for a linear function are in a straight line and the points for a nonlinear function are not in a straight line.

Review Example 7 on p. 269. Then complete Guided Practice on p. 269. For problem 2, even though the blanks for the rates of change are not given, you still need to find the rates of change. If they are constant, the function is linear. If they are not constant, the function is nonlinear.

### TEACHING NOTES

When a table is presented graphically, the change of output values can also be called the *rise*. The change of input values can also be called the *run*. Therefore, another way to remember the formula for the rate of change is in these terms: rate of change = riserun. Remembering the rate of change in these terms can also help your student from mistakenly putting the change in *x*-values in the numerator, which can be a common mistake.

**WATCH FOR THESE COMMON ERRORS**

Make sure your student finds the rate of change between *all* of the values in a given table before she determines if the rate of change is constant or not constant. Sometimes the rate of change between some of the values, but not all, will be constant. For a function to be linear, the rate of change needs to remain constant throughout the entire function.

### PRACTICE

Complete problems 1–4 of Practice 6.3 on p. 276 in Math in Focus 3A.

### TEACHING NOTES

Textbook Answer Key
WRAP-UP

Today you learned how to identify a linear function from a table of values. To determine this, you need to calculate the rate of change. The rate of change equals the change in output values divided by the change in input values. Use this graphic organizer to determine if a function is linear or nonlinear.

✅ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
LEARN

WARM-UP

Determine the rate of change between each pair of ordered pairs.

1. (4, 6) and (0, 9)
2. (0, 0) and (2, 12)
3. (7, −11) and (−3, 5)

TEACHING NOTES

WARM-UP ANSWERS
1. −3/4  2. 6  3. −8/5

INSTRUCTION

Read p. 270 in Math in Focus 3A. The root word of linear is line. Think about the root word when deciding if a graph is linear. Some mathematicians use the terms slope and rate of change interchangeably. Each term represents a ratio of the change in y-values to the change in x-values.

Review Example 8 on p. 271. To find the rate of change, you will use the same calculation used to find the slope because the slope is the rate of change for a straight line.

Complete Guided Practice on p. 272. For problem 3, choose points where the line intersects the grid lines on the coordinate plane shown.
Help your student understand the connection between a table and a graph. Have your student write the coordinate points from the graph in a table of values. Then she can see whether or not a graph is linear in the same way she determined if a function in a table of values is linear. For example, using the following graph, she can make the associated table as shown.

The constant rate of change shown by both the table and the graph is \( \frac{1}{2} \).

Complete problems 5–6 of Practice 6.3 on p. 276 in Math in Focus 3A.

Textbook Answer Key

Today you learned how to identify a linear function from a graph. This graphic organizer will help you identify whether a graph represents a linear function.
QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

If you answered incorrectly, remember to read the question carefully and review the definition of a function. You can also view the video Patterns and Nonlinear Functions.

Please go online to view this video ►
LEARN

WARM-UP

Tell whether each list of numbers increases or decreases. Then tell whether the increase or decrease is constant.

1. 4, 7, 10, 13
2. 9, 6.5, 4.3, 2.1
3. 11, 13, 15, 18, 23
4. 25, 19, 13, 7

TEACHING NOTES

WARM-UP ANSWERS
1. increases; constant  2. decreases; not constant  3. increases; not constant  4. decreases; constant

INSTRUCTION

Read p. 273 in *Math in Focus 3A*. Trace your finger on each of the first three graphs. Notice as your finger moves from left to right, your finger goes toward the top of the page. That means the functions are all *increasing*. Trace your finger on each of the next three graphs. Notice as your finger moves from left to right, your finger goes toward the bottom of the page. That means the functions are all *decreasing*. Read the *Math Note* at the bottom of p. 273.

Review Example 9 on p. 274. The graph shown is only a description of the situation because there are no values for x or y shown. It cannot be used to make any numerical calculations or predictions.
Complete **Guided Practice** on p. 274.

Complete **Hands-On Activity** on p. 275. After you have completed the steps a few times, you may see a pattern that indicates whether a function is increasing or decreasing.

### TEACHING NOTES

Your student can also determine if a function is increasing or decreasing when given the algebraic equation of the function. She must first write the equation in slope intercept form, $y = mx + b$. If the slope $m$ is positive, then the function is increasing. If the slope $m$ is negative, then the function is decreasing. For example, to determine whether the equation $2y = 4x - 6$ represents an increasing or decreasing function, write the function in slope-intercept form. Divide both sides by 2 to obtain $y = 2x - 3$. The slope of the function is 2. The slope is positive, so the function is increasing.

### PRACTICE

Complete problems 7–14 of **Practice 6.3** on pp. 276–277 in *Math in Focus 3A*.

### TEACHING NOTES

**Textbook Answer Key**

For problems 13 and 14, be sure your student understands the relationship between the table and the graph. Both formats show the same information. If it is easier for your student to use one form (table or graph) to determine whether a function is linear or nonlinear and whether it is increasing or decreasing, she can always make a table into a graph or a graph into a table.

### WRAP-UP

Today you learned how to describe a function as increasing or decreasing. Use these facts to determine whether a function is increasing or decreasing.
Increasing Function

- The graph rises from left to right.
- As the input values increase, the output values increase.
- The slope is positive.

Decreasing Function

- The graph falls from left to right.
- As the input values increase, the output values decrease.
- The slope is negative.

✔️ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
**LEARN**

**WARM-UP**

Determine the slope of each function.

1. 
   
<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

2. 

3. \( y = 4x + 6 \)

4. \( 6y = 2x + 1 \)

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. \(-1\)  2. \(1/2\)  3. \(4\)  4. \(1/3\)
INSTRUCTION

Read pp. 278–279 in *Math in Focus 3A*. The reason the initial output value is 0 is that neither time nor distance can be less than 0. Compare what you see visually in the graph (the line for Dion’s trip is steeper than the line for Lara’s trip) to what you see in the table (the rate of change for Dion’s trip is greater than the rate of change for Lara’s trip). This reinforces the connection between slope and rate of change.

The equations and verbal descriptions also show the same information, but in two different forms. When you compare two functions, it is helpful for them to be in the same form. Different forms reveal different information about the functions.

Review Example 10 on pp. 280–281. When asked to compare two functions, you usually want to compare the rates of change of the functions.


TEACHING NOTES

When your student is given the graphs of two increasing linear functions to compare, it may help to ask her which line is *steeper*. As long as the axes for each graph are labeled in the same manner, the steeper line will have a greater slope and a greater rate of change. This will help her understand the concept of comparing slopes and rates of change between two functions. If your student struggles with Example 10 and Guided Practice, have her make the graphs for these functions to reinforce the comparisons made.

PRACTICE

Complete problems 1–6 and 10 of Practice 6.4 on pp. 286–287 in *Math in Focus 3A*.

TEACHING NOTES

Textbook Answer Key
WRAP-UP

Today you learned how to compare two functions presented in the same form, whether that form is an equation, a table, or a graph. Use this list of questions as a guide to compare two functions.

- Is the function linear or nonlinear?
- Is it increasing or decreasing?
- Which function has a greater rate of change, or slope?
- Are the initial input and output values the same?
- Can you draw any conclusions from the input and output values?
- What algebraic equation represents each function?

QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

If you answered incorrectly, make sure to read each question before answering. You might also want to try graphing the lines to help you visualize the functions. You might also want to revisit the material in this lesson.
LEARN

WARM-UP

Write an algebraic equation to describe the function in each table.

1.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>5</td>
<td>1</td>
<td>-3</td>
<td>-7</td>
</tr>
</tbody>
</table>

2.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>4</th>
<th>16</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

3.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-18</td>
<td>3</td>
<td>17</td>
<td>31</td>
</tr>
</tbody>
</table>

TEACHING NOTES

WARM-UP ANSWERS

1. $y = -2x + 5$  
2. $y = x/4$  
3. $y = 7x - 18$

INSTRUCTION

Read Compare Two Linear Functions Represented in Different Forms on p. 283 in Math in Focus 3A.
Review Example 10 on pp. 284–285. This example shows one function as a table and another function as an equation. Read the Think Math question at the bottom of p. 284 and consider how you would compare the initial values and rates of change of two functions using a table and a graph.


**TEACHING NOTES**

When given two functions in different forms to compare, help your student determine which form she is most comfortable using. Present her with the same function as an equation, a graph, and a table. Ask your student to describe the function and the rate of change, and to make an analysis about the input and corresponding output values. Which form allows your student to give you the most information about the function? Although you want your student to understand descriptions of functions given in any form, initially starting with a form most comfortable for her will help her succeed.

**PRACTICE**

Complete problems 7–9 and 11–12 of Practice 6.4 on pp. 287–288 in Math in Focus 3A.

**TEACHING NOTES**

Textbook Answer Key

**WRAP-UP**

Today you learned how to compare two functions presented in different forms. The functions should first be put in the same form and then compared. You can choose the form that you are most comfortable working with.

An equation for the total fee, \( y \), for renting a car from Rental A is \( y = 0.35x + 100 \), where \( x \) is the total number of miles the car is driven.

The table shows the total fee for renting a car from Rental B.

<table>
<thead>
<tr>
<th>( x ) (total miles driven)</th>
<th>0</th>
<th>100</th>
<th>200</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y ) (total rental fee in dollars)</td>
<td>150</td>
<td>175</td>
<td>200</td>
<td>225</td>
</tr>
</tbody>
</table>
Since the functions need to be compared in the same form, write an algebraic equation to represent the function in the table.

Slope of the function: \( m = 0.25 \)

\( y \)-intercept: 150

Equation for the function: \( y = 0.25x + 150 \)

Both functions are increasing linear functions. Because 150 > 100, Rental B has a greater initial cost. A comparison of the rates of change shows that Rental A increases at a greater rate than Rental B because 0.25 < 0.35.

✅ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Linear and Nonlinear Equations - Part 6

Objectives
- Use knowledge of functions to solve real-world problems.

Books & Materials
- Math in Focus 3A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-Up.
- Read and complete Brain @ Work in Math in Focus 3A.
- Complete Use For Mastery.

---

LEARN

WARM-UP

Write an equation to represent the function in each table.

1.

<table>
<thead>
<tr>
<th>$x$</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>3</td>
<td>9</td>
<td>18</td>
<td>24</td>
</tr>
</tbody>
</table>

2.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>11</td>
</tr>
</tbody>
</table>

3.

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>8</td>
<td>-4</td>
<td>-16</td>
<td>-28</td>
</tr>
</tbody>
</table>

---

TEACHING NOTES

WARM-UP ANSWERS
1. $y = 3x - 12$  
2. $y = \frac{3}{5}x + 2$  
3. $y = -6x + 20$

---

INSTRUCTION

Read and complete Brain @ Work on p. 289 in Math in Focus 3A. To help compare the different packages, represent the functions in the same form.
Create a table to compare the values of the bracelets in this problem.

Shanise is asked to compare the costs of three charm bracelets. Which bracelet is the least expensive if she purchases 1 bracelet and 5 charms?

Bracelet A: $35 for the bracelet and $12 for each charm
Bracelet B: $50 for the bracelet and $10 for each charm
Bracelet C: $40 for the bracelet and $16 for each charm

**Practice Answers:**

<table>
<thead>
<tr>
<th></th>
<th>0 charms</th>
<th>1 charm</th>
<th>2 charms</th>
<th>3 charms</th>
<th>4 charms</th>
<th>5 charms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bracelet A</td>
<td>35</td>
<td>47</td>
<td>59</td>
<td>71</td>
<td>83</td>
<td>95</td>
</tr>
<tr>
<td>Bracelet B</td>
<td>59</td>
<td>69</td>
<td>79</td>
<td>89</td>
<td>99</td>
<td>109</td>
</tr>
<tr>
<td>Bracelet C</td>
<td>49</td>
<td>56</td>
<td>72</td>
<td>88</td>
<td>104</td>
<td>120</td>
</tr>
</tbody>
</table>

**Wrap-up**

Today you learned how to use your knowledge of functions to solve real-world problems. These steps will guide you in solving this type of problem.

**Step 1:** Determine what the problem is asking.

**Step 2:** If the functions in the problem are in different forms, put them in the same form so they can be compared. If the problem does not specify the form the functions need to be in, decide which form will work best to solve the problem. If a problem is asking you to substitute values into the functions, an algebraic equation may be the best form.

**Step 3:** Solve the problem by finding values, comparing, or writing verbal descriptions.
1. Trey needs to rent a car for one day. The rental company offers two plans. Both plans involve paying a fixed amount and then paying an additional charge per mile driven. For each plan, the cost of driving the car, \( y \) dollars, is a function of the number of miles traveled. The graph shows the car rental cost for Plan A. Plan B offers a fixed payment of $30 plus 25¢ per mile.

a. Write an algebraic equation to represent the function for Plan B.
b. Graph the linear function for Plan B on the same coordinate plan as Plan A.

c. Use a verbal description to compare the two functions. Then describe a scenario that would benefit Trey to rent the car using Plan A. Then describe a scenario that would benefit Trey using Plan B.
d. Describe a situation where either rental plan results in the same total cost.

USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information given to help you answer ALL the questions?
- Write an algebraic equation to represent the function for Plan B?
- Graph the linear function for Plan B on the graph as Plan A?
- Use a verbal description to compare the two functions?
- Describe a scenario that would benefit Trey to rent the car using Plan A?
- Describe a scenario that would benefit Trey using Plan B?
- Describe a situation where either rental plan results in the same total cost?
- Show your work in an organized, logical manner?
SHOW

For each plan, write a function to model the cost. Use your tables to help you calculate the slope and \(y\)-intercept. Write the functions in your Math Notebook or add them to the tables you created in a word processing program.

<table>
<thead>
<tr>
<th>Gym A</th>
<th>Gym B</th>
<th>Gym C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100) enrollment fee</td>
<td>$100) enrollment fee</td>
<td>$70) enrollment fee</td>
</tr>
<tr>
<td>$40 per month ($10 per week)</td>
<td>$60 per month ($15 per week)</td>
<td>$100 per month ($25 per week)</td>
</tr>
</tbody>
</table>

PROJECT PROGRESS

How ready do you feel to complete this part of the project?

- I feel very ready to complete this part of the project; I have learned everything I need to know to do it.
- I feel somewhat ready to complete this part of the project but I am unsure that I have learned everything I need to know to do it.
- I do not feel ready to complete this part of the project.
- I feel very unprepared to complete this part of the project.

How excited do you feel to complete this part of the project?

- I feel very excited to complete this part of the project.
- I feel somewhat excited to complete this part of the project.
- I do not feel excited to complete this part of the project.
- I feel completing this part of the project will be very boring.
Now it is time to create graphs for each of your functions. You may graph them by hand, using coordinate grid paper. Be sure to use a straightedge to make precise graphs. You may also use an online graphing tool like Desmos. Graph them on the same plane so that you will be able to determine the points of intersection. (Hint: You will be able to see the points of intersection more clearly if you mark the months at every fourth line on the x-axis.)

RATE YOUR PROGRESS

Please go online to view and submit this assessment.
Now it is time to use your functions to determine which gym offers the best price long term. Compare the graphs, two at a time, to see in which month the cost will be the same. You can enter this in your Math Notebook like this:

Gym A and Gym B will cost the same in the ___ month.

Gym B and Gym C will cost the same in the ___ month.

Gym A and Gym C will cost the same in the ___ month.

Then think about what this means in terms of the best deal. Which of the three gyms will offer you the best price in the long run? Write a paragraph in which you tell your choice and explain your reasoning. Refer to the graphs to support your ideas.

FINAL PROJECT

1. Upload all your work for the project here. Be sure to include the following:
   - the three tables showing the cost of each gym for 12 months, with the function written for each one
   - the graph of the three functions
   - your paragraph telling the months in which the costs will be the same, which of the gyms is the best choice, and your explanation supporting your choice
Now it is time to use your functions to determine which gym offers the best price long term. Compare the graphs, two at a time, to see in which month the cost will be the same. You can enter this in your Math Notebook like this:

Gym A and Gym B will cost the same in the ___ month.
Gym B and Gym C will cost the same in the ___ month.
Gym A and Gym C will cost the same in the ___ month.

Then think about what this means in terms of the best deal. Which of the three gyms will offer you the best price in the long run? Write a paragraph in which you tell your choice and explain your reasoning. Refer to the graphs to support your ideas.

What method do you think was the best method to find the number of months where the costs were the same for each pair of gyms? Why did you choose this method? Respond to two of your peers.

COLLABORATION

What method do you think was the best method to find the number of months where the costs were the same for each pair of gyms? Why did you choose this method? Respond to two of your peers.
Unit Quiz: Fitness/ Activity Center

Books & Materials
- Math in Focus - Teacher Edition
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

☑️ UNIT QUIZ

Please go online to view and submit this assessment.
Unit 4 - Pythagorean Theorem
LEARN

WARM-UP

Subtract.

1. $-4 - 5$
2. $6 - (-4)$
3. $-10 - (-3)$
4. $17 - 21$

WARM-UP ANSWERS

1. $-9$  2. $10$  3. $-7$  4. $-4$

INSTRUCTION

Today’s lesson covers important ideas you need to understand before beginning the lessons in this chapter. Read pp. 2–3 in Math in Focus 3B.

When squaring or cubing a number, you use the base as a factor the number of times indicated by the exponent.

$4^2 = 4 \cdot 4 = 16$

$4^3 = 4 \cdot 4 \cdot 4 = 64$
To find the square root of a number, think of a value you can multiply by itself to get the number.

\[ 4 \cdot 4 = 16 \text{ and } -4 \cdot (-4) = 16 \]

The square roots of 16 are 4 and −4.

You can find the cube root of a number in the same way. Think of a value you can use as a factor three times to get the number.

\[ 4 \cdot 4 \cdot 4 = 64 \]

The cube root of 64 is 4.

Read p. 4. Length is always positive. Therefore, the length of a horizontal or vertical segment is the absolute value of the difference in the coordinates that are not the same.

---

**TEACHING NOTES**

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the Quick Check sections.

---

**SKILLS CHECK**

Complete the Quick Check sections on pp. 3–4 in Math in Focus 3B.

---

**TEACHING NOTES**

Textbook Answer Key

Review your student’s answers to the Quick Check sections, noting the problems that he answered incorrectly. Click on the link to access the appropriate Reteach activity that your student should complete for the remainder of this lesson.

**RETEACH**

After your student completes the Quick Check in the Recall Prior Knowledge lesson of this chapter, review the questions that were answered incorrectly. If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him or her complete the activity.

Note that this chapter opener spans two lessons.
### Quick Check

<table>
<thead>
<tr>
<th>Question(s)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–12</td>
<td>Square Roots and Cube Roots</td>
</tr>
<tr>
<td>13–16</td>
<td>Length of a Line Segment</td>
</tr>
</tbody>
</table>

### WRAP-UP

In this lesson, you reviewed important concepts that will help you to be successful in this chapter.

- To square a number, multiply the number by itself. To find the square root of a number, find the value(s) that, when multiplied by itself, equals the number.
- To cube a number, use the number as a factor three times. To find the cube root of a number, find the value that, when used as a factor three times, equals the number.
- You can find the length of a horizontal or vertical line segment on the coordinate plane by counting units or finding the absolute value of the difference of the x- or y-coordinates.

### PRACTICE QUESTIONS

Please go online to view and submit this assessment.
LEARN

WARM-UP

Multiply.

1. \( \frac{1}{3} \cdot 5^2 \cdot 6 \)

2. \( \frac{1}{3} \cdot 12 \cdot 8.1 \)

3. \( 1.5^2 \cdot 2.3 \)

4. \( 7^2 \cdot 1.7 \)

TEACHING NOTES

WARM-UP ANSWERS

1. 50  2. 32.4  3. 5.175  4. 83.3

INSTRUCTION

Read Finding the volume of a solid on p. 5 in Math in Focus 3B, focusing on the volume of a prism and the volume of a cylinder. These can both be found by multiplying the area of the base by the height.

Look at the following diagram of a rectangular prism. This prism has a base that is 2 units by 3 units and a height of 4 units. You can think of the volume of the prism as layers of the base, 1 unit high. Since each layer is 1 unit, the volume of each layer is the same as the area of the base. The number of layers is equal to the height. In this prism there are 4 layers.
The volume is the area of the base times the number of layers:

\[(2 \cdot 3) \cdot 4 = 6 \cdot 4 = 24 \text{ units}^3\]

Note that for volume, the units are always cubed.

The following diagram for a cylinder shows the same idea. You can think of the volume of the cylinder as layers of the circular base 1 unit high. The volume is the area of the base times the number of layers, or the height of the cylinder.

The volume is the area of the base times the number of layers:

\[\pi(4)^2 \cdot 5 = 16\pi \cdot 5 \approx 251.3 \text{ units}^3\]

In this lesson, your student reviews formulas for finding the volume of prisms and cylinders. He can find the volume of these figures by multiplying the area of the base of the figure by its height. The formula for the area of the base will be different, depending on the shape. Your student should use \(A = s^2\) for square bases, \(A = lw\) for rectangular bases, \(A = \frac{1}{2}bh\) for triangular bases, and \(A = \pi r^2\) for circular bases.

**SKILLS CHECK**

Complete problems 18 and 20 of the Quick Check on p. 5 in Math in Focus 3B.
WRAP-UP
Today you reviewed how to find the volume of prisms and cylinders.

Volume of a prism = Area of base • Height

Volume of a cylinder = Area of base • Height = \pi r^2 • Height

QUICK CHECK
Please go online to view and submit this assessment.

MORE TO EXPLORE
View the Instructional Video Volume of Cylinders and Prisms to explore how to find the volume of solid shapes.
The volume is the area of the base times the number of layers:

\[(2 \times 3) \times 4 = 6 \times 4 = 24 \text{ units}^3\]

Note that for volume, the units are always cubed.

The following diagram for a cylinder shows the same idea. You can think of the volume of the cylinder as layers of the circular base 1 unit high. The volume is the area of the base times the number of layers, or the height of the cylinder.

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Complete problems 18 and 20 of the Quick Check on p. 5 in Math in Focus 3B Textbook Answer Key.

Today you reviewed how to find the volume of prisms and cylinders.

Volume of a prism = Area of base \(\times\) Height

Volume of a cylinder = Area of base \(\times\) Height = \(\pi r^2 \times\) Height

**INTERACTIVE ACTIVITY**

Follow these instructions for the activity shown below.

Click [here](#) to view the activity in a new window.

To begin, be sure Rectangle (under Shape of base) and Drag to rotate are selected. You will see a rectangular prism spinning on the right side of the screen. Drag the Height slider back and forth. How does the prism change? Drag the Base length and Base width sliders. How does the prism change? Share your observations with your Learning Guide.

Now set the Height to 1 unit, the Base length to 9 units, and the Base width to 6 units. Find the area of the base and write the answer in your Math Notebook, using the correct unit of measure. Turn on Show area of base to check your answer. Now select Show volume. What is the volume of this prism? Explain to your Learning Guide why the units used for area of the base and volume of the prism are different.

Copy this table into your Math Notebook.

<table>
<thead>
<tr>
<th>Height ((h))</th>
<th>Base length ((l))</th>
<th>Base width ((w))</th>
<th>Base area ((B))</th>
<th>Volume ((V))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 unit</td>
<td>9 units</td>
<td>6 units</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

Fill in the first row of the table below for the prism you created. Then create four more rectangular prisms of your choice and fill in the rest of the table (including the units). What is the relationship between the base area and the volume? Discuss your answers with your Learning Guide.
See if this relationship is also true for other solid shapes. Select **Triangle** from the **Shape of base** dropdown menu. Turn on **Drag to rotate** and **Show area of base**. Set **Height** to 3 units and **Base edge** to 10 units. What is the area of the base of this triangular prism? What do you think the volume of this prism is? Explain your answers to your Learning Guide.

Finally, select **Circle** under **Shape of base** to make a cylinder. Set the cylinder’s height to 2 units and the radius to 5 units. A. Find the exact area of the base. (Do not use a numerical equivalent for \( \pi \); simply leave it in the answer.) Turn on **Show area of base** to check. What is the volume of the cylinder? Click on **Show volume** to check.

If you would like, you can click on **Lesson Info** and download the **Student Exploration Sheet** to try more activities with the Gizmo.

---

### TEACHING NOTES

**Sample answers:** Changing the height makes the prism taller or shorter; changing the base length or width makes the prism thinner or longer. The area of the base is 54 square units, but the volume is 54 cubic units. When you find area, you multiply two dimensions, resulting in units of measure that are square, but when you find volume, you multiply three dimensions, giving you units of measure that are cubes.

**Answers for table:** Check student work for accuracy. The volume is the area of the base times the height.

**Sample answers for other solids:** The area of the base of the triangular prism is 43.3 square units, and the volume is 129.9 cubic units, which is 3 times the area of the base \( Bh \). The area of the base of the cylinder is \( 25\pi \) square units, and the volume is \( 50\pi \) cubic units, which is 2 times the area of the base.

If you would like, you can click on **Lesson Info** and download the **Student Exploration Sheet** and **Exploration Sheet Answer Key** to have your student try some other activities with the Gizmo.

---

### RATE YOUR ENTHUSIASM

Please go online to view and submit this assessment.
Volume - Part 4

Objectives
- Find the volume of a prism, square pyramid, cone, cylinder, and sphere.

Books & Materials
- *Math in Focus 3B*
- *Math in Focus - Teacher Edition*
- net of square prism
- net of square pyramid
- net of cylinder
- net of cone
- scissors
- tape
- small beans or rice
- scientific calculator (Optional)

Assignments
- Complete Warm-Up.
- Complete the assigned pages in *Math in Focus 3B*.
- Complete Quick Check.

LEARN

WARM-UP
Simplify.

1. \( \frac{4}{3} \cdot 3^2 \cdot 12 \)
2. \( \frac{1}{3} \cdot 5.6 \cdot 9 \)
3. \( \frac{4}{3} \cdot 5^2 \)
4. \( \frac{1}{3} \cdot 6^2 \cdot 11 \)

WARM-UP ANSWERS
1. 144  2. 16.8  3. 33\(\frac{1}{3}\)  4. 132

TEACHING NOTES

INSTRUCTION
Reread Finding the volume of a solid on p. 5 in *Math in Focus 3B*. Use what you know about finding the volume of prisms and cylinders to find the volume of pyramids and cones.

A pyramid with the same base and height as a prism can be thought of as corresponding to the prism.
The volume of a prism and its corresponding pyramid have a special relationship. Complete this activity to explore their relationship.

1. Cut out the nets of the square prism and the square pyramid in the Appendix. Notice that these figures have the same base and height.
2. Fold and tape the nets to make the solids. Put tape along the entire length of each edge.
3. Turn the figures upside down, leaving their bases (the faces with the dotted lines) open as shown in the diagram.
4. Fill the pyramid with beans or rice.
5. Pour the beans or rice from the pyramid into the prism. Keep track of the number of times it took to fill the prism.

Notice that it took 3 pours from the pyramid to fill the prism. The volume of the prism is 3 times the volume of the corresponding pyramid. This means that the volume of a pyramid is $\frac{1}{3}$ the volume of the corresponding prism. This relationship holds true for all prisms and pyramids, regardless of the shape of the base.

Volume of a prism = Area of base • Height

Volume of a pyramid = $\frac{1}{3}$ • Area of base • Height

Repeat the activity, but use the nets of the cylinder and the corresponding cone in the Appendix. Use the cone to fill the cylinder.

Notice that it takes 3 pours from the cone to fill the cylinder.

Volume of a cylinder = Area of base • Height

Volume of a cone = $\frac{1}{3}$ • Area of base • Height
The volume of a prism and its corresponding pyramid have a special relationship. Complete this activity to explore their relationship.

1. Cut out the nets of the square prism and the square pyramid in the Appendix. Notice that these figures have the same base and height.
2. Fold and tape the nets to make the solids. Put tape along the entire length of each edge.
3. Turn the figures upside down, leaving their bases (the faces with the dotted lines) open as shown in the diagram.
4. Fill the pyramid with beans or rice.
5. Pour the beans or rice from the pyramid into the prism. Keep track of the number of times it took to fill the prism.

Notice that it took 3 pours from the pyramid to fill the prism. The volume of the prism is 3 times the volume of the corresponding pyramid. This means that the volume of a pyramid is \( \frac{1}{3} \) the volume of the corresponding prism. This relationship holds true for all prisms and pyramids, regardless of the shape of the base.

\[
\text{Volume of a prism} = \text{Area of base} \times \text{Height}
\]
\[
\text{Volume of a pyramid} = \frac{1}{3} \times \text{Area of base} \times \text{Height}
\]

Repeat the activity, but use the nets of the cylinder and the corresponding cone in the Appendix. Use the cone to fill the cylinder.

Notice that it takes 3 pours from the cone to fill the cylinder.

\[
\text{Volume of a cylinder} = \text{Area of base} \times \text{Height}
\]
\[
\text{Volume of a cone} = \frac{1}{3} \times \text{Area of base} \times \text{Height}
\]

The activities your student completes in this lesson help him explore the relationship between the volume of a prism and its corresponding pyramid, as well as the volume of a cylinder and its corresponding cone. Corresponding in this context indicates figures that have the same size base and height.

Complete problems 17, 19, and 21 of the Quick Check on p. 5 in Math in Focus 3B.

**TEACHING NOTES**

The activities your student completes in this lesson help him explore the relationship between the volume of a prism and its corresponding pyramid, as well as the volume of a cylinder and its corresponding cone. Corresponding in this context indicates figures that have the same size base and height.

**SKILLS CHECK**

Complete problems 17, 19, and 21 of the Quick Check on p. 5 in Math in Focus 3B.

**TEACHING NOTES**

Textbook Answer Key

RETEACH

After your student completes the Quick Check in the Recall Prior Knowledge lesson of this chapter, review the questions that were answered incorrectly.

If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.

Note that this chapter opener spans three lessons.

<table>
<thead>
<tr>
<th>Quick Check Question(s)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>Volume of a Cone</td>
</tr>
<tr>
<td>21</td>
<td>Volume of a Pyramid</td>
</tr>
</tbody>
</table>

Textbook Answer Key
WRAP-UP

Today you reviewed how to find the volume of cones, pyramids, and spheres.

Volume of a cone = \( \frac{1}{3} \) \cdot Area of base \cdot Height = \( \frac{1}{3} \pi r^2 \cdot \) Height

Volume of a pyramid = \( \frac{1}{3} \) \cdot Area of base \cdot Height

Volume of a sphere = \( \frac{4}{3} \pi r^3 \)

☑️ QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

If you answered incorrectly, remember to read the question carefully and write down the given information. You should also write down the formula needed to solve the problem and then rewrite the formula, substituting the given values into the equation. Finally, solve for the missing information. To review how to find the volume of a sphere, revisit the material in this lesson.
Volume - Part 5

Books & Materials
- Math in Focus - Teacher Edition

Assignments
- Complete Interactive Activity.
- Complete Use for Mastery.

LEARN

INTERACTIVE ACTIVITY

Follow these instructions for the activity shown below.

Click here to view the activity in a new window.

To begin, be sure Square (under Shape of base) and Drag to rotate are selected. You will see a square pyramid spinning on the right side of the screen. Drag the Height slider right and left. How does the pyramid change? Drag the Base edge slider. How does the pyramid change? Share your observations with your Learning Guide.

Now set the Height to 8 units and the Base edge to 6 units. Find the area of the base and write the answer in your Math Notebook, using the correct unit of measure. Turn on Show area of base to check your answer. Now use the formula $V = Bh$ to find the volume of a rectangular prism with the same height and base edge. Check your answer by clicking on Show prism/cylinder volume. Now select Show pyramid/cone volume. What is the volume of this pyramid? How does it compare to the volume of a prism with the same dimensions? Explain this to your Learning Guide.

Copy this table into your Math Notebook.

<table>
<thead>
<tr>
<th>Base edge (s)</th>
<th>Base area (B)</th>
<th>Height (h)</th>
<th>Prism volume (V)</th>
<th>Pyramid volume (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 units</td>
<td>8 units</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Books & Materials

INTERACTIVE ACTIVITY

Assignments

Complete Interactive Activity.
Complete Use for Mastery.
Fill in the first row of the table below for the pyramid you created. Then create four more square pyramids of your choice and fill in the rest of the table (including the units). What is the relationship between the base area and the volume? Discuss your answers with your Learning Guide.

Finally, select Circle under Shape of base to make a cone. Set the height of the cone to 6 units and the radius to 5 units. A. Find the exact area of the base. (Do not use a numerical equivalent for π; simply leave it in the answer.) Turn on Show area of base to check. What is the volume of the corresponding cylinder? Click on Show prism/cylinder volume to check. What is the volume of the cone? Click on Show pyramid/cone volume to check. Then share your observations with your Learning Guide.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet to try more activities with the Gizmo.

**TEACHING NOTES**

**Sample answers:** Changing the height makes the pyramid taller or shorter; changing the base edge makes the pyramid thinner or wider. The area of the base is 36 square units, but the volume is 96 cubic units. The volume of a prism with the same dimensions is 288 cubic units. The volume of the pyramid is \( \frac{1}{3} \) the volume of the prism.

**Answers for first table:** Check student work for accuracy. The ratio in the last column should always be 1:3.

**Sample answers for the cone:** The area of the base of the cylinder is \( 25\pi \) square units, and the volume of the corresponding cylinder is \( 150\pi \) cubic units. The volume of the cone is one-third that amount, or \( 50\pi \) cubic units. The volumes of pyramids and cones are both found by dividing the volume of the corresponding prism or cylinder by 3.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

**USE**

**USE FOR MASTERY**

1. A cylindrical-shaped cup has a height of 7 centimeters and a volume of \( 112\pi \) cubic centimeters. Henry fills the cup completely full of water. He then pours the water from the cup and completely fills a cone. If the cone has the same radius as the cup, what is the height of the cone?
A cylindrical-shaped cup has a height of 7 centimeters and a volume of $112\pi$ cubic centimeters. Henry fills the cup completely full of water. He then pours the water from the cup and completely fills a cone. If the cone has the same radius as the cup, what is the height of the cone?

If you are able, use the text box to show your work and enter your final answer to the question. If not, complete your work on paper and upload it below.

**USE FOR MASTERY GUIDELINES & RUBRIC**

Did you:

- Use the information given to help you calculate the height of the cone?
- Use the text box to show your work and enter your final answer OR complete your work on paper and upload it?
OBJECTIVES

- Use the Pythagorean Theorem to find unknown lengths in a right triangle.

BOOKS & MATERIALS

- Math in Focus 3B
- Math in Focus - Teacher Edition
- Scientific calculator (optional)

ASSIGNMENTS

- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete Practice Questions.

LEARN

WARM-UP

Find each positive square root.

1. \(\sqrt{121}\)
2. \(\sqrt{256}\)
3. \(\sqrt{81}\)
4. \(\sqrt{676}\)

TEACHING NOTES

WARM-UP ANSWERS

1. 11 2. 16 3. 9 4. 26

INSTRUCTION

Read Discover the Pythagorean Theorem on p. 6 in Math in Focus 3B.

Read and complete Hands-On Activity on pp. 7–8. Side \(c\) in each triangle is the hypotenuse. The hypotenuse is the longest side of a right triangle. The Pythagorean Theorem shows that the sum of the squares of the lengths of the two shorter legs of a right triangle equal the square of the length of its hypotenuse.

Review Example 1 on p. 8. Then complete Guided Practice on p. 9. Since the answer represents a length, use only the positive square root.

Review Example 2 on p. 9. It might be helpful to trace triangle \(WXZ\) without \(\overline{WY}\). Since \(XZ\) is a leg of the right triangle, you can solve for \(a\) or \(b\) in the Pythagorean Theorem.
Discuss the **Think Math** question on p. 9 with your Learning Guide. Notice that triangle $WXY$ is also a right triangle with legs $WX$ and $XY$ and hypotenuse $WY$.

Complete **Guided Practice** on p. 10.

**HELPFUL ONLINE RESOURCES**

**Instructional Video:** BrainPOP: [Pythagorean Theorem](https://www.brainpop.com/math/geometryspatial.relationships/pythagoreantheorem/)  

---

**TEACHING NOTES**

If your student knows the lengths of two sides of a right triangle, he can use the Pythagorean Theorem to find the length of the third side. If your student has a scientific calculator, encourage him to use it to find the square root.

**WATCH FOR THESE COMMON ERRORS**

Your student might solve for the wrong variable in the Pythagorean Theorem. Either leg can be labeled $a$ or $b$, but the hypotenuse must always be $c$. Remind him that $c$ is always the side opposite the right angle.

---

**PRACTICE**

Complete problems 1–10 of **Practice 7.1** on pp. 16–17 in *Math in Focus 3B*.

---

**TEACHING NOTES**

[Textbook Answer Key](https://www.mathinfocus.com/courses/grade-8-calvert-

---

**WRAP-UP**

Today you learned how to use the Pythagorean Theorem to find the unknown side length of a right triangle.
The length of the third side is about 27.5 ft.

\[
\begin{align*}
\text{PRACTICE QUESTIONS} \\
\text{Please go online to view and submit this assessment.}
\end{align*}
\]
The Pythagorean Theorem - Part 2

Objectives
- Use models to develop an understanding of the Pythagorean Theorem.

Books & Materials
- Math in Focus - Teacher Edition
- Pythagorean Theorem Worksheet
- scissors

Assignments
- Complete Warm-Up.
- Complete Pythagorean Theorem Worksheet.
- Complete Practice Questions.

LEARN

LEARN

WARM-UP

Find the unknown side length of the following triangle.

\[
\begin{align*}
8\text{ km} & \quad x\text{ km} \\
6\text{ km} &
\end{align*}
\]

WARM-UP ANSWER

10 km

TEACHING NOTES

WARM-UP ANSWER

10 km

INSTRUCTION

Read the Pythagorean Theorem Worksheet. In Step 2, place the edge of a sheet of paper next to the shortest side of the triangle and make tick marks to show the length of the side. Use this width to cut a strip of paper.

Then put the long edge of the strip next to the short side and mark the length. Use this marking to cut the paper into a square that has a length and width matching the length of the shortest side. Repeat with the other sides.
In Step 5, cut out the small square and cut along the dotted lines. You should have 5 shapes: 1 square and 4 irregular quadrilaterals. Fit together the irregular quadrilaterals so they make a square. Put that square on top of the midsized square. Notice they are exactly the same size.

In this example, \(a\) is the shorter leg, and \(b\) is the longer leg. The relationship is still true if you reverse the legs and make the shorter leg \(b\) and the longer leg \(a\). It is still the small square that is traced inside the largest one.

Complete the **Your Turn** section. Notice that this right triangle is a different shape than the previous one.

Regardless of how similar or different the lengths of the two legs of a right triangle are, the relationship \(a^2 + b^2 = c^2\) still holds true.

---

**TEACHING NOTES**

**Worksheet Answers:**

**Your Turn**

1 squares 2 smallest; largest 3 smallest

4 \(a^2\) 5 \(b^2\) 6 \(c^2\) 7 \(a^2; b^2; c^2\)

This lesson is designed to help your student understand the Pythagorean Theorem. By exploring different-sized right triangles, your student will develop a sense that the special relationship among the sides of a right triangle is always true. Your student should not use a ruler to measure the sides of the triangles presented in the worksheet. By not attaching actual measurements to the sides, your student can more readily see the general proof of the Pythagorean Theorem for any right triangle.
Regardslessofhowsimilarordifferentthelengthsofthetwolegsofarighttriangleare,therelationship 
\[ a^2 + b^2 = c^2 \] still holdstrue.

Worksheet Answers:

Your Turn

1 squares

2 smallest; largest

3 smallest

4

5

6

7

8

ThislessonisdesignedtohelpyourstudentunderstandthePythagoreanTheorem. Byexploring
different-sizedrighttriangles,yourstudentwilldevelopasensethatthespecialrelationshipamong
thesidesofarighttriangleisalwaystrue. Yourstudentshouldnotusearulertomeasurethesides
ofthetrianglespresentedintheworksheet. Bynotattachingactualmeasurementstothesides,your
studentcanmorereadilyseethegeneralproofofthe

Pythagorean Theorem foranyrighttriangle.

Complete the Practice section of the Pythagorean Theorem Worksheet.

Worksheet Answers:

Practice

2 Sample answer: Each square in my drawing has an area of the side length squared. For example,
the square on the a side has an area of \( a \cdot a = a^2 \). When I draw the smallest square inside the
largest square, I can see that the largest square is equal to the area of the small square \( (a^2) \) with
pieces left over. When I cut apart the largest square, I can see that the leftover pieces have the
same area as the mid-sized square \( (b^2) \). So, the area of the largest square \( (c^2) \) is equal to the areas
of the other two squares combined \( (a^2 + b^2) \), and \( c^2 = a^2 + b^2 \).

For problem 1, your student should draw a triangle that is different than the two shown in the activity.
He may choose a triangle that is short and wide or one that is narrow and tall. Your student might
also wish to draw a triangle that has two legs that are the same length.

INTERACTIVE ACTIVITY

Follow these instructions for the activity shown below.

Click here to view the activity in a new window.
With **Show values** selected on the PYTHAGOREAN tab, drag the vertices to create a variety of triangles. What is true about all these triangles? How do you know? Where is the longest side of a right triangle located, relative to the right angle? Share your answers with your Learning Guide.

Now select **Show labels**. Drag the vertices again to explore more right triangles. The sides labeled $a$ and $b$ are the legs of the triangle, and the side labeled $c$ is the triangle’s hypotenuse. Explain to your Learning Guide where the legs and hypotenuse are located, relative to the right angle.

Copy this table into your Math Notebook:

<p>| | | | | | |</p>
<table>
<thead>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$b$</td>
<td>$c$</td>
<td>$a^2$</td>
<td>$b^2$</td>
<td>$a^2 + b^2$</td>
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</tr>
</tbody>
</table>

In the Gizmo, create four different right triangles. Click on **Show squared side lengths** to help you fill in the table for each triangle. Describe the relationship you see in the table to your Learning Guide.

**TEACHING NOTES**

**Sample answers:** All the triangles created in this Gizmo will be right triangles because the angle at point C is always a right angle (90°). The longest side of a right triangle (the hypotenuse) is always opposite (not forming) the right angle. The two sides that form the right angle are the legs. If you square the measures of the legs and add them, the sum will equal the square of the hypotenuse.

If you would like, you can click on **Lesson Info** and download the **Student Exploration Sheet** and **Exploration Sheet Answer Key** to have your student try some other activities with the Gizmo.

**WRAP-UP**

Today you learned one proof for the Pythagorean Theorem.

Given a right triangle with legs $a$ and $b$ and hypotenuse $c$, the sum of the areas of a square with side lengths $a$ and a square with side lengths $b$ is equal to the area of the square with side lengths $c$.

$$a^2 + b^2 = c^2$$
In the Gizmo, create four different right triangles. Click on Show squared side lengths to help you fill in the table for each triangle. Describe the relationship you see in the table to your Learning Guide.

Sample answers:
All the triangles created in this Gizmo will be right triangles because the angle at point \( C \) is always a right angle (90°). The longest side of a right triangle (the hypotenuse) is always opposite (not forming) the right angle. The two sides that form the right angle are the legs. If you square the measures of the legs and add them, the sum will equal the square of the hypotenuse.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

Today you learned one proof for the Pythagorean Theorem. Given a right triangle with legs \( a \) and \( b \) and hypotenuse \( c \), the sum of the areas of a square with side lengths \( a \) and a square with side lengths \( b \) is equal to the area of a square with side lengths \( c \).

\[ a^2 + b^2 = c^2 \]

Please go online to view and submit this assessment.
The Pythagorean Theorem - Part 3

Objectives
- Use the Pythagorean Theorem to solve real-world problems.

Books & Materials
- Math in Focus 3B
- Math in Focus - Teacher Edition
- grid paper
- scientific calculator (Optional)

Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete Quick Check.

LEARN

WARM-UP
Simplify.
1. $4^2 + 16^2$
2. $23^2$
3. $3^2 + 12^2$
4. $15^2$

WARM-UP ANSWERS
1. 272  2. 529  3. 153  4. 225

TEACHING NOTES

INSTRUCTION
Complete Hands-On Activity on p. 11 in Math in Focus 3B. It can be difficult to determine the length in grid squares of the hypotenuse because it is on a diagonal. Before starting the activity, cut a strip of grid paper 20 units long and 1 unit wide, as shown.

Use this grid paper strip to measure in grid squares the hypotenuse of your triangles.
Read **The Converse of the Pythagorean Theorem** on p. 11. Then review **Example 3** on p. 12. You can use the Pythagorean Theorem to determine if a triangle is a right triangle.

The longest side of a right triangle is the hypotenuse, \(c\). If you do not know if a triangle is a right triangle, use the longest side for \(c\) and the other two sides for \(a\) and \(b\). It does not matter which of the shorter sides you use for \(a\) and which you use for \(b\).

Complete **Guided Practice** on p. 12.

---

**TEACHING NOTES**

If a triangle is a right triangle, then the Pythagorean Theorem describes the relationship of the sides. If your student substitutes the two shorter sides for \(a\) and \(b\) and the longest side for \(c\) in the Pythagorean Theorem, he can determine whether the triangle is a right triangle. If the two sides of the equation are equal, the triangle is a right triangle. If the sides are not equal, the triangle is not a right triangle.

---

**PRACTICE**

Complete problems 11, 24, and 25 of **Practice 7.1** on p. 17 and p. 19 in *Math in Focus 3B*.

---

**TEACHING NOTES**

**Textbook Answer Key**

---

**WRAP-UP**

Today you learned how to use the Pythagorean Theorem to determine if a triangle is a right triangle.

\[
a^2 + b^2 = c^2 \\
28^2 + 45^2 = 53^2 \\
784 + 2025 = 53^2 \\
2029 = 2809
\]

The triangle is a right triangle, and 28, 45, and 53 is a Pythagorean triple.
View the video, *Pythagorean Theorem* (03:48) to explore how the Pythagorean Theorem can be used to identify a right triangle.

Please go online to view this video ►

Please go online to view and submit this assessment.
The Pythagorean Theorem - Part 4

**Objectives**
- Use the Pythagorean Theorem to solve real-world problems.

**Books & Materials**
- *Math in Focus 3B*
- *Math in Focus - Teacher Edition*
- Scientific calculator (Optional)

**Assignments**
- Complete Warm-Up.
- Complete the assigned pages in *Math in Focus 3B*.
- Complete Use for Mastery.

---

**LEARN**

**WARM-UP**

Simplify. Round the answer to the nearest tenth if necessary.

1. \(\sqrt{224}\)
2. \(34^2\)
3. \(\sqrt{107}\)
4. \(21^2\)

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. 15.0  
2. 1,156  
3. 10.3  
4. 441

---

**INSTRUCTION**

Read *Solve Real-World Problems Involving the Pythagorean Theorem* on p. 12 in *Math in Focus 3B*.

Review *Example 4* on p. 13. The distance along the ground from the pole to the banner is one of the shorter sides of the right triangle, so substitute this unknown value for \(a\) or \(b\) in the Pythagorean Theorem.

Complete *Guided Practice* on p. 13.
Review Example 5 on p.14. It does not appear that there is a right triangle in this problem. However, if you draw a line from the nail to the center of the top of the picture frame, you form a right triangle. Look at the following diagram.

Since the distance from the top of the frame to the nail is 10 cm, this leg of the triangle is 10 cm. The dotted line divides the picture frame in half. Since the picture frame is 15 cm wide, the other leg of the right triangle is $15 \div 2 = 7.5$ cm.

Discuss the Think Math question on p.14 with your Learning Guide. If there are 20 cm of wire, there will be 10 cm on each side of the nail. In this case, you know the hypotenuse and you have to find the leg of the triangle that shows the distance from the frame to the nail. Redraw the diagram above and label it with the new information before solving the problem.

Complete Guided Practice on p.15.

The Pythagorean Theorem is a useful tool when solving problems involving right triangles. Sometimes a problem involving another shape can be solved using the Pythagorean Theorem if the shape can be divided in such a way as to form right triangles. Throughout this lesson, it will help your student to copy the pictures so that he can use them to solve the problems.

Complete problems 12–23 of Practice 7.1 on pp.17–19 in Math in Focus 3B.

For problems 17, 18, 20, and 22, your student should draw a picture to help him visualize what is described in the problem. For problem 17, the diagonal length is the distance from the upper corner
of the television to the opposite lower corner. This diagonal length is the hypotenuse of a right triangle formed by the length and width of the television.

WRAP-UP

Today you learned how to use the Pythagorean Theorem to solve real-world problems.

USE FOR MASTERY

1. A textbook has a length of 6 inches, a height of \( y \) inches, and a width of \( x \) inches. If the length of the diagonal of the front cover is 8 inches and the length of the diagonal of the width is 7 inches, find the values of \( x \) and \( y \).

If you are able, use the text box to show your work and enter your final answer. If not, complete your work on paper and upload it below.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information given to find the values of $x$ and $y$?
- Use the text box to show your work and enter your final answer OR complete your work on paper and upload it to the site?

A textbook has a length of 6 inches, a height of $y$ inches, and a width of $x$ inches. If the length of the diagonal of the front cover is 8 inches and the length of the diagonal of the width is 7 inches, find the values of $x$ and $y$.
The Distance Formula - Part 1

Objectives
- Use the Pythagorean Theorem to derive the distance formula.

Books & Materials
- *Math in Focus 3B*
- *Math in Focus - Teacher Edition*
- scientific calculator (Optional)

Assignments
- Complete Warm-Up.
- Complete the assigned pages in *Math in Focus 3B*.
- Complete Practice Questions.

LEARN

WARM-UP

Find the length of each segment.

1.

2.

WARM-UP ANSWERS

1. 9 units  
2. 5 units

TEACHING NOTES

WARM-UP ANSWERS

1. 9 units  
2. 5 units
INSTRUCTION

Read p. 20 in *Math in Focus 3B*. Notice that $\overline{AC}$ and $\overline{BC}$ are the legs of a right triangle and $\overline{AB}$ is the hypotenuse. To find $AC$ and $BC$, you can count the units.

Review **Example 6** on p. 21. To find the length of a segment, you can subtract coordinates. The value is a length, so take the absolute value of the difference.

- In a vertical line, the $x$-coordinates are the same, so subtract the $y$-coordinates to find the length: $|y_2 - y_1|$.
- In a horizontal line, the $y$-coordinates are the same, so subtract the $x$-coordinates to find the length: $|x_2 - x_1|$.

Complete **Guided Practice** on p. 22.

Read **Understand the Distance Formula** on p. 23. For a right triangle on a grid, the length of one leg is the horizontal distance $|x_2 - x_1|$. The length of the other leg is the vertical distance $|y_2 - y_1|$. If we say the length of the hypotenuse is $c$, the Pythagorean Theorem tells us $c^2 = a^2 + b^2$. If $a = |x_2 - x_1|$ and $b = |y_2 - y_1|$, then:

$$c^2 = a^2 + b^2$$

$$c^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Notice that the absolute value bars are not included in the calculations for $c$. Because each difference is squared and the square of a number is never negative, the absolute value bars are not needed in the formula.

If the hypotenuse of the triangle is $\overline{AB}$, then:

$$\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This formula is the *distance formula* and can be used to find the distance between any two points on a coordinate grid.

TEACHING NOTES

Your student already knows how to find the lengths of vertical and horizontal segments on the coordinate plane. In this lesson, he learns how to use the Pythagorean Theorem to find the length of any segment by thinking of the segment as the hypotenuse of a right triangle.
The distance formula, \( AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \), can be derived from the Pythagorean Theorem. If \( |x_2 - x_1| \) is the length of leg \( a \), \( |y_2 - y_1| \) is the length of leg \( b \), and \( AB \) is the hypotenuse \( c \), then the distance formula is simply \( c = \sqrt{a^2 + b^2} \), which is how your student would solve for \( c \) given \( a \) and \( b \).

**PRACTICE**

Complete problems 1–2 of Practice 7.2 on p. 28 in *Math in Focus 3B*.

**TEACHING NOTES**

Textbook Answer Key

**WRAP-UP**

Today you learned how to use the Pythagorean Theorem to find the length of a segment on a coordinate grid. You also learned how to use the Pythagorean Theorem to write the distance formula.

\[
\begin{aligned}
  c^2 &= a^2 + b^2 \\
  c^2 &= 9^2 + 9^2 \\
  c^2 &= 162 \\
  c &= \sqrt{162} \approx 12.7 \text{ units}
\end{aligned}
\]

\[
\begin{aligned}
  c^2 &= a^2 + b^2 \\
  AB^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\
  AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\end{aligned}
\]

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
## The Distance Formula - Part 2

### Objectives
- Use the distance formula to solve geometry problems.

### Books & Materials
- Math in Focus B
- Math in Focus - Teacher Edition
- grid paper
- colored pencils
- scientific calculator (Optional)

### Assignments
- Complete Warm-Up
- Complete the Assigned Pages in Math in Focus 3B
- Complete Quick Check

## LEARN

### WARM-UP
What is the distance between point Y and point Z to the nearest tenth?

![Graph showing points Y and Z on a coordinate plane.]

### WARM-UP ANSWER
1. 8.5 units

### TEACHING NOTES

#### WARM-UP ANSWER
1. 8.5 units

### INSTRUCTION
Review Example 7 on p. 24 in Math in Focus 3B. You can use the distance formula to find the length of any line segment given the coordinates of its endpoints. Since a triangle has three sides, you will need to use the distance formula three times to find the length of each side.
Remember, a triangle with all sides the same length is an *equilateral triangle*, a triangle with at least two sides the same length is an *isosceles triangle*, and a triangle with no sides the same length is a *scalene triangle*.

Complete **Guided Practice** on p. 25. To help you keep track of the endpoints of each side of the triangle, copy the triangle on grid paper. Then use a different colored pencil to trace over each line segment and circle the coordinates of each endpoint. Each ordered pair will be circled twice because it is the endpoint of two line segments.

Review **Example 8** on p. 26. Then complete **Guided Practice** on p. 27.

**TEACHING NOTES**

Once your student has learned the distance formula, he can use it to find the distance between any two points on a coordinate grid. If these points are joined by a line segment, the distance found by the distance formula is the length of the segment. The number of times your student has to use the distance formula in a problem involving a geometric figure depends on how many sides there are in the figure. If the figure is a triangle, he will need to use the distance formula three separate times.

**PRACTICE**

Complete problems 3–11 of **Practice 7.2** on pp. 28–30 in *Math in Focus 3B*.

**TEACHING NOTES**

For problem 8b on p. 30, your student needs to remember the relationship among the sides of a right triangle, \(a^2 + b^2 = c^2\) and determine whether that relationship applies to the sides of triangle \(RST\).

For problem 10, your student needs to realize that every point on a circle is the same distance from the center of the circle. Drawing a picture that represents this problem may help your student to better understand it.

**WRAP-UP**

Today you learned to use the distance formula to solve geometry problems.
Once your student has learned the distance formula, he can use it to find the distance between any two points on a coordinate grid. If these points are joined by a line segment, the distance found by the distance formula is the length of the segment. The number of times your student has to use the distance formula in a problem involving a geometric figure depends on how many sides there are in the figure. If the figure is a triangle, he will need to use the distance formula three separate times.

Complete problems 3–11 of Practice 7.2 on pp. 28–30 in Math in Focus 3B.

Textbook Answer Key

For problem 8 on p. 30, your student needs to remember the relationship among the sides of a right triangle, \(a^2 + b^2 = c^2\) and determine whether that relationship applies to the sides of triangle RST.

For problem 10, your student needs to realize that every point on a circle is the same distance from the center of the circle. Drawing a picture that represents this problem may help your student to better understand it.

Today you learned to use the distance formula to solve geometry problems. Please go online to view and submit this assessment.

View the video, How to Find the Distance Between Two Points (04:35), to find out how to calculate the distance between two points on a coordinate plane.
The Distance Formula - Part 3

**Books & Materials**
- Math in Focus - Teacher Edition

**Assignments**
- Complete Interactive Activity.
- Complete Rate Your Enthusiasm.

**LEARN**

**INTERACTIVE ACTIVITY**

Follow these instructions for the activity below.

Click [here](#) to view the activity in a new window.

To begin, you will use coordinates to measure the distance between points that are lined up horizontally and vertically. With **Show values** selected, drag point A to (6, 2) and point B to (–2, 2). To find the distance between the points, click **Show ruler** and drag each circle over one of the endpoints. You can also use the x-coordinates of the points (6 and –2) to find this distance. Remember, absolute value is an expression of distance. Find |6 – (–2)| or |–2 – 6|. Both expressions give you a distance of 8 units.

With point A still at (6, 2), drag point B to (6, –3). What is the distance between the points now? Use the ruler to check. You can also use the y-coordinates of the points (2 and –3) to find this distance. Share your ideas with your Learning Guide about how you might do this.

Now click off the ruler and drag point A to (2, 6) and point B to (14, 1). On a piece of graph paper, sketch the points and the segment between them. Sketch segments to represent the horizontal and vertical distances between the points, as shown here.
Where these segments meet, draw a third point C and label its coordinates \((2, 1)\). You have drawn a right triangle! Label the long leg \(a\), the short leg \(b\), and the hypotenuse \(c\). (The labels \(a\), \(b\), and \(c\) represent the lengths of the sides.) Use absolute values to find length \(a\) (12 units) and length \(b\) (5 units). Use the Pythagorean Theorem \(a^2 + b^2 = c^2\) to find \(c\), the length of the hypotenuse of the right triangle. Show your work on the graph paper next to your drawing. Use the Gizmo ruler to check your calculation. You should have determined that \(c = 13\) units.

Now click off the ruler and click on Show labels. Suppose the coordinates of point \(A\) are \((x_1, y_1)\) and the coordinates of point \(B\) are \((x_2, y_2)\). In your Math Notebook, write absolute value expressions to find \(a\) and \(b\). Turn on Show triangle and Show labels to check your answers. (The order in which you write the variables does not matter, but you must subtract \(x\) values from each other and \(y\) values from each other.) The Pythagorean Theorem can also be written \(c^2 = a^2 + b^2\). Substitute the values you found for \(a\) and \(b\) above into this equation to write an expression for \(c^2\) in terms of \(x_1, y_1, x_2,\) and \(y_2\). Substitute \(d\) (distance) for \(c\) (length of hypotenuse). Then solve for \(d\) by taking the square root of both sides. This is called the distance formula. Select Show distance computation to check your formula. Make corrections if necessary.

Select Show values and turn off everything else. Drag point \(A\) to \((-6, -5)\) and point \(B\) to \((-1, 4)\). On your graph paper, make a labeled sketch of the triangle that can be used to find the distance between the points. Next to your sketch, use the distance formula to find \(AB\). Round to the nearest hundredth if necessary. Turn on Show distance computation to check your work above. Why is it not necessary to use the absolute value of the differences of the coordinates in the distance formula? Share your ideas with your Learning Guide.

Use the distance formula to find the distance between each pair of points. Round to the nearest hundredth if necessary. Do all your work in your Math Notebook. When possible, check your answers in the Gizmo.

1. \((-7, 9)\) and \((3, -5)\)
2. \((12, -8)\) and \((-4, 1)\)

**TEACHING NOTES**

**Sample answers:** To find the distance between \((6, 2)\) and \((6, -3)\), find one of these absolute values: \(|2 - (-3)|\) or \(|(-3) - 2|\). The distance between the points is 5 units.

**Distance formula computation:**

\[
c^2 = a^2 + b^2
\]

\[
c^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2
\]

\[
c^2 = (x2 - x1)^2 + (y2 - y1)^2
\]
Activity answers: It is not necessary to use the absolute value symbols because the square of any number is always a positive number. 1 approximately 10.77 units 2 approximately 18.36 units

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

⚠️ RATE YOUR ENTHUSIASM

Please go online to view and submit this assessment.
The Distance Formula - Part 4

Objectives

- Use the Pythagorean Theorem to find unknown measures in solids.

Books & Materials

- Math in Focus 3B
- Math in Focus - Teacher Edition
- scientific calculator (Optional)
- nets of solid figures (Optional)

Assignments

- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete Practice Questions.

LEARN

WARM-UP

Find the distance between each pair of points. Round your answer to the nearest tenth.

1. $(-1, 3)$ and $(5, 6)$
2. $(9, 2)$ and $(-7, 1)$
3. $(1, 1)$ and $(0, 4)$

WARM-UP ANSWERS

1. 6.7 units  2. 16.0 units  3. 3.2 units

TEACHING NOTES

INSTRUCTION

Read p.31 in Math in Focus 3B. Slant height is the length from the vertex to the center of the base of the slanted face on a pyramid or cone. In the diagram, $\overline{AC}$ is the slant height.

Notice the hidden right triangle in the pyramid. Point $B$ is in the center of the pyramid, halfway between one side and the other. $\overline{AB}$ is perpendicular to the base because it forms a right angle with the base.

The slant height of the pyramid is the length of the hypotenuse of a triangle with legs that are the height and half the length of the side of the base of the pyramid. $\overline{AB}$ is one leg (the height), $\overline{BC}$ is the other leg, and $\overline{AC}$ is the hypotenuse.

For the Think Math question, think about the properties of a square pyramid. The base is a square, so the sides of the base are all the same length and the triangular faces are the same size. The slant height gives the height of the triangular face. The lateral surface area is the area of all of the faces of
the pyramid except the base. In this case, the lateral faces are 4 triangles. To find the lateral surface area, find the area of one triangular face \( \left( \text{Area} = \frac{1}{2}bh \right) \) and multiply it by 4.

Review Example 9 on p. 32. Complete Guided Practice on p. 32.

Review Example 10 on p. 33. The central diagonal \( \overline{DF} \) is the hypotenuse of triangle \( DEF \). The legs are \( \overline{DE} \) and \( \overline{EF} \). Since the solid is a cube, you know that the length, width, and height of the figure are the same length, 15 in. Complete Guided Practice on p. 33.

Hidden right triangles exist in solid figures. Often, the height of the solid forms a leg of the hidden triangle. For the height of the solid to be part of this right triangle, it must be perpendicular to the base, or intersect with the base at a right angle. If your student knows two sides of this hidden triangle, he can find the third side by using the Pythagorean Theorem.

When solving for \( a, b, \) or \( c \), sometimes your student is asked to evaluate the square root by rounding it to the nearest tenth. Otherwise, an exact value can be found by leaving the answer as a square root.

Three-dimensional figures can often be hard to visualize. If you have objects around the house that are shaped like the solids in this lesson, allow your student to use them to help him visualize the faces and bases.

Complete Practice 7.3 on pp. 34–35 in Math in Focus 3B.

Nets can help your student visualize the faces of solids. For example, for problem 6, the net of a cone can help your student understand what is meant by the lateral surface. For problem 7, your student needs to find the diameter of the glass and divide that value by 2 to find the radius.
WRAP-UP
Today you learned to use the Pythagorean Theorem to find unknown measures in solids.

\[
\begin{align*}
5^2 + h^2 &= 7^2 \\
25 + h^2 &= 49 \\
h^2 &= 49 - 25 \\
h^2 &= 24 \\
\sqrt{h^2} &= \sqrt{24} \\
h &= \sqrt{24} \approx 4.9
\end{align*}
\]

PRACTICE QUESTIONS
Please go online to view and submit this assessment.
The Distance Formula - Part 5

Objectives
- Use the Pythagorean Theorem to find the volume of composite solids.

Books & Materials
- Math in Focus 3B
- Math in Focus - Teacher Edition
- scientific calculator (Optional)

Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete Quick Check.

LEARN

WARM-UP
What is the exact value of \( m \)?

![Diagram of a pyramid with dimensions 12 cm, 15 cm, and \( m \) cm.]

TEACHING NOTES

WARM-UP ANSWER
1. 9

INSTRUCTION
Read the instructional section on pp. 36–37 in Math in Focus 3B. If you visualize the hidden right triangle in the square pyramid that makes up the top of the birdhouse, you can use the Pythagorean Theorem to find the height of the pyramid.

Notice that the length of the edge of the pyramid is given, not the slant height. To make the right triangle, you need the length of the diagonal of the base of the pyramid. This is the same as the diagonal of the bottom of the square prism.
Look at the solution on p. 37. You could complete these steps in a different order. For example, you can find the volume of the square prism first. Then you can use the Pythagorean Theorem to find the volume of the pyramid. Remember to add the volumes of both figures as the final step.

Review Example 11 on p. 38. Since there is a cylindrical hole in the middle of the triangular prism, you need to subtract the volume of the cylinder from the volume of the triangular prism.

Complete Guided Practice on p. 39.

**TEACHING NOTES**

Finding volumes of composite figures can be a difficult skill to master because there are so many steps. A crucial step is to be sure that you have all the necessary measurements to find the volume of each part of the solid, and, if not, to determine which measurements are unknown.

In this part, your student will need to look for hidden right triangles in the solids and use the Pythagorean Theorem. Sometimes he will need to use the Pythagorean Theorem more than once. It will be important for your student to note whether he is adding or subtracting the volumes. Working slowly and carefully through each problem is the best way to ensure success.

**PRACTICE**

Complete Practice 7.4 on pp. 40–43 in *Math in Focus 3B*.

**TEACHING NOTES**

*Textbook Answer Key*

**WRAP-UP**

Today you learned to use the Pythagorean Theorem to find the volume of composite solids.

**Step 1:** Use the Pythagorean Theorem to find the diagonal of the base, $d$.

$13^2 + 9^2 = d^2$

$169 + 81 = d^2$

$250 = d^2$

$\sqrt{250} = d$

Half the diagonal base is $\frac{\sqrt{250}}{2}$.
Step 2: Find the height of the pyramid.
\[
\left(\frac{\sqrt{250}}{2}\right)^2 + h^2 = 15^2 \\
62.5 + h^2 = 225 \\
h^2 = 162.5 \\
h = \sqrt{162.5}
\]

Step 3: Find the volume.
rectangular prism: \(13 \times 9 \times 2 = 234 \text{ ft}^3\)
rectangular pyramid:
\[
\frac{1}{3} \times 13 \times 9 \times \sqrt{162.5} \approx 497.2 \text{ ft}^4
\]
approximate total volume:
\(234 + 497.2 = 731.2 \text{ ft}^3\)

Quick Check

Please go online to view and submit this assessment.

More to Explore

View the video, Calculating the Volume of Composite Solids (02:35), to see how volume is solved for composite shapes.

Please go online to view this video ▶
The Distance Formula - Part 6

Objectives
- Use the Pythagorean Theorem and distance formula to solve problems.

Books & Materials
- *Math in Focus 3B*
- *Math in Focus - Teacher Edition*
- online graphing calculator (Optional)

Assignments
- Complete Warm-Up.
- Complete the Assigned Pages in *Math in Focus 3B*.
- Complete the Practice Activity.
- Complete Use for Mastery.

LEARN

WARM-UP

Find the volume of the following figure to the nearest cubic foot.

![Volume Figure](image)

1. 2,336 ft³

TEACHING NOTES

INSTRUCTION

Read *Brain @ Work* on p. 43 in *Math in Focus 3B*. Note that in problem 1, the width and height of a staircase form a right angle as shown in the following picture.

![Staircase Diagram](image)
In problem 2, draw a diagram to help you solve the problem. Assume that Brad and John start at the same point on one riverbank. Find the distance between the two points where they land on the opposite bank.

**TEACHING NOTES**

**Textbook Answer Key**

The Pythagorean Theorem and distance formula can help your student solve the problems in this lesson. It is helpful for him to use diagrams and drawings to model the problems.

For problem 1, the height of the staircase is the total of the heights of the steps, and the width is the total of the widths of the steps. The measurement of the railings is the hypotenuse.

**PRACTICE**

Write a constructed response to explain how you solved problem 2 in Brain @ Work in Math in Focus 3B.

Then solve the following problems.

1. The sides of the following hexagon are all the same length. AE and BD are both about 26 cm. The height of triangle BCD is about 7.5 cm. What is the perimeter of the hexagon?

2. In the following right triangle, \( BX + BC = AX + AC \). If \( AC = 10 \) in. and \( BC = 26 \) in., what is BX? (Hint: The placement of \( X \) on \( AB \) in the figure may not be its actual location.)
Today you learned how use the Pythagorean Theorem and the distance formula to solve problems.
Find the distances. Round any decimals to the nearest tenth. Type your answers in the boxes.

a. The actual distance between the library and Town A is \( \) km.

b. The approximate distance between the library and Town B is \( \) km.

c. Amelia traveled from Town A to the library to return her books. She then traveled to Town B to meet her friend. She and her friend then traveled to Town C and had dinner. How far did Amelia travel?

If you are able, use the text box to show your work and enter your final answer to the question. If not, complete your work on paper and upload it below.

---

**Did you:**
- Use the information given to help you answer parts A, B, and C?
- Round decimals to the nearest tenth?
- Label your answers correctly?
- Use the text box to show your work and enter your final answer OR complete your work on paper and upload it below?

---

Supported file formats: PDF, JPG, GIF, PNG

0 / 12 File Limit
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information given to help you answer parts A, B, and C?
- Round decimals to the nearest tenth?
- Label your answers correctly?
- Use the text box to show your work and enter your final answer OR complete your work on paper and upload it to the site?
Unit Quiz: Pythagorean Theorem

Books & Materials

- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

✓ UNIT QUIZ

Please go online to view and submit this assessment.
Unit 5 - Transformations, Congruence, and Similarity
Project: Be an Architect

Books & Materials
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B
- Coordinate grid transparency (this can be made by printing a coordinate grid on plastic transparency paper)
- Pictures of buildings from magazines, online, or from a camera
- Geometry software or grid paper
- A ruler and a protractor to trace over pictures

PROJECT DESCRIPTION

An architect plans, designs, and reviews the construction of many types of buildings. Can you think of some famous building you like? Can you think of any famous architects? Frank Lloyd Wright and Frank Gehry are two examples of architects whose work is well known. Click on their names to learn more about them and see some of their work.

Before constructing a building, architects sometimes draw a sketch or scale drawing and sometimes make a model out of cardboard. Architects can also use software to create 2D and 3D building plans. In this activity, you are going to examine some of the similarities between different buildings and ways that they think out their designs. You will take pictures or find pictures online of buildings. You will gather at least five pictures with buildings that seem to have figures that are the same size and shape (congruent) and another five pictures of buildings that are the same shape, but not the same size (similar). In this activity you will apply concepts in the lesson to analyze and verify congruence or similarity.

PROJECT DETAILS

In this project, you will need to:

- Find images of buildings you believe are congruent and similar. You may compare the entire building or parts of the building.
- Copy these images onto a coordinate plane.
- Measure the sides and angles of these images to prove they are similar and congruent.
The Project Rubric will help you understand how your project will be scored. Your goal should be to earn all possible points for each part.

Think of a building that you have seen that you felt was well designed. What shapes would have been drawn when designing the building? Share what you like about these buildings. You can even upload pictures of these buildings. Respond to two of your peers.

Please go online to view and submit this assessment.
Translators, Reflections, Rotations - Part 1

Objectives
- Find a point symmetric to a given point on a coordinate grid
- Identify directly proportional quantities
- Identify perpendicular bisectors

Books & Materials
- Math in Focus 3B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B
- grid paper

Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete Practice Questions.

LEARN

WARM-UP

Solve each problem.

1. Aiden’s toy race track just fits diagonally on his bed. His bed is 72 inches long and 52 inches wide. How long is Aiden’s track? Give your answer in feet and inches to the nearest inch.
2. Find the distance between the points (–1, 4) and (5, 0). Round your answer to the nearest tenth.

WARM-UP ANSWERS

1. 7 ft 5 in. 2. 7.2 units

INSTRUCTION

Today’s lesson covers important ideas you need to understand before beginning the lessons in this chapter. Read pp. 48–50 in Math in Focus 3B.

Reflections are a type of geometric transformation. The review focuses on the reflection of a point. On the coordinate plane, you can reflect points about any line. Usually, you will reflect points about the x-axis or the y-axis. You will be able to see this clearly if you
use grid paper. Draw your coordinate axes on the paper and then draw Point \( A \) \((-3, -2)\). To reflect the point about the \( x \)-axis, fold the paper along the \( x \)-axis and hold it up to the light. Mark the new point as if you are tracing it, on the clean side of the paper. Point \( B \), which you just drew, is the reflection of Point \( A \) about the \( x \)-axis.

Points \( A \) and \( B \) are said to be symmetric about the \( x \)-axis, which means that they are the same distance from the line. Find the distance between the two points (4 units). Now draw the line \( y = x \) on the same graph. Fold the paper along the line and mark the reflection of Point \( A \) about the line \( y = x \). Label this Point \( C \). Find the distance between Point \( A \) and Point \( C \) (\( \sqrt{2} \) units).

You may have studied directly proportional relationships using ratios such as 10:2 and 20:4. A directly proportional relationship can be described by the equation \( y = kx \), where \( k \) is called the constant of proportionality. Note that \( k \) can be positive or negative and \( y = kx \) is the equation of a line with a \( y \)-intercept of 0.

If the relationships are written as \( y:x \), you can find the constant of proportionality by substituting the values into the equation \( y = kx \). In the example of 10:2, \( 10 = k \cdot 2 \). Solving gives \( k = 5 \). The ratios 10:2 and 20:4 are both directly proportional relationships with 5 as the constant of proportionality.

The statement at the end of Recognizing Perpendicular Bisectors is very important. A point on the perpendicular bisector of a line segment is equidistant from the two endpoints of the line segment. To find the distance between an endpoint of the line segment and a point on the perpendicular bisector, you can use either the Pythagorean Theorem (since the perpendicular bisector makes a right angle with the line) or the distance formula. In either case, it is helpful to draw a picture of the problem first.

---

### TEACHING NOTES

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the Quick Check.

Your student may not be able to visualize where a point will be after a reflection. He can always fold the sheet of paper and trace the original point to locate the reflection.

After your student completes several reflection problems, guide her to visualize simple reflections (about the \( x \)-axis or the \( y \)-axis) without having to fold the paper.

---

**SKILLS CHECK**

Complete the Quick Check sections on pp. 49–50 in *Math in Focus 3B*. 
Textbook Answer Key

After your student completes the Quick Check in the Recall Prior Knowledge lesson of this chapter, review the questions that were answered incorrectly. If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.

Quick Check

<table>
<thead>
<tr>
<th>Question(s)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–4</td>
<td>How to Reflect a Line Segment over the x-Axis</td>
</tr>
<tr>
<td>5–7</td>
<td>Using Proportions to Find Missing Values</td>
</tr>
<tr>
<td>8–10</td>
<td>Perpendicular Bisector Definition and Theorem</td>
</tr>
</tbody>
</table>

WRAP-UP

Today you reviewed how to reflect points across a line. You also reviewed direct proportions, which can be written as the equation of a line, \( y = kx \), with a \( y \)-intercept of 0. A perpendicular bisector cuts a line segment into two equal lengths. Any point on the perpendicular bisector of a line segment is equidistant from the two endpoints of the line segment. You can usually find distances in perpendicular bisector problems by using the Pythagorean Theorem or the distance formula.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Translations, Reflections, Rotations - Part 2

Objectives
- Identify a translation.
- Translate a point and a line segment.

Books & Materials
- Math in Focus 3B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B
- grid paper
- geometry software (Optional)
- construction or colored paper (Optional)

Assignments
- Complete Warm-Up.
- Complete assigned pages in Math in Focus 3B.
- Complete Quick Check.

LEARN

WARM-UP

Use grid paper to solve each problem.

1. On a piece of grid paper, mark point \( P \) \((-3, -4)\). Now mark a new point that is 2 units to the right and 3 units up from point \( P \). What are the coordinates of the new point?
2. On a piece of grid paper, mark point \( C \) \((3, -1)\). Mark point \( D \) \((-2, 5)\). Describe how to move from point \( C \) to point \( D \) in terms of horizontal and vertical units. (For example, move 3 units to the right and 7 units down.)

WARM-UP ANSWERS

1. \((-1, -1)\)  
2. Move 5 units to the left and 6 units up.

TEACHING NOTES

INSTRUCTION

Read Understand the Concept of a Translation on pp. 51–52 in Math in Focus 3B. You can think of translations as a way of directing a person how to move from one place to another. When you say: Walk three steps forward and then two steps to the right, you are describing a translation.

In the picture on p. 51, you can see that the dashed arrows form the legs of a right triangle. The hypotenuse is \( PA \). If you started at point \( P \) and walked 2 units to the right and 3 units up, you would end up at point \( A \) after having walked 5 units. In math, specific vocabulary is used to describe these movements. The translation 2 units to the right and 3 units up maps point \( P \) to its image, point \( A \).

Review Example 1 on p. 53. Then complete Guided Practice on p. 53.
Review Example 2 on p. 54. When translating a line segment, translate each endpoint and then draw the new line segment. Be sure to label each endpoint as you translate it. Notice the image is parallel to the original line segment. If your image is not parallel after a translation, check your work.

Complete Guided Practice on p. 54. Translate point $H$ and label it $H'$. Then translate point $L$ and label it $L'$. Remember, $H'L'$ should be parallel to $HL$.

If you have access to geometry software, complete Technology Activity on p. 55. You will most likely find the Translate function within the Transform menu because a translation is one type of a transformation. You have translated a line segment by hand. You can also translate a triangle or a rectangle by hand by translating each vertex of the figure, then connecting the vertices.

Read Draw Images After Translations on p. 55. The shape and size of an object are not affected by translations. Neither is the orientation of the object. Later in this chapter, you will learn about different types of transformations that change these properties.

If your student has difficulty translating objects, she can cut a small triangle out of colored paper. She should trim the triangle until each of its vertices, when laid on a piece of grid paper, is on an intersection of the grid lines. Then she can label the vertices. From here, she can physically slide the triangle in a translation and mark the images of the vertices. She can then draw the image of the triangle. Stress to your student that she must only slide the triangle left and right and up and down. If she changes the orientation of the triangle, she is performing a different type of transformation—not a translation.

In this lesson and several others in this chapter, your student will be asked to use geometry software. You may choose to purchase geometry software or use a free website, such as Desmos, to complete the assignments. Many of the lessons have been designed so that your student can complete the assignments by hand.

PRACTICE

Complete problems 1–6 of Practice 8.1 on p. 59 in Math in Focus 3B.

Textbook Answer Key
WRAP-UP

Today you learned how to translate a point or several points. When you translate an object, sliding it left and right or up and down, you do not change the object’s size, shape, or orientation. You only change the object’s location.

✅ QUICK CHECK

Please go online to view and submit this assessment.

👉 MORE TO EXPLORE

If you struggled with this question, use a coordinate grid to help you visualize the transformations.
LEARN

Translations, Reflections, Rotations - Part 3

Objectives
- Translate a polygon.
- Identify coordinates of points after translation.

Books & Materials
- Math in Focus 3B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B
- grid paper

Assignments
- Complete Warm-Up.
- Read and complete pages in Math in Focus 3B.
- Complete Practice Questions.

LEARN

WARM-UP

Map each point to its image. Write the coordinates of the image.

1. Point C (1, 1) translated 1 unit to the left and 1 unit down.

2. Point G (8, 4) translated 2 units to the right and 4 units down.

TEACHING NOTES

WARM-UP ANSWERS

1. (0, 0) 2. (10, 0)

INSTRUCTION

Review Example 3 on p. 56 in Math in Focus 3B. There are two images being created; one is designated as rectangle $A'B'C'D'$ and the other is $A''B''C''D''$. These are both images of the original rectangle $ABCD$. To find each image, begin with rectangle $ABCD$.

Look at point $A$ and point $A'$. You can see that point $A'$ is 5 units left and 1 unit down from point $A$. Each of the other three image points is 5 units left and 1 unit down from its respective original point.

Notice also that the image rectangles are the same size, shape, and orientation of the original rectangle.

Complete Guided Practice on p. 57. Draw and label figure $EFGH$ on grid paper as your first step. Choose a labeled point and find its image. Proceed one point at a time, labeling the image points, until you have translated all four points. Then draw the image of the figure.
Read Find the Coordinates of Points after Translations on p. 57. Just as a number line was a useful tool as you were learning to add and subtract, the coordinate plane is useful for learning to translate shapes. You learned to move left on a number line when you subtracted a number or added a negative number. The same concept holds for translating, except you are now working with both an x- and a y-axis.

When an object moves to the left, add a negative number to the x-coordinate. If it moves down, add a negative number to the y-coordinate. Moving to the right is the same as adding a positive number to the x-coordinate and moving up is the same as adding a positive number to the y-coordinate.

Review Example 4 on p. 58. Then complete Guided Practice on p. 58.

### TEACHING NOTES

If your student has difficulty calculating with the coordinates instead of simply using the coordinate grid, go back through Examples 1–3 from the previous lesson with her. Look at the graphical representations of the translations and determine what numbers to add to or subtract from the x- and y-coordinates of each point to achieve the same result.

**WATCH FOR THESE COMMON ERRORS**

Your student may confuse the translations in the x- and y-directions. Use the following table to help her keep track of which value in the coordinate pair is changing.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>Right (+)</td>
<td>Up (+)</td>
</tr>
<tr>
<td>Left (−)</td>
<td>Down (−)</td>
</tr>
</tbody>
</table>

### PRACTICE

Complete problems 7–11 of Practice 8.1 on p. 60 in Math in Focus 3B.

### TEACHING NOTES

Textbook Answer Key
Today you learned how to translate an object by adding and subtracting values instead of drawing the object and counting units on a grid.

The point (1, 5) is translated 2 units to the right and 4 units down. What are the coordinates of the image?

\[
x\text{-coordinate: } 1 + 2 = 3 \\
y\text{-coordinate: } 5 - 4 = 1 \\
The image is (3, 1).
\]

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Translions, Reflections, Rotations - Part 4

Objectives
- Identify a reflection.
- Reflect a point and a line segment.

Books & Materials
- *Math in Focus 3B*
- *Math in Focus - Teacher Edition A*
- *Math in Focus - Teacher Edition B*
- Geometry software (Optional)

Assignments
- Complete Warm-Up.
- Read and complete pages in *Math in Focus 3B*.
- Complete Practice Activity.
- Complete Quick Check.

WARM-UP
Write your name in large print on a sheet of paper. Hold it up to a mirror. Read your name from left to right, as you would a book. Answer each question.

1. Which letter do you see first—the first letter of your name or the last?

2. What do you notice about the letters—are they right side up? Are they correct from left to right?

WARM-UP ANSWERS
1. last  
2. They are right side up, but they are backward from left to right.

INSTRUCTION
Read Understand the Concept of a Reflection on pp. 61–62 in *Math in Focus 3B*. If you have ever folded a piece of paper and cut out shapes to make snowflakes, you have used lines of symmetry and made reflections.

In the picture on p. 61, the clouds in the reflection on the water look the same as the clouds in the sky, but they are upside down. Ms. Jenkins folded the paper to make the dot A’. Dots A and A’ are equidistant from the fold line or the *line of symmetry*. The line of symmetry is the line about which the reflection happens; the original shape and the reflection are symmetric about this line. Because the shape here is just a dot, you cannot tell if dot A’ is upside down from dot A.

Review Example 5 on p. 62. Then complete Guided Practice on p. 62.
If you have access to geometry software, read and complete Technology Activity on p. 63. To find the Reflections function, you may need to look under Transformations. Reflections are a type of transformation, just as translations are a type of transformation. Label the vertices of your shapes so you can see exactly what happens. Draw a parallelogram with angles other than 90°. Repeat steps 1 to 3.

Read Draw Images After Reflections on p. 63. Recall that translations preserve shape, size, parallelism, and perpendicularity. Translations also preserve orientation. Reflections preserve shape, size, parallelism, and perpendicularity. You saw in the Warm-up that reflections do not preserve orientation; the letters in your name were backward, as was their order. You will learn why reflections do not preserve orientation in the following lessons.

**PRACTICE**

Draw a shape that represents half an object. It can be complex, like a snowflake. Fold the paper on the line of symmetry and trace the reflection. Open the paper and compare the two sides of the shape—the original and the image. How are they similar and different?

**TEACHING NOTES**

Textbook Answer Key

**WRAP-UP**

Today you learned how to reflect a point about a line. The line is called the line of reflection or the line of symmetry. One way to draw a reflection is to fold the paper on the line of symmetry and trace the original shape; this gives the reflected image.

**QUICK CHECK**

Please go online to view and submit this assessment.
MORE TO EXPLORE

If you had difficulty with this question, try sketching to visualize the situation.
**Translations, Reflections, Rotations - Part 5**

**Books & Materials**
- Math in Focus - Teacher Edition
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

**Assignments**
- Complete Interactive Activity.
- Complete Rate Your Enthusiasm.

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**LEARN**

---

**INTERACTIVE ACTIVITY**

Use this activity to explore reflections. Click [here](#) if you would like to see the activity in a new window.

Go to the Lesson Info and download the Student Exploration Sheet. Use the tool to help you complete the sheet.

---

**TEACHING NOTES**

Go to Lesson Info to access the Exploration Sheet Answer Key. You may also wish to explore the other teaching resources in the activity.

---

**RATE YOUR ENTHUSIASM**

Please go online to view and submit this assessment.
**Translations, Reflections, Rotations - Part 6**

**Objectives**
- Reflect a polygon in the x- and y-axes.
- Identify coordinates of points after reflection.

**Books & Materials**
- *Math in Focus 3B*
- *Math in Focus - Teacher Edition A*
- *Math in Focus - Teacher Edition B*
- *grid paper*

**Assignments**
- Complete Warm-Up.
- Complete the assigned pages in *Math in Focus 3B*.
- Complete Practice Questions.

**LEARN**

**WARM-UP**
Solve each problem.

1. Julio is cutting paper masks by folding colored sheets of paper in half and cutting out one half of each mask. Then he unfolds the paper to reveal the full mask. If his masks need to be 10 inches wide, how wide is each mask from the fold line to the edge?

2. In the following diagram, \( EF \) is the perpendicular bisector of \( VW \). \( FW = 8 \), \( EG = 7 \), and \( GW = 9 \). What is \( GV \)?

![Diagram of VWFW and EF as the perpendicular bisector]

**TEACHING NOTES**

**WARM-UP ANSWERS**
1. 5 in.  2. 9

**INSTRUCTION**

Review Example 6 on pp. 64–65 in *Math in Focus 3B*. \( \overline{AB} \) is on the sheet of newspaper that was folded under the sheet shown. You can fold a sheet of paper in half and cut it along \( \overline{AB} \) to see how \( \overline{AB} \) looks, or you can draw the original line segment and trace the image instead of cutting the paper. Either method will show you the original line segment and its reflected image.
In part b, notice that \( \overline{AB} \) makes the same angle with the fold line as \( \overline{A'B'} \) does. Also, the distance from point \( A \) to the fold is the same as the distance from point \( A' \) to the fold. The distance from point \( B \) to the fold is the same as the distance from point \( B' \) to the fold.

Complete Guided Practice on p. 66. Each point on each line segment is the same distance from \( \overline{MN} \) as its reflection or image point. Label each image point using the prime notation.

Review Example 7 on pp. 66–67. Note that Susan placed the cup on top of the cardboard upside down. It is the reflection of the cup underneath the cardboard.

Complete Guided Practice on p. 67. Find the reflection of the points \( P \), \( Q \), and \( R \). Then find the reflection of the unlabeled points where the lines change direction. Note that points \( P \) and \( R \) are invariant.

Review Example 8 on p. 68. Then complete Guided Practice on p. 68. Identify the invariant points after you construct the drawing.

Read Find the Coordinates of Points After Reflections on p. 68. The key statement here is: The line of reflection is the perpendicular bisector of the segment joining each point to its image. You will need to be able to identify the line of reflection if you are given an object and its image.

Review Example 9 on p. 69. In this case, the line of reflection is the \( y \)-axis. If you imagine a line segment joining a point and its image, you can see that the perpendicular bisector of each segment is the \( y \)-axis. The distance between point \( A \) and the \( y \)-axis is the same as the distance between point \( A' \) and the \( y \)-axis; both distances are 1, but \( A' \) is on the left side of the \( y \)-axis. The coordinates of \( A' \) are \((-1, 1)\). There are no invariant points in this example.

Complete Guided Practice on p. 69. The birdhouse on the right is the original, since it has positive values for the \( x \)-coordinates of its vertices. Begin by identifying the vertices. (Hint: Point \( P \) is the bottom of the birdhouse.)

Guide your student to understand the similarities and differences between finding the coordinates of points after translations and finding the coordinates of points after reflections. For translations, both coordinates can change; whereas with reflections, only one coordinate changes. For translations, all points of the object move the same distance, whereas all points are not required to move the same distance for reflections. It is possible, however, to map both transformations by adding numbers to or subtracting numbers from the values of the coordinates.

Complete Practice 8.2 on pp. 70–72 in Math in Focus 3B.
In this activity, you will examine the reflections of different figures. To begin, be sure **Point** is selected from the **Figure type** menu and that **Show reflection** is checked. Drag point **A** upward. How does point **E** (the reflection of point **A**) move? Drag point **A** downward toward the line. Where does point **E** go? Drag point **A** below the line. What happens to **E**? Share your observations with your Learning Guide.

The line shown on the screen is the line of reflection. Drag the line upward by the left point to rotate it clockwise. What happens to the distance between point **E** and the line as the line gets closer to point **A**? Continue to rotate the line clockwise. What happens to point **E**? Discuss your ideas with your Learning Guide.

Recall that point **E** is the reflection (image) of point **A** (the preimage). Select **Show ruler**. Attach one circle to point **A** and one to point **E**. Then drag one of the points on the line of reflection until it is on line segment **AE**. What is the distance from **A** to **E**? Write this measurement in your Math Notebook. Then use the other ruler to measure the distances from points **A** and **E** to the line of reflection. Record these distances in your Math Notebook. Compare all the distances you found above. What do you notice? Report your observations to your Learning Guide.

Turn off the ruler and select **Show angle measure tool** to open a protractor. Attach the circle in the middle to the point on the line; then attach one of the end circles to point **A** and the other end circle to the other point on the line. Open the second protractor to measure the angle that point **E** makes with the line of reflection. What do you notice about these angle measures? Turn off all the measurement tools and click on **Show connecting line** and **Show midpoint**. Use the tools to help you describe the relationship between the connecting line and the line of reflection to your Learning Guide.

Finally, select **Segment** from the **Figure type** menu. Check that **Show reflection** is turned on. Drag the endpoints of line segment **AB**. Watch line segment **EF** as you do. Which point is the image (reflection) of point **A**? Point **B**? Compare the lengths of both line segments. What appears to be true? Use the Gizmo measuring tools to check; then share your answers with your Learning Guide.
Sample answers for the reflection of a point: Points $A$ and $E$ always remain the same distance from the line of reflection. This remains true even when the line of reflection is moved. The line that connects a point and its reflection is always perpendicular to and bisected by the line of reflection.

Sample answers for the reflection of a line: Point $E$ is the reflection of point $A$, and point $F$ is the reflection of point $B$. The line segments are congruent.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

WRAP-UP

Today you learned how to reflect an object about a line. You can do this by drawing the object and reflecting each point about the line of symmetry. You can also find the coordinates of points after a reflection by recognizing that each point’s image is the same distance from the line of reflection as the point itself. A point is invariant if it is unchanged by a transformation.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Transitions, Reflections, Rotations - Part 7

Objectives
- Identify a rotation.
- Rotate a point and a line segment.

Books & Materials
- Math in Focus 3B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B
- analog clock (Optional)
- geometry software (Optional)
- protractor

Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete Quick Check.

LEARN

WARM-UP

Solve the following problems.

1. Give three examples of things that rotate. If you are having difficulty, think of children’s toys.

2. You are facing due west. You turn to face due south. Complete this sentence: I turned __________ degrees __________ (choose: clockwise or counterclockwise).

3. How many degrees are in one complete circle or rotation?

WARM-UP ANSWERS

1. Answers will vary. Examples: a top, a ball, the planets, the hands of an analog clock, a dancer, a compass, a wheel. 2. 90, counterclockwise 3. 360°

TEACHING NOTES

INSTRUCTION

Read Understand the Concept of a Rotation on p. 73 in Math in Focus 3B. The terms clockwise and counterclockwise come from clocks that have hands (not digital clocks). Watch the movement of the minute or second hand on an analog clock to understand these terms.

All rotations have a center of rotation; this is the point about which the rotation occurs. On a clock, the center of rotation is the point at which the hands are attached to the clock face. For the Ferris wheel shown on p. 73, it is the point O. For a car wheel, it is the axle. When you specify the angle of rotation,
you will use the center of rotation as the middle point. For example, the angle of rotation for the Ferris wheel is $\angle AOA'$.

Rotations produce images that are the same distance from the center of rotation as the object. The radius remains constant. Look at the Ferris wheel in the picture; the radius of the Ferris wheel does not change. Points $A$ and $A'$ are the same distance from point $O$, but they are in different physical locations.

Review Example 10 on p. 74. If the fruit platter rotates counterclockwise, it must go through a much larger rotation to move the strawberry to point $R'$. The counterclockwise angle is $270^\circ$.

Complete Guided Practice on p. 75. For problem 1, make a point at the origin. This is the center of rotation. You can use the distance formula to find the distance between point $P$ and the origin. You can find the distance between point $P'$ and the origin the same way. You will see that the distances are the same. You can also use a ruler or a piece of string to measure the distance.

For problem 2, draw a line segment between $W$ and $W'$. The center of rotation must be halfway between the two points.

Example 11 on p. 76. The center of rotation is the origin.

Complete Guided Practice on p. 76. To find the angle of rotation, think about the face of the clock. There are 12-hour intervals, and a full rotation is $360^\circ$.

If you have access to geometry software, complete Technology Activity on p. 77. Notice that Rotations are included under Transformations in the geometry software. Rotations are the third type of transformation you have studied. The other two are translations and reflections.

Read Draw Images After Rotations on p. 77.

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TEACHING NOTES

Encourage your student to draw one line segment between the center of rotation and the point and a different line segment between the center of rotation and the image. This will make it much easier for her to recognize and measure the angle of rotation.
PRACTICE
Complete problems 1–4 of Practice 8.3 on pp. 83–84 in Math in Focus 3B.

WRAP-UP
Today you learned how to draw images after rotations. Rotations preserve shape, size, parallelism, and perpendicularity. They are described by stating the amount an object is rotated, measured in degrees, and the direction of the rotation, which is either clockwise or counterclockwise. All rotations occur about a center of rotation; often this is the origin or the point at which an object is fixed. For example, a windshield wiper rotates about where it is attached to the car.

SUPPLEMENTAL
- BrainPOP: Transformation

QUICK CHECK
Please go online to view and submit this assessment.

MORE TO EXPLORE
If you struggled with this question, revisit the material in this lesson and view the video Rotations.
Translations, Reflections, Rotations - Part 8

**Objectives**
- Rotate an image about the origin.
- Identify coordinates of points after rotation.

**Books & Materials**
- *Math in Focus 3B*
- *Math in Focus - Teacher Edition A*
- *Math in Focus - Teacher Edition B*
- grid paper
- protractor
- ruler
- colored paper (Optional)

**Assignments**
- Complete Warm-Up.
- Read and complete pages in *Math in Focus 3B*.
- Complete Practice Questions.

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**LEARN**

**WARM-UP**

Define each term.

1. center of rotation
2. angle of rotation

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. the point about which an object rotates  
2. the angle through which an object rotates

**INSTRUCTION**

Review Example 12 on pp. 78–79 in *Math in Focus 3B*. It might be easiest to examine the rotation of point C first. If point C rotates 90° clockwise, it will end up on the negative y-axis. The same is true of point B.

Look at the right triangle OCA and consider point A. To move from point O to point A, you move 2 units along the x-axis to point C, and then move 1 unit down. Once you have points C’ and B’ mapped, you can see that, to find A’, you move 2 units down from point O, then 1 unit to the left.

When rotating an object, begin with whichever point you prefer.

For part b, think about the 90° rotation in part a. From there, rotate another 90° clockwise.

Complete Guided Practice on p. 79. Rotate each point by itself. To rotate point A, construct \( \overline{OA} \).

Rotate the line segment 90° clockwise about point O. To rotate point B, construct \( \overline{OB} \) and do the same thing.
Read **Find the Coordinates of Points After Rotations** on p. 80. To visualize Emma’s rotation, draw a line segment from the center of the Ferris wheel to Emma’s starting point (2, 0). Rotate the line segment about the center of the Ferris wheel for each successive position.

Review **Example 13** on pp. 80–81. In part a, all four points are rotated 180° about the origin. Check the table to convince yourself that all four points follow the description in the comment bubble.

Complete **Guided Practice** on p. 82. Notice that problem 5 is a 180° rotation about the origin.

**TEACHING NOTES**

Rotations can be difficult to visualize, and your student may inadvertently flip an object. For instance, in **Example 13**, part a, she may try to put the image on the same side of line $\overline{AA'}$ as the original flag.

To help your student visualize rotations, encourage her to cut a triangle out of colored paper for **Example 12**. She can hold the triangle next to any straight object such as a pencil. Then she should rotate the flag made by the pencil and colored triangle together through the angles in the example. For **Example 13**, she will cut a square out of colored paper and hold it next to a straight object representing $\overline{OB}$. Then she should move her square flag through the various angles in the example to see the orientation of the image.

Visualization skills are important for transformations. Using physical objects and actually moving them to see what happens and to understand their final orientation is very helpful.

**PRACTICE**

Complete problems 5–8 of **Practice 8.3** on p. 85 in **Math in Focus 3B**.

**TEACHING NOTES**

**Textbook Answer Key**

**WRAP-UP**

Today you learned how to draw images of objects after rotations. When rotating objects that are more complex than individual points, it is helpful to draw a line segment from the origin to each vertex of the polygon (or labeled point on the object). Rotate each line segment about the center of rotation and label the image point. When point $P(x, y)$ is rotated 180° about the origin, the image is point $P'(−x, −y)$. 
PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Translations, Reflections, Rotations - Part 9

Objectives
- Identify images that are formed by transformations as congruent.

Books & Materials
- Math in Focus 3B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B
- grid paper
- ruler
- Transformations Worksheet

Assignments
- Complete Warm-Up.
- Complete Transformations Worksheet.
- Complete Use For Mastery.

LEARN

WARM-UP

Look at Figure A.

Tell how each of the following images relates to Figure A. (It could be a translation, reflection, or rotation, or it could be none of these.) Explain your reasoning.

1.  
2.  
3.  
4.  

TEACHING NOTES

WARM-UP ANSWERS

1. rotation  2. none (It is not the same shape.)  3. reflection or 180° rotation  4. none (It is not the same size.)

INSTRUCTION

In this lesson, you have explored translations, reflections, and rotations and the characteristics that they have in common. Now you will see how they transform geometric figures into images that are congruent.
You have already learned that translations, reflections, and rotations all preserve shape, size, parallelism, and perpendicularity. Look at this set of figures.

Figure $ABCD$ has gone through a series of transformations to create the image $A'B'C'D'$. First it was reflected across the x-axis, resulting in $A'B'C'D'$; then it was translated 7 units to the left and 1 unit up to map to its new position ($A''B''C''D''$). Rotating the image 90° clockwise brought it to its final position.

Even though three different transformations were used, some aspects remain the same. Consider, for example, the length of line segment $AB$. In the original figure, it is 3 units long. In the first image, line segment $A'B'$ is also 3 units long, as are the lengths of line segments $A''B''$ and $A'''B'''$. No matter how you transform the image, each line segment maintains its original length. Now look at angles $A$, $A'$, $A''$, and $A'''$. All these angles are right angles, which means they measure 90°. The angles and their measures are also preserved in every transformation. You will also notice that line segments $AB$ and $CD$ are parallel. This relationship is also maintained as the figure is reflected, translated, and rotated. Since the figure and its images have line segments and angles with the same measures, as well as parallel sides, all four images are also congruent. Figures that are obtained by translating, reflecting, or rotating another are always congruent to the original image.

As your student studies the four images, make sure he understands and can explain how each transformation occurred. Help your student to find the points and line segments that were mapped onto each other. Have him explain to you why all four figures are congruent.

**PRACTICE**

Complete the [Transformations Worksheet](#).
Today you reviewed how translations, reflections, and rotations maintain size, shape, parallelism, and perpendicularity. You used this concept to see how line segments and angles map onto figures with the same measures. This means that figures that are related to each other by one or more transformations are congruent.
1. Shanti is making a patchwork quilt. Each piece of fabric has a black star on it. She uses the grid shown to plan the locations of the different fabric squares.

If you are able, use the text box to show your work and enter your final answer to each of the following questions. If not, complete your work on paper and upload it below.

a. What transformation maps square 1 onto square 2?

b. What transformation maps square 2 onto square 3?

c. What transformation maps square 1 onto square 5?
Shanti is making a patchwork quilt. Each piece of fabric has a black star on it. She uses the grid shown to plan the locations of the different fabric squares.

If you are able, use the text box to show your work and enter your final answer to each of the following questions. If not, complete your work on paper and upload it below.

a. What transformation maps square 1 onto square 2?

b. What transformation maps square 2 onto square 3?

c. What transformation maps square 1 onto square 5?

**USE FOR MASTERY GUIDELINES & RUBRIC**

Did you:

- Use the information given to answer parts A, B, and C?
- Use the text box to show your work and enter your final answer OR complete your work on paper and upload it?
SHOW

Gather at least five pictures with buildings that seem to have congruent figures either as part of their design or the entire building and another five pictures of buildings that have similar figures. Look at the figures in their pictures. Have students place a coordinate grid transparency over the picture. You can also trace figures that seem to be identical (congruent) onto a coordinate plane. Now try to identify a series of transformations that would map one figure onto another. Consider if there is more than one possible series of transformations. If there is more than one possible series, compare the series. How are they the same? How are they different? Repeat the process for another set of figures that seem congruent on the same picture or one of the other pictures.

Note these transformations and the pictures involved in your Math Notebook.

RATE YOUR PROGRESS

Please go online to view and submit this assessment.
LEARN

WARM-UP
Answer the following questions.

1. It is 2:00. What time will it be when the hour hand rotates 180° clockwise?
2. It is 3:15. What time will it be when the minute hand rotates 90° clockwise?

WARM-UP ANSWERS

1. 8:00  2. 3:30

INSTRUCTION

Read Understand the Concept of a Dilation on p. 86 in Math in Focus 3B. When you shrink or expand a photo on a computer screen or a smart phone, you are working with a dilation. A dilation does not change the shape of the object; it merely changes its size. The scale factor \( k \) is the factor by which the object shrinks or expands.

Review Example 14 on p. 87. Two sides of \( \Delta FGH \) (\( FG \) and \( FH \)) are the same length. This is not true of the other two triangles. \( \Delta FGH \) could not be a dilation of the other two triangles, even if it shared a point with them.
Complete **Guided Practice** on p. 87. Look for pairs of triangles such that the ratio of the pairs of the sides is the same for all three sides. For one triangle to be a dilation of another, they must share a center of dilation.

**HELPFUL ONLINE RESOURCE**

**Instructional Video:** *Dilations*

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**TEACHING NOTES**

Your student can expand or contract photos or maps on a smart phone or computer screen to understand dilations.

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**PRACTICE**

Complete problems 1–2 of **Practice 8.4** on p. 96 in *Math in Focus 3B*.

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**TEACHING NOTES**

[Textbook Answer Key](#)

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**INTERACTIVE ACTIVITY**

Follow these instructions for the activity shown below.

Click [here](#) if would like to see the activity in a new window.

To begin, be sure **Point** is selected from the **Figure type** dropdown menu. Drag point A to (8, 6). Look at the matrix (rectangular array of numbers) under **Preimage matrix**. Notice how the coordinates of point A form the numbers in the matrix, with the x value on the top and the y value on the bottom. Where will point A be if the preimage matrix is \[
\begin{pmatrix}
-2 \\
7
\end{pmatrix}
\]? Check your answer in the Gizmo by entering the numbers under **Preimage matrix** to the left of the graph. (Click on a number in the text field, type in a new value, and press *Enter*.)

Now select **Segment** from the **Figure type** menu. With A at (–7, 5) and B at (10, 9), look at the **Preimage matrix** and notice how the coordinates of both points are shown. Drag the **Scale factor** slider. Line segment JK is the dilated image of line segment AB (the preimage). The ratio of JK to AB is the scale factor. A. What happens to line segment JK when the scale factor is greater than 1? What
happens when the scale factor is less than 1? Experiment with a variety of segments to see if this is always true. Then share your observations with your Learning Guide.

Drag point $A$ to $(-4, 5)$ and point $B$ to $(9, -8)$. Set Scale factor to 2.0. Mouse over points $J$ and $K$ to see their coordinates. Notice how these coordinates are also shown in the second matrix. How can you use the scale factor of 2 and the endpoints of line segment $AB$ to find the endpoints of $JK$? Experiment with a variety of scale factors and watch how the endpoints of line segment $JK$ change. Determine a rule for using the coordinates of the preimage points to find the coordinates of the image points and share it with your Learning Guide.

Finally, set Scale factor back to 2.0. A. How do you think $JK$ compares to $AB$? Use the Gizmo rulers to check your answer. (Select Show ruler. Attach one circle to point $A$ and one to point $B$; then click the other ruler and set the circles on points $J$ and $K$.) Repeat with several other segments and different scale factors. In general, how can you use the scale factor to find the length of the image? Share your answer with your Learning Guide.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet to try some other activities with the Gizmo.

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### TEACHING NOTES

Go to Lesson Info to access the Exploration Sheet Answer Key. You may also wish to explore the other teaching resources in the activity.

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### WRAP-UP

Today you learned about another type of transformation, dilations. Dilations change the size of a figure, making it larger or smaller by a scale factor $k$.

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### PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Dilations and Comparing Transformation - Part 2

Objectives
- Find the dimensions of a figure after dilation.

Books & Materials
- Math in Focus 3B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete the Practice Questions.

LEARN

WARM-UP
Complete the following problems.

1. A ruler is 12 inches long. A yardstick is 36 inches long. How many times longer than the ruler is the yardstick?

2. Samantha is 3 feet and 2 inches tall. Her father is 6 feet and 4 inches tall. How many times taller is her father than she?

3. A photo is 3 inches by 5 inches. Julio scans, expands, and prints the photo. The photo he prints is 18 inches by 30 inches. How many times wider and longer is the photo Julio prints than the original?

TEACHING NOTES

WARM-UP ANSWERS
1. 3 times longer  2. 2 times taller  3. 6 times wider and longer

INSTRUCTION
Read Find the Dimensions of Figures After Dilations on p. 88 in Math in Focus 3B. The image formed by a dilation can be either larger or smaller than the object. The factor by which the object shrinks or expands is a ratio called the scale factor $k$. 
Scale factors involve three quantities:

- the number $k$ itself
- a measured quantity, such as the length, for the object
- the similar measured quantity for the image

No matter which two quantities you are given, you should be able to find the third quantity.

Review Example 15 on p. 89. You can recognize the pancake expands by looking at the scale factor. Since $k > 1$, you know the diameter of the pancake should be greater than 4 inches.

Complete Guided Practice on p. 89. Draw a quick sketch of the rectangle $ABCD$ and label the points. Then you can find the length and width of the rectangle.

In part b, $k > 1$, so you know the image will be larger than the original rectangle.

For part d, multiply each coordinate of each point by the scale factor. To check your work, use the image coordinates to find the lengths and widths for the expanded and contracted rectangles and compare those values to your answers in parts b and c.
WRAP-UP

Dilations change the sizes of objects, making them bigger or smaller by a scale factor $k$. Today you learned how to find the dimensions of figures after dilations. If $0 < k < 1$, the image is smaller than the object. If $k > 1$, the image is larger than the object.

✅ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Dilations and Comparing Transformation - Part 3

**Objectives**
- Draw a dilation, given coordinates and a scale factor.

**Books & Materials**
- *Math in Focus 3B*
- *Math in Focus - Teacher Edition A*
- *Math in Focus - Teacher Edition B*
- geometry software (Optional)

**Assignments**
- Complete Warm-Up.
- Complete the assigned pages in *Math in Focus 3B*.
- Complete Quick Check.

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**LEARN**

**WARM-UP**

Answer the following questions.

1. You have learned about four types of transformations: translations, rotations, reflections, and dilations. Which of these preserve shape and size?
2. If \(0 < k < 1\), what can you say about the size of the image as compared to the size of the object?
3. Point \(Q(x, y)\) maps to point \(Q'(x', y')\) after a 180° rotation. What are the coordinates of point \(Q'\)?

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. translations, rotations, and reflections  
2. The image is smaller than the object.  
3. \((-x, -y)\)

**INSTRUCTION**

If you have access to geometry software, complete *Technology Activity* on p. 90 in *Math in Focus 3B*. The geometry software automatically sets the center of dilation to the origin. Notice what happens to the rectangle when the scale factor is less than 1 \((0 < k < 1)\).

Read *Draw Images After Dilations* on p. 90. If \(k = 1\), you multiply the coordinates of the points of the object by 1 to find the coordinates of the points of the image. Anything multiplied by 1 is itself, so when \(k = 1\), the image is the same size as the object.

Review *Example 16* on pp. 91–92. In part b, \(|k| = 1\), so the size of the image is the same as the size of the object. The negative sign on \(k\), however, changes the sign of each coordinate for each point.
other words, when point \( A \) \((x, y)\) is dilated with a scale factor \( k = -1 \), it maps to point \( A' (-x, -y) \). Refer to Warm-up problem 3. If you do not remember what a 180° rotation does to the coordinates, look back to Section 8.3.

Complete **Guided Practice** on p. 92. *Note:* To find the coordinates of the image of point \( S \), multiply its coordinates by the scale factor. Follow the same process to find the coordinates of the image of point \( T \).

The negative \( x \)-axis represents the water in the pool; the surface of the pool is along the axis, so the blue shading represents the water in the pool. The positive \( x \)-axis represents the ground; the springboard will sit above the ground. *(Note: The textbook calls the positive \( x \)-axis the *floor*. This means the surface around the pool.)*

### TEACHING NOTES

Your student may benefit from further exploration of dilations using geometry software. Other scale factors to investigate include \( k = 1 \), \( k = -1 \), and \( k = -\frac{1}{2} \).

**WATCH FOR THESE COMMON ERRORS**

Your student may think that the image is smaller than the object if \( k < 0 \). Reiterate that \( k \) is a ratio comparing the image to the object. If the image is smaller than the object, then \(|k| < 1\).

### PRACTICE

Complete problems 4–6 of **Practice 8.4** on pp. 96–97 in *Math in Focus 3B*.

### TEACHING NOTES

*Textbook Answer Key*

Remind your student that the coordinates of the image can be found by multiplying the coordinates for the object by the scale factor *as long as the center of dilation is the origin*. Tell her to predict aspects of the solution prior to completing the problem. She should determine if the image’s orientation will be different from the object’s orientation \((k < 0)\) and whether the image will be larger than the object \((|k| > 1)\) or smaller than the object \((|k| < 1)\).

### WRAP-UP

Today you learned how to draw images after dilations. If \( k < 0 \), the object both dilates and rotates 180°. If the center of dilation is the origin, multiply the coordinates of the original figure by the scale factor to obtain the coordinates of the image.
Please go online to view and submit this assessment.

If you had difficulty with this question, try drawing the images so you can visualize the dilation. You might also wish to revisit the material in this lesson.
Dilations and Comparing Transformation - Part 4

**Objectives**
- Find the center of a dilation.

**Books & Materials**
- *Math in Focus 3B*
- *Math in Focus - Teacher Edition A*
- *Math in Focus - Teacher Edition B*

**Assignments**
- Complete Warm-Up.
- Complete the assigned pages in *Math in Focus 3B*.
- Complete Practice Questions.

**LEARN**

**WARM-UP**
Complete the following.

1. If the coordinates of the image can be found by multiplying the coordinates of the object by the scale factor, the center of dilation is ________.

2. The scale factor of a dilation is defined as the length of the ________ divided by the length of the ________.

3. Describe the relationship among the object, the origin, and the image for a dilation about the origin when $k < 0$.

**TEACHING NOTES**

**WARM-UP ANSWERS**
1. the origin 2. image, object 3. The object and image are on opposite sides of the origin.

**INSTRUCTION**

Read **Find the Center of a Dilation** on p. 93 in *Math in Focus 3B*. Until now, you have only considered dilations that have their centers at the origin. Dilations can occur with different centers.

Review **Example 17** on p. 94. A line through points C and C’ also passes through point P. To describe a dilation, include both the scale factor and the center of dilation.

In part a, you cannot find the coordinates of $A'$, $B'$, and $C'$ by multiplying the coordinates of $A$, $B$, and $C$ by the scale factor, 2. This is because the center of dilation is *not* the origin.
In part b, the center of dilation is the point (1, 3), so again, you cannot multiply the coordinates of the object by the scale factor to find the coordinates of the image. Notice that you must use logical reasoning to recognize the scale factor is negative. If you simply divide one length by another, you will always get a positive scale factor because lengths are always positive.

Complete Guided Practice on p. 95. Choose any two pairs of points to find the center of dilation.

**TEACHING NOTES**

Recommend to your student that she find the scale factor using all six ratios for Example 17, part a. All six ratios should yield the same scale factor. This will help your student recognize the relationship between objects, images, and centers of dilations.

**PRACTICE**

Complete problems 7–8 of Practice 8.4 on p. 97 in Math in Focus 3B.

**TEACHING NOTES**

Textbook Answer Key

**WRAP-UP**

Today you learned how to find the center of dilations. If the center of dilation is the origin, multiply the coordinates of the original figure by the scale factor to obtain the coordinates of the image. Sometimes, however, the center of dilation is not the origin.

To find the center of dilation that is not the origin, draw a line through a point on the object and the corresponding point on the image. Draw a second line through a different point on the object and the corresponding point on the image. The intersection of the two lines is the center of dilation. If the object and image are on opposite sides of the center of dilation, \( k < 0 \).

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
LEARN

WARM-UP

Fill in the blanks.

1. Translations, reflections, rotations, and dilations are usually found under the _________ menu in geometry software.

2. One type of transformation, a(n) _________, never changes the orientation of the object.

TEACHING NOTES

WARM-UP ANSWERS

1. transformations  2. translation

INSTRUCTION

Read and complete Technology Activity on pp. 98–99 in *Math in Focus 3B*.

In step 2, there are two locations in the *Translation* column for “before” measurements. They are the first and third rows. At this point, you can fill in rows 1 and 3 completely.

In step 3, choose the amount you will use to translate the object. You will need to translate it in both the *x*-direction and the *y*-direction. Write the amount you will use to translate the object in the top row under *Translation*. Example: +4 in the *x*-direction; −2 in the *y*-direction.
In step 4, choose which axis you will reflect the object over. Make a note of that in the top row of the Reflection column.

Continue choosing transformations and noting them for steps 5 and 6.

This activity lends itself to good note-taking and critical thinking. Emphasize to your student the importance of making notes about what she is doing for each transformation. Otherwise, when she studies the table she may not remember what she did to arrive at each image. Also emphasize the importance of choosing transformations to illustrate certain attributes, not just choosing the easiest way to complete the activity.

If you do not have access to online geometry software, your student can complete this lesson by hand. Have her create an angle using a protractor and a ruler on graph paper. Then have your student follow the remainder of the directions using the same angle. She may want to draw the angle on a separate piece of paper and cut it out and use the edge of the cutout to trace along to recreate the angle for each transformation.

Complete the Math Journal problem on p. 99 in Math in Focus 3B.

Textbook Answer Key

Today you reviewed and compared the four types of transformations: translations, reflections, rotations, and dilations. Only one type, dilations, can make an image that is a different size from the object.

Please go online to view and submit this assessment.
Dilations and Comparing Transformation - Part 6

**Objectives**
- Compare translations, reflections, rotations, and dilations.

**Books & Materials**
- *Math in Focus 3B*
- *Math in Focus - Teacher Edition A*
- *Math in Focus - Teacher Edition B*

**Assignments**
- Complete Warm-Up.
- Complete the assigned pages in *Math in Focus 3B*.
- Complete Quick Check.

---

**LEARN**

**WARM-UP**
Fill in the blanks.

1. __________ are transformations that can be seen by using a mirror.

2. A(n) __________ moves an object vertically, horizontally, or both.

3. A transformation that can change the size of an object is a(n) __________.

4. A dilation with scale factor $k = -1$ __________ (does/does not) change the size of the object.

---

**TEACHING NOTES**

**WARM-UP ANSWERS**
1. reflections 2. translation 3. dilation 4. does not

---

**INSTRUCTION**

Read Compare Translations, Reflections, Rotations, and Dilations on p. 99 in *Math in Focus 3B*.

*Isometry* is a word that has Greek origins. The prefix *iso*- means “equal,” and the suffix *-metry* means “measuring.” An *isometry* is something with equal measure. Dilations *can* make an image that is the same size as the original, in the cases of $k = -1$ and $k = 1$, but they usually make an image that is a different size from the original. Dilations are not isometries.
For the Think Math question, first consider what a $90^\circ$ rotation does to the length of the line segment. Then consider what the dilation does to the length of the segment.

Review Example 18 on pp. 100–102. In part a, since the translation is a dilation, you must give both the scale factor and the center of dilation.

In part b, compare a point on the object to the corresponding point on its image. In the textbook, the point where the hypotenuse intersects the long leg is used. (The textbook calls this point $A_1$ on the object and point $C_1$ on the image.) The point forming the right angle could also have been used, as could the third vertex of the triangle. Be sure you compare only corresponding points. To describe a rotation, you must give the center of rotation, the angle, and the direction the object is rotated.

Complete Guided Practice on p. 102. In part a, the size and orientation of the triangle are preserved. The triangle has merely been moved. For part b, remember what happens when you look in a mirror.

---

**TEACHING NOTES**

Encourage your student to list the four types of transformations, describe them, and indicate what information is necessary to describe each type of transformation.

---

**PRACTICE**

Complete Practice 8.5 on pp. 103–105 in Math in Focus 3B.

**TEACHING NOTES**

Textbook Answer Key

---

**WRAP-UP**

Today you reviewed and compared the four types of transformations: translations, reflections, rotations, and dilations. All four transformations preserve shape, angle, measures, parallel lines, and perpendicular lines. Only one type, dilations, can change the size of the object.

---

**QUICK CHECK**

Please go online to view and submit this assessment.
MORE TO EXPLORE

View the video, *Reflection, Rotation, Translation* (09:11) to explore how images can be moved on a coordinate plane.

Please go online to view this video ►
Dilations and Comparing Transformation - Part 7

**Objectives**
- Use knowledge of transformations to solve problems.

**Books & Materials**
- *Math in Focus 3B*
- *Math in Focus - Teacher Edition A*
- *Math in Focus - Teacher Edition B*
- grid paper

**Assignments**
- Complete Warm-Up.
- Complete the assigned pages in *Math in Focus 3B*.
- Complete Use For Mastery.

---

**LEARN**

---

**WARM-UP**

Fill in the blanks.

1. If an object undergoes a dilation and the image is smaller than the object, then the absolute value of the scale factor must be between _____ and _____.
2. Two line segments joining the center of rotation and corresponding points on the object and image must be of _____ length.

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. 0, 1  
2. equal

---

**INSTRUCTION**

Read and complete *Brain @ Work* on p. 106 in *Math in Focus 3B*. Show your work.

For problem 2, draw the two lines on a coordinate grid. You can draw the lines by constructing tables of values or by using the slope and y-intercept of both lines. Use the graph to help you answer parts a and b.

For problem 3, first indicate which point maps to which. For instance $A \rightarrow X$.

For problem 4, the scale factor is the same for both days. In other words, if the scale factor is $\frac{4}{5}$ the first day, it is $\frac{4}{5}$ again on the second day.
**PRACTICE**

Write a constructed response to explain your answer to problem 4 of *Brain @ Work*.

Then solve the following problems.

1. In a coordinate grid, point $A$ is located at $(8, -6)$. It is reflected across the $y$-axis to point $B$. Point $B$ is reflected across the $x$-axis to point $C$. Find the length of $AC$.
2. If $OM$ is rotated counterclockwise, at what point will it intersect the $y$-axis?
WRAP-UP

Today you solved problems using transformations. In some cases, you performed two step transformations to find the final image. When working with transformations, the following process can be useful:

1. Identify the type of transformation (translation, reflection, rotation, dilation).
2. Clearly label the points on the object and on the image.
3. Ensure that you have provided all of the information for that type of transformation.

USE FOR MASTERY

1. A toy car represented as $ABCD$ is drawn on the coordinate plane.
a. Johnny translated $ABCD$ 3 units to the right and 4 units up to a new position, $EFGH$. Draw and label $EFGH$.

b. Tom rotated $ABCD$ to a new position, $IJKL$, $90^\circ$ clockwise about the origin, $O$. Draw and label $IJKL$. 
c. Tony placed a smaller car, represented as \( MNOP \), on the coordinate plane. \( MNOP \) is a dilation of \( ABCD \) with its center at the origin and a scale factor of -0.5. Draw and label \( MNOP \).

**USE FOR MASTERY GUIDELINES & RUBRIC**

Did you:

- Translate \( ABCD \) 3 units to the right and 4 units up to a new position, \( EFGH \)?
- Draw and label \( EFGH \)?
- Rotate \( ABCD \) to a new position, \( IJKL \), 90° clockwise about the origin, \( O \)?
- Draw and label \( IJKL \)?
- Represent \( MNOP \) as a dilation of \( ABCD \) with its center at the origin and a scale factor of -0.5?
- Draw and label \( MNOP \)?
Dilations and Comparing Transformation - Part 8

Books & Materials
- Math in Focus - Teacher Edition
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

SHOW

Now look at the figures in their pictures again. This time you are looking for figures that are the same shape but a different size, which means they are similar. Place a coordinate grid transparency over the picture or trace figures that seem to be similar onto a coordinate plane, or draw a coordinate plane on a copy of the picture.

RATE YOUR PROGRESS

Please go online to view and submit this assessment.
Angle Relationships - Part 1

LEARN

WARM-UP
Solve mentally for $x$.

1. $x \times 6 = 10$
2. $x \times 2 = 60$
3. $x \times 4 = 40$

WARM-UP ANSWERS
1. $x = 60$ 2. $x = 120$ 3. $x = 160$

INSTRUCTION
Today’s lesson covers important ideas you need to understand before beginning the lessons in this chapter. In this chapter, you will study similarity and congruence. Congruent figures have the same shape and size. Similar figures have the same shape, but they may be different in size. Congruent figures are a subset of similar figures.

In this chapter, you will also learn how to apply proportional relationships to find measurements indirectly. Read pp. 112–113 in Math in Focus 3B. You will use proportional reasoning to solve real-world problems. Recall that two quantities are directly proportional when they increase or decrease at the same rate. This rate is called a constant of proportionality.

In this chapter, you will also use what you have learned previously about scale drawings. Read p. 114. The scale factor is a ratio.

$$scale \ factor = \frac{\text{length of line segment in scale drawing}}{\text{actual length of object}}$$
Think about the area of a rectangle, which is found by multiplying its length by its width. If both the length and width are increased by a scale factor of 2, the new area is $A = 2l \cdot 2w$ or $A = 4(l \cdot w)$, which is 4 times the original area.

**TEACHING NOTES**

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the Quick Check sections.

If your student needs additional review, have him practice calculating the scale factor for various objects by giving him the actual length of an object and the hypothetical length of the line segment in a scale drawing.

**SKILLS CHECK**

Complete the Quick Check sections on pp. 113–114 in *Math in Focus 3B*.

**TEACHING NOTES**

Textbook Answer Key

Review your student’s answers to the Quick Check sections, noting the problems that your student answered incorrectly. Click on the link to access the appropriate Reteach activity that your student should complete for the remainder of this lesson.

**RETEACH**

After your student completes the Quick Check in the Recall Prior Knowledge lesson of this chapter, review the questions that were answered incorrectly.

If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.

Note that this chapter opener spans two lessons.

<table>
<thead>
<tr>
<th>Quick Check Question(s)</th>
<th>Activity</th>
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</thead>
<tbody>
<tr>
<td>1–6</td>
<td>Scale Drawings</td>
</tr>
</tbody>
</table>
In this lesson, you reviewed how to find the scale factor, given the actual length of an object and the length of the line segment that represents that length in a scale drawing.

The scale factor is a ratio:

\[
\text{scale factor} = \frac{\text{length of line segment in scale drawing}}{\text{actual length of object}}
\]

If you are comparing an area in real life to the area in a scale drawing, the ratio of the two areas is equal to the square of the scale factor. For example, if two figures have lengths related by a scale factor of 2, their areas are related by a scale factor of \(2^2\), or 4.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Angle Relationships - Part 2

**Objectives**
- Find the measures of the interior and exterior angles of a triangle.
- Find the measures of angles formed by parallel lines and a transversal.

**Books & Materials**
- *Math in Focus 3B*
- *Math in Focus - Teacher Edition A*
- *Math in Focus - Teacher Edition B*

**Assignments**
- Complete Warm-Up.
- Complete the assigned pages in *Math in Focus 3B*.
- Complete the Quick Check.

**LEARN**

**WARM-UP**
Solve mentally for \( x \).

1. \( x + 60 + 40 = 180 \)
2. \( 50 + 40 + x = 180 \)
3. \( 90 + x + 45 = 180 \)

**TEACHING NOTES**

**WARM-UP ANSWERS**
1. \( x = 80 \)  2. \( x = 90 \)  3. \( x = 45 \)

**INSTRUCTION**

In this chapter, you will use what you have learned about the measures of the interior and exterior angles in a triangle.

Read the first instructional section on **p. 115** in *Math in Focus 3B*. The sum of the measures of the interior angles in any triangle is 180°. The measure of the exterior angle is equal to the sum of the measures of the two interior angles in the triangle that are not adjacent to the exterior angle.
Read the second instructional section on p. 115. There are three different types of angles formed when two parallel lines are cut by a transversal. The angles formed are alternate interior angles, alternate exterior angles, and corresponding angles. Review the location of each angle type as you read this section.

**TEACHING NOTES**

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the Quick Check sections.

**SKILLS CHECK**

Complete the Quick Check sections on p. 115 in *Math in Focus 3B*.

**TEACHING NOTES**

After your student completes the Quick Check in the Recall Prior Knowledge lesson of this chapter, review the questions that were answered incorrectly. If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.

Note this chapter opener spans two lessons.

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<tbody>
<tr>
<td>7–11</td>
<td>Angles of a Triangle: Sum of Their Measures</td>
</tr>
<tr>
<td>12–13</td>
<td>Transversal Lines: Alternate and Corresponding Angles</td>
</tr>
</tbody>
</table>

**WRAP-UP**

In this lesson, you reviewed how to find the measures of the interior and exterior angles in a triangle. The measures of the interior angles in a triangle always add up to 180°. The measure of an exterior angle is equal to the sum of the measures of the two nonadjacent interior angles in the triangle.

You also reviewed the types of angles formed and the locations of those angles when two parallel lines are cut by a transversal: alternate interior angles, alternate exterior angles, and corresponding angles.
Solve mentally for \( x \).

1. \( x + 60 + 40 = 180 \)
2. \( 50 + 40 + x = 180 \)
3. \( 90 + x + 45 = 180 \)

**WARM-UP ANSWERS**

1. \( x = 80 \)
2. \( x = 90 \)
3. \( x = 45 \)

In this chapter, you will use what you have learned about the measures of the interior and exterior angles in a triangle.

Read the first instructional section on p. 115 in Math in Focus 3B. The sum of the measures of the interior angles in any triangle is 180°. The measure of the exterior angle is equal to the sum of the measures of the two interior angles in the triangle that are not adjacent to the exterior angle.

Read the second instructional section on p. 115. There are three different types of angles formed when two parallel lines are cut by a transversal. The angles formed are alternate interior angles, alternate exterior angles, and corresponding angles.

**Objectives**

- Find the measures of the interior and exterior angles of a triangle.
- Find the measures of angles formed by parallel lines and a transversal.

**Books & Materials**

- Math in Focus 3B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

**Assignments**

- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete the Quick Check.

**QUICK CHECK**

Please go online to view and submit this assessment.

**MORE TO EXPLORE**

If you answered incorrectly, remember that the measures of supplementary angles add up to 180°. Draw 2 parallel lines and a transversal. Then write in one angle measure. Next, find the other angle measures. You might also want to revisit the material in this lesson.
Angle Relationships - Part 3

Objectives
- Find the measures of the interior and exterior angles of a triangle.
- Find the measures of angles formed by parallel lines and a transversal.

Books & Materials
- Math in Focus - Teacher Edition B
- Exploring Angle Relationships Worksheet

Assignments
- Complete Warm-Up.
- Read and Complete Exploring Angle Relationships Worksheet.

LEARN

WARM-UP

Complete the following statements.

1. The sum of the angle measures in a triangle is _________ degrees.
2. The measures of the alternate interior angles formed when two parallel lines are cut by a transversal are _________.
3. An _________ angle of a triangle has a measure equal to the sum of the measures of the two interior angles not adjacent to that angle.

TEACHING NOTES

WARM-UP ANSWERS
1. 180 2. equal 3. exterior

INSTRUCTION

Review Example 1 and Example 2 on the Exploring Angle Relationships Worksheet. If you know the measures of two interior angles in a triangle, you can calculate the measure of the third interior angle. You can also find the measure of each exterior angle in the triangle.

Complete the first Your Turn section.

Review Example 3. When two parallel lines are cut by a transversal, eight angles are formed. Study the example to learn about the relationships between the angles. Remember, every straight line forms an angle that measures 180°. Also, recall that there are 360° in a circle. You can use this information to find the measures of the angles in the diagram.

Complete the second Your Turn section.
The problems in this lesson involve one of two geometric concepts: triangles and parallel lines cut by a transversal. If your student is having difficulty processing and applying the information, have him first determine which type of problem he is solving. If the problem is a triangle problem, your student needs to determine if he is working with interior angles or with both interior and exterior angles so he knows which rule to apply.

If the problem involves parallel lines cut by a transversal, he needs to determine if he is working with alternate interior angles, corresponding angles, or alternate exterior angles. He may even be working with vertical angles. Encourage him to identify the type of problem to be solved, the information given, and the information he needs to find so he can choose the appropriate rule(s).

**Exploring Angle Relationships Worksheet Answers:**

**Example 2**

Your Turn

1 60; 70; 130; 50 2 60; 70; 130 3 70; 50; 120 4 60; 50; 110

**Example 3**

Your Turn

1 70 2 70 3 supplementary; 110 4 4; 110 5 110; 70; 110

**PRACTICE**

Complete the Practice problems in the Exploring Angle Relationships Worksheet.

**TEACHING NOTES**

Practice Answers:

1a 65°  b 115°  c 115°  d 65°  e 65°  f 115°  g 65°  2a 70°  b 135°  c 45°  3 Answers will vary.
In this lesson, you practiced finding the measures of interior and exterior angles in a triangle. You also found the measures of the angles formed when two parallel lines are cut by a transversal.

USE FOR MASTERY

1. Solve for each variable.
a = °

b = °

c = °

d. Tell how you found the value of each variable.

USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information given to solve for each variable?
- Enter your answers for parts A, B, and C?
- Tell how you found the value of each variable?
- Show your work?
Congruence and Similarity - Part 1

Objectives
- Identify figures that are congruent.
- Write the statement of congruence for a given pair of figures.

Books & Materials
- Math in Focus 3B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete Practice Questions.

LEARN

WARM-UP
Identify each angle pair as alternate interior, alternate exterior, or corresponding.

1. \( \angle 1 \) and \( \angle 3 \)
2. \( \angle 2 \) and \( \angle 3 \)
3. \( \angle 1 \) and \( \angle 4 \)

WARM-UP ANSWERS
1. corresponding 2. alternate interior 3. alternate exterior

TEACHING NOTES

INSTRUCTION
Read p.116 in Math in Focus 3B. Congruent figures have the same shape and size. They may have different orientations or positions, meaning they can be positioned differently in space or on a plane.
Example 1 on p. 117. Then complete Guided Practice on p. 117. Imagine laying one figure on top of another to determine if the two figures are congruent.

Read p. 118 to learn how to identify congruent triangles. The order in which you name the angles in a triangle is important. For example, if triangle ABC is congruent to triangle DEF, then that means angle A is congruent to angle D, angle B is congruent to angle E, and angle C is congruent to angle F. It also means that segment AB is congruent to segment DE, segment BC is congruent to segment EF, and segment CA is congruent to segment FD.

The symbol used to state that two figures are congruent is \( \cong \). For example, you can write the following, which means triangle ABC is congruent to triangle DEF:

\[
\triangle ABC \cong \triangle DEF
\]

Tick marks are used on figures to show which sides are congruent and which angles are congruent. Be sure you use the same number of marks for congruent sides but different numbers of marks for sides that are not congruent. The marks will help you name the triangles in the correct order.

Congruent quadrilaterals have four pairs of congruent sides and four pairs of congruent angles. For any two polygons that are congruent, all of their corresponding sides are congruent, and all of their corresponding angles are congruent.

Review Example 2 on p. 119 in Math in Focus 3B. There are many different ways to name two congruent triangles. Notice the first step in the example is to match the corresponding angles (or sides) and write this information down to help you keep things in order. As long as the corresponding angles are listed in order, you can write the triangle name in any order. For example, you can write \( \triangle ABC \cong \triangle DEF \), or you can write \( \triangle BCA \cong \triangle EFD \).

Complete Guided Practice on p. 120.

TEACHING NOTES

Make sure your student understands how to identify figures that are congruent, as well as how to accurately write a statement of congruence for a given pair of congruent figures. Stress the importance of order in naming the figures.

PRACTICE

Complete problems 1–2 of Practice 9.1 on p. 126 in Math in Focus 3B.
Today you learned how to identify figures that are congruent. **Congruent figures** are the same shape and size.

You also learned how to write a statement of congruence for a given pair of figures. You must write the letters of the corresponding angles and the corresponding sides in order when writing a statement of congruence.

Please go online to view and submit this assessment.
LEARN

Triangle ABC is congruent to triangle EFG. Identify the angles and line segments that are congruent.

WARM-UP ANSWER
Segment AB is congruent to segment EF, AC is congruent to EG, and BC is congruent to FG. Angle A is congruent to angle E, B is congruent to F, and C is congruent to G.

INSTRUCTION
Review Example 3 on p.120 in Math in Focus 3B. Recall that the sum of the measures of the angles in any triangle is 180°.

When working with congruent triangles, always identify the corresponding angles and corresponding sides before you begin. This will help you accurately find the unknown angle measures and line segments. List the pairs of corresponding angles and the pairs of corresponding sides. Then you can use the information given in the diagrams to write equations that can be solved to find the missing information.
Remember to use tick marks and arcs to indicate the pairs of corresponding sides and angles. In a triangle, there are three pairs of corresponding angles and three pairs of corresponding sides. Use three types of arcs to mark each pair of congruent angles and three different types of tick marks to mark each pair of congruent sides. Refer to the markings on the triangles on pp. 118–119, if needed.

Complete **Guided Practice** on p. 121.

**TEACHING NOTES**

Make sure your student understands how to identify the pairs of corresponding angles and corresponding sides in a figure, such as a triangle. If he needs help, walk through an example, identifying corresponding angles and corresponding sides, while labeling them with arcs and tick marks.

If needed, encourage your student to overlay figures one on top of the other to literally see the congruent angles and congruent sides.

**PRACTICE**

Complete problems 5–7 of **Practice 9.1** on p. 127 in *Math in Focus 3B*.

**TEACHING NOTES**

Textbook Answer Key

**WRAP-UP**

In this lesson, you learned how to find unknown measures in congruent figures.

You learned that it helps to first list the pairs of corresponding angles and pairs of corresponding sides. You also learned how to use the information given in diagrams to write and solve equations to find missing information.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
## Objectives
- Identify congruent triangles.

## Books & Materials
- *Math in Focus 3B*
- *Math in Focus - Teacher Edition A*
- *Math in Focus - Teacher Edition B*
- geometry software (Optional)
- *grid paper* (Optional)

## Assignments
- Complete Warm-Up.
- Complete the assigned pages in *Math in Focus 3B*.
- Complete Quick Check.

## LEARN

### WARM-UP

Multiply mentally.

1. \(2 \cdot 5^2\)
2. \(3 \cdot 4^2\)
3. \(2 \cdot 7^2\)
4. \(2 \cdot 5^2 \cdot 3\)

### TEACHING NOTES

#### WARM-UP ANSWERS

1. 50  2. 48  3. 98  4. 150

## INSTRUCTION

Read *Use Tests for Congruent Triangles* on pp. 122–123. This section summarizes the minimum conditions that must hold true to confirm that two triangles are congruent. You should memorize all four tests.

Review *Example 4* on p. 124. Here you will use the tests for congruence to identify congruent triangles. Be sure to name the angles in order when you write statements of congruence.

Complete *Guided Practice* on p. 125.
Review the tests for congruent triangles with your student to make sure he understands each of them.

Complete problems 3–4 and 8–10 of Practice 9.1 on p. 126 and p. 128 in Math in Focus 3B.

Textbook Answer Key

In this lesson, you learned ways to prove that two triangles are congruent. The following are the tests you can use to confirm two triangles are congruent:

- **Side-Angle-Side Test (SAS):** If two pairs of corresponding sides are congruent, and the angle between them is congruent, then the two triangles are congruent.

- **Angle-Angle-Side or Angle-Side-Angle Test (AAS or ASA):** If two pairs of corresponding angles are congruent, and any one pair of corresponding sides is congruent, then the two triangles are congruent.

- **Side-Side-Side Test (SSS):** If all three pairs of corresponding sides are congruent, then the two triangles are congruent.

- **Hypotenuse-Leg Test (HL):** In a right triangle, if the hypotenuse of one triangle is congruent to the hypotenuse of the other triangle, and if one other pair of corresponding sides is congruent, then the two triangles are congruent.

Please go online to view and submit this assessment.

If you struggled with this question, revisit the material in this lesson.
LEARN

WARM-UP
Answer each question.

1. Are all squares rectangles?
2. Are all rectangles squares?
3. Are all rectangles parallelograms?
4. Are all parallelograms rectangles?

WARM-UP ANSWERS
1. yes 2. no 3. yes 4. no

INSTRUCTION
Read Understand the Concept of Similarity on p. 129 in Math in Focus 3B. Then review Example 5 on p. 129. Notice that figures A and E are the same color but not the same shape, so they are not considered similar figures in the mathematical sense. Although the two similar figures (B and E) are both squares, not all pairs of similar figures are squares. Pairs of similar figures can be any shape.

Complete Guided Practice on p. 130. Identify each type of triangle before you begin. For instance, triangle A is a right triangle.

Complete Hands-On Activity on p. 130. Draw a large triangle so you can place one triangle on top of another as you complete the activity. Use a straightedge to draw the triangles.
Read **Apply the Concept of Similarity** on pp. 131–132. In similar polygons, corresponding angles have the same measure. Additionally, the ratios of corresponding side lengths are equal. Even though all rectangles have the same angle measures, not all rectangles are similar. Two rectangles are similar only if their corresponding side lengths are proportional. The ratio of the corresponding lengths of similar figures is called the *scale factor* or *constant of proportionality*.

Discuss the **Think Math** question on p. 131 with your Learning Guide. Congruent figures are a special case of similar figures, a case in which the ratio of corresponding side lengths is 1. All congruent figures are similar.

The symbol used to indicate that two shapes are similar is ~. For example, you can write the following, which means triangle $ABC$ is similar to triangle $DEF$:

$$\triangle ABC \sim \triangle DEF$$

Write the letters of the vertices in order so that you list the corresponding angles and corresponding sides in the same order. For example, you can write $\triangle ABC \sim \triangle DEF$, or you can write $\triangle BCA \sim \triangle EFD$.

If two figures are similar and the ratio of their corresponding side lengths is $k$, then the ratio of their corresponding areas is $k^2$.

---

**TEACHING NOTES**

Make sure your student understands the difference between similarity and congruence. Review the requirements for similarity and congruence with your student as needed.

Remind your student that the order in which he writes the vertices to name similar polygons is important when writing statements about similarity and congruence. Check to see that your student knows how to accurately write a statement of similarity for a given pair of similar figures.

---

**PRACTICE**

Complete problems 1–5 of **Practice 9.2** on p. 140 in *Math in Focus 3B*.

---

**TEACHING NOTES**

[Textbook Answer Key]
WRAP-UP

Today you learned how to identify figures that are similar. In similar polygons, the ratios of corresponding side lengths are equal and their corresponding angles have the same measure.

The ratio of the corresponding side lengths of similar figures is called the scale factor or constant of proportionality.

SUPPLEMENTAL

- BrainPop: Similar Figures

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
LEARN

WARM-UP
Evaluate each expression.

1. $k^2$, if $k = 4$
2. $k^2$, if $k = 7$
3. $k^2$, if $k = 5$
4. $k^2$, if $k = 10$

TEACHING NOTES

WARM-UP ANSWERS
1. 16  2. 49  3. 25  4. 100

INSTRUCTION
Review **Example 6** on pp. 132–133 in *Math in Focus 3B*. The ratios of the corresponding sides in a pair of similar figures are equal. To find the unknown length of a side of a figure, set up a proportion. You may be able to use mental math to solve some of these problems.

When you write the ratios, make sure each fraction refers to the relationship between two corresponding sides. Since the scale factor is the same for each pair of corresponding sides in two similar figures, write an equation to state that the ratio between one pair of corresponding sides is equal to the ratio between another pair of corresponding sides. Then solve that equation to find the unknown piece of information.
Complete Guided Practice on p. 133.

Review Example 7 on pp. 134–135. For similar figures, the ratios of corresponding sides are equal, so you can write a proportion equation and then solve it to find the unknown information.

In part b, remember that corresponding angles of similar triangles have the same measure. Again, you can use this fact to write and solve an equation to find the unknown or unknown information.

As you read part c, recall that if two figures are similar and the ratio of their corresponding side lengths is \( k \), then the ratio of their corresponding areas is \( k^2 \).

Complete Guided Practice on pp. 135–136. For problem 3, first determine which line segment spans the river; that segment is the width of the river and the quantity you are looking for. For problem 5, you can find \( k \) by using the corresponding sides of the similar triangles. Once you know \( k \), you can find \( k^2 \).

### TEACHING NOTES

Make sure your student knows how to set up the proportion to find unknown lengths in similar figures. For two similar figures, if the ratio of the side lengths is \( k \), then the ratio of the areas is \( k^2 \). This is because length is a measure of one dimension, while area is a measure of two dimensions.

### PRACTICE

Complete problems 6–9 of Practice 9.2 on p. 141 in Math in Focus 3B.

### TEACHING NOTES

Textbook Answer Key

### WRAP-UP

In this lesson, you learned to apply what you know about similar figures to find missing information, such as side lengths and angle measures.

You have learned the following facts about similar figures:

- In similar figures, the ratios of corresponding side lengths are equal and their corresponding angles have the same measure.
- The ratio of the corresponding side lengths of similar figures is called the scale factor or constant of proportionality.
Please go online to view and submit this assessment.

If you chose the wrong answer, you might have incorrectly set up the proportion or made a calculation error when solving. Remember that 40-foot side is 4 times as long as the 10-foot side, so the other side lengths have the same relationship. You might also want to revisit the material from this lesson.
WARM-UP

Solve mentally for x.

1. \( \frac{1}{3} = \frac{x}{12} \)
2. \( \frac{3}{8} = \frac{x}{32} \)
3. \( \frac{5}{9} = \frac{25}{x} \)
4. \( \frac{6}{11} = \frac{x}{77} \)

WARM-UP ANSWERS

1. \( x = 4 \)  
2. \( x = 12 \)  
3. \( x = 45 \)  
4. \( x = 42 \)
is 180°. By knowing that two pairs of corresponding angles have the same measure, you know that all three pairs have the same measure. By definition, all corresponding angles of similar triangles are equal in measure.

Review Example 8 on p. 138. To confirm that two triangles are similar, you only need to show that two pairs of corresponding angles have equal measure. The third pair of corresponding angles must then also have the same measure.

Complete Guided Practice on p. 139. Be sure to write which of the three tests applies.

TEACHING NOTES
Make sure your student sees the relationship between the results he observed when he completed Hands-On Activity on p. 137 and the tests he learned in this lesson for proving that two triangles are similar. Your student can compare and contrast the tests for congruence and similarity.

PRACTICE
Complete problems 10–15 of Practice 9.2 on p. 142 in Math in Focus 3B.

TEACHING NOTES
Textbook Answer Key

WRAP-UP
In this lesson, you learned three tests you can use to determine if two triangles are similar. Only one test must be applied to indicate similarity.

1. If two pairs of corresponding angles have equal measure, then the two triangles are similar.

2. If all three pairs of corresponding sides have the same ratio, then the two triangles are similar.

3. If two pairs of corresponding side lengths have the same ratio and the included angles have the same measure, then the two triangles are similar. (Note: It must be the included angle, which is the angle between the two pairs of corresponding side lengths that have the same ratio.)

PRACTICE QUESTIONS
Please go online to view and submit this assessment.
Congruence and Similarity - Part 7

LEARN

INTERACTIVE ACTIVITY

Follow these instructions for the activity shown below.

Click here if you would like to see this activity in a new window.

In this activity, you will explore similar figures. Start by clicking the triangle button. Set Scale factor to 1.0 and Rotation, in degrees to 0. (To set the value of a slider, drag the slider or select the number in the text field, type in a new value, and press Enter.) Do the pink and green triangles appear to be the same size and shape? B. These triangles are congruent. You know that corresponding side lengths and angle measures of congruent triangles are equal. Select Show lengths and then Show angle measures to see that this is true. Drag the Rotation, in degrees slider. Are the triangles still congruent? Tell your Learning Guide how you know.

Set Rotation, in degrees to 0. Drag the Scale factor slider. Notice that the size of ΔEFG (the image) changes, but ΔABC (the preimage) stays the same. How do the image and preimage compare when the scale factor is greater than 1? How do the image and preimage compare when the scale factor is less than 1? Explain your ideas to your Learning Guide.

Now set the Scale factor to 3.0 and Rotation, in degrees to 0. Make sure the triangle button is selected. Similar figures have pairs of corresponding angles and pairs of corresponding sides, just like congruent figures. In your Math Notebook, name the part of ΔEFG that corresponds to each of the following parts of ΔABC: ∠ABC, ∠BCA, ∠BCA, side AB, side BC, and side CA. Turn on Show angle measures. What is true about the measures of the corresponding angles of these similar triangles? Select Show lengths. Find the ratio of the measures of each pair of corresponding sides in simplest form: AB EF , BC FG , and CA GE . Compare the simplest forms of the ratios to each other and to the scale factor. What do you notice? Share your observations with your Learning Guide.

Finally, use the buttons at the top left of the Gizmo to explore other types of figures. For each type, create a variety of figures and vary the scale factor and rotation. What do you notice about the angle measures? What do you notice about the side lengths? Explain your findings to your Learning Guide.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet to try some other activities with the Gizmo.
Sample answers: If one of two congruent figures is rotated, they remain congruent because the side lengths and angle measures do not change. When the scale factor is greater than 1, the image is larger; when the scale factor is less than 1, the image is smaller. If two figures are similar, the measures of their corresponding angles are equal, and the lengths of their sides are in the same ratio as the scale factor.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

RATE YOUR ENTHUSIASM

Please go online to view and submit this assessment.
WARM-UP

Solve mentally for \( y \).

1. \( \frac{y}{3} = \frac{10}{15} \)
2. \( \frac{4}{y} = \frac{36}{81} \)
3. \( \frac{5}{y} = \frac{25}{y} \)
4. \( \frac{8}{12} = \frac{y}{60} \)

WARM-UP ANSWERS

1. \( y = 2 \)  2. \( y = 9 \)  3. \( y = 35 \)  4. \( y = 40 \)

INSTRUCTION

When working with triangles, you must name the corresponding angles in order. If two triangles are similar, then the corresponding angles are congruent. If you know the measures of two angles in a triangle, you can find the measure of the third angle. The angle measures for any triangle sum to 180°. If the angle measures are the same in two triangles, then the triangles are similar.

If two triangles are similar, you know that the corresponding sides are proportional. That means the ratio of the corresponding sides is the same for all three pairs of corresponding sides.

You can use these facts to solve problems with similar triangles. For example, you can use the given information to write ratios to find the lengths of the sides.
The figure shows two triangles. It helps to draw the two triangles separately and label each side of each triangle with the given information. Name the triangles in the same order so the corresponding sides are listed in order. Then write a ratio to find the values of $x$ and $y$.

$\Delta PQR \sim \Delta PST$, so the ratio of the corresponding sides is the same.

\[
\frac{PS}{PQ} = \frac{ST}{QR} = \frac{PT}{PR}
\]

\[
\frac{x + 4.5}{x} = \frac{y}{12} = \frac{9}{6}
\]

Solve for $y$.

\[
\frac{y}{12} = \frac{9}{6}
\]

\[
6y = 9 \quad \text{Multiply both sides by 6.}
\]

\[
y = 9 \quad \text{Simplify the left side of the equation.}
\]

\[
y = 18 \quad \text{Multiply both sides by 2.}
\]

The length of side $ST$ is 18 inches.

Solve for $x$.

\[
\frac{x + 4.5}{x} = \frac{9}{6}
\]

\[
6(x + 4.5) = 9 \quad \text{Use the first and last ratios to solve for } x.
\]

\[
6x + 27 = 9x \quad \text{Multiply both sides by 6.}
\]

\[
6x + 27 = 9x \quad \text{Multiply both sides by } x.
\]

\[
x = 9 \quad \text{Distribute the 6.}
\]

\[
27 = 3x \quad \text{Subtract 6x from both sides.}
\]

\[
x = 9 \quad \text{Divide both sides by 3.}
\]

The length of side $PQ$ is 9 inches.

---

**TEACHING NOTES**

Make sure your student understands how to write the corresponding sides in order so he sets up the equivalent ratios correctly when solving problems with similar triangles.
**PRACTICE**

Complete problems 16–20 of Practice 9.2 on p. 143 in *Math in Focus 3B*.

---

**TEACHING NOTES**

Textbook Answer Key

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**WRAP-UP**

In this lesson, you solved problems with similar triangles. Since the ratios of the corresponding sides are equivalent in similar triangles, you can write the ratios, substitute the information, and solve the equations to find the missing information.

---

**USE FOR MASTERY**

1. Elvin has three sails of different sizes for his model ships. Which of the sails are congruent, and which are similar?

   ![Sail I](image1.png)  
   ![Sail II](image2.png)  
   ![Sail III](image3.png)

If you are able, use the text box to show your work and enter your final answer to the question. If not, complete your work on paper and upload it below.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information given to help you tell which of the sails are congruent?
- Use the information given to help you tell which of the sails are similar?
- Use the text box to show your work and enter your final answer OR complete your work on paper and upload it?
Congruence, Similarity, and Transformations - Part 1

Objectives
- Relate congruent and similar figures using transformations.

Books & Materials
- Math in Focus 3B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B
- Grid paper

Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete the Practice Questions.

LEARN

WARM-UP
Complete each unit conversion mentally.

1. 10 feet = __________ inches
2. 3 yards = __________ feet
3. __________ inches = 6 feet
4. $2\frac{1}{3}$ yards = __________ feet

TEACHING NOTES

WARM-UP ANSWERS
1. 120  2. 9  3. 72  4. 7

INSTRUCTION
Read p. 144 in Math in Focus 3B. Translations, reflections, and rotations are isometries, meaning they preserve the shape and size of the figure. Dilations do not preserve the size of the figure unless the scale factor is 1 or −1. Draw a triangle on grid paper and dilate it three times: first by 1, then by −1, and then by 2 to see the results.

Translations, reflections, and rotations result in congruent figures because the size and shape of the original figure is preserved during the transformation. If two figures are congruent, then you can always map one figure onto another by performing a series of these three isometric transformations.
Review Example 9 on p. 145. This example shows a reflection. You can use grid paper to plot each point and draw the two triangles. Then fold the paper along the line \( y = 1 \) to see that this is a reflection. Recall that a reflection is what you see when you look in a mirror.

Complete Guided Practice on p. 145. Remember, the order matters when naming triangles and completing translations. \( A \) must be mapped to \( A' \), \( B \) must be mapped to \( B' \), and \( C \) must be mapped to \( C' \).

Read the instructional section at the top of p. 146. A dilation of a figure results in a similar figure because the corresponding sides of the figure are increased or decreased by the same scale factor, and the before-and-after angle measures are the same. A dilation is the only transformation that can result in a similar, but not congruent, image.

Review Example 10 on p. 146. You know this transformation is a dilation because the resulting sphere is similar to the original one but not congruent to it. The scale factor is a ratio, and ratios have no units. In this example, the snowball measurements are given in different units of measure: 3 inches and 3 feet. You must convert them to the same unit of measure before finding the scale factor of the dilation, which is a ratio.

Complete Guided Practice on p. 147.

---

**TEACHING NOTES**

Make sure your student understands why a dilation is the only transformation that results in a figure that is not congruent to the original figure.

**Looking Forward:** Have your student begin collecting pictures and/or taking pictures of buildings with congruent and similar shapes. Your student will need the pictures for the unit project.

---

**PRACTICE**

Complete problems 1–5 of Practice 9.3 on p. 154 in *Math in Focus 3B*.

**TEACHING NOTES**

Textbook Answer Key
WRAP-UP

Today you learned how to relate congruent or similar figures using geometric transformations. Translations, reflections, and rotations are transformations that result in congruent figures.

Dilations are transformations that result in similar figures, unless the scale factor is 1 or −1, in which case the dilation results in a congruent figure.

When completing a dilation, the scale factor is a ratio, which has no units. Be sure the units are the same before you write a ratio to report the scale factor.

SUPPLEMENTAL

- BrainPop: Transformation

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
LEARN

WARM-UP

Determine whether the expressions in each pair are equivalent.

1. $6(x + 2)$ and $6x + 2$
2. $6(x + 2)$ and $6x + 12$
3. $8(2x + 4)$ and $16x + 12$
4. $8(2x + 4)$ and $16x + 32$

TEACHING NOTES

WARM-UP ANSWERS

1. no 2. yes 3. no 4. yes

INSTRUCTION

Read Describe a Sequence of Transformations on p. 148 in Math in Focus 3B. A sequence is a particular order in which events take place or in which things are listed. Transformations are completed in a sequence; they take place in a particular order. The order in which transformations take place matters. For example, if you complete two transformations in a specific order (e.g., translate the object first, and then rotate the object), you may get a different result if you complete the transformations in the opposite order (e.g., rotate the object first, and then translate it). Draw an object on grid paper and try it yourself.
Review Example 11 on p. 149. You can often find more than one way to transform a figure to get the same result. Try to find another way to transform pitcher A to the same location and position as pitcher B. Use grid paper and a ruler to graph pitcher A, and then test your transformation(s).

Complete Guided Practice on p. 149. Use grid paper and a ruler to draw the food package GHIJ. Then complete the transformations described in the problem to find the line of reflection.

After you determine the answer, try it again. This time, complete the transformations in the opposite order. Reflect the image first; then translate it to see if the results are the same.

⚠️ TEACHING NOTES

Make sure your student understands that the order in which a sequence of transformations takes place impacts the result. If the order is changed, the result may be different. Make sure your student understands, too, that the results may also be the same, as in the Guided Practice problem.

Practice

Complete problems 6–10 of Practice 9.3 on pp. 154–157 in Math in Focus 3B.

⚠️ TEACHING NOTES

Textbook Answer Key

Wrap-Up

In this lesson, you learned how to describe a sequence of transformations. The order in which you transform an object matters. You also learned that there is often more than one way to transform an object to get the same result.

Sometimes, one transformation can accomplish the same result as multiple transformations.

✔️ Practice Questions

Please go online to view and submit this assessment.
**Congruence, Similarity, and Transformations - Part 3**

### Objectives
- Determine the relationship between congruent and similar figures based on a sequence of transformations.

### Books & Materials
- *Math in Focus 3B*
- *Math in Focus - Teacher Edition A*
- *Math in Focus - Teacher Edition B*
- Geometry software (Optional)
- Grid paper
- Ruler

### Assignments
- Complete Warm-Up.
- Complete the assigned pages in *Math in Focus 3B*.
- Complete Quick Check.

---

**LEARN**

**WARM-UP**

Determine whether each sequence of transformations results in the same location.

1. Walk east for 1 mile, then walk south for 1 mile.
   Walk south for 1 mile, then walk east for 1 mile.

2. Rotate 360° clockwise, then walk forward 3 steps.
   Walk forward 3 steps, then rotate 360° clockwise.

3. Hop to your right 6 times; rotate 90° counterclockwise; hop backwards once.
   Rotate 90° counterclockwise; hop to your right 6 times; hop backwards once.

### TEACHING NOTES

**WARM-UP ANSWERS**

1. yes  2. yes  3. no

---

**INSTRUCTION**

Complete Technology Activity on p. 150 in *Math in Focus 3B*. Notice which transformations result in congruent figures and which ones result in similar figures.

Read the instructional section at the bottom of p. 150. When completing a sequence of transformations, the order in which you perform the transformations impacts the result. For example, if you tell someone...
to walk 10 steps forward and then to turn around, she will end up in a different position than if you tell her to turn around and then walk 10 steps forward.

Review Example 12 on p. 151. Then complete Guided Practice on p. 152. As you complete problem 5a, keep in mind that, to determine whether the sequence of transformations affects the position of the final image, you must examine the labels on the vertices of the final image.

Review Example 13 on p. 153. Remember, a dilation is the only transformation you have studied that can change the size of the image.

Complete Guided Practice on p. 153. Note which figures are congruent and which ones are similar.

**TEACHING NOTES**

If your student does not have access to geometry software to complete Technology Activity on p. 150, he could complete it using grid paper and pencil. This will take more time than using the software, but it will help to reinforce the concepts.

**PRACTICE**

Complete problems 11 and 14 of Practice 9.3 on p. 157 in Math in Focus 3B. (Problems 12 and 13 are optional; you may complete these problems involving similarity of three-dimensional figures as an additional challenge.)

**TEACHING NOTES**

Textbook Answer Key

**WRAP-UP**

Today you explored sequences of transformations involving both congruent and similar figures. Translations, reflections, and rotations are transformations that result in congruent figures. A sequence of these transformations also results in a congruent image. Dilations are transformations that result in similar figures. If the scale factor is 1 or −1, the dilation results in a congruent figure.

Sometimes, a single geometric transformation can accomplish the same result as a sequence of transformations. There may be more than one sequence of transformations that will yield the same result.

The order in which you complete a sequence of transformations may impact the result.
Quick Check

Please go online to view and submit this assessment.

More to Explore

If you struggled with this question, revisit the material in this lesson and view the video Transformations.
Follow these instructions for the activity shown below.

Click here if you would like to see the activity in a new window.

The Rock Art Gizmo will help you analyze and then recreate ancient paintings by transforming six images. The art is made by transforming images. There are three transformations you can try: translations, reflections, and rotations. To begin, click on the dog so that it is selected. (It should have a yellow glow.) Click each transformation button (Translate, Reflect, and Rotate) and drag the dog around. Which transformation(s) make the dog face a new direction? Which ones change the dog’s location? Share your answers with your Learning Guide.

Turn on Show old paintings and check that painting A is selected. Under Art symbols, use the checkbox to add the dog. Then transform the dog to match the painting. (If you would like to try something again, click Remove last. To start over, click New rock.) Tell your Learning Guide what you did to match the painting. Then try paintings B and C.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet to try some other activities with the Gizmo.

Go to Lesson Info to access the Exploration Sheet Answer Key. You may also wish to explore the other teaching resources in the activity.

Please go online to view and submit this assessment.
LEARN

WARM-UP

For each ratio, write an equivalent ratio.

1. 5:7
2. 24:36
3. 12:60
4. 33:77

TEACHING NOTES

WARM-UP ANSWERS


INSTRUCTION

Read and complete Brain @ Work on p. 158 in Math in Focus 3B. Use drawings to explain your reasoning and show your work.

For problem 1, the shortest distance across the river is perpendicular to the riverbank. The original bridge was not perpendicular to the riverbank.

For problem 2, each dimension of the pattern for the box must be scaled down by the same factor.
Encourage your student to use grid paper and a straightedge to draw each image. For problem 1, the original bridge would be the longest side of a right triangle.

Write a constructed response to explain how you found the answer to Brain @ Work problem 1 on p. 158.

Then complete the following problems.

1. Trapezoid $ABCD$ has vertices as follows: $A (3, 4)$, $B (6, 4)$, $C (7, 1)$, and $D (2, 1)$.
   
   a. Reflect the trapezoid over the $y$-axis and then reflect it over the $x$-axis. What are the coordinates of the resulting image?
   
   b. What happens if you reverse the sequence of transformations in part a? Is the result the same or different? Explain.
   
   c. Is there a single transformation that would give the same result as the sequence of transformations in part a? If so, what is it?

2. A rectangular poster has a length of 30 inches and a perimeter of 100 inches. A similar poster has a perimeter of 60 inches.
   
   a. What is the width of the similar poster?
   
   b. What is the area of the similar poster?

3. A survey marker is placed perpendicular to the ground and stands 4 feet tall. A flagpole nearby also stands perpendicular to the ground. It is 32 feet tall. The flagpole casts a shadow that is 56 feet long. How long is the shadow of the survey marker at that same time?

Practice Answers:

1a $A'' (-3, -4)$, $B'' (-6, -4)$, $C'' (-7, -1)$ and $D'' (-2, -1)$. b The result is the same. In this case, the order in which you complete the sequence of transformations does not change the result. c Yes, you could rotate the original figure $180^\circ$ about the origin. 2a 12 inches b 216 square inches 3 7 feet
WRAP-UP
Today you learned how to apply mathematical skills and concepts to solve real-world problems. You completed transformations and used ratios to solve problems.

✅ PRACTICE QUESTIONS
Please go online to view and submit this assessment.
Congruence, Similarity, and Transformations - Part 6

LEARN

WARM-UP
Solve the following problems.

1. Two mixing bowls are similar. One has a diameter of 9 inches and a height of 6 inches. The other has a height of 8 inches. What is its diameter?

2. A package of foldable gift boxes contains boxes that are all similar. One is 16 inches long by 10 inches wide. Another is 24 inches long. How wide is it?

3. Two pairs of corresponding sides from a pair of triangles have the same ratio, and a pair of corresponding angles has the same measure. What can you say about the two triangles?

TEACHING NOTES

WARM-UP ANSWERS
1. 12 inches  2. 15 inches  3. You cannot determine anything conclusive. To know they are similar, the angles that have the same measure must be the included angles.

INSTRUCTION
Read Chapter Wrap Up on p. 159 in Math in Focus 3B. Use the notes in the concept map to review each skill. Review the chapter vocabulary and memorize the tests for congruence and similarity that you learned.
You may wish to quiz your student on the tests for congruence and similarity.

**PRACTICE**

Complete Chapter Review/Test on pp. 160–165 in *Math in Focus 3B*.

**WRAP-UP**

In this chapter, you learned how to identify congruent and similar figures. You also learned how to prove that two figures are congruent or similar. You studied transformations and sequences of transformations and determined whether the images were congruent or similar to the original objects.

**USE FOR MASTERY**

1. $\triangle ABC$ is mapped onto $\triangle A'B'C'$ which is then mapped onto $\triangle A''B''C''$. $\triangle ABC$ and $\triangle A''B''C''$ are shown in the diagram.
If you are able, use the text box to show your work and enter your final answer to each question. If not, complete your work on paper and upload it below.

a. Describe the sequence of transformations from $\Delta ABC$ to $\Delta A'B''C''$.

b. Describe a single transformation from $\Delta ABC$ to $\Delta A''B''C''$.

c. What is the relationship between $\Delta ABC$ and $\Delta A''B''C''$? Explain your reasoning.
Did you:

- Describe the sequence of transformations from $\triangle ABC$ to $\triangle A'B'C'$?
- Describe a single transformation from $\triangle ABC$ to $\triangle A'B'C'$?
- Show the relationship between $\triangle ABC$ and $\triangle A''B''C''$?
- Explain your reasoning for Part C?
- Use the text box to show your work and enter your final answer OR complete your work on paper and upload it?
Show: Be an Architect

Books & Materials
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

SHOW

Refer to your pictures and pick one pair that you believe are congruent and one pair that you believe are similar based on the copies you have made on the coordinate plane. Using a protractor and a ruler, prove that the pair is congruent and the other pair is similar. Remember, you will need to use the side lengths and the angles measures. Make sure to include the scale factor between your two similar shapes.

FINAL PROJECT

1. Which pictures did you decide were congruent? Provide all of your information and copies of your pictures to prove they are congruent.

Which pictures did you decide were similar? Provide all of your information and copies of your pictures to prove they are similar.

Attach all your evidence.

Supported file formats: PDF, JPG, GIF, PNG, Word, Powerpoint
Describe the buildings that you found to be congruent and similar. What did you notice about the shapes? Why do you think that architects often use similar or congruent shapes in their designs? Reply to two peers.

Attach all your evidence.

COLLABORATION

Describe the buildings that you found to be congruent and similar. What did you notice about the shapes? Why do you think that architects often use similar or congruent shapes in their designs? Reply to two peers.
Unit Quiz: Be an Architect

Books & Materials

- Math in Focus - Teacher Edition
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

UNIT QUIZ

Please go online to view and submit this assessment.
Unit 6 - Bivariate Data
Project: Movie Profits

Do you enjoy going to the movies? Most people do! Movie companies have been known to make quite a bit of money from moviegoers, and, historically, the opening weekend has been used to provide insight as to just how much money a movie will make. Imagine that you are a movie investor, and you are being asked to invest in one of four movies based on their opening weekend profits. In this project, you are going to do some research to compare the opening weekend gross with the lifetime gross for at least 50 movies. Once you have gathered your data, you will create a scatterplot and define the correlation. You will then create a line of best fit and use the equation of that line to predict the estimated lifetime gross profits for at least four movies.

PROJECT DETAILS

In this project, you will need to:

- Research online to find the weekend gross and lifetime gross of at least 40 movies. (Be sure to note your sources to include in the final project.)
- Create a scatter plot with the weekend gross on the horizontal axis and the lifetime gross on the vertical axis.
- Define the correlation between the data.
- Draw the line of best fit.
- Determine the equation for the line of best fit.
- Use the equation of the line of best fit to predict the lifetime gross profits of at least four movies based on their opening weekend profits.

PROJECT RUBRIC

The Project Rubric will help you understand how your project will be scored. Your goal should be to earn all possible points for each part.
COLLABORATION

The gross profit means the money made after the costs and expenses are deducted. What are some of the costs associated with making a movie? Make some predictions regarding how much you think it would cost to make a movie. What is the basis for this decision? Respond to at least two of your peers.

RATE YOUR EXCITEMENT

Please go online to view and submit this assessment.
Today’s lesson covers important ideas you need to understand before beginning the lessons in this chapter. Read p. 172 in *Math in Focus 3B*.

The information about cycling in a bike race discusses two variables: cadence and speed. These two variables are an example of bivariate data. Notice the prefix bi-, meaning *two*, in the word *bivariate*. A set of numerical bivariate data can be represented visually in a scatter plot. You will learn about scatter plots and lines of fit in this chapter.
Read Recall Prior Knowledge on p. 173. To find the relative frequency for each item, divide the number of each type of snack ordered by the total number of snacks ordered. When the relative frequencies for all five items are added, the sum is 1.

Look at the row for hamburgers. The table shows that 36 of the 90 items ordered are hamburgers. Hamburgers make up 40% of all the items ordered during a noontime rush hour.

### TEACHING NOTES

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for Quick Check.

Have your student find the sum of the numbers listed in the middle column, Number Ordered. Point out that the sum, 90, represents the total number of items ordered and that this number is used as the divisor to find the relative frequency for each item.

### SKILLS CHECK

Complete Quick Check on p. 173 in Math in Focus 3B.

### TEACHING NOTES

Textbook Answer Key

After your student completes the Quick Check in the Recall Prior Knowledge lesson of this chapter, review the questions that were answered incorrectly. If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him or her complete the activity.

### Quick Check

<table>
<thead>
<tr>
<th>Question(s)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Relative Frequency</td>
</tr>
</tbody>
</table>
WRAP-UP

Today you reviewed how to find relative frequencies.

The table shows the number of books checked out at a local library one day. By dividing the number of books checked out in each genre by the total number of books checked out, you can find the relative frequency for each genre.

<table>
<thead>
<tr>
<th>Genre</th>
<th>Number Checked Out</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiction</td>
<td>28</td>
<td>( \frac{28}{70} = 0.4 )</td>
</tr>
<tr>
<td>Historical Fiction</td>
<td>21</td>
<td>( \frac{21}{70} = 0.3 )</td>
</tr>
<tr>
<td>Nonfiction</td>
<td>14</td>
<td>( \frac{14}{70} = 0.2 )</td>
</tr>
<tr>
<td>Biography</td>
<td>7</td>
<td>( \frac{7}{70} = 0.1 )</td>
</tr>
</tbody>
</table>

The sum of the relative frequencies is 1.

\[ 0.4 + 0.3 + 0.2 + 0.1 = 1 \]

✔ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Objectives
- Identify bivariate data.
- Construct a scatter plot to represent bivariate data.

Books & Materials
- Math in Focus B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B
- grid paper
- straightedge (Optional)

Assignments
- Complete Warm-Up
- Complete the assigned pages in Math in Focus 3B
- Complete Practice Questions

LEARN

WARM-UP
Plot each table of values on a coordinate grid.

1. 

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

2. 

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
</tr>
</tbody>
</table>

TEACHING NOTES

WARM-UP ANSWERS
INSTRUCTION

Read the information about scatter plots on pp. 174–175 in *Math in Focus 3B*. The data shown in the tables and in the scatter plot are *bivariate data* because they include two types of quantitative data: speed and angle of incline. Note that creating a scatter plot is similar to plotting points on a coordinate grid. One variable is represented on the horizontal axis, and the other is represented on the vertical axis.

Look at the information on p. 175 in the speech bubble and the enlarged section of the scatter plot. In this scatter plot, the value represented by each tick mark is 0.2 unit. The value represented by each tick mark varies depending upon the values assigned on the axes. It is best to use a unit difference of 1, 5, or 10 units on the axes. This makes it easier to read the tick mark values between units.

Review **Example 1** on p. 176. Before you begin constructing a scatter plot, look carefully at the data in the table. Keep in mind the range of data values that will appear on each corresponding axis. This will help you determine the scale. In this example, the scale is given for each axis. Be sure to label the axes and name the scatter plot.

As you plot data points, be sure to use the correct order of the variables (x: mass, y: length of spring). The data must be correctly represented on the coordinate plane in order to accurately identify and study the association between data sets. You may find using a straightedge helpful in constructing or reading points from scatter plots. Use the straightedge to help visually connect each point with the corresponding number on the horizontal or vertical axes.

Complete **Guided Practice** on p. 177.

TEACHING NOTES

This lesson introduces your student to constructing scatter plots. As she works through the examples, make sure she accurately draws the axes and understands the value each tick mark represents. You may want to point out to your student that a scatter plot is not a graph of a function because there can be multiple y-values for each x-value. She can use the vertical line test to confirm this for either of the scatter plots shown on pp. 175–176.

PRACTICE

Complete problems 1–3 of **Practice 10.1** on p. 183 in *Math in Focus 3B*.

TEACHING NOTES

*Textbook Answer Key*
Today you learned how to construct a scatter plot to represent bivariate data.

The table shows the amount of money a company spends on advertising and the amount earned in sales.

<table>
<thead>
<tr>
<th>Advertising Cost (In thousands of dollars)</th>
<th>37</th>
<th>46</th>
<th>35</th>
<th>49</th>
<th>25</th>
<th>65</th>
<th>55</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekly Sales (In thousands of dollars)</td>
<td>395</td>
<td>401</td>
<td>402</td>
<td>406</td>
<td>390</td>
<td>420</td>
<td>415</td>
<td>375</td>
</tr>
</tbody>
</table>

The table above shows the data for advertising cost and weekly sales. The scatter plot below represents this data.

![Scatter Plot](image)

Please go online to view and submit this assessment.
This lesson introduces your student to constructing scatter plots. As she works through the examples, make sure she accurately draws the axes and understands the value each tick mark represents. You may want to point out to your student that a scatter plot is not a graph of a function because there can be multiple y-values for each x-value. She can use the vertical line test to confirm this for either of the scatter plots shown on pp. 175–176.

Complete problems 1–3 of Practice 10.1 on p. 183 in MathinFocus3B Textbook Answer Key.

Today you learned how to construct a scatter plot to represent bivariate data.

It’s time to begin the project. Start by thinking about some of your favorite movies. Although most of them were probably made over the past 10 years, try to think of some older movies that you enjoy as well. Create a table listing the name of the movie, the gross opening weekend profits, and the lifetime gross profits. Make sure to write down your sources to create a Works Cited list for the Final Show.

Please go online to view and submit this assessment.
Using your data from the tables in the previous task, create a scatter plot. The horizontal axis should be the opening week gross profits in millions, and the vertical axis should be the lifetime gross profits in millions.
### LEARN

#### WARM-UP

1. Plot the data in the table on a coordinate grid.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
</tr>
</tbody>
</table>

2. Look at your graph from problem 1. As the $x$-values increase, what happens to the $y$-values?

#### WARM-UP ANSWERS

2 The $y$-values also increase.
INSTRUCTION

Read the instructional section on pp. 177–178 in *Math in Focus 3B*. In the previous lesson, you learned that scatter plots are used to identify association between bivariate data. In this lesson, you will examine how the data points are arranged on a coordinate plane to show whether the bivariate data has *strong*, *weak*, or *no association*.

Look at part a on p. 178. Note that data with a positive association are arranged in a cluster that slopes upward from left to right. Data with a negative association are arranged in a cluster that slopes downward from left to right.

Compare the general shape formed by the graphs in part b. Data with a linear association are arranged in a cluster along a straight line. If the data has a nonlinear association, the data clusters along a curved line.

Refer back to the graphs at the bottom of p. 177 for examples of how data may be arranged when there is a weak association or no association in the bivariate data.

Review Example 2 on p. 179. Use the three categories in the speech bubble to describe the association of bivariate data in a scatter plot. When there is a strong association between bivariate data, use all three categories to describe the association. When there is no association, the last two categories do not apply.

Complete Guided Practice on p. 179. Use the three categories shown to describe the association in each scatter plot.

TEACHING NOTES

Textbook Answer Key

In this lesson, your student observes the arrangement of data points in a scatter plot to determine the association. She can describe the association using three categories:

- strong, weak, or no association
- positive or negative association
- linear or nonlinear association

For Guided Practice, you may need to point out that the data in Graph E is nonlinear. When nonlinear data are arranged in this manner, it has neither a positive nor negative association.

PRACTICE

Complete problems 4–7 of Practice 10.1 on pp. 183–184 in *Math in Focus 3B*.
INTRODUCTION

You may need to point out that the data in Graph E is nonlinear. When nonlinear data are arranged in this manner, it has neither a positive nor negative association.

INTERACTIVE ACTIVITY

Follow these instructions for the activity shown below.

Click here to view the activity in a new window.

In this activity, you will explore different types of correlation. To begin, check that the slider is close to the Positive trend side. When there is a positive trend (or positive correlation), do the y-values of the points on the scatter plot tend to increase or decrease as the x-values increase? Now move the slider to the Negative trend side. When there is a negative trend (or negative correlation), how do the y-values change as the x-values increase? Move the slider to the middle, under No trend. What does the scatter plot look like now? Share your observations with your Learning Guide.

Below the graph, click on the Positive trend button several times to view different graphs with positive trends. When data has a strong positive or negative trend, the points often approximate a line. A trend line is a line that best fits the data and gives an estimate of how one variable is correlated to the other. Select Show actual trend line. In the current scatter plot, how many points are above the trend line? How many points are below the trend line? Drag the slider all the way to the right. What do you notice? Drag the slider to the left. What happens as you approach the No trend setting? Drag the slider all the way to the left, under Negative trend. What happens now? Study the equation of the trend line, shown on the left, and use the word slope to explain what happens to the trend line to your Learning Guide.

You can try fitting your own line to a data set. To do this, turn off Show actual trend line, click the Positive trend button, and then select Fit a line. A. Use the m slider to change the slope and the b slider to change the y-intercept of your estimated trend line. What is the equation of your estimated trend line? Turn on Show actual trend line. What is its equation? Compare your equation to the equation of the actual trend line and discuss the results with your Learning Guide.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet to try some other activities with the Gizmo.

TEACHING NOTES

Sample answers: When there is a positive trend, the y-values of the points on the scatter plot tend to increase as the x-values increase. When there is a negative trend, the y-values of the points on the scatter plot tend to decrease as the x-values increase. When there is no trend, some increase, and some decrease. (Your student may also notice that, the stronger the trend, the closer together the points are on the scatter plot.)

Sample answers for trend line: When there is a strong positive trend, most of the points lie on or close to the trend line, and the line has a positive slope. When there is a strong negative trend, most
of the points lie on or close to the trend line, and the line has a negative slope. When there is no
trend, most of the points are off the trend line, which is closer to horizontal.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and
Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

WRAP-UP

Today you learned how to determine association between bivariate data represented in a scatter plot.

Graph 1: Strong, positive, and nonlinear association
Graph 2: Strong, negative, and linear association

QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

If you chose the wrong answer, you might not have noticed the question asks for the answer that best
describes the association. Remember that when the question asks for the best answer, more than one
of the answer choices might be true, but you are asked to choose the answer that gives the best
description. You might also wish to revisit the material in this lesson.
LEARN

WARM-UP

1. Sketch a simple scatter plot that shows a strong positive association.

2. How do you know if a graph shows a negative association?

TEACHING NOTES

WARM-UP ANSWERS

1. Answers will vary. Check student's work. The graph should show points closely clustered and sloping upward from left to right. 2. A graph shows a negative association when the data points slope downward from left to right.

INSTRUCTION

Read Identify Outliers in a Scatter Plot on p. 180 in Math in Focus 3B. Notice that the x-axis is labeled with the number of minutes past 7:00 A.M. rather than actual times such as 7:30 A.M. or 8:00 A.M. The outlier occurs at 90 minutes past 7:00 A.M. You can name the outlier by its coordinates, (90, 15).

Review Example 3 on pp. 181–182. An exam score is likely dependent upon the time spent studying. If you ignore the outliers, the data points support this idea since they show a positive association between time spent studying and exam scores.

Identify the two outliers. They are located at (1, 90) and (4, 50). Think about what these outliers represent. Compare the y-values of the outliers with the y-values of other data points that have the same or similar x-values. For example, the other data points show that students who spent 4 or more hours studying earned a 90% or above on the exam. The outlier (4, 50) may represent a student who...
claims to have studied for 4 hours, but in fact spent much of that time multi-tasking by also watching television or texting. This may explain why the student did not retain much of the information and therefore scored only 50% on the exam.

Complete Guided Practice on p. 182.

**TEACHING NOTES**

**Textbook Answer Key**

This lesson introduces outliers. Encourage your student to ignore outliers as she identifies a general trend in a scatter plot. If your student seems confused by the wording of the first sentence of Guided Practice part c, explain that she is being asked to describe the relationship between the two variables: the height of the tomato plants and the amount of water given.

**PRACTICE**

Complete problems 8–17 of Practice 10.1 on pp. 184–185 in Math in Focus 3B.

**WRAP-UP**

Today you learned how to identify outliers, determine possible explanations for them, and compare them to the general trend shown in a scatter plot.

The scatter plot displays bivariate data on the outside temperature in degrees Fahrenheit, x, and the corresponding number of beachgoers, y, at a local beach.

Outliers appear to be located at (66, 45) and (85, 15).
The outlier (66, 45) represents 45 beachgoers visiting the beach on a day when the outside temperature was 65°F. The outlier (85, 15) represents 15 beachgoers at the beach on a day when the outside temperature was 85°F. Perhaps on the 85°F day, people attended a big event, such as a football game instead of the beach. This could explain why fewer people attended the beach. Likewise, maybe there was a special competition at the beach on the day that the temperature was 66°F. This could explain why more people went to the beach even though it was not very warm.

The strong, positive, and linear association indicates that there are more beachgoers on warmer days. The general trend shows that, for temperatures below 70°F, there are fewer than 20 beachgoers, and when the temperature is above 80°F, there are more than 50 beachgoers.

Scientists predict that there is a relationship between the latitude of a location and the atmospheric temperature. In an experiment to determine the truth of this hypothesis, the following sets of data are collected.

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>12</td>
<td>6</td>
<td>4</td>
<td>–3</td>
<td>–9</td>
<td>–17</td>
<td>–25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>–34</td>
<td>–42</td>
<td>–45</td>
<td>–49</td>
<td>–50</td>
<td>–68</td>
</tr>
</tbody>
</table>

Click [here](#) to download a graph. Construct the scatter plot on the graph. Each mark on the horizontal axis represents 1 kilometer of altitude, and each mark on the vertical axis represents 1°C of temperature. When you are finished, upload your work in the box below.
Scientists predict that there is a relationship between the latitude of a location and the atmospheric temperature. In an experiment to determine the truth of this hypothesis, the following sets of data are collected.

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>–3</td>
</tr>
<tr>
<td>4</td>
<td>–9</td>
</tr>
<tr>
<td>5</td>
<td>–17</td>
</tr>
<tr>
<td>6</td>
<td>–25</td>
</tr>
<tr>
<td>7</td>
<td>–34</td>
</tr>
<tr>
<td>8</td>
<td>–42</td>
</tr>
<tr>
<td>9</td>
<td>–45</td>
</tr>
<tr>
<td>10</td>
<td>–49</td>
</tr>
<tr>
<td>11</td>
<td>–50</td>
</tr>
<tr>
<td>12</td>
<td>–68</td>
</tr>
</tbody>
</table>

Click here to download a graph. Construct the scatter plot on the graph. Each mark on the horizontal axis represents 1 kilometer of altitude, and each mark on the vertical axis represents 1°C of temperature. When you are finished, upload your work in the box below.

Did you:
- Download and print the graph?
- Construct the scatter plot on the graph using the information given?
- Upload your finished scatter plot?
- Identify any outliers?
- Describe the association between the altitude and the temperature?
- Upload your work in the box?
Scatter Plots - Part 7

Books & Materials
- Math in Focus - Teacher Edition
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

Assignments
- Complete Rate Your Progress

SHOW

Based on the scatter plot you made for this project, how would you describe the correlation between the weekend and lifetime gross profits? What are the outliers? Note your thoughts in your Math Notebook for the Final Show.

✅ RATE YOUR PROGRESS

Please go online to view and submit this assessment.
Line of Best Fit - Part 1

Objectives
- Find the line of best fit for a scatter plot.

Books & Materials
- Math in Focus B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B
- grid paper

Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete Practice Questions.

LEARN

WARM-UP
1. Draw an example of a scatter plot that shows a strong, negative, and linear association.
2. Draw an example of a scatter plot that shows a strong, positive, and linear association.

TEACHING NOTES

WARM-UP ANSWERS
1 Answers vary. Sample:

![Graph with negative association](image)

2 Answers vary. Sample:

![Graph with positive association](image)
INSTRUCTION

Read Understand Line of Best Fit on p. 187 in Math in Focus 3B. A line of best fit is a best estimate for bivariate data that has a strong linear association. Outliers are ignored, and there should be about the same number of data points above and below the line. Recall that you can estimate a y-value for a given x-value by using the trend identified from data points in a scatter plot. A line of best fit aids in estimating data values for bivariate data with linear association.

Review Example 4 on pp. 188–189. A line of best fit is an approximation. Different people may draw lines that vary a bit, but the slope and general position of their lines should be quite similar. The outlier is ignored in the line of best fit because the line of best fit approximates the trend of the data. Outliers do not follow the trend.

Read and discuss the Think Math question on p. 189 with your Learning Guide. Use your own experience to think of reasonable explanations for the unusually low number of accidents.

Complete Guided Practice on p. 189. For part a, notice that a specific range is given for the horizontal axis. You must determine the x interval. For part b, look for a general trend in the data points, and determine whether the trend is linear. After you complete part c, think of possible explanations for the outlier. For example, perhaps bad weather kept most people away from the park, resulting in less litter.

TEXTBOOK ANSWER KEY

In this part, your student learns to identify a linear trend and draw a line of best fit. This line is used to approximate the trend of the data points in a single line. As your student draws a line of best fit, monitor to make sure she does not simply try to connect two or more data points. There should be about the same number of points above and below the line, and it is possible for the line to not pass through any data points.
INTERACTIVE ACTIVITY

Ask your Learning Guide to help you access Polynomial Functions and Scatter Plots. Try this activity to explore scatter plots.

TEACHING NOTES

Follow these steps to access the activity:

1. Go to the Eighth Grade Functions section of the activity list.
2. Click on Polynomial Functions and Scatter Plots.
3. Click on View Teacher Tool or View Teacher Tool in Spanish.
4. Read and agree to the Terms of Use. Click Continue.

PRACTICE

Complete problems 1–5 of Practice 10.2 on p. 196 in Math in Focus 3B.

WRAP-UP

Today you learned how to draw a line of best fit that represents the best approximation of the trend in a set of data points with a strong linear association.
Data from an analysis of the association between the amount spent on advertising, $x$, and the amount of weekly sales, $y$, is shown in the scatter plot. Sketch a line of best fit to represent the data.

There is a strong, positive, and linear association between the amount spent on advertising and the amount earned in sales. This indicates that more advertising is associated with more sales.

The data point (32, 375) is an outlier representing only $375,000 in sales when $32,000 was spent on advertising.

✅ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
LEARN

WARM-UP

1. Graph the line that goes through the points shown in the table.

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

2. Write an equation for the line you graphed in problem 1.

$2y = 2x + 1$

TEACHING NOTES

WARM-UP ANSWERS

INSTRUCTION

Read Write a Linear Equation for a Line of Best Fit on p. 190 in Math in Focus 3B. A line of best fit can be written as a linear equation in the form $y = mx + b$. Recall that $m$ represents the slope of the line and $b$ is the $y$-intercept. Both of these values can be obtained from the line of best fit on a scatter plot.

Review Example 5 on pp. 190–191. For part a, notice the zigzag at the bottom of the vertical axis on the graph. This indicates that the values between 0 and 54 have been skipped. Omitting these values keeps the graph to a manageable size. If all values between 0 and 54 were included, the graph would be quite large, and the part of the graph with the data points would be very small and hard to read.
Remember that a line of best fit is a best estimate of the trend of the data. Therefore it may or may not pass through any of the data points. Use the line of best fit, not data points, to find the slope and y-intercept. For calculating slope, choose points along the line that have integer values to make computation easier.

Complete **Guided Practice** on p.191. Look at the scale for both axes. Use a zigzag line if necessary. Check that you have plotted the data points correctly, and then draw a line of best fit. Use two points on the line to write an equation for the line of best fit.

### TEACHING NOTES

**Textbook Answer Key**

This lesson explains how to write an equation for a line of best fit. If necessary, review with your student the slope-intercept form of a linear equation: \( y = mx + b \). Help your student recall that the slope, \( m \), is found by identifying two points on the line and finding the difference between the y-coordinates and the x-coordinates:

\[
\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_1 - y_2}{x_1 - x_2}
\]

The y-intercept is the point where the line crosses the y-axis.

### PRACTICE

Complete problems 6–7 of **Practice 10.2** on p.197 in *Math in Focus 3B*.

### TEACHING NOTES

**Textbook Answer Key**

### WRAP-UP

Today you learned how to write an equation for a line of best fit on a scatter plot.
A sleep center collected data to find the association between a person's age, $x$, and nightly hours of sleep, $y$.

Find the slope. The points (16, 10) and (38, 8) are on the line.

$$m = \frac{10 - 8}{16 - 38} = \frac{2}{-22} = -\frac{1}{11}$$

M2019G8MIFOBL136.C08

Substitute to find the $y$-intercept.

$$y = mx + b$$
$$10 = -\frac{1}{11}(16) + b$$
$$10 = -\frac{16}{11} + b$$
$$10 + \frac{16}{11} = b$$
$$b = 11\frac{5}{11}$$

Therefore, a line of best fit is $y = -\frac{1}{11}x + 11\frac{5}{11}$.

✅ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Line of Best Fit - Part 3

**Objectives**
- Use the line of best fit to make estimations.

**Books & Materials**
- Math in Focus B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B
- grid paper
- online graphing calculator (Optional)

**Assignments**
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete Quick Check.

---

**LEARN**

**WARM-UP**

1. What does a line of best fit represent?
2. How do you find the line of best fit?

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. Sample answer: A line of best fit represents the general trend of the data points in a scatter plot.
2. Sample answer: In a scatter plot that shows strong linear association, draw a line through the data points so that about half the points are above the line and half are below the line.

---

**INSTRUCTION**

Read Use an Equation for a Line of Best Fit on p. 192 in Math in Focus 3B. Look at the prefixes in the words *interpolate* and *extrapolate*. The prefix *inter-* means *between*; the prefix *extra-* means *outside*. Knowing these prefixes will help you remember that *interpolate* means to predict a value *between* data points. *Extrapolate* means to predict a value *outside* the range of data.

Review Example 6 on pp. 192–193. Note that the increments along the horizontal axis represent the number of years since 1990, not actual years.

Part a is an example of interpolating an estimate. Part b is an example of extrapolating an estimate. Read the Caution box on p. 193.
Notice the opening clause in the last paragraph of the solution: “According to the trend observed in previous years.” This phrasing emphasizes that the estimate is based on a past trend and cannot be stated with certainty. The extrapolation assumes that the linear trend continues after the recorded years.

Complete Guided Practice on pp. 193–194. For part a, use the given points to find the slope, \( m \), of the line of best fit. Then use the value of \( m \) and the coordinates of one point to find the \( y \)-intercept and write the equation for the line of best fit in slope-intercept form. For part b, identify the \( y \)-value that corresponds to the \( x \)-value of 86 on the line of best fit in the graph. For part c, identify the \( y \)-value outside the data range that corresponds to the \( x \)-value of 65 on the line of best fit.

---

### TEACHING NOTES

**Textbook Answer Key**

In this lesson, your student uses a line of best fit to interpolate and extrapolate predicted values inside and outside the range of data. Discuss with her why interpolating is more reliable than extrapolating. She should understand that extrapolating assumes that an observed trend will continue outside the data range. Point out that other factors may come into play that would alter the trend. For example, new technology may cause newspaper readership to drop dramatically. A prediction becomes less accurate the farther the predicted value is from the range of data.

---

### PRACTICE

Complete problems 8–13 of Practice 10.2 on p. 197 in Math in Focus 3B.

---

### WRAP-UP

Today you learned how to use a line of best fit to make estimations.
A local drugstore keeps track of the amount of sunscreen sales, $y$, in relation to the outside temperature, $x$. The data is shown in the following scatter plot.

The line of best fit passes through (80, 520) and (65, 320).

$$m = \frac{520 - 320}{80 - 65} = \frac{200}{15} \approx 13.33$$

Find the $y$-intercept.

$$320 = 13.33(65) + b$$
$$320 = 866.45 + b$$
$$-546.45 = b$$

Therefore, the equation of the line of best fit is $y = 13.33x - 546.45$.

About $400 in sunscreen sales can be predicted if the outside temperature is 71°F. About $653 in sunscreen sales can be predicted if the outside temperature is 90°F.

**QUICK CHECK**

Please go online to view and submit this assessment.

**MORE TO EXPLORE**

If you struggled with this question, use a pencil or straightedge to help you line up data points with numbers on the axis. You might also wish to revisit the material in this lesson.
### Line of Best Fit - Part 4

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Books &amp; Materials</th>
<th>Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use technology to find the line of best fit.</td>
<td>Math in Focus B</td>
<td>Complete Warm-Up.</td>
</tr>
<tr>
<td></td>
<td>Math in Focus - Teacher Edition A</td>
<td>Complete the assigned pages in Math in Focus 3B.</td>
</tr>
<tr>
<td></td>
<td>Math in Focus - Teacher Edition B</td>
<td>Complete Practice Activity.</td>
</tr>
<tr>
<td></td>
<td>straightedge</td>
<td>Complete Use for Mastery.</td>
</tr>
<tr>
<td></td>
<td>online graphing calculator (Optional)</td>
<td></td>
</tr>
</tbody>
</table>

### LEARN

**WARM-UP**

1. In what form do you write an equation for a line of best fit?

2. What do the values of $m$ and $b$ represent in a linear equation?

### TEACHING NOTES

**WARM-UP ANSWERS**

1. In slope-intercept form: $y = mx + b$

2. $m$ represents the slope of the line, and $b$ is the $y$-intercept of the line.

### INSTRUCTION

Read through Technology Activity on pp. 194–195 in Math in Focus 3B. Refer back to the data you collected and the scatter plot you drew in Lesson 134. Draw a line of best fit on your scatter plot and write an equation for the line.

Then use your data to complete step 1. For step 4, you may find that the LinReg calculator feature draws a slightly different line of best fit that the one you drew by hand.

Compare your work to the results shown on the graphing calculator and describe the similarities and differences.
**TEACHING NOTES**

**Textbook Answer Key**

This activity will help your student learn the skills for making scatter plots and generating a line of best fit quickly by using a graphing calculator. If your student does not have access to a graphing calculator, then she can explore some of the graphing calculators that are available online.

---

**PRACTICE**

Continue using a graphing calculator to practice making a scatter plot and finding a line of best fit for some of the other problems you have completed in Chapter 10.

---

**WRAP-UP**

Today you learned how to use a graphing calculator to produce a scatter plot, find a line of best fit, and calculate the values of \( m \) and \( b \).

---

**USE**

---

**USE FOR MASTERY**

1. While it is often stated that the boiling point of water is 100°C (212°F), in reality water boils at different temperatures at different altitudes. Gerald collected data on the boiling point of water at different altitudes. His findings are as follows:

<table>
<thead>
<tr>
<th>Height (ft in 1,000s)</th>
<th>0</th>
<th>2.5</th>
<th>6.33</th>
<th>1.7</th>
<th>9.0</th>
<th>1.65</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boiling Point of Water (°F)</td>
<td>212</td>
<td>207</td>
<td>200</td>
<td>209</td>
<td>196</td>
<td>209</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Height (ft in 1,000s)</th>
<th>3.0</th>
<th>5.5</th>
<th>7.52</th>
<th>4.57</th>
<th>5.73</th>
<th>0.87</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boiling Point of Water (°F)</td>
<td>206</td>
<td>202</td>
<td>198</td>
<td>204</td>
<td>202</td>
<td>210</td>
</tr>
</tbody>
</table>
1. While it is often stated that the boiling point of water is 100°C (212°F), in reality water boils at different temperatures at different altitudes. Gerald collected data on the boiling point of water at different altitudes. His findings are as follows:

<table>
<thead>
<tr>
<th>Height (ft)</th>
<th>Boiling Point (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>212</td>
</tr>
<tr>
<td>2.5</td>
<td>207</td>
</tr>
<tr>
<td>6.33</td>
<td>200</td>
</tr>
<tr>
<td>1.7</td>
<td>209</td>
</tr>
<tr>
<td>9.0</td>
<td>196</td>
</tr>
<tr>
<td>1.65</td>
<td>209</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Height (ft)</th>
<th>Boiling Point (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>206</td>
</tr>
<tr>
<td>5.5</td>
<td>202</td>
</tr>
<tr>
<td>7.52</td>
<td>198</td>
</tr>
<tr>
<td>4.57</td>
<td>204</td>
</tr>
<tr>
<td>5.73</td>
<td>202</td>
</tr>
<tr>
<td>0.87</td>
<td>210</td>
</tr>
</tbody>
</table>

Download a blank graph [here](#). Construct a scatter plot for this data on the graph. Use 1 centimeter on the horizontal axis to represent 1,000 feet. Use 1 centimeter on the vertical axis to represent 2°F from the interval of 180°F to 220°F. Sketch a line that appears to best fit the data and label a point on the graph for 3,200 feet.

A. Upload your finished scatter plot here.

B. Write the equation for the line of best fit.

C. Find the point you labeled for 3,200 feet. What is the corresponding temperature?

D. Find the altitude if the boiling point of water is 190°F.

**USE FOR MASTERY GUIDELINES & RUBRIC**

Did you:

- Download a blank graph?
- Construct a scatter plot for this data on the graph?
- Sketch a line that appears to best fit the data and label a point on the graph for 3,200 feet?
- Write the equation for the line of best fit?
- Find the point you labeled for 3,200 feet and give the corresponding temperature at that height?
- Find the altitude if the boiling point of water is 190°F?
- Upload your finished scatter plot?
Two-Way Tables - Part 1

Objectives
- Read data in a two-way table.

Books & Materials
- Math in Focus B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete Practice Questions.

LEARN

WARM-UP
Find the relative frequency for each item in the table.

<table>
<thead>
<tr>
<th>Item</th>
<th>Number of Items</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shirt</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Pants</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Hat</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

WARM-UP ANSWERS

<table>
<thead>
<tr>
<th>Item</th>
<th>Number of Items</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shirt</td>
<td>5</td>
<td>$\frac{5}{12} \approx 0.42$</td>
</tr>
<tr>
<td>Pants</td>
<td>4</td>
<td>$\frac{4}{12} = 0.33$</td>
</tr>
<tr>
<td>Hat</td>
<td>3</td>
<td>$\frac{3}{12} = 0.25$</td>
</tr>
</tbody>
</table>

TEACHING NOTES

INSTRUCTION
Read the information about two-way tables on p. 198 in Math in Focus 3B. Categorical data, or qualitative data, cannot be measured numerically. In the sample, the categorical data include gender and whether or not a student wears glasses. Notice where the totals appear in the table: column totals.
appear in the bottom row, row totals appear in the right column, and the overall total appears in the bottom-right corner.

Review Example 7 on p. 199. The problem states that the number of adults polled is 100. The table shows that 60 men are polled. Use these numbers to find the number of women polled. You can complete the table by finding the sum across each row or down each column. Since the problem states that 100 people were polled, this is the number that should appear in the bottom-right entry. Both the row totals and the column totals should provide an overall total of 100. This will help you check your entries in the table.

Complete Guided Practice on p. 200. Remember to use information given in the problem as well as in the table to find the missing numbers.

PROGRESS CHECK

Review Example 7 on p. 199. The problem states that the number of adults polled is 100. The table shows that 60 men are polled. Use these numbers to find the number of women polled. You can complete the table by finding the sum across each row or down each column. Since the problem states that 100 people were polled, this is the number that should appear in the bottom-right entry. Both the row totals and the column totals should provide an overall total of 100. This will help you check your entries in the table.

Complete Guided Practice on p. 200. Remember to use information given in the problem as well as in the table to find the missing numbers.

Complete problems 1–9 of Practice 10.3 on p. 207 in Math in Focus 3B.

WRAP-UP

Today you learned how to read categorical data that is presented in a two-way table.

The following two-way table shows the results of a poll of 100 adults about whether they speak a foreign language. Some information is missing from the table.
<table>
<thead>
<tr>
<th>Gender</th>
<th>Only English</th>
<th>One Foreign Language</th>
<th>Two or More Foreign Languages</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>32</td>
<td>8</td>
<td>5</td>
<td>?</td>
</tr>
<tr>
<td>Women</td>
<td>41</td>
<td>10</td>
<td>?</td>
<td>55</td>
</tr>
<tr>
<td>Total</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>100</td>
</tr>
</tbody>
</table>

Total number of men: 100 – 55 = 45 men
Women who speak two or more foreign languages: 55 – 41 – 10 = 4
Total number of people who speak only English: 32 + 41 = 73
Total number of people who speak one foreign language: 8 + 10 = 18
Total number of people who speak two or more foreign languages: 5 + 4 = 9

✅ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
WARM-UP

The results of a poll of 100 people about whether they play a musical instrument are shown in the two-way table. Complete the table.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Instrument</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>Yes</td>
<td>28</td>
<td>24</td>
<td>52</td>
</tr>
<tr>
<td>Female</td>
<td>Yes</td>
<td>25</td>
<td>23</td>
<td>48</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>53</td>
<td>47</td>
<td>100</td>
</tr>
</tbody>
</table>

TEACHING NOTES

INSTRUCTION

Read *Construct and Interpret a Two-Way Table* on p. 201 in *Math in Focus 3B*. The two categories classified in this example are based on visual characteristics: color and shape. The sample table shows shapes placed in columns and colors placed in rows. The table could also be constructed with the shapes placed in rows and the colors in columns. It does not matter which category is placed in columns and which is placed in rows as long as the value entered in each cell is accurate. Notice the column totals, the row totals, and the overall total.
Review Example 8 on p. 202. This example includes a legend box to show what each letter represents in the tables that display the survey results. When data is presented in this manner, it may be helpful to use a tally chart to count and total the frequencies of each subcategory before constructing a two-way table.

Complete Guided Practice on p. 203.

**TEACHING NOTES**

**Textbook Answer Key**

In this lesson, your student will make a two-way table to represent given bivariate data. She will refer to a legend box to determine what each piece of data represents. She will then count and record the frequency for each subcategory in the appropriate cells of a two-way table. She will look for and describe any association between variables.

**PRACTICE**

Complete problems 10–12 of Practice 10.3 on p. 208 in *Math in Focus 3B*.

**WRAP-UP**

Today you learned how to make a two-way table to represent bivariate data.

A survey of 20 people was conducted regarding whether they are left- or right-handed and their preferred beverage.

<table>
<thead>
<tr>
<th>Handedness</th>
<th>R</th>
<th>L</th>
<th>R</th>
<th>R</th>
<th>R</th>
<th>L</th>
<th>R</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beverage</td>
<td>W</td>
<td>T</td>
<td>T</td>
<td>J</td>
<td>W</td>
<td>J</td>
<td>T</td>
<td>J</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Handedness</th>
<th>R</th>
<th>R</th>
<th>L</th>
<th>R</th>
<th>L</th>
<th>R</th>
<th>R</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beverage</td>
<td>J</td>
<td>W</td>
<td>T</td>
<td>W</td>
<td>W</td>
<td>T</td>
<td>W</td>
<td>T</td>
</tr>
</tbody>
</table>

L represents left-handed
R represents right-handed
J represents juice
T represents tea
W represents water

The two-way table displays the data.
Please go online to view and submit this assessment.

View the video Constructing a Two-Way Frequency Table to learn more about this kind of data display.
You can use spreadsheet software to create two-way frequency tables. You were given this information in the Wrap-up section of the last lesson part.

A survey of 20 people was conducted regarding whether they are left- or right-handed and their preferred beverage.

<table>
<thead>
<tr>
<th>Handedness</th>
<th>R</th>
<th>L</th>
<th>R</th>
<th>R</th>
<th>R</th>
<th>L</th>
<th>R</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beverage</td>
<td>W</td>
<td>T</td>
<td>T</td>
<td>J</td>
<td>W</td>
<td>J</td>
<td>T</td>
<td>J</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Handedness</th>
<th>R</th>
<th>R</th>
<th>L</th>
<th>R</th>
<th>L</th>
<th>R</th>
<th>R</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beverage</td>
<td>J</td>
<td>W</td>
<td>T</td>
<td>W</td>
<td>T</td>
<td>W</td>
<td>J</td>
<td>W</td>
</tr>
</tbody>
</table>

L represents left-handed
R represents right-handed
J represents juice
T represents tea
W represents water
Open your spreadsheet software and see if you can create this two-way table.

<table>
<thead>
<tr>
<th>Beverage</th>
<th>Water</th>
<th>Tea</th>
<th>Juice</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left-handed</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Right-handed</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>Total</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>20</td>
</tr>
</tbody>
</table>

Experiment with formatting your table by adding different colors of fill, using bold or italic type, and adding borders. If you are able, you can also try using formulas to find the totals.

✅ RATE YOUR ENTHUSIASM

Please go online to view and submit this assessment.
### Two-Way Tables - Part 4

#### Objectives
- Find relative frequencies in a two-way table.

#### Books & Materials
- Math in Focus B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

#### Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete Practice Questions.

---

#### LEARN

---

#### WARM-UP

Rewrite each fraction as an equivalent decimal.

1. \(\frac{8}{20}\)
2. \(\frac{6}{50}\)
3. \(\frac{24}{30}\)

---

#### INSTRUCTION

Read p.204 in Math in Focus 3B. Relative frequencies make comparisons easier and they make the association between variables more apparent. Recall that relative frequencies can be written as fractions, decimals, or percents.

The sample shows two ways of presenting relative frequencies: among columns or among rows. To show relative frequency among the columns, divide each frequency by its corresponding column total. To show relative frequency among rows, divide each frequency by its corresponding row total. The relative frequencies for any column or any row should always total 1.

Review Example 9 on p. 205. The relative frequencies for data in a two-way table vary depending upon which two categories in the table are being analyzed. Read each question carefully to determine whether to find the distribution among columns or rows.
In part a, you are finding the distribution of gender among sports, so the table is drawn to show the relative frequencies among columns. Divide the value in each data cell by the total number of people who prefer each sport to find the relative frequency.

In part b, you are finding the distribution of sports among genders, so the table is drawn to show the relative frequencies among rows. Divide the value in each data cell by the total number of people in each gender category to find the relative frequency.

Complete Guided Practice on p. 206. Read each question carefully to determine which relative frequencies you need to find. Check your answers by making sure the sum of the frequencies in each column and each row is 1. Keep in mind that due to rounding, your answer may be slightly greater than or less than 1.

### TEACHING NOTES

Textbook Answer Key

In this part, your student finds the relative frequencies of data within a two-way table. Finding the relative frequencies makes it easier to compare the distribution among categories. Monitor your student as she completes relative frequency tables to ensure she is correctly finding the distribution among columns or among rows as each question requires.

### PRACTICE

Complete problems 13–15 of Practice 10.3 on p. 208 in Math in Focus 3B.

### WRAP-UP

Today you learned how to find relative frequencies in two-way tables to compare the distribution among categories.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Activity</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reading</td>
<td>Gaming</td>
</tr>
<tr>
<td>Male</td>
<td>140</td>
<td>200</td>
</tr>
<tr>
<td>Female</td>
<td>170</td>
<td>120</td>
</tr>
<tr>
<td>Total</td>
<td>310</td>
<td>320</td>
</tr>
</tbody>
</table>

The following relative frequencies compare the distribution of the genders among each activity.
More female teens prefer reading than male teens.
More male teens prefer gaming than female teens.
More female teens prefer television than male teens.
Two-Way Tables - Part 5

Objectives
- Use knowledge of two-way tables to solve problems.

Books & Materials
- Math in Focus B
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 3B.
- Complete Practice Activity.
- Complete Use for Mastery.

LEARN

WARM-UP
1. Draw a scatter plot that shows a weak, negative, and nonlinear association.

2. Draw a scatter plot that shows a strong, positive, and linear association.

TEACHING NOTES

WARM-UP ANSWERS
1 Answers vary. Sample:

2 Answers vary. Sample:

INSTRUCTION
Read and complete Brain @ Work on p. 209 in Math in Focus 3B.
Compare the scatter plots shown in problem 1 with those presented earlier in Chapter 10. Analyze the scatter plots that show linear association, looking for differences from those presented in problem 1. Think about why it is incorrect to conclude linear association for either scatter plot in Brain @ Work.

In problem 2, look carefully at the information. Think of strategies you can use to find the information you need. For instance, you know that 500 students are surveyed and that 200 students are learning a second language. Use this information to find the number of students not learning a second language. Then you can use the column totals and the relative frequencies in each column to find the value for each cell.

Then think of how you can use what you have found so far to determine the number of students who are and are not learning music. Record all your new data in a two-way data table.

Textbook Answer Key

For problem 2 on p. 209, your student is essentially working backward to construct a two-way table. She should analyze the information given and devise strategies to find needed data values. She should recognize that she could calculate the frequencies for a two-way table once she finds the total number of students who are learning and are not learning a second language.

Write a constructed response to explain your answer to problem 1, part b on p. 209 in Math in Focus 3B.

Then complete the following problems.

1. Refer to problem 1 of Brain @ Work. Plot additional data points on each scatter plot so that the data show a nonlinear trend.

2. Make your own table showing relative frequencies for either the columns or the rows, similar to the one shown in problem 2 of Brain @ Work. Be sure that the sum of relative frequencies for each column or row is 1. Assign a total for one of the categories and an overall total. Then work backward to complete the two-way frequency table.

If your student has difficulty with Practice problem 2, walk through the steps she took to complete the second problem of Brain @ Work. Point out that she will follow a similar procedure to convert her relative frequency table into a two-way table with actual data values.
Today you applied mathematical concepts and skills to solve problems.

**USE FOR MASTERY**

1. Of the 400 students taking a second language, 70% are girls. Among the girls, \( \frac{1}{5} \) of them take German, 100 girls take Spanish, and the rest take French. Among the boys, \( \frac{1}{4} \) of them take French, 36 take Spanish, and the rest take German.

a. Complete the two-way table below to display the data.

<table>
<thead>
<tr>
<th></th>
<th>German</th>
<th>Spanish</th>
<th>French</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Find the relative frequencies among the rows. Round your answers to the nearest hundredth, where necessary.

<table>
<thead>
<tr>
<th></th>
<th>German</th>
<th>Spanish</th>
<th>French</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
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<td></td>
</tr>
<tr>
<td>Male</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
c. Interpret the meaning of the relative frequencies across the rows.

d. Find the relative frequencies among the columns. Round your answers to the nearest hundredth, where necessary.

<table>
<thead>
<tr>
<th></th>
<th>German</th>
<th>Spanish</th>
<th>French</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

e. Interpret the meaning of the relative frequencies across the columns.
Did you:

- Use the information given to complete the two-way table to display the data?
- Find the relative frequencies among the rows? Did you round your answers to the nearest hundredth, where necessary?
- Interpret the meaning of the relative frequencies across the rows?
- Find the relative frequencies among the columns? Did you round your answers to the nearest hundredth, where necessary?
- Interpret the meaning of the relative frequencies across the columns?
- Show all your work in an organized, logical manner?

**USE FOR MASTERY GUIDELINES & RUBRIC**
Show: Movie Profits

Books & Materials
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

SHOW

Draw the line of best fit in the scatter plot. What is the equation of the line of best fit? You are going to use this equation to predict the gross lifetime profits of four movies that opened this weekend. Go online and find four movies that opened this weekend and their gross profits for the weekend. Input this amount to determine what their estimated lifetime gross profits will be. Consider checking back in 4–6 months to see how much money the movie has made within the time.

FINAL PROJECT

1. Tell which four movies you picked, their gross profit for the weekend, and the predicted lifetime gross profit. Make sure to show all your work and include your sources.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>

0 / 10000 Word Limit
Upload all your work throughout the profit to show your data, your scatter plot, and how you found your answer.

COLLABORATION

Which of the four movies do you think will make the most money based on your scatter plot? Is this the movie that you think you would like the best? Which movie do you want to see the most? Respond to at least two of your peers.
Unit Quiz: Movie Profits

Books & Materials
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

☑️ UNIT QUIZ

Please go online to view and submit this assessment.
Appendix
This form is to be used when completing Use for Mastery assessments or Projects offline. Your assessment can then be scanned and uploaded into the correct lesson online.

Please Fill In This Form Completely

Student’s Name

Grade

Course Name

Lesson Title

Provide your answer in the space below.
## Fitness/Activity Center

**Student Facing Project Rubric**

Read the chart below to understand how your project will be scored. Your goal should be to earn all 4 points for each part.

<table>
<thead>
<tr>
<th>CRITERIA</th>
<th>4 POINTS</th>
<th>3 POINTS</th>
<th>2 POINTS</th>
<th>1 POINT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Creating a Table</td>
<td>You correctly created three tables showing the cost of each gym as a function of the number of months for the first 12 months.</td>
<td>You created three tables showing the cost of each gym as a function of the number of months for the first twelve months, but you made 1–3 computation errors.</td>
<td>You created three tables showing the cost of each gym as a function of the number of months for the first twelve months. Your table shows a consistent change equal to the cost per month, but you forgot to include the initial down payment.</td>
<td>You correctly created three tables showing the cost of each gym as a function of the number of months, but you did not include all 12 months, OR you correctly created only two tables showing the cost of each gym as a function of the number of months</td>
</tr>
<tr>
<td>Linear Equation</td>
<td>You wrote three linear equations that correctly reflect the enrollment fee as the ( y )-intercept and the cost per month as the slope.</td>
<td>You wrote three linear equations that reflect the down payment as the ( y )-intercept and the cost per month as the slope, although you made one error.</td>
<td>You wrote three linear equations, but you reversed the slope and the ( y )-intercept.</td>
<td>You wrote two linear equations that correctly reflect the down payment as the ( y )-intercept and the cost per month as the slope.</td>
</tr>
<tr>
<td>Graph</td>
<td>You correctly created a graph for each of the equations using straight lines that clearly define the points of intersection.</td>
<td>You created a graph for each of the equations using straight lines that clearly define the points of intersection, but you made one error.</td>
<td>You created a graph for each of the equations using straight lines that clearly define the points of intersection, but your graphs show the slope as the ( y )-intercept and the ( y )-intercept as the slope.</td>
<td>You correctly created a graph for two of the equations using straight lines that clearly define the points of intersection.</td>
</tr>
<tr>
<td>Determining the Number of Months in Which the Cost Is the Same</td>
<td>You correctly determined the month in which each pair of gyms would cost the same.</td>
<td>You determined the month in which each pair of gyms would cost the same, but you made one error.</td>
<td>You determined the month in which each pair of gyms would cost the same, but you made two or three errors.</td>
<td>You determined the month in which each pair of gyms would cost the same, but you made more than three errors.</td>
</tr>
<tr>
<td>Explanation</td>
<td>You showed all of your work, and you correctly explained how to find the answers using the tables, the equations, and the graphs.</td>
<td>You showed all of your work, and you correctly explained how to find the answers using two of the following methods: the tables, the equations, and the graphs.</td>
<td>You showed all of your work, and you correctly explained how to find the answers using one of the following methods: the tables, the equations, and the graphs.</td>
<td>You showed your work, but you did not explain how you found your answers.</td>
</tr>
</tbody>
</table>

**Total Possible Points: 20**
### Be an Architect

#### Student Facing Project Rubric

Read the chart below to understand how your project will be scored. Your goal should be to earn all 4 points for each part.

<table>
<thead>
<tr>
<th>CRITERIA</th>
<th>4 POINTS</th>
<th>3 POINTS</th>
<th>2 POINTS</th>
<th>1 POINT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copying on a Coordinate Plane</td>
<td>You correctly identified at least one pair of buildings that appear similar using a straightedge or ruler. You prove the points of at least one pair of buildings that appear similar using a straightedge or ruler.</td>
<td>You correctly identified at least one pair of buildings that appear similar using a straightedge or ruler, but you made one error.</td>
<td>You identify a pair of buildings that appear similar using a straightedge or ruler, but you made one error.</td>
<td>You did not use a straightedge or ruler to identify the points of at least one pair of buildings that appear similar using a straightedge or ruler.</td>
</tr>
<tr>
<td>Shape Copying</td>
<td>You correctly identified at least one pair of buildings that are similar. You provided a proof that shows each angle is congruent and the sides are proportional.</td>
<td>You correctly identified at least one pair of buildings that are similar, but you made one error. You provided a proof that shows each angle is congruent and the sides are proportional.</td>
<td>You correctly identified a pair of buildings that are similar. You provided a proof that shows each angle is congruent and the sides are proportional, but your measurements show 1–2 errors.</td>
<td>You correctly identified a pair of buildings that are similar, but you made more than 4 errors.</td>
</tr>
<tr>
<td>Congruency</td>
<td>You correctly identified a pair of buildings that are congruent. You provided a proof that shows that each side and angle are congruent.</td>
<td>You correctly identified a pair of buildings that are congruent. You provided a proof that shows that each side and angle are congruent, but your measurements show 1–2 errors.</td>
<td>You identified a pair of buildings that are congruent. You provided a proof that shows that each side and angle are congruent, but your measurements show 3–4 errors.</td>
<td>You identified a pair of buildings that are congruent. You provided a proof that shows that each side and angle are congruent, but your measurements show more than 4 errors.</td>
</tr>
<tr>
<td>Scale Factor</td>
<td>You correctly identified the scale factor between the two similar shapes. You showed the ratio that led you to the answer.</td>
<td>You identified the scale factor between the two similar shapes. You showed the ratio that led you to the answer, but you made one error.</td>
<td>You correctly identified the scale factor between the two similar shapes, but you made one error.</td>
<td>You correctly identified the scale factor between the two similar shapes, but you made more than 4 errors.</td>
</tr>
</tbody>
</table>

**Total Possible Points: 20**
## Movie Profits
### Student Facing Project Rubric

Read the chart below to understand how your project will be scored. Your goal should be to earn all 4 points for each part.

<table>
<thead>
<tr>
<th>CRITERIA</th>
<th>4 POINTS</th>
<th>3 POINTS</th>
<th>2 POINTS</th>
<th>1 POINT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Research</td>
<td>You created a table accurately showing the gross weekend profit along with the gross lifetime profit for at least 40 movies. You also included the resources used to find the data.</td>
<td>You created a table showing the gross weekend profit along with the gross lifetime profit for at least 40 movies, although you made a few errors. You also included the resources used to find the data.</td>
<td>You created a table showing the gross weekend profit along with the gross lifetime profit for at least 40 movies.</td>
<td>You created a table accurately showing the gross weekend profit along with the gross lifetime profit for 20–39 movies. You also included the resources used to find the data.</td>
</tr>
<tr>
<td>Scatter Plot</td>
<td>You correctly created a scatterplot of 40 data points with the gross opening weekend profits represented on the horizontal axis and the gross lifetime profits represented on the vertical axis.</td>
<td>You created a scatterplot of 40 data points with the gross opening weekend profits represented on the horizontal axis and the gross lifetime profits represented on the vertical axis, but you made 1–3 errors.</td>
<td>You created a scatterplot of 40 data points with the gross opening weekend profits represented on the horizontal axis and the gross lifetime profits represented on the vertical axis, but you made 4–6 errors.</td>
<td>You created a scatterplot of 40 data points with the gross opening weekend profits represented on the horizontal axis and the gross lifetime profits represented on the vertical axis, but you made 4–9 errors.</td>
</tr>
<tr>
<td>Correlation</td>
<td>You correctly described the correlation between the data points and identified any outliers.</td>
<td>You correctly described the correlation between the data points. You identified some, but not all of the outliers.</td>
<td>You correctly described the correlation between the data points, but you did not identify any outliers.</td>
<td>You incorrectly described the correlation between the data points, but you identified the outliers.</td>
</tr>
<tr>
<td>Line of Best Fit</td>
<td>You drew a line of best fit that is consistent with your scatter plot. You correctly wrote the equation for this line.</td>
<td>You drew a line of best fit that is consistent with your scatter plot. You wrote the equation for this line, but you made a slight error.</td>
<td>You forgot to draw the line of best fit on your scatter plot. The equation of the line that you drew would correctly represent the line of best fit.</td>
<td>You drew a line of best fit that is consistent with your scatter plot. You forgot to write the equation of the line.</td>
</tr>
<tr>
<td>Predictions</td>
<td>You correctly predicted the lifetime gross profits for at least four movies based on the equation of the line of best fit.</td>
<td>You predicted the lifetime gross profits for at least four movies based on the equation of the line of best fit, but you made 1–2 computation errors.</td>
<td>You predicted the lifetime gross profits for at least three movies based on the equation of the line of best fit.</td>
<td>You predicted the lifetime gross profits for at least three movies based on the equation of the line of best fit, but you made 1–2 computation errors.</td>
</tr>
</tbody>
</table>

Total Possible Points: 20
Exploring Angle Relationships Worksheet

Example 1: The Sum of Interior Angles
The sum of the interior angles in any triangle is 180°. If you know the measures of two of the interior angles, you can subtract the sum of those two angle measures from 180° to find the measure of the third interior angle.

\[ m\angle 1 = 180° - (70° + 55°) \]
\[ m\angle 1 = 180° - 125° \]
\[ m\angle 1 = 55° \]

Example 2: The Measure of an Exterior Angle
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

\[ m\angle 1 = m\angle A + m\angle C \]
\[ = 80° + 50° \]
\[ = 130° \]

To find the measure of angle 2, use the fact that the sum of the measures of the interior angles of a triangle is 180°.

\[ m\angle A + m\angle 2 + m\angle C = 180° \]
\[ 80° + m\angle 2 + 50° = 180° \]
\[ 130° + m\angle 2 = 180° \]
\[ m\angle 2 = 50° \]

You can check your work by using the fact that \( \angle 1 \) and \( \angle 2 \) are supplementary and the sum of the measures of supplementary angles is 180°.

\[ m\angle 1 + m\angle 2 = 180° \]
\[ 130° + 50° = 180° \]
Your Turn

Find the missing angle measures.

1. \( m\angle 1 = 180° - (\text{_______}° + \text{_______}°) = 180° - \text{_______}° = \text{_______}° \)

2. \( m\angle 2 = (\text{_______}° + \text{_______}°) = \text{_______}° \)

3. \( m\angle 3 = (\text{_______}° + \text{_______}°) = \text{_______}° \)

4. \( m\angle 4 = (\text{_______}° + \text{_______}°) = \text{_______}° \)

Example 3: Transversals

Three different types of angles are formed when two parallel lines are crossed by another line called a transversal. The angles formed are alternate interior angles, alternate exterior angles, and corresponding angles. Vertical, or opposite, angles are also formed.

If line \( x \) is parallel to line \( y \), then:

- The alternate interior angles are equal in measure. In the diagram, \( \angle 1 \) and \( \angle 2 \) and are alternate interior angles. So are \( \angle 4 \) and \( \angle 5 \).

- The alternate exterior angles are equal in measure. In the diagram, \( \angle 1 \) and \( \angle 8 \) are alternate exterior angles. So are \( \angle 2 \) and \( \angle 7 \).
The **corresponding angles** are congruent. In the diagram, ∠4 and ∠8 are corresponding angles. There are three other pairs of corresponding angles.

The **vertical angles** (also called opposite angles) are congruent. In the diagram, ∠1 and ∠4 are opposite angles. So are ∠2 and ∠3.

Any pair of angles between the two parallel lines and on the same side of the transversal are supplementary angles—the sum of their measures is 180°. In the diagram, ∠4 and ∠6 are supplementary.

**Your Turn**

In the diagram, lines x and y are parallel, and m∠2 = 70°. Find the unknown angle measures.

```
1. ∠2 and ∠3 are opposite angles. m∠3 = _______°
2. ∠3 and ∠6 are alternate interior angles. m∠6 = _______°
3. ∠3 and ∠4 are _______________ angles. m∠4 = _______°
4. ∠1 and ∠_______ are opposite angles. m∠1 = _______°
5. Find the remaining angle measures.
   m∠5 = _______°
m∠7 = _______°
m∠8 = _______°
```
Practice

1. Lines $x$ and $y$ are parallel, and $m\angle 7 = 115^\circ$. Find the measures of angles 1–6 and 8.

   \[
   \begin{array}{ccc}
   & x & \\
   y & 1 & 2 \\
   & 3 & 4 \\
   & z & \\
   & 5 & 6 \\
   & 7 & 8 \\
   \end{array}
   \]

   a. $m\angle 1 = \underline{\hspace{2cm}}$
   b. $m\angle 2 = \underline{\hspace{2cm}}$
   c. $m\angle 3 = \underline{\hspace{2cm}}$
   d. $m\angle 4 = \underline{\hspace{2cm}}$
   e. $m\angle 5 = \underline{\hspace{2cm}}$
   f. $m\angle 6 = \underline{\hspace{2cm}}$
   g. $m\angle 8 = \underline{\hspace{2cm}}$

2. Find the measures of angles 1, 2, and 3.

   \[
   \begin{array}{ccc}
   & \underline{\hspace{2cm}} & 25^\circ \\
   & 110^\circ & \\
   & 2 & 3 \\
   \end{array}
   \]

   a. $m\angle 1 = \underline{\hspace{2cm}}$
   b. $m\angle 2 = \underline{\hspace{2cm}}$
   c. $m\angle 3 = \underline{\hspace{2cm}}$

3. Draw and label a pair of parallel lines cut by a transversal. Label each angle that is formed. Then write as many true statements as you can about the angle measures.
Pythagorean Theorem Worksheet

Proving the Pythagorean Theorem

One way to prove the Pythagorean Theorem is to think of each side of a right triangle as the side of a square. To explore this proof, follow these steps.

1. Trace this right triangle.

![Trace the right triangle](image)

2. Use a straightedge to draw squares with sides that are the same lengths as the sides of the triangle. You can use the side of the triangle to measure each side of the square. You can use the corner of a sheet of paper to ensure that your squares have four right angles.

![Draw squares](image)
3. Make a copy of the smallest square and trace around it in the middle of the largest square.

4. Use your ruler to draw lines from each corner of the smallest square to the edge of the largest square.
5. Cut apart the largest square and rearrange the pieces. Because of how you constructed it, the square piece is the same size as the smallest square. The other pieces fit together to make a square that is the same size as the mid-sized square.
If you label the sides of the triangle: $a$, $b$, and $c$:

The area of the smallest square is $a^2$.
The area of the mid-sized square is $b^2$.
The area of the largest square is $c^2$.

Therefore, the area of the largest square is equal to the areas of the other two squares put together:

\[ a^2 + b^2 = c^2 \]
Your Turn

Fill in the blanks to complete the steps you just learned to show the Pythagorean Theorem. Then trace the following triangle and follow the steps.

1. Draw ______________ that have the same side lengths as the sides of the triangle.

2. Copy the ______________ square inside the ______________ one.

3. Draw lines from the corners of the ______________ square.

Cut out pieces of the largest square and rearrange them. Label the sides of the triangle: \( a \) for the shortest side, \( b \) for the mid-sized side, and \( c \) for the longest side.

4. The area of the smallest square is ________.

5. The area of the mid-sized square is ________.

6. The area of the largest square is ________.

7. Therefore, the area of the largest square is equal to the sum of the areas of the other two squares: 
   ________ + ________ = ________
Practice

1. Draw a right triangle that looks different than the ones on this sheet. Follow the steps of the activity to show the Pythagorean Theorem. Label your sides $a$, $b$, and $c$ and label your squares $a^2$, $b^2$, and $c^2$.

2. Explain how your drawing in problem 1 shows the Pythagorean Theorem.
A. Finding the Slope-Intercept Form Equation for a Line

The slope formula, \( m = \frac{y_2 - y_1}{x_2 - x_1} \), can also be written \( m = \frac{y - y_1}{x - x_1} \), where \((x, y)\) and \((x_1, y_1)\) are two points on the line. Any subscripts can be used to distinguish the two points on the line.

You can use the slope formula to write the equation of a line in slope-intercept form. For the following problem, you first use the point where the line crosses the \( y \)-axis, \((0, 2)\), and another point on the line, \((2, 3)\), to calculate the slope.

The line crosses the \( y \)-axis at \((0, 2)\).
\( x_1 = 0 \) and \( y_1 = 2 \)

The line also crosses point \((2, 3)\).
\( x = 2 \) and \( y = 3 \)

\[
m = \frac{y - y_1}{x - x_1} = \frac{3 - 2}{2 - 0} = \frac{1}{2}
\]

The slope is \( \frac{1}{2} \), so \( m = \frac{1}{2} \).

Now that you know the slope, \( m \), use the slope formula again and substitute the values for \( m, x, \) and \( y \).

\[
m = \frac{y - y_1}{x - x_1}
\]

\[
\frac{1}{2} = \frac{y - 2}{x - 0}
\]

\[
\frac{1}{2} = \frac{y - 2}{x}
\]
Then rewrite the equation in terms of $y$.

\[ \frac{1}{2} x = y - 2 \]
\[ \frac{1}{2} x = y - 2 \]
\[ \frac{1}{2} x + 2 = y \]
\[ y = \frac{1}{2} x + 2 \]

Notice that $m = \frac{1}{2}$ and $b = 2$, and this equation is in the form $y = mx + b$.

**Specific equation** | **General equation**
---|---
$y = \frac{1}{2} x + 2$ | $y = mx + b$

**B. Writing the General Form of the Slope-Intercept Equation**

Recall that the $y$-intercept is the point where a line crosses the $y$-axis. This point is written as $(0, b)$, where $b$ represents the $y$-intercept. Therefore, the slope of any line that passes through points $(x, y)$ and $(0, b)$ can be written in the following way:

\[ m = \frac{y - y_1}{x - x_1} = \frac{y - b}{x - 0} \]

You use this formula to write the equation for a specific line in slope-intercept form. You can also use it to write the general form of the equation for any nonvertical line that crosses the $y$-axis.

The line crosses the $y$-axis at $(0, b)$, where $b$ is the $y$-intercept.
Rewrite the equation in terms of $y$.

$$m = \frac{y - b}{x}$$

$$mx = y - b$$

$$mx + b = y$$

$$y = mx + b$$

This is the general form of the slope-intercept equation.

**Your Turn**

Use the slope formula to write the slope-intercept form for a direct proportion.

The line crosses the $y$-axis at $(0, b)$, where $b$ is the $y$-intercept.

1. For a direct proportion, $b = \underline{\phantom{1}}$.

2. So, $x_1 = \underline{\phantom{1}}$ and $y_1 = \underline{\phantom{1}}$.

Write the missing numbers.

$$m = \frac{y - y_1}{x - x_1}$$

3. $m = \underline{\phantom{1}}$

4. $m = \underline{\phantom{1}}$
Rewrite the equation in terms of \( y \).

5. \( m = \) \\

6. \( m = \) \\

or \( y = mx \)

**General equation for direct proportion**

\[
y = mx
\]

slope \( y \)-intercept = 0

**Practice**

Determine if the following equations are in slope-intercept form. Write *yes* or *no*. If not, rewrite the equation in slope-intercept form. Then identify the slope and \( y \)-intercept.

1. \( 4y = 3x + 5 \)
   - a. slope-intercept form? 
   - b. If not, rewrite. 
   - c. slope: \( m = \) 
   - d. \( y \)-intercept: \( b = \)

2. \( -2x = -2y - \frac{2}{3} \)
   - a. slope-intercept form? 
   - b. If not, rewrite. 
   - c. slope: \( m = \) 
   - d. \( y \)-intercept: \( b = \)

3. \( y = \frac{5}{2}x + (-4) \)
   - a. slope-intercept form? 
   - b. If not, rewrite. 
   - c. slope: \( m = \) 
   - d. \( y \)-intercept: \( b = \)

4. \( y = 5 - \frac{1}{7}x \)
   - a. slope-intercept form? 
   - b. If not, rewrite. 
   - c. slope: \( m = \) 
   - d. \( y \)-intercept: \( b = \)
5. Use the slope formula to write an equation for the following graph in slope-intercept form. Include specific values for \( m \) and \( b \). Show your work.

![Graph of a line](image)

a. What is the slope? ____________________

b. Write the equation in slope-intercept form. Show your work.
6. Gerrard found the slope and $y$-intercept of a line using an equation for the line. Is his answer correct? If not, explain what he did wrong and provide the correct answer. Write a constructed response to explain your answer.

Write the slope and $y$-intercept of this equation: $-3y = \frac{3}{4}x + (-2)$

Answer: slope: $\frac{3}{4}$  
$y$-intercept: -2