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*The Assessment/File Upload Form and many worksheets in the appendix will be used multiple times throughout this course. Please make additional copies of these pages.
Getting Started
WELCOME TO CALVERT!
We are glad you have selected our curriculum. Please take the time to read the information that follows.

Note: This lesson part, "Welcome to Calvert," is identical for all courses. Once it is finished, it will be marked complete for each course.

If you are the Learning Guide, please make sure you are logged in and have the Teaching Notes enabled. You can do this by clicking on the Teaching Notes toggle, as shown here:

CALVERT’S PLUS CURRICULUM
You will learn using Calvert's PLUS curriculum framework. Our framework is designed to motivate and engage you by using a research-based, digitally supported instructional approach.

WHY DO WE CALL THIS THE PLUS FRAMEWORK?
Our PLUS framework includes Project-Based Learning, Active Learning, Use for Mastery, and Show elements. Details on each element appear below.

Project - Projects are designed to give you fun, engaging, real-world opportunities to creatively show what you have learned. You can also collaborate with other students in the same course.
Learn - Our courses contain a variety of active learning opportunities, including interactive digital activities designed to encourage you to think independently and Quick Checks to assess your understanding.

Use - You will complete a Use for Mastery assessment at the end of each lesson to make sure you have achieved a deeper knowledge (and have "mastered" the concepts).

Show - We offer many creative and exciting opportunities for you to showcase what you have learned. You can submit audio, images, and videos from your computer or mobile device for a teacher to evaluate.

You can view the following video to learn more about the PLUS framework.

Your course is divided into units. Units are made up of lessons, and a lesson is split into lesson parts. Each lesson part is planned to be a day's work.

Please go online to view this video ▶

WHAT YOU WILL FIND IN YOUR COURSE

PROJECT OPENER

Some units in your course are built around a project. When there is a project in your unit, you will see an introduction and description in the beginning of the unit that will tell you:

• What the project will be about
• What you will be doing as part of the project
• How the project will be graded
• Any work that needs to be created or submitted as part of the project

Projects often encourage you to be creative by adding audio, video, or images to make your presentation more interesting and informative. For hints and tips on creating and uploading your projects, click here.

LESSON PARTS

Each unit is made up of lessons. Each lesson helps you learn a new idea in the unit. The lessons are divided into parts. Each part makes up one day's work.

SHOW

“Show” lessons are places in the unit that focus on your project. They give you a chance to show what you have
learned so far and help you make progress on your project. You can check to see where you are in the project and how your work will be scored.

UNIT QUIZ

At the end of every unit, a unit quiz checks your understanding of all the concepts from the unit. Some questions will be scored by the computer, and some will be marked by your teacher.

In lower grades, the Learning Guide will need to help Grade K and Grade 1 students by reading assessments aloud in cases where Text-to-Speech is not available and taking dictation to submit students’ answers online or helping them to upload responses completed using paper and pencil.

You can view the following video to learn more about what you will find in a course.

[Please go online to view this video]

WHAT YOU WILL FIND IN A LESSON

At the beginning of each lesson, you will see a lesson title and part number at the top of the screen. You will also see resource buttons to the right of the screen. These resource buttons will identify what you will be working on for your project (if applicable) and will also include lesson objectives, books and materials, assignments, as well as the ability to use Text-to-Speech and print the lesson.
RESOURCE BUTTONS
Here's what each resource button will include:

- **Project** – The Project button provides a short description of the project you are doing as part of the lesson.

- **Objectives** – Objectives are statements that describe what you will be learning. The objective will be your goal for the lesson across all lesson parts.

- **Assignments** – The Assignments list highlights the lesson's work at a glance. This list includes reading assignments, labs, activities, and exercises.

- **Books & Materials** – All books and materials needed for the day's lesson are listed here. You may find it helpful to review this list before each day's lesson part.

- **Standards** show how each lesson is aligned with national or state standards.

- **Text-to-Speech** will read the page text aloud or allow you to look up the definition of a word that appears in the lesson.

- **Print** allows you to print the lesson, unit, or course you are currently viewing.

You can view the following video to learn more about what your course and lessons will look like.

Please go online to view this video ▶

COLORS AND CARD TYPES

COLORS
Each lesson card is color-coded.

- **Green** refers to Learn sections.
- **Purple** refers to Use sections.
- **Orange** refers to Project/Show sections.
CARD TYPES

All content in a lesson part is laid out as a series of cards. Each card indicates a distinct activity that you will do as part of your daily work. Here are the different types of cards:

- **Collaboration** is a way you can share information, data, or projects with other Calvert students in your school. Calvert uses an online collaborative tool to allow you to chat with other students in the classes in specifically designed lessons.

- **Final Project** cards will be a place to showcase what you have learned at the end of your project. You can be creative and submit audio, images, or video from your computer or from your mobile device.

- **Interactive Activities** are fun digital tools that will help you learn more about a topic. Interactive Activities are digital activities that may include virtual labs, simulations, videos, and more.

- **More to Explore** is additional content that can help you either learn more about a concept or help you understand a new concept. More to Explores can include videos, additional readings, or digital activities that help you apply knowledge of a concept a different way.

- Some projects are designed to be completed one piece at a time. **Project Progress** cards provide the opportunity to share pieces of project work for feedback in advance of pulling all the pieces together for the final Show.

- **Quick Checks** are short assessments that will help you clarify what topics you have mastered and what concepts you may need to review. After you complete a Quick Check, you will be given the correct answer and a resource to help you review the concept in a new way.

- We want to check in with you to see how you're feeling about your lessons. **Rate Your Enthusiasm** will appear periodically after your lessons, so you can give us real-time feedback during your course.

- We want to check in with you to see how excited you are to begin a project. **Rate Your Excitement** will appear periodically after your lessons so you can give us real-time feedback while you complete each course.
We want to check in with you to see how you are progressing through your project. Rate Your Progress will appear on some of the days you are working on a project so you can let us know where you are in the project and how things are going.

We want to check in with you to see how ready you feel for the course. Rate Your Readiness will appear in lessons in the Getting Started unit.

We want to check in with you to see how you are understanding each lesson part. Rate Your Understanding will appear periodically after your lessons so you can give us real-time feedback while you complete each course.

At the end of every unit, we provide a Unit Quiz where you will be assessed on your understanding of all the key concepts learned in that unit. The concepts that are tested are based on the key standards identified by your state.

Each lesson has a Use for Mastery assessment. These open-ended response questions help assess how well you understood the lesson concepts. The 'Use For Mastery Guidelines & Rubric' below each question will provide helpful information on how and what to submit for your response. You may be asked to type into a text box or upload a document.

ONLINE PLATFORM ACCESS

You can complete our course using a fully online approach with access to a computer or with a hybrid approach, with the help of printed materials. When online, you can use our content in one of two ways:


2. If you are viewing the Calvert product through your school's LMS, please contact your school for how to get access.

Please review our Technology Requirements to make sure your computer is set up to allow full access to our courses.
SUGGESTED DAILY SCHEDULE

The following is a suggested daily schedule as it displays in CTN. Although each subject can be studied in a designated order, know that you can adapt the schedule and pace to meet your individual educational needs.

A complete course is planned for an average school year of about nine months. There are 160–180 daily lesson parts in a course. The number of lesson parts and tests for individual subjects will vary based on the amount of material that must be covered in the course during the school year.

Each day, we recommend that you spend approximately 120-150 minutes in grades K-2 and 100-120 minutes in grades 3-8 on English Language Arts, 45 minutes on Math, 45 minutes on Science, 45 minutes on Social Studies, and 30 minutes reading independently.

You can view the following video to learn more about the Suggested Daily Schedule.

Please go online to view this video ▶

KNOW YOUR ROLE

ROLE OF THE LEARNING GUIDE

The Learning Guide is a responsible adult (usually a parent) who guides the student through his or her academic journey.
Your certified school teacher directs the instruction, determines the pacing, and makes decisions for intervention and enrichment. However, the Learning Guide has an essential role in helping you on the road to academic success.

The Learning Guide has access to all the course materials. Additionally, teacher-specific instructions (Teaching Notes) written specifically to the Learning Guide or instructor give information, directions, and suggestions for leading you through a lesson.

When Teaching Notes are enabled, teacher-specific instructions for a card will appear just below that card.

You can view the following video to learn more about the role of Teaching Notes and the Learning Guide.

Please go online to view this video ▶

ROLE OF THE STUDENT

While the lessons in this curriculum are written to you, the student, that does not mean you are expected to work completely on your own. Keep in mind that your Learning Guide is here to support and help you. You and your Learning Guide will work as partners. Together you will decide which assignments you will work on independently and which you will do jointly. During the course, there will be times when you will be directed to read a selection aloud for your Learning Guide, share information you have learned, or take part in a discussion.

When working on your own, ask for your Learning Guide's assistance if you have any questions or if directions do not seem clear. You should also check with your Learning Guide before linking to any of the websites listed in the lessons or activities.

ROLE OF THE CALVERT SUPPORT STAFF

At Calvert, we understand the importance of having support when you need it. We offer many resources to help you along the way. If you have a question about our curriculum, our Education Counselors are available to help you Monday through Friday, 9:00 a.m. to 5:00 p.m. Eastern time, by phone at 1-888-487-4652, or email at support@calvertservices.org.

Please go online to view and submit this assessment.
PRINT VS. DIGITAL EXPERIENCE

If you plan to do this course exclusively online, you will have access to all the course material digitally.

If you are going to complete some of this course offline, you might have already received a printed version of the lesson manual. If not, you can print at any time using our Print-On-Demand functionality. Using this functionality, you can print a single lesson, an entire unit, or the entire course.

Print-On-Demand does not print the textbooks that you will need as part of your course. Please contact your school directly to have the textbooks shipped to you.

As part of your project work or assessment, you may be required to submit a file, image, or video to your teacher. To do this, you will need access to a computer and a camera-equipped mobile phone.

WORKSHEETS

If you are working in the print version of our lessons, all the worksheets that are needed to complete the course are provided in the Appendix as part of the printed packet. Otherwise, PDFs of all worksheets will be linked to the individual lessons. You will need Adobe Reader® to use these worksheets. Most of these worksheets are fillable, and you can use your computer keyboard to type directly in them and save them on your computer.

NOTEBOOKS AND JOURNALS

You may be directed to use a notebook throughout this course. The Math Notebook should be used to reflect on your learning and can serve as a single place to record information as you move through the course. You can take notes in your physical notebook or even digitally by using an application such as Evernote®.

ONLINE ACTIVITIES

Your course may include interactive digital activities, videos from publishers such as YouTube®, virtual simulations, and digital assessments that cannot be completed without going online.
BOOKS AND MATERIALS

MATH IN FOCUS TEXTBOOK

You will find textbook page numbers in the lesson that are underlined. We refer to this as hyperlinking. Clicking directly on the link opens the corresponding page of the textbook. You can then scroll through the pages of your textbook.

The e-text will not allow you to directly type into any blanks.

INSTRUCTIONAL VIDEOS

The Math in Focus course is based on the Singapore Math method, which may be new to some Learning Guides. For this reason, Calvert Learning has produced a series of instructional videos to provide training in the basics of this method. These videos will be linked directly in the appropriate lessons, but for your convenience they are listed and linked here as well:

How to Teach Problem Solving
How to Teach Bar Models, Part 1
How to Teach Bar Models, Part 2
How to Teach Bar Models, Part 3

Calvert Learning instructional videos for students are also available to provide review of important skills in preparation for learning new material. These videos are directly linked to the lessons where appropriate.
BRAINPOP®

Calvert Learning is pleased to offer BrainPOP®, an engaging web-based interactive program that supports the core curriculum. BrainPOP® activities include animated video tutorials, interactive activities, and assessments that provide a rich, multisensory experience designed to improve learning. These research-based activities were developed in accordance with national and state academic standards. These engaging activities are accessed through the online course. When a BrainPOP® activity is appropriate for a lesson, the link is located with the online lesson for that day. Click on the link, and you will be directed to the instructional activities.

DISCOVERY EDUCATION™ VIDEOS

Your course may include videos from Discovery Education™, which provides thousands of subject and grade specific videos to enrich your learning experience. Discovery EducationTM videos have been aligned to lessons throughout the Calvert curriculum to reinforce lesson objectives. These videos can be accessed through the online lessons in Grades K–8. If a video has been aligned to a lesson, you will find a link to that video in the online lesson.

ADDITIONAL MATERIALS

We have included many resources designed to provide additional help and support as you complete your course. These supplementary resources are provided to you in the appropriate lessons as downloadable PDFs that you can print as needed.

Your course may also use these materials that are commonly found throughout your home.

Please go online to view this video ➤

RATE YOUR READINESS

Please go online to view and submit this assessment.
Unit 1 - Rational Number System
Rational Numbers - Part 1

Objectives
- Find the absolute value of rational numbers.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition
- tape or string (Optional)

Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 2A.
- Complete Practice Questions.

WARM-UP
Evaluate.

1. $18^2$
2. $9^3$
3. $\sqrt{169}$
4. $\sqrt{125}$

LEARN

TEACHING NOTES

WARM-UP ANSWERS
1. 324 2. 729 3. 13 4. 5

INSTRUCTION

Read page 6 in Math in Focus 2A. The sign in front of a number indicates its location with respect to 0 on a number line. If a number has no sign, you can assume it is positive. Negative numbers are to the left of 0, and positive numbers are to the right. The absolute value of a number tells you how far the number is from zero. Since distances are always positive, the absolute value of any number is positive. What is the absolute value of 0? Since 0 is already at 0 on the number line, its absolute value is zero.

Complete Quick Check on page 6.

Read Find the Absolute Values of Positive Fractions on page 7. Just as with integers, the absolute value of a fraction is its distance from zero on a number line.
Then read p. 8. The opposite of an integer is the number on the opposite side and same distance away from zero on the number line. In the same way, the opposite of a fraction is the fraction on the opposite side and same distance away from zero on the number line. List two pairs of fractions that are opposites. One example is $\frac{1}{2}$ and $-\frac{1}{2}$.

Read Example 1 on p. 9. Notice that the farther a number is from zero on the number line, the greater its absolute value. Even if one number is positive and one is negative, if the negative number is farther from zero, its absolute value is greater.

Complete Guided Practice on p. 9.

HELPFUL ONLINE RESOURCE

Instructional Video: Integers, Rational Numbers, and Absolute Value

---

### TEACHING NOTES

Textbook Answer Key:

WATCH FOR THESE COMMON ERRORS

Some students confuse finding the absolute value of a number with finding the opposite of a number. For example, your student may say that the absolute value of $-\frac{2}{3}$ is $\frac{2}{3}$ and that the absolute value of $\frac{1}{5}$ is $-\frac{1}{5}$. He must understand that the absolute value of any number is positive.

---

### PRACTICE

Complete problems 1–4 and 28–35 of Practice 1.1 on pp. 14–15 in Math in Focus 2A.

---

### WRAP-UP

Today you learned how to find the absolute value of fractions and decimals.

\[
\left| \frac{4}{5} \right| = \left| \frac{4}{5} \right|
\]
\[
\left| -\frac{15}{6} \right| = \left| \frac{15}{6} \right|
\]
\[
\left| -2.7 \right| = \left| 2.7 \right|
\]

You also learned to compare numbers’ distances from zero.

$-1.86$ is closer to zero than $2\frac{6}{11}$ because $|-1.86| < \left| 2\frac{6}{11} \right|$. 
Please go online to view and submit this assessment.
Rational Numbers - Part 2

**Objectives**
- Identify rational numbers.
- Represent rational numbers in mnform.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition
- decimal place-value chart (Optional)

**Assignments**
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 2A.
- Complete Practice Questions.

---

### LEARN

#### WARM-UP

Solve.

1. Write $3\frac{8}{11}$ as an improper fraction.
2. Write $\frac{76}{8}$ as a mixed number in simplest form.
3. For the number 5.13, in what place is the digit 3?

---

### TEACHING NOTES

**WARM-UP ANSWERS**
1. $4\frac{1}{11}$
2. $9\frac{1}{2}$
3. hundredths

---

### INSTRUCTION

Read **Express Integers and Fractions in $\frac{m}{n}$ Form** on p. 9 in Math in Focus 2A. Notice that the term rational number contains the root word ratio. A rational number can be written in fractional form, like a ratio. Be sure to read and discuss the Think Math section on p. 9 with your Learning Guide. Remember that a fraction can also be renamed as a division problem.

$$\frac{4}{5} = 4 \div 5$$

Review **Example 2** on p. 10. Then complete **Guided Practice** on p. 10.

Read **Express Decimals in $\frac{m}{n}$ Form** on p. 11. Remember that you can use place value to write decimals as fractions.

Read **Example 3** on p. 11. Then complete the Guided Practice. You can check your work by working backward and rewriting your answers as equivalent decimals. The decimal should match the original number.

---

Grade 7 Calvert Math in Focus 20 Unit 1
Read **Locate Rational Numbers on the Number Line** on p. 12. Then read **Example 4** on p. 12.

Complete **Guided Practice** on p. 13. Remember that writing improper fractions as mixed numbers can help you determine where the numbers lie on the number line.

---

**TEACHING NOTES**

**Textbook Answer Key**

If your student struggles with expressing decimals in $\frac{m}{n}$ form, review the concept of place value and have him write each decimal number using a place-value chart. Ask him to read the number aloud using the place-value chart and then write the number as a fraction. Then have your student simplify his answer.

Point out to your student that when writing a negative number in $\frac{m}{n}$ form, the negative sign can be placed in front of the entire fraction, in the numerator, or in the denominator. Make sure he places the sign in only one of these three locations.

---

**PRACTICE**

Complete problems 5–27 of **Practice 1.1** on pp. 14–15 in **Math in Focus 2A**.

---

**WRAP-UP**

Today you learned how to express integers and fractions in $\frac{m}{n}$ form.

$$24 = \frac{24}{1}$$

$$-\frac{18}{27} = -\frac{2}{3}$$

You learned how to express decimals and mixed numbers in $\frac{m}{n}$ form.

$$6.7 = \frac{67}{10}$$

$$-7\frac{3}{5} = -\frac{38}{5}$$

You also learned how to locate rational numbers on a number line.

---

[Diagram showing rational numbers on a number line]
PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Rational Numbers - Part 3

**Objectives**
- Use long division to rename fractions as repeating or terminating decimals.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition
- Calculator
- Spreadsheet program or grid paper (Optional)

**Assignments**
- Complete Warm-up.
- Read and complete pages in Math in Focus A.
- Complete problems 1-20, in Math in Focus A.
- Complete the Quick Check.

---

**WARM-UP**
Solve.

1. $54 \div 2$
2. $324 \div 3$
3. Which is greater, $\frac{2}{3}$ or $\frac{5}{8}$? How can you tell?

---

**TEACHING NOTES**

**WARM-UP ANSWERS**
1. 27  2. 108  3. $\frac{2}{3}$; Sample answer: I can use equivalent denominators or use a model.

---

**INSTRUCTION**

Read p.16 in *Math in Focus 2A*. Notice that any fraction can be written as a division problem by dividing the numerator by the denominator. Read and discuss the Think Math question with your Learning Guide. Remember that decimal place value is based on powers of ten.

Review Example 5 on p.17. Then complete Guided Practice on p.17. Check that your answers are reasonable. For example, if the given fraction is less than 1, then your answer should be a decimal less than 1.

Read the instructional section on p.18 (in the purple box). Think about how the steps of the long division process indicate whether a fraction is equivalent to a repeating or terminating decimal.

Review Example 6 on pp.18–19. Then complete Guided Practice on p.19. For problem 7, be careful when identifying the repeating digits.
Read **Write Repeating Decimals Using Bar Notation** on p. 20. When using bar notation, it is very important to only draw the bar over digits that repeat. For example, to write the decimal 2.01111… using bar notation, the bar is only placed over the 1, not the zero: \( 2.0\overline{1} \). Writing \( 2.0\overline{1} \) means 2.01010101010101…

Complete **Guided Practice** on p. 20.

---

### TEACHING NOTES

**Textbook Answer Key**

If your student struggles with expressing fractions as decimals, have him solve a basic long division problem. Point out that these same steps are used to write a fraction as a decimal.

Your student may also find it helpful to write a fraction using the ÷ symbol before setting up the long division. For example, he might write \( \frac{5}{6} = 5 ÷ 6 \) and then divide 6 ÷ 5.

If you would like, have your student complete the **Technology Activity** and **Math Journal** on pp. 20–21.

---

### PRACTICE

Complete problems 1–20 of **Practice 1.2** on p. 25 in *Math in Focus 2A*.

---

### WRAP-UP

Today you learned how to write a rational number as a terminating decimal using long division.

\[
\frac{1}{5} = \begin{array}{c} 0.2 \\ \hline 1.0 \\ 10 \\ 0 \end{array}
\]
You learned how to write a rational number as a repeating decimal using long division.

\[
\frac{1}{6} = \frac{0.166\ldots}{6) \overline{1.000}}
\]

\[
\begin{array}{c}
6 \\
\underline{-6} \\
\hline
40 \\
\underline{-36} \\
\hline
4 \\
\end{array}
\]

You also learned how to write a repeating decimal using bar notation.

\[0.166\ldots = 0.1\overline{6}\]

✔ QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

A fraction is a way to represent division. View the Discovery video Decimals: Terminating and Repeating to learn how fractions can be represented as decimals.
Rational Numbers - Part 4

**Objectives**
- Compare rational numbers.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition

**Assignments**
- Complete Warm-up.
- Read and complete pages in *Math in Focus 2A*.
- Complete problems 21–23 in *Math in Focus 2A*.
- Complete the Quick Check.

---

**LEARN**

**WARM-UP**

Compare the numbers using < or >.

1. 5469  <  5482
2. −5469  <  −5482
3. $\frac{2}{7}$  <  $\frac{5}{7}$
4. $\frac{8}{11}$  >  $\frac{6}{11}$

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. <
2. >
3. <
4. >
Read Compare Rational Numbers on p. 21 in Math in Focus 2A. Another way to compare rational numbers is to rewrite the fractions with a common denominator and then compare the numerators. For example, $\frac{3}{4} = \frac{15}{20}$ and $\frac{4}{5} = \frac{16}{20}$. Since $15 < 16$, $\frac{15}{20} < \frac{16}{20}$, or $\frac{3}{4} < \frac{4}{5}$. Note that this method may not be the best choice, particularly if the denominators are very large.

Review Example 7 on p. 22. Then complete Guided Practice on p. 22. For problem 12, decide whether you prefer to write each fraction as a decimal or to rewrite the fractions with a common denominator.

**TEACHING NOTES**

Textbook Answer Key

Check your student’s understanding by asking him to explain how he compared the numbers in problems 13–15 of Guided Practice on p. 24. Discuss which method of comparing numbers your student prefers and why. Listen for conceptual understanding and explanations that one method may be preferred in one situation and another method in a different situation.

**PRACTICE**

Complete problems 21–23 of Practice 1.2 on p. 25 in Math in Focus 2A.

**TEACHING NOTES**

When working through Hands-On Activity, Technology Activity, and Practice sections, you will notice that some of the problems are designated as Math Journal activities. These will require longer written responses, so you may choose to have your student keep these in a separate Math Notebook, or he may write the responses directly on his work page.
You also learned how to compare two negative rational numbers.

\[
\frac{12}{5} = 2.4 \\
\frac{22}{9} = 2.444... \\
= 2.\overline{4}
\]

2.4 lies to the left of 2.\overline{4}, so \( \frac{12}{5} < \frac{22}{9} \).

You also learned how to compare two positive rational numbers.

\[-\frac{45}{8} = -5.625 \\
-\frac{37}{6} = -6.1\overline{6}
\]

\[|-5.625| < |-6.1\overline{6}|\]

\[-5.625 > -6.1\overline{6} \\
-\frac{45}{8} > -\frac{37}{6}\]

You also learned how to compare two negative rational numbers.

\[
\frac{12}{5} = 2.4 \\
\frac{22}{9} = 2.444... \\
= 2.\overline{4}
\]

2.4 lies to the left of 2.\overline{4}, so \( \frac{12}{5} < \frac{22}{9} \).

You also learned how to compare two positive rational numbers.

\[-\frac{45}{8} = -5.625 \\
-\frac{37}{6} = -6.1\overline{6}
\]

\[|-5.625| < |-6.1\overline{6}|\]

\[-5.625 > -6.1\overline{6} \\
-\frac{45}{8} > -\frac{37}{6}\]
Follow these instructions for the activity shown below.

Click [here](#) to see the activity in a new window.

First practice using the Gizmo. Click the “zoom in” (+) button to the right of the number line twice so that the number line only shows the interval between –1 and 1. Drag the purple dot to the left of the number line to 0, and the green dot to the right of the number line to 0.5. How does the position of the dots on the number line show which value is greater? In your Math Notebook, write an inequality for these two numbers. To check, turn on Compare numbers. You will see that 0.5 is greater than 0.

Click “Reset” and click off Compare numbers. Compare each of the following pairs of numbers. Write inequalities in your Math Notebook; then check your work in the Gizmo.

1. \(-\frac{3}{4}\) and \(\frac{5}{12}\)
2. \(\frac{7}{12}\) and \(-\frac{1}{3}\)
3. \(-\frac{2}{3}\) and \(-\frac{5}{6}\)

Now press “zoom out” (–) once so the range is –2 to 2. Move the purple dot to \(2\frac{1}{4}\). Click on Show opposites. You can see that \(-2\frac{1}{4}\) is the opposite of \(2\frac{1}{4}\). Opposites are the same distance away from zero but on different ends of the number line. Now turn on Show absolute values. The symbol for absolute value is vertical bars. For example, the equation \("|2\frac{1}{4}| = 2\frac{1}{4}\"\) means “the absolute value of \(2\frac{1}{4}\) is \(2\frac{1}{4}\).” The absolute value represents the distance from zero on the number line, regardless of direction.
For each of the following numbers, find its opposite and its absolute value. Check your work in the Gizmo.

1. $\frac{7}{12}$
2. $-\frac{11}{12}$
3. $-1\frac{3}{4}$
4. $2\frac{1}{12}$

**Answers:**

1. $-\frac{3}{4} < \frac{5}{12}$
2. $\frac{7}{12} > -\frac{1}{3}$
3. $-\frac{2}{3} < -\frac{5}{6}$

You can download the Exploration Sheet Answer Key, as well as other helpful teaching resources, by clicking on Lesson Info.

Please go online to view and submit this assessment.
USE FOR MASTERY

Write two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, where $a$, $b$, $c$, and $d$ are the digits 2, 3, 4, or 5, such that no digit can be used more than once and that the two rational numbers are the greatest distance apart on the number line.

Explain how you got this answer.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Only use each digit once when figuring out the possible pairs of rational numbers?
- Show which two rational numbers are the farthest apart?
- Show how many units apart those two numbers are on the number line?
- Provide an explanation how you found the answer?
Adding Integers - Part 1

Objectives

- Compare rational numbers.
- Use the order of operations to simplify an expression.
- Rename improper fractions and mixed numbers.
- Add, subtract, multiply, and divide fractions.

Books & Materials

- Math in Focus - Teacher Edition
- Math in Focus 2A

Assignments

- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 2A.
- Complete the Practice Questions.

LEARN

WARM-UP

Rename as a decimal.

1. \( \frac{3}{5} \)
2. \( \frac{5}{12} \)

WARM-UP ANSWERS

1. 0.375  2. 0.416 (with the 6 repeating)

TEACHING NOTES

INSTRUCTION

Today’s part covers important ideas you need to understand before beginning the lessons in this chapter. Read pp. 52–56 (top) in Math in Focus 2A. You will compare rational numbers, use the order of operations to simplify an expression, rename improper fractions and mixed numbers, and compute with fractions.

To compare rational numbers, first look at the integer’s sign. Any positive number is greater than every negative number. To compare mixed numbers or decimals, look at the whole-number parts first. If they are equal, compare the fractional parts or decimal parts. Remember, > means is greater than, and < means is less than.

Review the order of operations on p. 54. Some students use a memory device, such as PEMDAS or Please Excuse My Dear Aunt Sally, to remember the order.
To rename an improper fraction as a mixed number, divide the numerator by the denominator. The quotient is the whole-number part, the remainder forms the numerator of the fraction, and the denominator remains as is. For example, to write $\frac{13}{4}$ as a mixed number, divide 13 by 4 to get a quotient of 3 with a remainder of 1. So, $\frac{13}{4} = 3\frac{1}{4}$. To rename a mixed number as an improper fraction, multiply the whole-number part by the denominator and then add the numerator. The denominator remains as is.

To add mixed numbers with unlike denominators, use a common multiple of the denominators to write equivalent fractions. Then add the whole numbers and the fractions separately before adding their sums. To subtract fractions with unlike denominators, follow the same steps to convert before subtracting. Simplify if needed. Fractions should be proper or mixed numbers and in lowest terms.

When multiplying two proper fractions, the product will be less than either factor.

To divide fractions, multiply the dividend by the reciprocal of the divisor. Find the reciprocal by reversing the numerator and denominator.

**TEACHING NOTES**

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the Quick Check sections.

**SKILLS CHECK**

Complete the Quick Check sections on pp. 53–56 (top) in Math in Focus 2A.

**TEACHING NOTES**

Textbook Answer Key

After your student completes the Quick Check in the Recall Prior Knowledge lesson of this chapter, review the questions that were answered incorrectly. If, after the review, you feel that your student
needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him or her complete the activity.

Note that this chapter opener spans two lesson parts.

**Quick Check**

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</table>

**WRAP-UP**

Today you reviewed comparing two rational numbers.

\[2.72 > 2.27\quad -9 < -6\]

You reviewed the order of operations.

\[(24 + 15) - 12 \cdot 3\]
\[= 39 - 12 \cdot 3\]
\[= 39 - 36\]
\[= 3\]

You reviewed renaming improper fractions as mixed numbers and mixed numbers as improper fractions.

\[
\frac{17}{5} = \frac{15}{5} + \frac{2}{5} \\
= 3 \frac{2}{5}
\]

\[
4 \frac{1}{3} = \frac{4 \times 3}{3} + \frac{1}{3} \\
= \frac{13}{3}
\]
You also reviewed computing with fractions.

\[
\begin{align*}
\frac{5}{8} + \frac{2}{3} &= \frac{15}{24} + \frac{16}{24} \\
\frac{3}{4} - \frac{1}{3} &= \frac{9}{12} - \frac{4}{12} \\
\frac{3}{4} \cdot \frac{1}{3} &= \frac{3}{4} \cdot \frac{1}{3} \\
\frac{3}{5} \div \frac{2}{3} &= \frac{3}{5} \cdot \frac{3}{2} \\
&= \frac{31}{24} \\
&= \frac{5}{12} \\
&= \frac{3}{12} \\
&= \frac{9}{10} \\
&= 1\frac{7}{24} \\
&= 1\frac{1}{4}
\end{align*}
\]

☑️ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Adding Integers - Part 2

### Objectives
- Add integers with the same sign.

### Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition
- counters
- number line

### Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 2A.
- Complete Quick Check.

---

## LEARN

### WARM-UP

Find each absolute value.

1. $|32|$
2. $|–136|$
3. $|115|$
4. $|–45|$

### WARM-UP ANSWERS

1. 32   2. 136   3. 115   4. 45

---

### TEACHING NOTES

#### WARM-UP ANSWERS

1. 32   2. 136   3. 115   4. 45

---

### INSTRUCTION

Read Add Integers with the Same Sign on pp. 58–60 in Math in Focus 2A.

You may not realize it, but you use number lines in daily life. Thermometers are vertical number lines. They show temperatures below zero as negative numbers and temperatures above zero as positive numbers. Elevators can be thought of as vertical number lines, too, with the floors above street level representing positive integers and those below street level representing negative integers. The farther you are from zero, the greater the absolute value of the number.

In this part, you will learn how to add integers with the same sign. One way to model this is with counters. Use one color to represent positive integers and another to represent negative integers. In the text, red and yellow counters are used. If red represents negative numbers and you want to add $–5$ and $–3$, use 5 red counters and then add 3 more red counters. Altogether you have 8 red counters. The sum is $–8$. 
You can also model integer addition with a number line. To add $-5$ and $-3$, for example, begin at $-5$ and move 3 spaces to the left on the horizontal number line or 3 spaces down on the vertical number line. You should be at $-8$. Notice that positive numbers indicate a move to the right or up, and negative numbers indicate a move to the left or down.

A third way to add integers with the same sign is to find the absolute value of each number first. Add the absolute values and then apply the sign. For example, to add $-5$ and $-3$, first find the sum of the absolute values. Then apply the correct sign. Both addends are negative, so the sum is $-8$.

The Distributive Property explains why you can do this. Think about how you model $-4$ with counters. You have 4 counters of the same color, each of which represents $-1$. This means you have 4 groups of $-1$, or $4(-1)$. Similarly, $-6$ can be represented as $6(-1)$. You can think of adding $-4$ and $-6$ this way:

$$-4 + -6 = (-1)4 + (-1)(6)$$

The Distributive Property allows you to rename this:

$$(-1)4 + (-1)(6) = (-1)(4 + 6) = (-1)10 = -10$$

In other words, you can think of the signs as representing either $-1$ or $+1$. Make sure you understand this concept, as you will use it again later in this course.

Review Example 1 on page 61. Then complete Guided Practice on page 62. Use counters to represent positive and negative numbers to help you model the problems in this section.

**TEACHING NOTES**

**Textbook Answer Key**

Encourage your student to model the problems in this section as she completes them, either with different colored counters or with number lines. While she may feel this is unnecessary, inform her that she may find the models helpful in tackling more challenging problems later in the chapter and should therefore practice using them now.
PRACTICE

Complete problems 1–2 and 7–9 of Practice 2.1 on p. 73 in Math in Focus 2A.

SUPPLEMENTAL

Instructional Videos:

- Commutative and Associative Properties
- Distributive Property

WRAP-UP

Today you learned how to add integers of the same sign using a number line, counters, and absolute value. You can add the numbers in any order using the Commutative Property of Addition. When you add integers with the same sign, the sum will also have the same sign. The Distributive Property explains why this is true.

QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

If you did not answer this question correctly, try using a number line or counters to help you add. You may also think of the problem as "6 negatives plus 7 more negatives." Revisit Adding Integers, Part 2.
Adding Integers - Part 3

**Objectives**
- Add integers and their opposites.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition
- counters
- number line

**Assignments**
- Complete Warm-up.
- Read and complete pages in *Math in Focus 2A*.
- Complete the Quick Check.

**LEARN**

**WARM-UP**

Add.
1. \(-15 + -23\)
2. \(-95 + -17\)
3. \(38 + 112\)
4. \(46 + 35\)

**WARM-UP ANSWERS:**
1. \(-38\)
2. \(-112\)
3. \(150\)
4. \(81\)

**TEACHING NOTES**

**WARM-UP ANSWERS:**

1. \(-38\)
2. \(-112\)
3. \(150\)
4. \(81\)

**INSTRUCTION**

Read *Add Integers to Their Opposites* on pp.63–64 in *Math in Focus 2A*. Think about what happens when the temperature rises during the day and then falls at night. Suppose when you wake up one winter morning, the thermometer reads 0°C. During the day, the temperature rises to 15°C. When the sun sets, the temperature falls by 15°C. What is the temperature after the sun sets? (It is 0°C again.)

The rising and falling temperature is a model of *additive inverses*. When you add additive inverses, or opposites, such as 5 and \(-5\), their sum is zero. 5 and \(-5\) make a *zero pair*. This happens because the absolute value of both integers is the same. If you move 5 spaces to the right of zero on the number line and then move 5 spaces to the left, you end up in the same spot you started.
Review Example 2 on p. 64. Notice how the addition is modeled on the number line and how it is modeled with counters. Complete Guided Practice on p. 64.

Complete Hands-On Activity on p. 65. Use counters to accomplish steps 1 and 2. You will use the gray and white sides of the counters in place of the yellow and red sides shown in the book.

Complete the Math Journal activity on p. 65. Explain what you observed in Hands-On Activity about adding integers with two different signs.

Textbook Answer Key

Have your student work with counters and the number line to complete this part. Help her understand that the sum of each pair of additive inverses equals zero and encourage her to use both counters and number lines to develop a mental picture of what is happening when she adds opposites. Hands-On Activity builds a concrete understanding of addition with integers that will be developed in the next part.

PRACTICE

Complete problems 3–5 of Practice 2.1 on p. 73 in Math in Focus 2A.

WRAP-UP

Today you learned how to find zero pairs by adding integers and their opposites. The sum of opposite integers is always zero.

QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

If you needed help with this question, try this activity to model addition of integers. Click here to see the activity in a new window.
To begin, select “Explore.” Drag four of the yellow 1 chips and three of the red –1 chips into the Modeling Area. You have modeled 4 + (–3). The sum of these two numbers is 1, shown at the top of the screen. Drag a zero pair (a 1 chip and a –1 chip) into the area. Does this change the total value of chips in the area? Explain this to your Learning Guide. Remove all the zero pairs you can by dragging 1 chips over –1 chips. (You can also remove zero pairs by selecting a whole region of zero pairs and then clicking inside the region.) Explain to your Learning Guide why removing zero pairs does not change the sum.

Now click the button for Addition. Check that under Evaluate: the problem is 5 + (–2). (If not, refresh your browser to restart the Gizmo.) Follow the instructions to show 5 and –2 in the modeling area. Recall that adding and removing zero pairs does not change the sum. Remove all the zero pairs you can by dragging 1 chips over –1 chips. How many zero pairs did you remove? What remains after all the zero pairs have been removed? Click Continue. Type the sum in the space above the modeling area and click Enter. What is 5 + (–2)?

Now find these sums. Use the Gizmo to check your work.

1. (–7) + 3
2. –6 + (–5)
3. 7 + (–12)
4. –22 + 23

Answers:

In the problem 5 + (–2), two zero pairs are removed, leaving three 1 chips. 5 + (–2) = 3

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.
LEARN

INTERACTIVE ACTIVITY

Follow these instructions for the activity shown below.

Click here to view the activity in a new window.

The number line is another way to model the addition of integers. To begin, set slider a to 3 (3.0) and slider b to 7 (7.0). (To set the value of a slider quickly, click on the number in the text field next to the slider, type a new value, and press Enter.) Notice that the red arrow that represents a points 3 units to the right of zero, and the blue arrow that represents b goes 7 units beyond that. The arrows end at 10 on the number line. Also notice that both arrows point to the right. How does the Gizmo model the sum of the positive integers 3 and 7? Explain this to your Learning Guide.

Now click Reset. Move the a slider back and forth to model –6 + 5. Notice how the arrow points to the left when the integer is negative. What is the sum of –6 and 5?

Find these sums. Then check your work with the Gizmo.

1. 10 + (−7)
2. (−5) + 9
3. 1.2 + (−2.6)
4. 5.3 + 6.7 + (−9.5) (Hint: Click “Add three numbers” at the top of the screen before working.)
**TEACHING NOTES**

**Answers:**

\(-6 + 5 = 1\)

1 3 2 4 3 \(-1.4\) 4 2.5

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

**RATE YOUR UNDERSTANDING**

Please go online to view and submit this assessment.
Adding Integers - Part 5

**Objectives**
- Add integers with different signs.

**Books & Materials**
- Math in Focus 2A
- *Math in Focus - Teacher Edition*
- counters
- number line

**Assignments**
- Complete Warm-Up.
- Complete the assigned pages in *Math in Focus 2A*.
- Complete the Quick Check.

---

**LEARN**

**WARM-UP**
Which number goes in each blank to make the equation true?

1. $-15 + _____ = 0$
2. $95 + (-95) = _____$
3. _____ + 112 = 0
4. $365 + (-365) = _____$

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. 15
2. 0
3. –112
4. 0

**INSTRUCTION**
Read *Add Integers with Different Signs* on pp. 66–67 in *Math in Focus 2A*. Consider parking in a parking garage on level $-2$, which is 2 stories below ground level. You can represent that on a number line as $-2$. Now imagine you get into the elevator and go to the eighth floor. If you start at $-2$, how many stories do you travel to get to the eighth floor? After 2 floors, the elevator reaches 0, or ground level, and 8 floors after that, you are on the eighth floor. You have traveled 10 stories. On a vertical number line, if you start at $-2$ and move 10 spaces up, you arrive at 8.

$-2 + 10 = 8$
When the integers have different signs, use one of two methods to find the sum.

**Method 1**: Use a number line. From your starting number, move right (or up) on the number line if you are adding a positive integer or left (or down) on the number line if you are adding a negative integer.

You can illustrate this by using counters to indicate the value of each integer and then removing the zero pairs. The number and color of the remaining counters indicate both the sign and the value of the sum.

**Method 2**: Subtract the absolute value of the lesser integer from the absolute value of the greater integer. The sign of the integer with the greater absolute value is the sign of your answer.

Review Example 3 on p. 68. Then complete Guided Practice on p. 68.

Review Example 4 on pp. 69–70. In this example, you will add more than two integers of different signs. According to the Associative Property of Addition, you can add three or more numbers by regrouping them. You may want to rearrange the numbers, adding those with like signs first, to calculate more efficiently.

In this case, \(-5 + 6 + (-2)\) can be rewritten as \(-5 + (-2) + 6\). Add the like integers first.

\[-5 + (-2) + 6 = -7 + 6 = -1\]

Complete Guided Practice on p. 70.

---

**TEACHING NOTES**

Textbook Answer Key

Encourage your student to work with the counters and number line to understand the concept of zero pairs and moving in the correct direction on the number line. Have her explain how to add integers with opposite signs by subtracting the absolute values.

---

**PRACTICE**

Complete problems 6 and 10–18 of Practice 2.1 on p. 73 in Math in Focus 2A.
WRAP-UP

Today you learned how to add integers with different signs. You can use a number line or counters to model the addition. You can also subtract the absolute values of the integers and give the sum the sign of the addend with the greater absolute value. You also learned how to add three integers with different signs.

QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

If you struggled with this problem, try modeling it on a vertical number line. Remember that −6 is only 6 below 0, so if more than 6 degrees is added, the temperature becomes positive. You can also use counters: one color for positive integers and one color for negative integers. Remember that one counter of each color becomes a zero pair, canceling each other out because they have a sum of 0. You may also want to review using the Instructional Video, Adding Integers.
### Adding Integers - Part 6

#### Objectives
- Solve real-world problems by adding or subtracting integers.

#### Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition
- counters (Optional)
- number line

#### Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 2A.
- Complete Use For Mastery.

## LEARN

### WARM-UP

Add.

1. $-12 + 16$
2. $-8 + 24 + (-15)$
3. $64 + (-92) + 12$
4. $-9 + (-16)$

### WARM-UP ANSWERS

1. 4  
2. 1  
3. $-16$  
4. $-25$

### TEACHING NOTES

#### WARM-UP ANSWERS

1. 4  
2. 1  
3. $-16$  
4. $-25$

### INSTRUCTION

Read Example 5 on pp. 71–72 in Math in Focus 2A. Remember that when adding two integers with opposite signs, the sum will have the sign of the addend with the greater absolute value.

Complete Guided Practice on p. 72. Look at the word below in the problem. Below tells you that the number is less than another value. Sea level has an elevation of 0 feet, so 400 feet below sea level is an elevation of $-400$ feet. Make sure you understand the problem situation before trying to solve.

You can use the Commutative Property of Addition to reorder the addends and the Associative Property of Addition to group together addends with the same sign. Find the sum of the two negative addends and then add the positive addend to find the elevation.
Encourage your student to use the properties of addition to add integers, such as the Commutative Property and the Associative Property. The Inverse Property allows her to eliminate addends that form zero pairs. Using a combination of these properties will allow your student to break down multistep addition problems into simpler problems.

**PRACTICE**

Complete problems 19–24 of **Practice 2.1** on p. 73 in **Math in Focus 2A**.

**WRAP-UP**

Today you learned how to solve a real-world problem by adding integers.

The Lions football team gained 3 yards on first down, lost 5 yards on second down, and lost 4 yards on third down. How many yards did the Lions gain?

**Step 1:** Write an expression to represent the problem.

\[3 + (-5) + (-4)\]

**Step 2:** Use the Associative Property to group the negative addends. Add inside the brackets.

\[3 + [(-5) + (-4)] = 3 + (-9)\]

**Step 3:** Add.

\[3 + (-9) = -6\]

The Lions gained -6 yards (or lost 6 yards).
In a certain office tower the ground level is indicated by 0, and the underground levels are indicated by –1, –2, –3, and so on. The above ground levels are indicated by 1, 2, 3, and so on.

a) Andy was at Level –1 and took the elevator to Level 21. How many levels did the elevator move upward?

b) Kate works on Level 12 and parks her car on the lowest level. If she has to travel 16 levels down to get her car, how many underground levels are in the office tower?

Use the box to answer the questions and explain how you found your answers.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Answer how many levels the elevator moved upward when Andy went from Level –1 to Level 21?
- Answer how many underground levels there are in the office tower where Kate works?
- Show your work and use logical steps to solve the problems?
- Label your answer to give meaning to the numerical values?
Operations with Integers - Part 1

**Objectives**
- Add, subtract, multiply, and divide decimals.
- Solve problems with percents.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition

**Assignments**
- Complete Warm-Up.
- Complete the assigned pages in *Math in Focus 2A*.
- Complete the Practice Questions.

---

**LEARN**

**WARM-UP**

Multiply or divide.

1. What is two-thirds of one-half?
2. \( \frac{3}{4} \div 1\frac{1}{2} \)

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. 2/6 or 1/3  
2. 1/2

---

**INSTRUCTION**

Today's part covers important ideas you need to understand before beginning the parts in this chapter. Read pp. 56–57 in *Math in Focus 2A*. You will multiply and divide decimals and use percents to compare quantities to 100.

To multiply decimals, ignore the decimal point as you multiply; then count the total number of decimal places in each factor to determine where to place the decimal point in the product.

One way to divide decimals is to rewrite the division expression as a fraction. Then multiply both the numerator and the denominator by 10, 100, 1,000, or another power of 10 to make the denominator a whole number. Simplify and divide.

A percent is a ratio that compares a quantity to 100. To find a percent of a number, remember that of indicates multiplication. For example, 10% of 20 can be rewritten as \( \frac{10}{100} \times 20 \). You can find the percent
of increase or percent of decrease by first finding the difference between the first and second numbers. Create a fraction with the difference as the numerator and the original number as the denominator; then multiply the new fraction by 100%.

TEACHING NOTES

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the Quick Check sections.

Your student should be able to recognize that the word *of* generally indicates multiplication. She should also be able to convert a fraction into a usable number for decimals or percents, either by multiplying by 10 or 100 or by choosing a denominator of 10 or 100.

SKILLS CHECK

Complete the Quick Check sections on pp. 56–57 in Math in Focus 2A.

TEACHING NOTES

Textbook Answer Key

Review your student's answers to the Quick Check sections, noting the problems that she answered incorrectly. Then go online through your student's portal to today's part. Click on the link to access the appropriate Reteach activity that your student should complete for the remainder of this part.

Reteach

After your student completes the Quick Check in the Recall Prior Knowledge part of this chapter, review the questions that were answered incorrectly.

If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.

Note this chapter opener spans two parts.

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WRAP-UP

Today you reviewed multiplication and division with decimals. You also reviewed how to find the percent of a number and how to find the percent of increase or decrease.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
LEARN

WARM-UP

Add.

1. \(-9 + 3\)
2. \(-10 + 4 + 9\)
3. \(15 + (-22) + (-7)\)
4. \(-7 + (-13) + 15\)

WARM-UP ANSWERS

1. \(-6\)  2. 3  3. \(-14\)  4. \(-5\)

INSTRUCTION

Read step 1a of Hands-On Activity on page 74 in Math in Focus 2A. In the top model, 2 positive counters are taken away from the 5 positive counters. This is shown by the expression \(5 - (+2)\). Copy the bottom model, \(5 + (-2)\), with your counters. Match each white counter with a gray counter to make a zero pair. Since there are 2 white counters, 2 zero pairs are formed. When they are removed, 3 gray counters are left, just as there were in the first model. This shows that the expression \(5 - (+2)\) is equivalent to the expression \(5 + (-2)\).

Read step 1b on page 75. Show \(-5\) with your counters. How can you subtract \(+2\) when there are no gray counters to remove? (Add 2 zero pairs to your model.) Because \(-5 + 0 + 0 = -5\), you have not changed the value of the expression. Now remove the 2 gray counters to show subtracting \(+2\), leaving you with 7 negative counters, or \(-7\). The bottom model shows the expression \(-5 + (-2)\). Two negative counters are added to the 5 negative counters to give 7 negative counters. The two expressions are equivalent because they both have a value of \(-7\).
Complete step 2 on p. 75.

Read step 3a on p. 75 and model +5 with your counters. To subtract −2 when there are no white counters to remove, you can add 2 zero pairs and not change the value of the expression. Two more positive counters are added to the 5 counters. The expression 5 − (−2) is equivalent to 5 + 2.

Read step 3b on p. 76. In the top model, 2 negative counters are removed. In the bottom model, zero pairs are used to remove the negative counters because 2 is being added. Copy these models with your counters to show that the expressions are equivalent.

Complete step 4 on p. 76. Then complete the Math Journal activity at the bottom of the page.

TEACHING NOTES
Your student can add as many zero pairs as necessary to a model and not change the value of the expression that is shown. Adding a zero pair is like adding zero; it doesn't change the value.

PRACTICE
Use counters to model and complete problems 1–5 of Practice 2.2 on p. 84 in Math in Focus 2A.

TEACHING NOTES
Textbook Answer Key

WRAP-UP
Today you learned how to use counters to model subtraction with positive and negative integers.

PRACTICE QUESTIONS
Please go online to view and submit this assessment.
Operations with Integers - Part 3

**LEARN**

**WARM-UP**

Add.

1. $-5 + 2$
2. $-8 + (-6)$
3. $7 + (-9)$
4. $6 + (-6)$

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. $-3$  
2. $-14$  
3. $-2$  
4. $0$

**INSTRUCTION**

Read *Subtract Integers by Adding Their Opposites* on pp. 76–77 in *Math in Focus 2A*. Because addition and subtraction are inverse operations, you can subtract an integer by adding its opposite. If the *subtrahend* (the number being subtracted) is positive, you can rewrite the problem as adding a negative number. If the subtrahend is negative, you can rewrite the problem as adding a positive number.

A number line or counters can be used to model subtraction problems. On a horizontal number line, if you are subtracting a positive integer, move to the left. If you are subtracting a negative integer, move to the right.

Review *Example 6* on p. 78. Then complete *Guided Practice* on p. 79. Directional words can help you determine how to solve a word problem involving integers. For example, in problem 2, the word *below*...
indicates that the net is 35 feet below the surface, or at −35 feet, and the word lowers means to subtract. You can also draw a diagram to help you understand what is happening in a given problem.

Review Example 7 on p. 80. Then complete Guided Practice at the top of p. 81. To subtract negative integers, rewrite the expression as an addition problem. You can use the Associative Property of Addition to group three or more numbers in a way that makes the problem easier to solve. You may want to use a number line to check your work.

**TEACHING NOTES**

**Textbook Answer Key**

As your student is working through the problems for this lesson, make sure she is rewriting them correctly. She should change both the subtraction sign to addition and the subtrahend to its opposite. If she changes only one, the expressions will not be equivalent.

**PRACTICE**

Complete problems 6–12 and 19–23 of Practice 2.2 on p. 84 in *Math in Focus 2A*.

**INTERACTIVE ACTIVITY**

Ask your Learning Guide for help in accessing this activity to improve your skill with subtracting integers.
Follow these steps to access the activity:

1. Go to the Seventh Grade Number Sense and Proportional Reasoning section of the activity list.
2. Click on Subtracting Integers.
3. Click on View Teacher Tool or View Teacher Tool in Spanish.
4. Read and agree to the Terms of Use. Click Continue.

WRAP-UP

Today you learned how to subtract integers by adding their opposites.

Subtracting a positive integer is like adding a negative integer:

\[ a - b = a + (-b) \]

Subtracting a negative integer is like adding a positive integer:

\[ a - (-b) = a + b \]

Evaluate. \( 4 - 6 \)
\[ 4 - 6 = 4 + (-6) = -2 \]

Evaluate. \( 4 - (-6) \)
\[ 4 - (-6) = 4 + 6 = 10 \]

Please go online to view and submit this assessment.

Quick Check
MORE TO EXPLORE

View the video, *Subtracting Integers*, to help you with this skill.
Operations with Integers - Part 4

Follow these instructions for the activity shown below.

Click here to view the activity in a new window.

You can use a number line to model subtraction of integers. To begin, click “Subtract integers” at the top of the screen. Set the Value of the first integer slider to 5 and the Value of the second integer slider to 3. (To set the value of a slider quickly, click on the number in the text field next to the slider, type a new value, and press Enter.) Notice that the red arrow that represents the first integer points 5 units to the right to show that its value is positive. The value of the second integer (3) is also positive, but the arrow points to the left. This is because the value is being subtracted, and subtraction is the opposite of addition. Therefore, the arrow points in the opposite direction and brings the line indicating the difference to 2. This shows that $5 - 3 = 2$.

Now click Reset. Move the sliders to show $5 - 8$. Look at the directions of the arrows. What does $5 - 8$ equal? Now model $(-5) - 8$. Did you get the same difference? Why or why not? Finally, model $2 - (-7)$. What is the difference? Discuss your observations with your Learning Guide. Can you come up with a rule for subtracting integers? Write it in your Math Notebook.

Find these differences and write them in your Math Notebook. Check your work with the Gizmo.

1. $3 - 10$
2. $7 - (-9)$
3. $(-4) - (-9)$
4. $(-2) - 12$
Answers:

5 – 8 = 3; –5 – 8 = –13; 2 – (–7) = 9

To subtract integers, change the subtrahend to its opposite value and add.

1 7 2 16 3 5 4 –14

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

RATE YOUR UNDERSTANDING

Please go online to view and submit this assessment.
Operations with Integers - Part 5

Objectives
- Find the distance between two integers on a number line.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition
- number line

Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 2A.
- Complete the Practice Questions.

LEARN

WARM-UP

Subtract.

1. 6 − 10
2. 3 − (−5)
3. −7 − 4
4. −2 − (−4)

WARM-UP ANSWERS
1. −4  2. 8  3. −11  4. 2

TEACHING NOTES

INSTRUCTION

Read Find the Distance Between Two Integers on a Number Line on p. 81 in Math in Focus 2A. When you subtract two integers, the difference could be negative. The distance between two integers on a number line is never negative, so finding the distance between two integers is different from subtracting integers.

When you find the distance between two integers on a number line, it does not matter if you count from left to right or from right to left.
Remember that absolute value is the distance from 0 on a number line. Since absolute value is a distance, it is never negative.

\[|3| = 3 \text{ and } |-3| = 3\]

You can find the distance between two integers using absolute value.

Review Example 8 on p. 82. Notice how distance can be found by using a number line or by calculating the absolute value of the difference.

Complete Guided Practice on p. 83. Use either method to find the distances.

**TEACHING NOTES**

Textbook Answer Key

As your student works through the problems, make sure that she is representing numbers accurately. For example, she should represent a distance 5 feet below sea level as \(-5\), and a distance 12 feet above sea level as \(+12\).

**PRACTICE**

Complete problems 13–18 and 24–30 of Practice 2.2 on pp. 84–85 in Math in Focus 2A.

**WRAP-UP**

Today you learned two ways to find the distance between integers by using a number line or absolute value.

Find the distance between 4 and \(-3\). On a number line, count the units.
You can also find the distance between integers by finding the absolute value of their difference.

\[ |4 - (-3)| = |4 + 3| \]

\[ = 7 \]

The distance between 4 and -3 is 7 units.

---

☑️ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Operations with Integers - Part 6

Objectives
- Multiply integers.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B
- number line (Optional)

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete the Quick Check.

LEARN

WARM-UP

Compute.
1. 12 \cdot 12
2. 15 \cdot 9
3. −5 + (−5)
4. −7 + (−7)

WARM-UP ANSWERS
1. 144  2. 135  3. −10  4. −14

TEACHING NOTES

INSTRUCTION

Read step 1a of Hands-On Activity on p. 86 in Math in Focus 2A. Remember that multiplication is a shortcut for repeated addition. The example shows adding +2 three times. The positive sign indicates that you need to move right on the number line when you add.

Read step 1b on p. 86. This time you are multiplying by −2, so you must move left on the number line.

Complete step 2 and the Math Journal activity on p. 87.

Read step 3a on p. 87. Remember that a negative sign also means the opposite of. Multiplying -3 and 2 will give a result that is the opposite of multiplying 3 and 2. Compare the number line shown with the one shown in step 1a to see that this does indeed occur.
Similarly, for step 2b, you can expect the product \(-3 \cdot (-2)\) to have the opposite value of \(3 \cdot (-2)\). Comparing the number lines shown for steps 3b and 2b, you can see that this is the case.

Complete step 4 and the Math Journal activity on p. 88. Read Multiply Integers on pp. 88–89. Think about how you can use the flowchart shown at the bottom of p. 88 to help you multiply integers. Review Example 9 on p. 90 and complete Guided Practice on p. 90. Remember, you can use the Commutative and Associative Properties of Multiplication to reorder and group factors. For example, for problem 3, you may want to reorder the factors to put the positive factors together. Discuss your answers to Think Math with your Learning Guide.

Review Example 10 on p. 90 and complete Guided Practice on p. 91. Look for words or phrases to tell you if a factor is positive or negative when solving a word problem.

**TEACHING NOTES**

Your student may want to use the Commutative and Associative Properties of Multiplication to reorder factors to help her find a product.

**PRACTICE**

Complete problems 1–21 and 30–31 of Practice 2.3 on p. 93 in *Math in Focus 2A*.

**TEACHING NOTES**

[Textbook Answer Key]

**WRAP-UP**

Today you learned how to multiply nonzero integers. If the signs are the same, the product is positive. If the signs are different, the product is negative.

Two positive factors: \(5 \cdot 7 = 35\)

One negative factor: \(5 \cdot (-7) = -35\) and \(-5 \cdot 7 = -35\)

Two negative factors: \(-5 \cdot (-7) = 35\)
To multiply more than two nonzero factors, count the number of negative signs. If the number of negative signs is an even number, the product is positive. If the number of negative signs is an odd number, the product is negative. Use the properties of multiplication to reorder and group factors.

\[
4(-7)(5) = 4(5)(-7)
= (4 \cdot 5)(-7)
= (20)(-7)
= -140
\]

**QUICK CHECK**

Please go online to view and submit this assessment.

**MORE TO EXPLORE**

If you struggled with this question, view the Instructional Video, *Multiplying Integers*. 
Operations with Integers - Part 7

Objectives
- Divide integers.

Books & Materials
- Math in Focus 2A
  - Math in Focus - Teacher Edition

Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 2A.
- Complete the Quick Check.

LEARN

WARM-UP

Multiply.

1. $6 \cdot (-7)$
2. $8 \cdot 9$
3. $-15 \cdot (-5)$
4. $-12 \cdot 8$

WARM-UP ANSWERS

1. $-42$  2. $72$  3. $75$  4. $-96$

TEACHING NOTES

INSTRUCTION

Read Divide Integers on pp. 91–92 in Math in Focus 2A. Divide integers in the same way that you divide whole numbers. Then look at the signs of the dividend and divisor. If one of the signs is negative, the quotient is negative. If neither sign is negative or if both signs are negative, the quotient is positive. Be sure to read the Math Note at the bottom of p. 91 to see how division problems can be written in fractional form.

Review Example 11 on p. 92.

Complete Guided Practice on p. 92. When solving a word problem, look for a word or phrase that would indicate that either the dividend or divisor is negative. For example, for problem 9, the word descended indicates a negative value.
Remind your student that division is the opposite (inverse) of multiplication. As with whole numbers, the quotient of a division problem can be checked by multiplying the quotient and the divisor. If the product equals the dividend, the quotient is correct.

Complete problems 22–29 and 32–35 of Practice 2.3 on p. 93 in Math in Focus 2A.

Today you learned how to divide integers. To divide integers, divide as you would with whole numbers. Then follow these rules to determine the sign of the quotient:

• If the dividend and divisor have the same sign, the quotient is positive.

• If the dividend and divisor have different signs, the quotient is negative.

28 ÷ 7 = 4

28 ÷ (−7) = −4

−28 ÷ 7 = −4

−28 ÷ (−7) = 4

Please go online to view and submit this assessment.

This video presents a song that can help you remember how to multiply and divide integers.
**Operations with Integers - Part 8**

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Books &amp; Materials</th>
<th>Assignments</th>
</tr>
</thead>
</table>
| - Solve real-world problems by adding or subtracting integers.  
- Use all four operations to evaluate expressions with integers. | - Math in Focus 2A  
- Complete the assigned pages in Math in Focus 2A.  
- Complete the Practice Questions. |

### LEARN

### WARM-UP

Solve.

1. \( 12 \cdot (-8) \)
2. \( -360 \div (-12) \)
3. An airplane ascends 18,000 feet in 9 seconds. Find the change of height in feet per second.

### WARM-UP ANSWERS

1. \(-96\)  
2. \(30\)  
3. \(2,000 \text{ ft/s}\)

### TEACHING NOTES

#### WARM-UP ANSWERS

1. \(-96\)  
2. \(30\)  
3. \(2,000 \text{ ft/s}\)

### INSTRUCTION

Read p. 94 in *Math in Focus 2A*. In previous lessons, you learned how to multiply and divide integers. In this lesson, you will learn how to simplify expressions involving multiple operations. Recall that the correct order of operations is parentheses, exponents, multiplication or division (left to right), and then addition or subtraction (left to right). You may want to use the abbreviation *PEMDAS* to remember the order of operations. (Parentheses, Exponents, Multiplication or Division, Addition or Subtraction)
Now read **Example 12** on p. 95. Notice in part a, since there are no parentheses or exponents, division is the first operation you perform. Complete **Guided Practice** on p. 95. Remember to follow the order of operations when you evaluate each expression.

Read **Example 13** at the top of p. 96. Notice how drawing a diagram can be helpful in solving a problem. Then complete **Guided Practice**. Recall that the formula for the area of a triangle is \( A = \frac{1}{2}bh \). It will help you if you copy the diagram and label the dimensions of the triangles. Remember to follow the order of operations when solving the problem.

**TEACHING NOTES**

**Textbook Answer Key**

**WATCH FOR THESE COMMON ERRORS**

Your student may find it helpful to work down the page, evaluating the expression one step at a time. If your student struggles, encourage her to rewrite the expression after each step as a way to more easily see the problem-solving process.

Ensure that your student is correctly following the order of operations as she evaluates expressions. It is a common misconception that all multiplication must be performed before any division, and similarly that all addition must be performed before any subtraction. Encourage your student to pay close attention to steps 3 and 4 of the order of operations presented on p. 94, which indicates that these pairs of operations should be performed in order from left to right.

**PRACTICE**

Complete **Practice 2.4** on p. 97 in **Math in Focus 2A**.

**SUPPLEMENTAL**

Instructional Video: **Order of Operations**
WRAP-UP
Today you learned how to use the order of operations to solve equations and real-life problems. First evaluate anything within parentheses. If there are any exponents, evaluate them next. Next, multiply or divide from left to right. Finally, add or subtract, again working from left to right.

\((2 - 10) \cdot (-7) + (-99) \div 3\)

Evaluate expressions inside the parentheses.

\((-8) \cdot (-7) + (-99) \div 3\)

Next multiply or divide.

56 + (-33)

Finally, add or subtract.

23

✔️ PRACTICE QUESTIONS
Please go online to view and submit this assessment.
Integers were used in the 15th century by European traders to label barrels of grain or flour. For example, a barrel that was 1 pound too heavy was labeled +1, and a barrel that was 1 pound too light was labeled −1.

A trader checked twenty 50-pound barrels and found the labels to be: five −3s, nine −5s, two +4s, and four −8s. Find the actual total weight of the barrels.

Upload your work to show how you got the answer.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information given to help you determine how much total weight was in the barrels?
- Show the use of logical steps in your problem solving?
- Label your answer to give meaning to the numerical values?
- Organize your work neatly?
- Upload your work when finished?
Operations with Rational Numbers - Part 1

LEARN

WARM-UP

Solve.

1. \( \frac{1}{3} + \frac{1}{2} \)
2. \( \frac{2}{17} - \frac{1}{34} \)
3. \( \frac{2}{7} + \frac{5}{6} \)

TEACHING NOTES

WARM-UP ANSWERS

1. \( \frac{5}{6} \)  2. \( \frac{3}{34} \)  3. \( 1-\frac{5}{42} \)

INSTRUCTION

Read **Add Rational Numbers** on p. 98 in *Math in Focus 2A*. Recall that you can only add or subtract fractions if they have the same denominator. If the denominators are not the same, use the least common denominator to create equivalent fractions and then add or subtract the fractions.

If the fractions have signs, follow the same rules for adding and subtracting integers to help you evaluate the expression.

Now read **Example 14** on pp. 99–100. In part a, notice how the least common denominator is used to create equivalent fractions prior to adding. Here, the fractions have the same sign (negative), so the absolute values of the fractions are added, and the sum has the common sign.

The same procedure is used for part b; however, this time the addends have different signs, so the absolute values are subtracted when finding the sum. You may use the Commutative Property to first add or subtract the integer parts of the mixed numbers and then add or subtract the fractions.
Remember, however, that in a negative mixed number, both the integer part and the fractional part are negative.

In part c on p. 100, notice the two different methods for adding three rational numbers.

Now complete Guided Practice on p. 101. Remember to find the least common denominator when necessary. You may also want to rewrite each rational number as the sum of an integer and a fractional part. Be careful with the signs when you rewrite a negative mixed number.

### TEACHING NOTES

Adding rational numbers is very similar to adding fractions; however, your student will need to consider the effect of the signs when completing the addition. When adding rational numbers that involve a negative mixed number (that is, a negative integer and a fraction), remind your student that both the integer part and the fractional part are negative.

### PRACTICE

Complete problems 1–5 of Practice 2.5 on p. 110 in Math in Focus 2A.

### TEACHING NOTES

Textbook Answer Key

### WRAP-UP

Today you learned how to add rational numbers. If the expression includes mixed numbers, you can separate the integer parts and the fractional parts.

\[
2\frac{1}{2} + 3\frac{1}{8} = 2\frac{4}{8} + 3\frac{1}{8} = (2 + 3) + \left(\frac{4}{8} + \frac{1}{8}\right) = 5\frac{5}{8}
\]
PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Ask your Learning Guide to access this activity. Complete the activity to practice adding fractions and decimals.

Follow these steps to access the activity:

1. Go to the *Seventh Grade Number Sense and Proportional Reasoning* section of the activity list.
2. Click on *Fraction and Decimal Sums*.
3. Click on *View Teacher Tool* or *View Teacher Tool in Spanish*.
4. Read and agree to the Terms of Use. Click Continue.
RATE YOUR UNDERSTANDING

Please go online to view and submit this assessment.
Operations with Rational Numbers - Part 3

**Objectives**
- Subtract rational numbers.
- Add rational numbers.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition

**Assignments**
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 2A.
- Complete the Quick Check.

---

**LEARN**

---

**WARM-UP**
Add. Write the sum in simplest form.

1. \( \frac{3}{5} + \left( -\frac{1}{2} \right) \)
2. \( -\frac{3}{5} + \left( -\frac{2}{3} \right) \)

---

**TEACHING NOTES**

---

**WARM-UP ANSWERS**
1. \(-\frac{1}{8}\)  2. \(-1 - \frac{4}{15}\)

---

**INSTRUCTION**

Read Subtract Rational Numbers on p. 102 in Math in Focus 2A. Subtracting negative fractions combines the skills you learned for subtracting fractions and subtracting integers. You can find equivalent fractions with like denominators and then subtract or add the opposite. Take note of the information in the Caution box at the bottom of the page. Be sure that the numbers are in the correct order when writing an expression with subtraction.

Review Example 15 on pp. 103–104. Complete Guided Practice on p. 104. You may also want to rewrite each subtraction expression as adding the opposite. For example, for problem 6, you could rewrite \(3\frac{1}{4} - 7\frac{5}{6}\) as \(3\frac{1}{4} + -7\frac{5}{6}\).

Review Example 16 on p. 105 and complete Guided Practice.
Be sure to review the Caution box at the bottom of p. 102 in Math in Focus 2A with your student. As she works through this part, check to see that she is writing the numbers in subtraction expressions in the correct order.

**PRACTICE**

Complete problems 6–9 and 33–36 of Practice 2.5 on pp. 110–111 in Math in Focus 2A.

**WRAP-UP**

Today you learned how to subtract rational numbers that include negative fractions and mixed numbers.

\[
\begin{align*}
7 \frac{1}{6} - 10 \frac{1}{4} &= 7 \frac{2}{12} - 10 \frac{3}{12} \\
&= 7 + \frac{2}{12} - 10 - \frac{3}{12} \\
&= 7 - 10 + \left(\frac{2}{12} - \frac{3}{12}\right) \\
&= -3 + \left(-\frac{1}{12}\right) \\
&= -3 \frac{1}{12}
\end{align*}
\]

\[
\begin{align*}
3 \frac{1}{8} - 1 \frac{5}{12} &= \frac{9}{24} - \frac{6}{24} - \frac{10}{24} \\
&= \left(\frac{9}{24} - \frac{6}{24}\right) - \frac{10}{24} \\
&= \frac{3}{24} - \frac{10}{24} \\
&= -\frac{7}{24}
\end{align*}
\]
Quick Check

Please go online to view and submit this assessment.

More to Explore

If you struggled with this question, watch the BrainPop video Adding and Subtracting Fractions to review adding and subtracting fractions with unlike denominators.
Operations with Rational Numbers - Part 4

Books & Materials
  - Math in Focus - Teacher Edition

Assignments
  - Complete Interactive Activity.
  - Complete Rate Your Enthusiasm.

LEARN

INTERACTIVE ACTIVITY

Go to the Fractions activity to model multiplying positive and negative fractions.

Interactive Activity

1. Example: $-1\frac{1}{2} \times 3$

2. Click on the + icon at the bottom to bring up a fraction bar. Click OK to leave it as a whole.

3. Click on the fraction bar again and choose a color from the palette at the bottom of the screen. Click on the bar to color it.

4. Bring up another fraction bar. Click 2 and click OK. Move the second bar next to the first one.

5. Color one half of the bar. You have now made a model for the mixed number $-1\frac{1}{2}$. Use your mouse to draw a circle around both fraction bars.

6. Then click on the Duplicate icon at the bottom of the screen 2 more times. You have now made 3 copies of your model to show multiplying by 3.
7. Use the models to write the final product. You can use the text tool or the drawing tool at the bottom of the screen.

Practice modeling other problems from this lesson. Remember, if the signs are the same, the product is positive. If they are different, the product is negative.

✅ RATE YOUR ENTHUSIASM

Please go online to view and submit this assessment.
Multiply or divide. Express each answer in simplest form.

1. \( \frac{3}{4} \cdot \frac{2}{9} \)
2. \( 1 \frac{5}{6} \cdot 3 \frac{1}{3} \)
3. \( \frac{7}{8} \div \frac{5}{6} \)
4. \( 2 \frac{1}{4} \div \frac{3}{5} \)

WARM-UP ANSWERS
1. \( \frac{1}{6} \)
2. \( 6 \frac{1}{9} \)
3. \( 1 \frac{1}{20} \)
4. \( 3 \frac{3}{4} \)

INSTRUCTION
Read Multiply Rational Numbers on p. 106 in Math in Focus 2A. Multiplying negative fractions combines the skills you learned for multiplying fractions and multiplying integers. Multiply the fractions as you would if they were positive and then check the signs, according to the information from the Math Note. If the signs are the same, the product is positive. If the signs are different, the product is negative.

Review Example 17 on p. 106. Then complete Guided Practice on p. 107. You can use the greatest common factor (GCF) to simplify the terms before multiplying. For example, for problem 9, you can divide the denominator of \( -\frac{4}{5} \) and the numerator of \( \frac{20}{21} \) by 5 to obtain \( -\frac{4}{1} \cdot \frac{4}{21} \).

If you simplify before multiplying, the fractional part will already be in simplest form. You still may have to rename an improper fraction as a mixed number.
Read **Divide Rational Numbers** on pp. 107–108. Dividing with negative fractions combines the skills you learned for dividing fractions and dividing integers. You will divide by multiplying by the reciprocal of the divisor. Remember, two numbers are reciprocals if their product is *positive*. If your divisor is *negative*, its reciprocal is also negative. Once you have completed the division by multiplying by the reciprocal, determine the sign of the quotient using the rules from the **Math Note** on p. 107.

Take note of the complex fractions on p. 108. A *complex fraction* has a fraction in the numerator, in the denominator, or in both places. You can simplify a complex fraction by renaming it as a division expression.

Review **Example 18** on pp. 108–109 and complete **Guided Practice** on p. 109. Be sure to rewrite any complex fractions as division expressions.

---

**TEACHING NOTES**

Make sure your student is correctly rewriting division expressions as multiplication by the reciprocal of the divisor. Encourage her to simplify before multiplying whenever possible; this will not only help her calculate more quickly and efficiently, but it will also provide a strong foundation for higher levels of mathematics.

---

**PRACTICE**

Complete problems 10–32 and 37–42 of **Practice 2.5** on pp. 110–111 in *Math in Focus 2A*.

---

**TEACHING NOTES**

[Textbook Answer Key](#)

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**WRAP-UP**

Today you learned how to multiply and divide rational numbers that include negative fractions and mixed numbers. After you calculate, check the signs. If the signs are the same, the product or quotient is positive. If the signs are different, the product or quotient is negative.

<table>
<thead>
<tr>
<th>Multiply Fractions</th>
<th>Divide Mixed Numbers</th>
<th>Simplify Complex Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{5}{12} \times \left( \frac{3}{8} \right) = \frac{5}{4} \times \left( \frac{1}{8} \right) )</td>
<td>( -2 \frac{1}{4} \div -1 \frac{3}{5} = \frac{9}{4} \div \left( \frac{8}{5} \right) )</td>
<td>( -1 \frac{2}{3} \div 2 \frac{1}{5} = -\frac{1}{2} \div 2 \frac{1}{5} )</td>
</tr>
<tr>
<td>= \frac{5}{32}</td>
<td>= \frac{9}{4} \times \left( \frac{5}{8} \right)</td>
<td>= -\frac{5}{3} \div \frac{11}{5}</td>
</tr>
<tr>
<td></td>
<td>= \frac{45}{32}</td>
<td>= \frac{5}{3} \cdot \frac{5}{11}</td>
</tr>
<tr>
<td></td>
<td>= 1 \frac{13}{32}</td>
<td>= -\frac{25}{33}</td>
</tr>
</tbody>
</table>
Please go online to view and submit this assessment.

If you answered incorrectly, remember to rename the mixed numbers as fractions (ignoring the signs at first) before multiplying. If you are having problems multiplying fractions, try to practice with positive fractions.

Your student may find it helpful to rewrite subtraction expressions as adding the opposite. Doing this may help her to determine more easily whether a difference is positive or negative. For example, she can rewrite $0.75 - (-1.3)$ as $0.75 + 1.3$ or rewrite $2.4 - 5.6$ as $2.4 + (-5.6)$. Because the sign of the sum takes the sign of the addend with the greater absolute value, choosing the correct sign then becomes a matter of comparing two absolute values.

Complete problems 1–5, 15–18, and 21 of Practice 2.6 on p. 120 in Math in Focus 2A.

Today you learned how to add and subtract positive and negative decimals. To add decimals with different signs, subtract the lesser absolute value from the greater absolute value. The following table summarizes the rules for addition.

For subtraction, the difference can be positive or negative if the signs are alike. If the minuend (the number from which another number is to be subtracted) is greater than the subtrahend, the difference will be positive. If the minuend is less than the subtrahend, the difference will be negative.
Operations with Rational Numbers - Part 6

Objectives

- Add rational numbers.
- Subtract rational numbers.

Books & Materials

- Math in Focus 2A
- Math in Focus - Teacher Edition

Assignments

- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 2A.
- Complete the Practice Questions.

LEARN

WARM-UP

Add or subtract.

1. \(-7 + (-15)\)
2. \(2.83 + 3.5\)
3. \(6 - (-8)\)
4. \(5.7 - 3.42\)

TEACHING NOTES

WARM-UP ANSWERS

1. \(-22\)  
2. \(6.33\)  
3. \(14\)  
4. \(2.28\)

INSTRUCTION

Read Add and Subtract Decimals on pp. 112–113 of Math in Focus 2A. As with adding integers, if all addends are negative, the sum will be negative, and if all addends are positive, the sum will be positive. If there is one positive addend and one negative addend, the sum will take the sign of the addend with the greater absolute value. To subtract decimals, you can rewrite the problem as addition by adding the opposite of the subtrahend (the number being subtracted).

Review Example 19 on p. 114. Then complete Guided Practice on p. 114. You can rewrite the problems horizontally and align the decimal points. If necessary, attach zeros to ensure that the numbers have the same number of decimal places.

Review Example 20 on p. 115. Then complete Guided Practice on p. 115. Look for key words to help you determine whether a value is positive or negative.
Your student may find it helpful to rewrite subtraction expressions as adding the opposite. Doing this may help her to determine more easily whether a difference is positive or negative. For example, she can rewrite $0.75 - (-1.3)$ as $0.75 + 1.3$ or rewrite $2.4 - 5.6$ as $2.4 + (-5.6)$. Because the sign of the sum takes the sign of the addend with the greater absolute value, choosing the correct sign then becomes a matter of comparing two absolute values.

**PRACTICE**

Complete problems 1–5, 15–18, and 21 of Practice 2.6 on p. 120 in Math in Focus 2A.

**TEACHING NOTES**

Textbook Answer Key

**WRAP-UP**

Today you learned how to add and subtract positive and negative decimals. To add decimals with different signs, subtract the lesser absolute value from the greater absolute value. The following table summarizes the rules for addition.

<table>
<thead>
<tr>
<th>Addend 1</th>
<th>Addend 2</th>
<th>Sum</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>+</td>
<td>$2.3 + 4.8 = 7.1$</td>
</tr>
<tr>
<td>+</td>
<td>-</td>
<td>+ or - depending on the greater absolute value</td>
<td>$3.6 + (-4.5) =</td>
</tr>
<tr>
<td>-</td>
<td>+</td>
<td>+ or - depending on the greater absolute value</td>
<td>$-4.25 + 5.1 =</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$-5.7 + (-3.9) = -9.6$</td>
</tr>
</tbody>
</table>

For subtraction, the difference can be positive or negative if the signs are alike. If the minuend (the number from which another number is to be subtracted) is greater than the subtrahend, the difference will be positive. If the minuend is less than the subtrahend, the difference will be negative.

<table>
<thead>
<tr>
<th>Minuend</th>
<th>Subtrahend</th>
<th>Difference</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>+ or - depending if the minuend is greater than or less than the subtrahend</td>
<td>$2.6 - 3.9 = 2.6 + (-3.9) = -1.3$</td>
</tr>
<tr>
<td>+</td>
<td>-</td>
<td>+</td>
<td>$2.4 - (-4.2) = 2.4 + 4.2 = 6.6$</td>
</tr>
<tr>
<td>-</td>
<td>+</td>
<td>-</td>
<td>$-3.4 - 4.8 = -8.2$</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>+ or - depending if the minuend is greater than or less than the subtrahend</td>
<td>$-3.2 - (-4.7) = -3.2 + 4.7 =</td>
</tr>
</tbody>
</table>
PRACTICE QUESTIONS

Please go online to view and submit this assessment.
### Operations with Rational Numbers - Part 7

**Objectives**
- Multiply and divide rational numbers.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition

**Assignments**
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 2A.
- Complete the Quick Check.

### LEARN

**WARM-UP**

Multiply or divide.

1. $-8 \cdot (-7)$
2. $2.7 \cdot 3.2$
3. $-72 \div 12$
4. $3.84 \div 1.6$

**WARM-UP ANSWERS**

1. 56   2. 8.64   3. $-6$   4. 2.4

### TEACHING NOTES

**WARM-UP ANSWERS**

1. 56   2. 8.64   3. $-6$   4. 2.4

### INSTRUCTION

Read Multiply Numbers in Decimal or Percent Form on p. 115 of Math in Focus 2A. Multiplying negative decimals follows the same rules as multiplying positive decimals, except that you need to check the signs. Just like when multiplying integers, if both signs are the same, the product is positive. If the signs are different, the product is negative.

Review Example 21 on p. 116. Then complete Guided Practice on p. 117. Remember, you can rename a percent as a decimal by dividing the percent by 100 and removing the percent sign. You can rename a percent as a fraction by writing the percent as a numerator over a denominator of 100.

Read Divide Numbers in Decimal or Percent Form on p. 117. Dividing negative decimals follows the same rules as dividing positive decimals, except that you need to check the signs. If the dividend and
divisor have the same sign, the quotient is positive. If the signs are different, the quotient is negative. When dividing by a decimal divisor, multiply the dividend and divisor by the same power of 10 so the divisor becomes a whole number.

Review **Example 22** on p. 117. Then complete **Guided Practice** on p. 117. When performing long division of a decimal dividend, place the decimal point in the quotient before dividing. You may also want to check your quotient by estimating to see if it is reasonable.

### TEACHING NOTES

When finding the percent of a number, your student should decide whether she wants to rename the percent as a fraction or as a decimal. She may discover that, if the percent can be renamed as a unit fraction, the reciprocal will be a whole number. This will allow your student to solve a simple division problem. For example, to find 25% of 248, she can rename 25% as 14. The reciprocal of 14 is 4, so she can simplify by finding 248 ÷ 4.

### PRACTICE

Complete problems 6–14, 22–24, and 29 of **Practice 2.6** on pp. 120–121 in *Math in Focus 2A*.

### TEACHING NOTES

[Textbook Answer Key](#)

### WRAP-UP

Today you learned how to multiply and divide positive and negative decimals, including calculations with percents. The steps are the same as multiplying or dividing decimals with the added step of checking the signs. If the signs are the same, the product or quotient is positive. If the signs are different, the product or quotient is negative.

\[
\begin{array}{c}
-6.2 \\
\times 5.6 \\
\hline
372 \\
3100 \\
-34.72 \\
\end{array}
\quad
-16\overline{28.8} \\
\overline{160\overline{288}} \\
16 \\
128 \\
0
\]
Please go online to view and submit this assessment.

If you struggled with this question, review the rules for multiplying and dividing signed numbers.
## Operations with Rational Numbers - Part 8

### Objectives
- Use the order of operations to simplify an expression.

### Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition

### Assignments
- Complete Warm-Up.
- Complete the assigned pages in Math in Focus 2A.
- Complete the Practice Questions.

---

## LEARN

### WARM-UP

Multiply or divide.

1. $-2.6 \cdot 4.2$
2. $12\% \text{ of } 280$
3. $-2.7 \div 0.9$
4. $-4.81 \div (-3.7)$

### WARM-UP ANSWERS

1. $-10.92$
2. $33.6$
3. $-3$
4. $1.3$

---

## TEACHING NOTES

### WARM-UP ANSWERS

1. $-10.92$  
2. $33.6$  
3. $-3$  
4. $1.3$

---

## INSTRUCTION

Review Example 23 on p. 118 of Math in Focus 2A. For parts a and b, there are two parts to consider when finding the solution. Remember that sales tax is the amount the government adds to the cost of an item. You can either multiply the cost of the item by the sales tax rate to find the sales tax, which is then added to the cost of the item, or you can add the sales tax to 100% and then multiply the cost of the item by the sum of the percents. For part c, the percent of change is first found by dividing the change by the original number; then the sign is determined by whether the change is an increase (+) or decrease (−).

Complete Guided Practice on p. 119. For problem 10, break the problem into parts. Because the question asks for the total decrease, the answer is a positive number. If the question had asked for the total change, the answer would be a negative number.
For problem 11, you may want to rename 20% as a fraction because it will be a unit fraction, which will then allow you to divide by a one-digit divisor.

For problem 12, make sure you follow the correct order of operations needed to solve the problem.

For problem 13, use the formula for finding the percent of change.

$$\text{Percent of Change} = \frac{\text{Final value} - \text{Original value}}{\text{Original value}}$$

**TEACHING NOTES**

Part of the process of solving a word problem is asking *What do I know?* and *What do I need to find?* This will help your student break down multistep problems and then perform the operations in the correct order.

**PRACTICE**


**TEACHING NOTES**

*Textbook Answer Key*
**WRAP-UP**

Today you learned how to solve multistep problems involving multiple operations.

A diver descends at 1.4 feet per second for 9 seconds. She then rises 0.8 foot per second for 5 seconds. What is the diver’s overall change in position?

**Step 1:** Write an expression.

\[-1.4 \cdot 9 + 0.8 \cdot 5\]

**Step 2:** Multiply.

\[-1.4 \cdot 9 + 0.8 \cdot 5 = -12.6 + 4\]

**Step 3:** Add.

\[-12.6 + 4 = -8.6\]

The overall change is -8.6 feet.

You also learned how to find the percent change.

The cost of a jacket was $64. Now it is $48. What is the percent change?

Use the formula for percent change.

\[
\text{Percent of Change} = \frac{48 - 64}{64}
\]

\[
= \frac{-16}{64}
\]

\[
= \frac{-1}{4}
\]

\[
= -25%
\]

The cost of the jacket changed by -25%.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
LEARN

WARM-UP
Evaluate each expression.

1. \(-0.7 + 2.6 - 1.2\)
2. \(0.9 \cdot 4 - 0.2 \cdot 3\)
3. \(-1.6 \cdot 2.5 + (-0.75) \cdot 4\)
4. \(-2.4 ÷ 3.2 \cdot 3.5\)

WARM-UP ANSWERS
1. 0.7  2. 3  3. -7  4. -2.625

TEACHING NOTES

INSTRUCTION
Read and solve Brain @ Work on p. 122 in Math in Focus 2A. For the first problem, think about what you know about averages. You may want to experiment with a group of seven numbers to review finding the average. Think about how the average relates to the sum of the numbers and how you can use this information to find the missing seventh number.

You may choose to solve problem 2 in several ways. For example, you may want to draw a vertical number line or diagram. If you are familiar with proportions, you can write one to help you find the amount of change. Pay attention to the units that are given and make sure you are using the same units every time.
If you do not understand what Julie did in the third problem, take some time to act out the process, showing Julie's work on paper. You might find it helpful to write the numbers in order first. Then use her method to find the average of 32, 35, 38, and 36.

### TEACHING NOTES

If your student has difficulty with the first problem, use counters to model finding the average, or mean, of three numbers: 6, 5, and 1. Lay out a row of 6 counters, a row of 5 counters, and a single counter. Have your student move counters to form three rows that are the same length. She should finish with 4 counters in each row.

Now lay out a row of 2 counters, a row of 3 counters, and a row of 7 counters and have your student again make the rows even. Once again, she will discover that there are 4 counters in each row. Lead her to see that the original three numbers do not matter; the average (mean) is always 4, and the total number of counters (sum) is always 12. Then encourage her to apply this understanding to help her solve problem 1.

For the second problem, you may need to remind your student that she will have to convert to the same unit to solve the problems correctly.

Your student may need your help with the procedure used in problem 3. You may remind her that the average (mean) is generally somewhere between the greatest value and the least value in the set, so it may be helpful to write the numbers in order first.

### PRACTICE

Throughout the remainder of this course you will be asked to write constructed responses for some of your Brain @ Work assignments. When you answer a constructed response question, you not only solve the problem, but you also explain how you found the answer. The explanation of a constructed response can be in the form of a word sentence, a drawing, a chart or table, an equation, or any combination of these. The explanation of your constructed response answers the question *how do you know?* You may want to write your constructed responses in a separate notebook so that you can evaluate your progress as you continue through the course.

Write a constructed response to explain how you found the average in problem 3 on p. 122 in *Math in Focus 2A*. 
Then complete the following problems.

1. When five consecutive integers are added together, the sum is −5.
   
a. What are the five integers?
   
b. What is the greatest product that can be obtained by multiplying any two of these integers?

2. The points $A$, $B$, $C$, and $D$ lie on a number line in that order from left to right. Use the following clues to find each point:
   
   - The distance from $B$ to $D$ is 6.
   - The distance from $B$ to $C$ is 3.
   - The sum of $A$ and $B$ is −6.

---

**TEACHING NOTES**

**Textbook Answer Key**

**Practice Answers:**

1a -3, -2, -1, 0, 1  
2 Possible answers: A: -6, B: 0, C: 3, D: 6; A: -5, B: -1, C: 2, D: 5; A: -4, B: -2, C: 1, D: 4

For the first problem, you may need to remind your student that consecutive integers appear in order on the number line, as in −3, −2, −1 or 5, 6, 7. If she needs additional help getting started, ask what kind of integers would need to be added to obtain a negative sum. Lead her to see that at least some of the integers must be negative. She may want to draw a number line to help her as she experiments with combinations of integers that add up to −5. Once she has found these integers, she may wish to make an organized list to find all the possible products.

Your student may want to draw a number line to find the answer to the second problem. She can either determine the relative positions of the points without assigning actual number values, or she may wish to determine values for $A$ and $B$ and then plot the rest of the points accordingly. Once she has arrived at one set of values, help her see that there may be more than one correct solution to a math problem by having her find a second set of values that also meet the given criteria.

**Looking Forward:** Have your student prepare or reuse the counters from today’s part for the chapter test. She will need counters for the Constructed Response portion of the test.

---

**WRAP-UP**

Today you learned how to solve challenging problems involving rational numbers.
PRACTICE QUESTIONS

Please go online to view and submit this assessment.
The melting points of some chemical elements are shown below.

<table>
<thead>
<tr>
<th>Substance</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nickel</td>
<td>2,651</td>
</tr>
<tr>
<td>Mercury</td>
<td>-37.89</td>
</tr>
<tr>
<td>Helium</td>
<td>-458</td>
</tr>
<tr>
<td>Copper</td>
<td>1,984.32</td>
</tr>
</tbody>
</table>

a) Which two elements have melting points that are closest in value?

b) What is the average melting point of the substances? Round your answer to the nearest tenth.

Explain how you got to the answer.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information in the chart to determine which two elements have melting points that are closest in value?
- Provide the average melting point for the four elements given?
- Show your work with a neat, logical procedure for determining the answers to the questions?
- Label your answers appropriately?
- Upload your work when finished?
Unit Quiz: Rational Number System

Books & Materials
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

UNIT QUIZ
Please go online to view and submit this assessment.
Unit 2 - Algebraic Expressions and Equations
Project: Planning a Trip

Books & Materials
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

PROJECT DESCRIPTION

In this project, you are going to plan your dream vacation. You will apply your knowledge of equations to calculate the cost of a hotel stay based on the number of days. You will add in the cost of tickets or passes to different activities. Finally, you will apply your understanding of multiplying decimals to calculate some discount costs.

PROJECT DETAILS

In this project, you will:

- Write an expression to show the cost of 3 different hotels based on the number of nights
- Add the cost of any passes or tickets
- Write inequalities to reflect a budget of $3000
- Solve each inequality to determine how many nights you can stay at each hotel and remain within your budget
- Write down all of the URLs and titles of your sources
- Use your inequalities to reach a conclusion regarding the hotel at which you will stay and why

PROJECT RUBRIC

The Project Rubric will help you understand how your project will be scored. Your goal should be to earn all possible points for each part.
Think of all of the fun places you have vacationed. Have you ever helped plan one of your vacations? There are often many costs involved in traveling. In this project, you have won round-trip plane tickets for your entire family to anywhere in the world that you would like to visit. Since you won the tickets, you have been put in charge of finding the hotel and deciding what you will be doing. In this project you will use equations to calculate the cost of your hotel stay and your activities. Remember that you can vacation anywhere in the world!

**COLLABORATION**

Where have you traveled on vacation? Think about the different costs related to the trip. What are some of the costs associated with your last trip? How much do you think that the vacation ended up costing? Share your ideas with your peers and respond to two classmates.

**RATE YOUR EXCITEMENT**

Please go online to view and submit this assessment.
LEARN

WARM-UP

For problems 1–2, consider the following algebraic expression.

\[ 8 + 5a \]

1. What is the constant term? What is the coefficient?
2. Given that \( a = 2 \), what is the value of the expression?
3. Write an expression for \( 3 \) more than twice a number. Use \( y \) for the unknown number.

WARM-UP ANSWERS

1. 8; 5  
2. 18  
3. \( 2y + 3 \)

INSTRUCTION

Today’s part covers important ideas you need to understand before beginning the lessons in this chapter. Read p. 128 in Math in Focus 2A. The total cost for the field trip depends on the number of students. The number of students is variable, which means it can change, so a letter is used to represent this value. An algebraic expression, which contains a variable (letter) to stand for the unknown value, can be used to represent the total cost of the field trip.

Read the first instructional section on p. 129 in Math in Focus 2A. An algebraic expression typically contains a constant, a variable, and an operation. An expression does not include an equal sign.
Then read the second instructional section on p. 129. When the assigned value of the variable changes, so does the value of the expression.

Next read the first instructional section on p. 130. Sometimes you can simplify an algebraic expression by combining like terms before evaluating it. Read the second instructional section on p. 130. Expand an algebraic expression written in this form by using the Distributive Property to multiply the number outside the parentheses by each term inside the parentheses. Be especially carefully if any of the signs are negative. Look at these examples:

\[5(t - 9) = 5(t + -9) = 5t + -45 \text{ or } 5t - 45\]
\[-4(x + 2) = -4(x) + -4(2) = -4x + -8 \text{ or } -4x - 8\]
\[-7(y - 3) = -7(y + - 3) = -7(y) + -7(- 3) = -7y + 21\]

Read the first instructional section on p. 131. Factoring is the opposite of expanding. It separates an expression into factors. When factoring, be sure to use the greatest common factor of the terms.

Then read the second section on p. 131. When you expand or factor an algebraic expression, the resulting expression is equivalent to the original expression.

Finally, read the third instructional section on p. 131. Recall that a variable is used to represent a value that can change. You can write a phrase as an algebraic expression. For example, five less than a number \(t\) can be written as \(t - 5\).

### Teaching Notes

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the Quick Check sections.

### Skills Check

Complete the Quick Check sections on pp. 129–131 in Math in Focus 2A.
After your student completes the **Quick Check** in the **Recall Prior Knowledge** lesson of this chapter, review the questions that were answered incorrectly. If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.

<table>
<thead>
<tr>
<th>Quick Check</th>
<th>Question(s)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1–4</td>
<td>Algebraic Expressions</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Evaluating the Expression: Constants and Variables in Formulas</td>
</tr>
<tr>
<td></td>
<td>6–17</td>
<td>Simplifying Algebraic Expressions</td>
</tr>
<tr>
<td></td>
<td>18–22</td>
<td>Distributive Property</td>
</tr>
<tr>
<td></td>
<td>23–26</td>
<td>Expressing Algebra: Practice Problems</td>
</tr>
</tbody>
</table>

**WRAP-UP**

Today you learned about algebraic expressions. Evaluate an algebraic expression by replacing the variable with its assigned value. Simplify algebraic expressions by adding or subtracting like terms.

\[5n + 7n - 3 = 12n - 3\]

Expand algebraic expressions by applying the Distributive Property. Be sure to watch the signs.

\[-6(n + 2) = -6(n) + -6(2) = -6n + (-12) \text{ or } -6n - 12\]

Equivalent expressions are equal for any values of the variables. In the previous example, \(-6(n + 2)\) and \(-6n - 12\) are equivalent.

Factor an algebraic expression by writing it as a product of its factors.

\[3x + 15 = 3(x) + 3(5) = 3(x + 5)\]

When writing algebraic expressions, use a variable to represent the value that can change.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Simplifying Algebraic Expressions - Part 2

**Objectives**
- Draw bar models to represent algebraic expressions.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition

**Assignments**
- Complete Warm-up.
- Read and complete pages in *Math in Focus 2A*.
- Complete the Practice Questions.

---

**LEARN**

**WARM-UP**

Solve.

1. Write an example of an algebraic expression.
2. What are the two sets of like terms in the following list?
   - 3, 0.5x, 1\(\frac{1}{4}\), 9x, 1\(\frac{1}{3}\)x, 0.75, 4.6, 3.2x

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. Sample answer: 5x + 3  
2. 3, 1/4, 0.75, 4.6 and 0.5x, 9x, (1/3)x, 3.2x

---

**INSTRUCTION**

Read **Represent Algebraic Expressions Using Bar Models** on pp. 132–133 in *Math in Focus 2A*. This section shows how to use bar models to represent algebraic terms with fractional or decimal coefficients. Notice that each bar model is divided into sections, which are separated with dotted lines. When the coefficient is a fraction, as in \(\frac{2}{3}x\), the denominator tells you how many sections are in a whole. The numerator tells you how many of those sections to shade.

When the coefficient is a decimal written in tenths, as in 0.5y, use ten sections to represent a whole. The digit in the tenths place of the decimal tells you how many sections to shade in your model.

Look at the bar models in examples c and d. Each bar model represents a coefficient with a value greater than one. The dashed lines show breaks between equal parts of one whole. The solid lines represent a division between the whole and parts of another whole. Make sure to show parts and wholes in this way in your bar models.
If your student struggles with representing expressions with a bar model, give him an algebraic term, such as \( \frac{3}{4}x \). Ask him into how many sections he would divide a bar to model the term. Guide your student in understanding that the denominator 4 tells how many equal parts are in a whole. Then ask how many sections of the model should be shaded. Ensure that your student understands that \( \frac{3}{4} \) is less than one whole. Therefore, not all the sections of the model will be shaded.

HELPFUL ONLINE RESOURCES:

- Instructional Video: How to Teach Bar Modeling, Part 1
- Instructional Video: How to Teach Bar Modeling, Part 2
- Instructional Video: How to Teach Bar Modeling, Part 3

Draw a bar model to represent each algebraic term.

1. \( \frac{3}{5}x \)
2. \( 0.7n \)
3. \( \frac{5}{3}x \)
4. \( 1.3t \)
WRAP-UP

Today you learned how to make a bar model to represent an algebraic term with a fractional or decimal coefficient. A bar model shows the whole and parts. For fractions, use the denominator to determine how many equal parts make a whole. For decimals, use ten equal parts to show a whole.

\[ \frac{3}{4}k \]

![Bar model for \( \frac{3}{4}k \)]

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Simplifying Algebraic Expressions - Part 3

Objectives
- Simplify an algebraic expression by adding like terms.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition
- paper strips all the same size (optional)
- colored pencils (optional)

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete Quick Check.

LEARN

WARM-UP

Simplify the algebraic expressions.

1. $3s + 5s$
2. $4p + 3p - 2$
3. $2t + 6t + 4$

WARM-UP ANSWERS

1. $8s$  
2. $7p - 2$  
3. $8t + 4$

TEACHING NOTES

INSTRUCTION

Read Simplify Algebraic Expressions with Decimal Coefficients by Adding on p. 133 in Math in Focus 2A. Recall that you simplify an algebraic expression by combining like terms. In this case, the like terms both have a decimal coefficient. Therefore, the bar model representing $x$ is divided into ten equal parts.

Then study Example 1 on p. 134. Notice that the sum of the terms, $1.6p$, is greater than one whole. Therefore, the bar model used to represent the sum shows one whole and part of a second whole. The dotted lines show equal decimal parts of a whole. The solid line shows the whole. Recall that if a variable term has no coefficient written, the coefficient is 1 (not 0).

Complete problems 1–8 and 22 of Practice 3.1 on pp. 138–139 in Math in Focus 2A.

Today you learned how to use bar models to simplify algebraic expressions with decimal coefficients by adding.

Simplify the following expression.

\[0.5x + 0.4x = 0.9x\]
Please go online to view and submit this assessment.

Review simplifying expressions by viewing *Simplifying Expressions*.
LEARN

WARM-UP

Find the least common denominator (LCD) of each pair of fractions.

1. $\frac{1}{3}$ and $\frac{1}{6}$
2. $\frac{1}{2}$ and $\frac{1}{7}$
3. $\frac{1}{3}$ and $\frac{2}{5}$

WARM-UP ANSWERS

1. 6 2. 14 3. 15

INSTRUCTION

Read p. 135 in Math in Focus 2A. Look at the bar models. For the fractional coefficients $\frac{1}{2}$ and $\frac{1}{4}$, you need to find the least common multiple (LCM) of the denominators. This is known as the least common denominator (LCD). In this example, the least common denominator is 4, so the bar is divided into four equal parts. If the coefficients were $\frac{1}{2}$ and $\frac{1}{5}$, then the LCD would be 10, and the bar model would be divided into ten equal parts.

Notice in the bottom portion of the instructional section that the LCD is used to rewrite the fractional coefficients so that they have the same denominator. Once the coefficients have the same denominator, add the numerators. When you add the fractional terms, you simplify the algebraic expression.
Then study Example 2 on pp. 136–137 in *Math in Focus 2A*. Be sure to read the Math Note in the middle of p. 136. It is important to remember that the coefficient of a variable is never shown as a mixed number. Instead, use an improper fraction in lowest terms. This is why the resulting coefficient in part a is written as $\frac{5}{3}$ instead of $1\frac{2}{3}$.


### TEACHING NOTES

**Textbook Answer Key**

Guide your student to understand that, to simplify algebraic expressions with fractional coefficients, he first needs to find the LCD of the coefficients. Reinforce that the method to find the LCD is the same as finding the LCM of the denominators. If your student needs to visualize the concept, allow him to model the addition of the fractional coefficients with fraction strips.

**WATCH FOR THESE COMMON ERRORS**

Some students think that the LCD of two fractions is always the product of their denominators. To find the LCD of $\frac{1}{4}$ and $\frac{1}{6}$, for example, have your student list the multiples of 4 and 6. Multiples of 4: 4, 8, 12. Multiples of 6: 6, 12, 18. Since 12 is the LCM of 4 and 6, it is the LCD of $\frac{1}{4}$ and $\frac{1}{6}$.

Because students are used to writing mixed numbers when they simplify fractions, they may be tempted to do so with fractional coefficients. Make sure that your student leaves these coefficients as improper fractions.

### PRACTICE

Complete problems 9–21 and 23–26 of Practice 3.1 on pp. 138–139 in *Math in Focus 2A*.

### TEACHING NOTES

For problem 21, in *Math in Focus 2A*, some students may confuse the m in the rectangle dimensions with a variable. Explain that m is an abbreviation for meters—a unit of measurement. The italic x is a variable.
WRAP-UP

Today you learned how to simplify algebraic expressions with fractional terms by using bar models and by adding the coefficients.

\[
p + \frac{2}{5}p \\
= \frac{5}{5}p + \frac{2}{5}p \\
= \frac{7}{5}p
\]

👍 PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Simplifying Algebraic Expressions - Part 5

Objectives
- Simplify an algebraic expression by subtracting like terms.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition
- paper strips all the same size (Optional)
- colored pencils (Optional)
- scissors (Optional)

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete the Practice Questions.

LEARN

WARM-UP

Find the difference.

1. 0.8 − 0.4
2. 1.5 − 0.8
3. 1.0 − 0.9

TEACHING NOTES

**WARM-UP ANSWERS**
1. 0.4  2. 0.7  3. 0.1

INSTRUCTION

Read p. 140 in Math in Focus 2A. You have learned to simplify an algebraic expression with decimal coefficients by adding. In this part, you will simplify expressions with decimal coefficients by subtracting. In each case, you simplify the expression by combining like terms. Whether you combine by adding or subtracting depends on the operation in the expression.

Notice the top bar model representing \( x \). Recall that \( x \) is equivalent to \( 1x \). The coefficient \( 1 \) means 1 whole.

Then read through Example 3 on p. 141. Notice that subtracting 0.9\( y \) from 1.8\( y \) on the bar model is represented by removing nine sections from the shaded part. The remaining shaded part represents the difference of 1.8\( y \) − 0.9\( y \).
Complete Guided Practice on p. 141. After drawing each bar model, line up the decimal coefficients vertically by the decimal points and subtract to check your answers.

Textbook Answer Key

For Guided Practice problem 3, make sure your student understands that the coefficient for y is 1. He can then write it as 1.0 to subtract it from 1.2. Point out that bar models can help him visualize and understand how to simplify an expression by subtracting like terms. If he still has difficulty with the bar models, encourage him to model the problems with paper strips, cutting off the part that is subtracted.

Complete problems 1–6 and 20 of Practice 3.2 on p. 144 in Math in Focus 2A.

For problem 20, in Math in Focus 2A, you may need to remind your student that the formula for finding area is \( A = l \cdot w \). This problem is a three-step process. First, your student must find the area for each rectangle. Next he should write an algebraic expression to represent the difference in the areas of the rectangles. Then he should simplify the algebraic expression.

WRAP-UP

Today you learned how to simplify algebraic expressions with decimal coefficients by bar modeling and by subtracting the coefficients of like terms. The difference between the coefficients equals the coefficient of the simplified expression.

\[ 1.5x - 0.8x = 0.7x \]
PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Simplifying Algebraic Expressions - Part 6

Objectives
- Simplify an algebraic expression by subtracting like terms.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition
- fraction strips (Optional)

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete the Quick Check.

LEARN

WARM-UP

Solve.

1. \( \frac{5}{8} - \frac{2}{8} \)
2. What is the LCD of \( \frac{2}{6} \) and \( \frac{1}{4} \)?
3. \( \frac{5}{6} - \frac{2}{3} \)

TEACHING NOTES

WARM-UP ANSWERS
1. 3/8 2. 12 3. 1/6

INSTRUCTION

Read p. 142 in Math in Focus 2A. Because the denominators of the coefficients in the problem are different, both methods require finding the LCD.

For Method 1, use the LCD to determine how many equal parts each bar model should have. Subtracting like terms is shown by removing sections from the bar model. When making your own bar models, you can show the subtraction by crossing out the sections that need to be removed. For Method 2, rewrite the coefficients as equivalent fractions using the LCD. Then subtract.

Then read through Example 4 on p. 143. The LCD of the two fractional coefficients, 12, tells how many equal parts the bar should have. Once the coefficients are rewritten as fractions with a denominator of 12, they can be represented on the bar model.

Complete Guided Practice on p. 143. Remember that the fractional coefficients must have a common denominator before you can subtract.
If your student struggles with the concept, give him two fractions, such as $\frac{5}{6}$ and $\frac{1}{3}$. Have him identify the LCD. Then guide him to rewrite the second fraction so that both have the same denominator, and have him subtract. Remind him to always write the answer in simplest form. If he needs to visualize the concept, have him model the subtraction of fractions with fraction strips.

Complete problems 7–19 of Practice 3.2 on p. 144 in *Math in Focus 2A*.

Ask your student to simplify the expression in problem 19. Then have him compare his method and answer with Matthew's. He can use his work to help explain where Matthew went wrong.

Today you learned that algebraic expressions can be simplified by subtracting the fractional coefficients of like terms. Fractional coefficients need to have a common denominator before subtracting and simplifying.

$$\frac{9}{4}x - \frac{3}{8}x$$

$$= \frac{18}{8}x - \frac{3}{8}x$$

$$= \frac{15}{8}x$$

Please go online to view and submit this assessment.
MORE TO EXPLORE

If you needed help with this question, review adding and subtracting fractions by viewing Adding and Subtracting Fractions.
LEARN

WARM-UP

Solve.

1. Identify the two sets of like terms: $0.4x, 4, 3, 1.2x, 1, 0.2x$
2. $0.3x + 0.4x$
3. $\frac{3}{4}x - \frac{1}{2}x$

WARM-UP ANSWERS

1. $0.4x, 1.2x, 0.2x$ and $4, 3, 1$
2. $0.7x$
3. $(1/4)x$

INSTRUCTION

Read p. 145 in *Math in Focus 2A*. Before you can simplify an expression with more than two terms, you must first identify like terms. Once you have combined the like terms, and no like terms remain, you have simplified the expression.

Read Example 5 on p. 146 in *Math in Focus 2A*. Although they appear together on the bar model, there is no relation between the width of the constant term, 4, on the right and the width of the sections used to represent one whole $p$ and tenths of $p$. Since $p$ is a variable, it can represent any value. Its size is not related to the size of the constant term.

Complete Guided Practice on p. 146. Identify like terms before simplifying.
Next, read the instructional section at the bottom of p. 146. Sometimes there is more than one set of like terms in an expression. This expression has like constant terms and like algebraic terms. Simplify by adding or subtracting each set of like terms.

Study Example 6 on p. 147. Computations are performed separately for each set of like terms. The fractional coefficients of the x-terms are added. The constant terms are subtracted. You can also think of $-1 - 2$ as $-1 + (-2)$.

Complete Guided Practice on p. 147. Remember to rewrite fractional coefficients as fractions with a common denominator before adding or subtracting.

Read the instructional section at the top of p. 148. You can use the Associative Property of Addition to group numbers when adding.

\[ 1.5 + 0.2 + 0.5 = 1.5 + (0.2 + 0.5) \]
\[ = 1.5 + 0.7 \]
\[ = 2.2 \]

You can also use the Commutative Property of Addition to reorder numbers before you use the Associative Property to group.

\[ 1.5 + 0.2 + 0.5 = 1.5 + 0.5 + 0.2 \]
\[ = (1.5 + 0.5) + 0.2 \]
\[ = 2.0 + 0.2 \]
\[ = 2.2 \]

Study Example 7 on p. 148. Notice that you can work from left to right to combine three like terms, just as you work from left to right when you combine three numbers.

Complete Guided Practice on p. 148. Remember that you can use the Associative Property of Addition to group like terms when adding.

---

**TEACHING NOTES**

Textbook Answer Key

Give your student an expression, such as $0.5x + 0.3x + 3 - 2$. Ask him to identify like terms. He may benefit from copying the expression and shading sets of like terms with colored pencils or outlining them with different shapes to more clearly see which terms go together.
Make sure he understands that this expression contains algebraic terms and numeric terms, which must be considered separately. When working with expressions with more than two like terms, remind him to work from left to right. Point out that he can use the Associative Property to group like terms and the Commutative Property to reorder like terms as needed.

**PRACTICE**

Complete problems 1–8 of Practice 3.3 on p. 152 in Math in Focus 2A.

**WRAP-UP**

Today you learned how to simplify algebraic expressions with more than two terms. Begin by identifying like terms. Then add or subtract like terms. You may use the Associative Property and/or the Commutative Property of Addition when adding.

Simplify the algebraic expression.

\[0.3n + 0.8n + 5 - 2 = 1.1n + 3\]

After adding and subtracting, if there are no more like terms, then you have arrived at the final simplified expression.

\[0.3n + 0.8n + 5 - 2 = 1.1n + 3\]

**QUICK CHECK**

Please go online to view and submit this assessment.

**MORE TO EXPLORE**

If you answered incorrectly, remember that you should only add like terms and you should be careful with signs. Since 0.6a is being subtracted, you can treat that as adding a negative.
Elle is 6 years younger than Felicia. Marta is twice as old as Felicia. If Marta is $w$ years old,

a) find the age difference between Marta and Elle.

b) write an expression that shows the three girls' average age.

Show how you found the answers.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information given to find the age difference between Marta and Elle?
- Write an expression that shows the three girls’ average age?
- Show your work to explain how you found your answers?
- Simply your final algebraic expression?
Now it is time for you to start researching and planning your trip. The first thing you need to do is choose the hotel where you want to stay. Choose at least 3 different hotels. Although you will need to write down the cost for this activity, it is also important to look for a hotel that you will really like. Think about how close it is to what you want to do or what the view is like. Maybe you want a hotel with a pool or a gym. Take all of these ideas into account as you research. Once you have picked three hotels, create an expression, with \( x \) representing the number of days, to show the cost per day of staying at your hotel. Remember to consider the 20% discount you will receive for staying multiple nights.

Please go online to view and submit this assessment.
More Simplifying Algebraic Expressions - Part 1

LEARN

WARM-UP

Identify the two sets of like terms in each group.

1. $5, 3p, 8, 8p, 2p, 2$
2. $\frac{2}{6}, \frac{1}{4}b, \frac{5}{6}, \frac{3}{4}b, \frac{2}{4}b, \frac{1}{6}$
3. $3d, -3, -2d, 4, -6, 4d$

WARM-UP ANSWERS

1. 5, 8, 2 and 3p, 8p, 2p
2. $\frac{2}{6}, \frac{5}{6}, \frac{1}{6}$ and $(1/4)b, (3/4)b, (2/4)b$
3. $3d, -2d, 4d$ and $-3, 4, -6$

INSTRUCTION

Read Simplify Algebraic Expressions by Grouping Like Terms on p. 149 in Math in Focus 2A. You may find it helpful to rewrite the subtraction of a term in an algebraic expression as addition of the opposite. Then rearrange and group like terms.

$$2t + 4 - 12t = 2t + 4 + (-12t)$$

$$= 2t + (-12t) + 4$$

$$= -10t + 4$$
Next read through Example 8 on pp. 149–150. Notice that each example has two pairs of like terms. You can use the Commutative Property of Addition to change the order of the terms so that like terms are next to each other. Look at part b on p. 150. It is important to keep track of the signs of the terms when you reorder. You could rewrite the problem:

$$38b + 23 - 18b - 13$$

Then group like terms and simplify. Fractions should be written in lowest terms when you write the simplified expression.

Complete Guided Practice on p. 150. Group like terms before you simplify the expression by adding or subtracting.

Then read Simplify Algebraic Expressions with Two Variables on pp. 150–151. The bar model in Method 1 uses a different width and color for each variable. The model shows the $y$-terms wider than the $x$-terms. It could also have been drawn with the $x$-terms wider than the $y$-terms. The important point is to use one width and color for each variable.

Read through Example 9 on p. 151. You can use parentheses to help group like terms. Make sure the number of terms before and after the regrouping is the same.

Complete Guided Practice on p. 151. Be sure to use the correct signs when you reorder terms with negative coefficients.
PRACTICE

Complete problems 9–23 of Practice 3.3 on p. 152 in Math in Focus 2A.

WRAP-UP

Today you learned how to simplify expressions by grouping like terms.

\[4q + 3 + 2q = 4q + 2q + 3\]
\[= 6q + 3\]

You also learned how to simplify algebraic expressions with two variables.

\[3x + 4y - 2x - 2y = (3x - 2x) + (4y - 2y)\]
\[= x + 2y\]

✅ QUICK CHECK

Please go online to view and submit this assessment.

𬌈 MORE TO EXPLORE

If you had difficulty with this problem, try using models to help you simplify expressions with one variable. Then, move to two variables. Refer back to Simplifying Algebraic Expressions, Part 7 to review this skill.
More Simplifying Algebraic Expressions - Part 2

**Books & Materials**
- Math in Focus - Teacher Edition
- Math in Focus 2A

**Assignments**
- Complete the Interactive Activity.
- Complete Rate Your Enthusiasm.

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**LEARN**

**INTERACTIVE ACTIVITY**

In this activity, you will use algebra tiles to help you visualize simplifying expressions.

To begin, click on Hide All Frames. Then practice building a few expressions. The red square stands for +1, the yellow square stands for −1, the long red bar stands for +1x, and the long green bar stands for −1x. Therefore, the expression for 2x − 5 would be built with 2 red bars and 5 yellow squares.

---

To begin, click on Hide All Frames. Then practice building a few expressions. The red square stands for +1, the yellow square stands for −1, the long red bar stands for +1x, and the long green bar stands for −1x. Therefore, the expression for 2x − 5 would be built with 2 red bars and 5 yellow squares.
Suppose you want to simplify a more complex expression, like $2x + 5 + 3x - 4$. Drag over the correct algebra tiles as shown.

Group the bars together and the squares together. Since each red square stands for +1 and each yellow square stands for −1, you can make zero pairs. Match each red square to one yellow square and move them to the bottom of the screen. You now have one red square left, which represents +1, along with 5 red bars. The simplified expression is $5x + 1$.

Now use the tool to simplify these expressions. Write the simplified expressions in your Math Notebook.

1. $4x - 5 - 6x + 8$
2. $2(b + 3) + 5$ (Hint: Build $b + 3$ twice.)
3. $-3c - 1 + 7c$ (Hint: The negative sign in front of $3c$ means that you need to show the opposite, or use the opposite tile color.)
4. $3(y - 2) - 5y$
5. $2(d - 2) + 3(d + 1)$

If you would like additional practice, use the tool to simplify some of the expressions on p. 152 and p. 159 of *Math in Focus 2A*. You can also use this tool throughout the rest of this lesson if you find it helps you simplify expressions.
5. \((d-2) + 3(d+1)\)

If you would like additional practice, use the tool to simplify some of the expressions on p. 152 and p. 159 of Math in Focus 2A. You can also use this tool throughout the rest of this lesson if you find it helps you simplify expressions.

Activity Answer Key:

1. \(-2x + 3\)
2. \(2b + 11\)
3. \(4c - 1\)
4. \(2y - 6\)
5. \(5d + 1\)

Please go online to view and submit this assessment.
More Simplifying Algebraic Expressions - Part 3

Objectives
- Expand algebraic expressions with fractions.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition
- algebra tiles

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete the Quick Check.

LEARN

WARM-UP
Use the Distributive Property to simplify each expression.

1. \(3(2 + 5)\)
2. \(4(1 - 2)\)
3. \(2(2 - 3)\)

WARM-UP ANSWERS
1. 21
2. -4
3. -2

TEACHING NOTES

INSTRUCTION
Read p. 153 in Math in Focus 2A. To expand an expression, write an equivalent expression without parentheses. Use algebra tiles to model expanding the algebraic expression \(3(2x + 4)\), as shown in the book. Multiplying the expression in parentheses by 3 is equivalent to adding three groups of \((2x + 4)\). When you expand, the new expression is equivalent to the original expression.

You can also expand expressions with fractions such as \(\frac{1}{2}(2x + 4)\). Read the instructions for Method 1 at the top of p. 154 in Math in Focus 2A and consider the bar model for \(2x + 4\) as follows:

The book shows the bar for 4 separated into two equal bars because \(2x + 4\) is multiplied by \(\frac{1}{2}\). Each of these two bars represents 2 because one-half of 4 is 2.
Next, read the instructions for Method 2 in the middle of p. 154 to see how to expand the expression algebraically.

Then read Example 10 on p. 154. The expression 3x + 15 has been arranged into three groups to represent thirds. When you use the Distributive Property as shown in Method 2, be sure to distribute the number outside the parentheses to all terms inside the parentheses. Drawing arrows like the ones shown in the book can help you make sure that you are not overlooking a term when distributing.

Complete Guided Practice on p. 155. You may draw bar models or use algebra tiles to help solve the problems. Although your algebra tiles show the variable x, you can use the tiles to represent any variable in an expression, regardless of the letter that is used.

**TEACHING NOTES**

**Textbook Answer Key**

To help your student understand expanding with fractional factors, give him an expression, such as 14(8x + 16). Guide your student to use algebra tiles to model the expression. Explain that the fraction shows that the expression 8x + 16 will be separated into four equal groups. Reinforce with your student that the new expression, 2x + 4, is equivalent to the original expression, 14(8x + 16).

**PRACTICE**

Complete problems 1–8 of Practice 3.4 on p. 159 in *Math in Focus 2A*.

**TEACHING NOTES**

For problem 8 on p. 159 in *Math in Focus 2A*, encourage your student to notice that this fractional factor, 2 5, is not a unit fraction as the other examples have been. If he is using bar models to solve, he will need to separate the model for (k − 10) into five groups, but then consider two of those groups.

**WRAP-UP**

Today you learned how to expand algebraic expressions that include fractional factors.

\[
\frac{1}{3}(3x + 6) = \frac{1}{3}(3x) + \frac{1}{3}(6) = x + 2
\]
Please go online to view and submit this assessment.

View Distributive Property to review this important skill.
More Simplifying Algebraic Expressions - Part 4

**Objectives**
- Expand algebraic expressions with decimals.
- Expand algebraic expressions with negative factors.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition
- colored pencils (Optional)

**Assignments**
- Complete Warm-up.
- Read and complete pages in Math in Focus A.
- Complete problems 9–26, Math in Focus A.
- Complete the Practice Questions.

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**LEARN**

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**WARM UP**

Multiply.

1. 0.6 · 0.3
2. 0.2 · 4
3. 0.3 · 1.5
4. Complete the statement: the product of a positive number and a negative number is a ___ number.

---

**TEACHING NOTES**

**WARM-UP ANSWERS**
1. 0.18  2. 0.8  3. 0.45  4. negative

---

**INSTRUCTION**

Read Expand Algebraic Expressions with Decimal Factors on p. 155 in *Math in Focus, Course 2, Volume A, Chapter 3*. Recall that when multiplying with decimals, the number of decimal places in the product equals the sum of the decimal places in the factors. For example, 1 decimal place in the first factor + 1 decimal place in the second factor = 2 decimal places in the product.

\[0.4 \cdot 0.3 = 0.12\]

Then read Example 11 on p. 155 in *Math in Focus 2A*. The example uses brackets and parentheses. Like parentheses, brackets are used to group terms so that operations are performed in the correct
order. Use brackets to group parts of an expression that already include parentheses. Also note the placement of the decimal point in each product.

Read the Caution note at the bottom of p. 155, and then complete Guided Practice on p. 156. Rewrite any subtractions by adding the opposite before expanding.

Read Expand Algebraic Expressions with Negative Factors on p. 156. Remember the rules for multiplying numbers: multiplying two factors with the same sign yields a positive product, and multiplying two factors with different signs yields a negative product.

Then read through Example 12 on pp. 156–157. The method used for expanding algebraic expressions with fractional and decimal factors is also used to expand expressions with negative factors. Notice how the signs of the factors affect the sign of the product. In part b, notice how the negative sign outside of the parentheses is equivalently rewritten as −1.

Complete Guided Practice on p. 157. Pay attention to the signs when multiplying by negative numbers.

Textbook Answer Key

Remind your student that he can continue to use curved arrows to help ensure that the term outside the parentheses is distributed to each term inside the parentheses.

Draw your student’s attention to the Caution note on p. 155. Remind him to rewrite subtraction within parentheses as addition before expanding. Point out how this is accomplished in Example 11 using brackets and parentheses.

Point out part b in Example 12 on p. 157. Explain that the negative sign outside the parentheses means that each term inside the parentheses is multiplied by -1.

PRACTICE

Complete problems 9–26 of Practice 3.4 on p. 159 in Math in Focus 2A.
**WRAP-UP**

Today you learned how to expand algebraic expressions that involve decimals.

\[
0.5(0.3b - 2) = 0.5[0.3b + (-2)] \\
= 0.5(0.3b) + 0.5(-2) \\
= 0.15b + (-1) \\
= 0.15b - 1
\]

You also learned how to expand algebraic expressions that involve negative factors.

\[
-2(4c - 2) = -2[4c + (-2)] \\
= -2(4c) + (-2)(-2) \\
= -8c + 4
\]

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
**More Simplifying Algebraic Expressions - Part 5**

**Objectives**
- Identify when it is necessary to expand an expression before simplifying it.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition
- colored pencils (Optional)

**Assignments**
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete the Practice Questions.

**LEARN**

**WARM-UP**
Expand each expression.

1. 3(4x + 2)
2. −4(x + 5)
3. −2(0.6m − 1)

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. 12x + 6  
2. −4x − 20  
3. −1.2m + 2

**INSTRUCTION**

Read Expand and Simplify Algebraic Expressions on p.157 in Math in Focus 2A. Sometimes an algebraic expression contains a factor with terms in parentheses as well as an additional constant or variable term. In this case, the order of operations indicates that you need to clear the parentheses by expanding before you simplify. Use the Distributive Property to clear the parentheses by multiplying the factor by each term inside the parentheses using curved arrows to help you if needed. You can then simplify the expression by adding or subtracting like terms.
Then read Example 13 on p. 158 in Math in Focus 2A. Recall the order of operations.

1. Parentheses (and other grouping symbols, like brackets)
2. Exponents
3. Multiplication and Division (from left to right)
4. Addition and Subtraction (from left to right)

Complete Guided Practice on p. 158. Identify the part or parts of each expression that you need to expand. Then expand and simplify. Be careful with the minus sign when you expand the second part of the expression in problem 14. See Example 13 part c if you do not remember how to expand it.

![TEACHING NOTES]

Textbook Answer Key

Make sure your student correctly applies the rules for multiplying with negative signs. Make sure he also understands that \(-(2k - 1)\) in problem 14 must be expanded. If needed, guide him to see that \(- (2k - 1) = -1[2k + (-1)]\). Both terms inside the parentheses must be multiplied by \(-1\).

Watch For These Common Errors

Your student may try to combine terms with different variables. For example, he may try to combine \(4a + 11b\) as \(15ab\) in problem 12 of Guided Practice. Remind him that unlike terms cannot be combined. If your student finds the technique helpful, encourage him to use colors or shapes to highlight like terms.

![PRACTICE]

Complete problems 27–51 of Practice 3.4 on pp. 159–160 in Math in Focus 2A.

![WRAP-UP]

Today you learned how to expand algebraic expressions in order to then simplify them.

\[
3(2b + 4) - (a - 2) = 3(2b + 4) + (-1)[a + (-2)] \\
= 3(2b) + 3(4) + (-1)[a] + (-1)(-2) \\
= 6b + 12 + (-a) + 2 \\
= 6b + (-a) + 12 + 2 \\
= 6b - a + 14
\]
Please go online to view and submit this assessment.

3. Multiplication and Division (from left to right)

4. Addition and Subtraction (from left to right)

Complete Guided Practice on p. 158. Identify the part or parts of each expression that you need to expand. Then expand and simplify. Be careful with the minus sign when you expand the second part of the expression in problem 14. See Example 13 part c if you do not remember how to expand it.

Textbook Answer Key

Make sure your student correctly applies the rules for multiplying with negative signs. Make sure he also understands that \(- (2k - 1)\) in problem 14 must be expanded. If needed, guide him to see that 

\[-(2k - 1) = -1[2k + (-1)].\]

Both terms inside the parentheses must be multiplied by \(-1\).

Watch For These Common Errors

Your student may try to combine terms with different variables. For example, he may try to combine \(4a + 11b\) as \(15ab\) in problem 12 of Guided Practice. Remind him that unlike terms cannot be combined. If your student finds the technique helpful, encourage him to use colors or shapes to highlight like terms.

Complete problems 27–51 of Practice 3.4 on pp. 159–160 in Math in Focus 2A.

Today you learned how to expand algebraic expressions in order to then simplify them.

TEACHING NOTES

PRACTICE

WRAP-UP

PRACTICE QUESTIONS
LEARN

WARM-UP
Find the greatest common factor (GCF).
1. 3 and 9
2. 8, 12, and 20
3. 16, 40, and 56

TEACHING NOTES

WARM-UP ANSWERS
1. 3 2. 4 3. 8

INSTRUCTION
Read **Factor Algebraic Expressions with Two Variables** on pp. 161–162 in *Math in Focus 2A*. Model the problem with your algebra tiles and act out the solution. **Factoring** means rearranging the terms in the expression into identical groups. Factoring is the opposite of expanding. Therefore, you can use expanding to check whether you have factored correctly.

Read through **Example 14** on p. 162. To find the GCF of 3x and −9y, think of the factors for each number. You may wish to list them on a piece of paper.

Factors of 3x: 1, 3, x

Factors of −9y: −1, −3, −9, 1, 3, 9, y
Then find the greatest number that is a factor for both numbers. This is the greatest common factor, or GCF. As shown, the GCF of $3x$ and $-9y$ is 3.

Complete **Guided Practice** on p. 163. Use expanding to check your answers.

---

**TEACHING NOTES**

**Textbook Answer Key**

Remind your student that, when using algebra tiles, the $x$ tiles can be used to represent any variable. Therefore, to model the expression $2a - 4$ on p. 161, he can use two $x$ tiles and four $-1$ tiles.

Encourage your student to find the GCF, as shown in Method 2 at the bottom of p. 162, to complete the **Guided Practice** problems. If he struggles with identifying the GCF of two algebraic terms with coefficients, explain that the process is the same as finding the GCF of two whole numbers. Guide your student to find the GCF of $2x$ and $4y$.

Factors of $2x$: 1, 2, $x$

Factors of $4y$: 1, 2, 4, $y$

From the list, your student can see that the GCF of $2x$ and $4y$ is 2.

---

**PRACTICE**

Complete problems 1–12 of **Practice 3.5** on p. 165 in **Math in Focus 2A**.

---

**WRAP-UP**

Today you learned how to factor algebraic expressions with two terms.

$4m + 12n$

$= 4(m) + 4(3n)$ The GCF of $4m$ and $12n$ is 4.

$= 4(m + 3n)$

Always check your answer. Expand your factored expression to make sure it equals the original expression.

$4(m + 3n) = 4(m) + 4(3n)$

$= 4m + 12n$

$4m + 12n$ is factored correctly.
Quick Check

Please go online to view and submit this assessment.

More to Explore

View Simplifying Expressions to review this important skill.
More Simplifying Algebraic Expressions - Part 7

Objectives
- Factor algebraic expressions.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete the Practice Questions.

LEARN

WARM-UP
Find the greatest common factor (GCF).

1. 3x and 6
2. 2a and 5
3. x and 4

TEACHING NOTES

WARM-UP ANSWERS
1. 3  2. 1  3. 1

INSTRUCTION
Read Factor Algebraic Expressions with Negative Terms on p. 163 in Math in Focus 2A. In expressions such as \(-x - 2\), you need to factor \(-1\) from each term. After factoring and simplifying, the resulting expression is \(-1(x + 2)\), or \(-x - 2\).

Then read through Example 15 on p. 164 in Math in Focus 7A. When you factor algebraic expressions, find the GCF. If the coefficients of all of the terms are negative, factor out \(-1\), or just factor out \(-1\) times the GCF.

Complete Guided Practice on p. 164 in Math in Focus 7A. Remember to factor the GCF completely from every term. Factor out \(-1\) if possible. Expand your answers to make sure you have factored correctly.
WATCH FOR THESE COMMON ERRORS

Your student may not factor completely. For example, he may not completely factor \(-3\) from each term.

Error: \(-3f - 6 = -3(f + 6)\)

Your student may also factor incorrectly because he has forgotten how to multiply or divide with negative integers.

Error: \(-3f - 6 = -3(f - 2)\)

Complete problems 13–33 of Practice 3.5 on p. 165 in Math in Focus 2A.

Instructional Video: Simplifying Expressions Using the Distributive Property

Today you learned how to factor algebraic expressions with negative terms.

\[-4x - 5 = -4x + (-5)\]

\[= (-1)(4x) + (-1)(5)\]

\[= -1(4x + 5)\]

\[= -(4x + 5)\]

Expand the expression to check the factoring.

\[-(4x + 5) = (-1)(4x + 5)\]

\[= (-1)(4x) + (-1)(5)\]

\[= -4x - 5\]

\[-4x - 5\] is factored correctly.
PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Travis is playing a video game that has several levels. On Level 1, he earned $x + 1$ points. On Level 2, he earned $3x + 1$ points plus 2 bonus points.

a) How many times more points did Travis earn on the second level than the first level?

b) What is the average number of points Travis earned on both levels?

c) Explain how you found your answers.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information given to calculate how many more points Travis earned on the second level than he did on the first level?
- Calculate the average number of points Travis earned on both levels?
- Show your work to explain how you found your answers?
**Writing an Algebraic Expression - Part 1**

**Objectives**
- Write algebraic expressions to represent real-world situations.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition

**Assignments**
- Complete Warm-up.
- Read and complete pages in *Math in Focus 2A*.
- Complete the Practice Questions.

---

**LEARN**

**WARM-UP**

Factor each expression.

1. $6b + 3$
2. $5d - 20$

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. $3(2b + 1)$  
2. $5(d - 4)$

---

**INSTRUCTION**

Read the instructional section on pp. 166–167 in *Math in Focus 2A*. Read verbal descriptions carefully to look for key pieces of information. When you translate verbal descriptions into algebraic expressions, you may need to use fractional, decimal, or negative coefficients.

Read through Example 16 on pp. 167–168. Notice that key pieces of information are pulled out and translated into terms. These terms are then combined using the appropriate operations. Carefully read the verbal description to determine which operations to use.

For part a, only the amount of orange juice is changed. The amounts of other ingredients in the fruit punch remain unchanged.

For part b, keep in mind that $x + 0.03x = 1x + 0.03x$. When these like terms are combined, the sum is $1.03x$.

Note that in part c, the weight the basket can hold is shown in terms of $w$. 

---

Grade 7 Calvert Math in Focus
Complete **Guided Practice** on p. 169. For problem 1, recall that a percent can be written as a fraction or as a decimal. Reread the verbal descriptions carefully to make sure that you accurately translate by parts. Make sure you combine terms in the correct order.

**TEACHING NOTES**

**Textbook Answer Key**

Suggest that your student draw a diagram or bar model for each problem. For instance, for the problem shown at the top of p. 167, he could draw 7 bars divided into 28 equal parts to represent the amount each student receives. Each bar must be divided into 4 equal parts. Since each bar weighs $c$ grams, this means each student receives $c/4$ grams of clay.

**PRACTICE**

Complete problems 1–10 of **Practice 3.6** on p. 176 in *Math in Focus 2A*.

**WRAP-UP**

Today you learned how to translate verbal descriptions into algebraic expressions. Translate by parts, using variable terms that have decimal, fractional, or negative coefficients as needed.
Mr. Williams has a board that is $x$ feet long. He cuts off one fourth of the board. Write an expression for the remaining length of the board.

$$x \text{ minus one-fourth of } x$$

$$x - \frac{1}{4}x$$

$$= \frac{3}{4}x \text{ feet}$$

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Writing an Algebraic Expression - Part 2

LEARN

WARM-UP

Draw a diagram to represent each situation. Then solve.

1. Pedro’s rectangular yard is 14 ft wide and 20 ft long. What is the perimeter of Pedro’s yard?
2. A square table measures 2.5 m along each side. What is the perimeter of the table?

WARM-UP ANSWERS

1. 68 ft  2. 10 m

TEACHING NOTES

INSTRUCTION

Read the instructional section at the top of p. 170 in Math in Focus 2A. Drawing a diagram can often help you better visualize a problem and make it easier to solve. It is not necessary to draw a model to scale. The important thing is to draw a basic representation to help you visualize a problem.

Then read Example 17 on p. 170. The perimeter is the sum of the lengths of all the sides of the rectangle. In this example, each term contains a variable. Remember to add like terms to simplify the algebraic expression.

Complete Guided Practice on p. 170. Copy and label the diagram to help you solve the problem.

Read the instructional section on p. 171. The process for translating verbal descriptions into algebraic expressions with more than one variable is similar to translating into expressions with one variable. Just remember that unlike terms cannot be combined when simplifying.
Read **Example 18** on pp. 171–172. Several key pieces of information are presented in the problem. Organizing the information in a table makes it easier to keep track of. The column headings in the table name key information for each type of fruit.

Tables can be especially helpful for situations involving rates. The price per pound for each fruit is a unit cost. Unit cost is a type of unit rate. The total cost of the strawberries, 3.6x dollars, was obtained by multiplying 3 5x by 6 and then converting the product to a decimal. This makes the expression easier to simplify. Your answers may contain either fractions or decimals but should not contain both in the same expression.

Complete **Guided Practice** on p. 172. For problem 6, copy and complete the table to organize the information. This will help you write an algebraic expression to represent the situation. For problem 7, remember that you can use decimal coefficients for algebraic terms. Express your answer in dollars, not cents.

**TEACHING NOTES**

**Textbook Answer Key**

Discuss with your student how using diagrams or models can help him visualize problems. Tables are useful for organizing information. Remind your student that it is important to label a drawing of a geometric figure with algebraic expressions that represent the dimensions of that figure.

**Looking Forward:** Your student will complete part of the next lesson using spreadsheet software such as Microsoft Excel. It is strongly recommended that the activity be completed as given, but if you do not have this software available, your student can copy the table on paper and use a calculator for computation.

**PRACTICE**

Complete problems 11–15 of **Practice 3.6** on p. 176 in *Math in Focus 2A*.

**WRAP-UP**

Today you learned how to use diagrams, models, and tables to visualize information in a real-world problem. Using visual aids makes it easier to write algebraic expressions.

Gavin made two square tables. The side lengths of the larger table are $s$ inches. The side lengths of the smaller table are $(s - 5)$ inches. What is the total combined perimeter of the two tables?
The total combined perimeter of both tables is \((8s - 20)\) inches.

**QUICK CHECK**

Please go online to view and submit this assessment.

**MORE TO EXPLORE**

If you needed help with this question, you can draw a diagram to help visualize the problem.
In this game, *Who Wants to Be a Hundredaire?*, you will identify an algebraic expression from its verbal expression.

Please go online to view and submit this assessment.
Writing an Algebraic Expression - Part 4

Objectives
- Write algebraic expressions to represent real-world situations.
- Use spreadsheet software to enter and evaluate algebraic expressions.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition
- spreadsheet software (Optional)

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete the Practice activity.
- Complete the Practice Questions.

LEARN

WARM-UP
Translate the verbal descriptions into algebraic expressions.

1. Kaitlyn made \(d\) dozen cookies for a bake sale. She sold all but 6 cookies. How many cookies did Kaitlyn sell?
2. Owen deposited $150 to open a savings account. He deposits \(n\) dollars each month for 4 months. How much does Owen have in his saving account?

WARM-UP ANSWERS
1. \((12d - 6)\) cookies
2. \((150 + 4n)\) dollars

TEACHING NOTES

INSTRUCTION
Read the instructional section at the top of p. 173 in Math in Focus 2A. Parentheses can help you organize key information when you translate more complicated verbal descriptions into algebraic expressions.

Then read Example 19 on p. 173. The top bar model represents the situation at the beginning of the problem. The bottom model represents the situation after Nancy gives her cards away. After she gives 12 cards to her brother, she has \(n - 12\) left to give to her friends. Because she is dividing these among 4 friends, this part of the bar is divided into 4 sections. Notice that \((n - 12)\) must be placed in parentheses before it is multiplied by \(\frac{1}{4}\).
Complete **Guided Practice** on p. 174. Since this problem is similar to the one in **Example 19**, you can use a similar strategy to solve.

Then read through the **Technology Activity** on pp. 174–175. The **Math Note** shows the method used to calculate Maria's average of 130 minutes per day. To find the total number of minutes used over 4 days, multiply the average by 4. \((4 \cdot 130 \text{ min})\)

For step 4, enter the formula “\(=B2/5\)” to calculate the average number of minutes spent over 5 days.

For step 5, work in row 7. Under columns B and C, define formulas in the cells as described in steps 3 and 4. Then enter likely values in the cell under column A until the cells under columns B and C show 750 min and 150 min respectively.

---

**TEACHING NOTES**

**Textbook Answer Key**

To connect the spreadsheet activity to the work your student has completed in this chapter, lead him to see that a spreadsheet is a type of table, similar to those seen in the previous lesson. If you do not have access to spreadsheet software, your student can copy the table on paper and use a calculator for computation.

---

**PRACTICE**

Complete problems 16–23 of **Practice 3.6** on p. 177 in *Math in Focus 2A*.

Then read the following problem and follow the instructions.

Mr. Conroy purchased groceries for the first three weeks of the month. He spent an average of $175 each week. Mr. Conroy spent \(d\) dollars during the fourth week.

1. Write an algebraic expression for the average amount he spent on groceries over 4 weeks.
2. Use your spreadsheet software to make a table similar to the one you completed in the **Technology Activity** on pp. 174–175. First determine the calculations that need to be made based on the algebraic expression you wrote. Then enter formulas in the spreadsheet so it can perform the calculations. Refer to the formulas you entered in the **Technology Activity**.

**TEACHING NOTES**

**Practice Answer:**

\[(1/4)(d + 525)\]
If your student struggles with problem 18 on p. 177 in *Math in Focus 2A*, help him make a bar model to represent the situation.

**WRAP-UP**

Today you learned how to use parentheses when you translate verbal descriptions into algebraic expressions.

Teri made $b$ bookmarks. She gave 20 bookmarks to her teacher and divided the rest equally among 5 friends. How many bookmarks did each friend receive?

$$1 \frac{1}{5} (b - 20) = 1 \frac{1}{5} b - 4$$

Each friend received $(1 \frac{1}{5} b - 4)$ bookmarks.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Writing an Algebraic Expression - Part 5

Objectives
- Write algebraic expressions to solve real-world problems.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition
- colored pencils (Optional)

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A
- Complete the Quick Check.

LEARN

WARM-UP
Use the picture to solve each problem.

1. What is the cost of each pair of scissors?
2. What is the cost of each stapler?

TEACHING NOTES

WARM-UP ANSWERS

1. $8.00
2. $5.50

INSTRUCTION
Read p. 178 in Math in Focus 2A. You can use algebraic reasoning to solve real-world problems. You use algebraic reasoning when you represent unknown amounts with symbols and manipulate the symbols using step-by-step logic. You use it when you use visual aids such as diagrams, models, and tables to visualize and organize information.

Read Example 20 on p. 179. The ratio 2:3 (Aaron's and Barry's share of the apples) is represented by 2 parts for Aaron and 3 parts for Barry. These sections are only included in the part of the model that represents ripe apples because they only eat the ripe apples. When you work to solve real-world problems, it is important to show your work in a clear and logical way.
Complete **Guided Practice** on p. 180. Use the diagram and bar model to help you visualize each problem. Remember to use the order of operations when you expand and simplify algebraic expressions.

Then read Example 21 on p. 181. The large number of terms and operations in this example demonstrates why it is important to work carefully, step-by-step, through a problem. You may forget a term or write a term or operation incorrectly if you work too quickly.

Complete **Guided Practice** on p. 181. In exercises such as this, it is helpful to list and label given information before you attempt to solve the problem.

### Number of Comic Books

**Amy:** \( x \)

**Melvin:** \( \left( \frac{2x}{5} + 1 \right) \)

**Joel:** \( \frac{2x}{5} + 1 \) \(-\) \( \frac{x}{10} \)

---

**TEACHING NOTES**

**Textbook Answer Key**

If your student struggles with problem 1 of **Guided Practice** on p. 180 in *Math in Focus 2A*, point out the reasoning in the thought bubble. The ratio of the unshaded region to the shaded region is 1 : 3. This ratio compares one *part* of the area of the whole triangle to another *part* of the area of the whole triangle. Your student needs to express the area of the shaded region in terms of \( u \). Because \( u \) is part of the expression \((u + 10)\), which describes the area of the whole triangle, this means he must describe *part* of the triangle in terms of the *whole* triangle. Therefore, the fraction he should multiply \((u + 10)\) by is 34.

If your student finds the length and complexity of these expressions challenging, encourage him to use colored pencils to highlight individual terms as he simplifies.

---

**PRACTICE**

Complete problems 1–6 of **Practice 3.7** on p. 183 in *Math in Focus 2A*. 

WRAP-UP

Today you learned how to solve real-world problems by using algebraic reasoning.

A rope measuring \((x + 5)\) feet in length is cut into two pieces in the ratio 4 : 6. What is the length of the longer piece?

\[
\frac{6}{10} (x + 5) = \frac{6}{10} \cdot x + \frac{6}{10} \cdot 5 = \frac{6}{10}x + 3 = \frac{3}{5}x + 3
\]

The longer piece is \(\left(\frac{3}{5}x + 3\right)\) feet long.

QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

View *Simplifying Expressions Using the Distributive Property* to review this important skill.
WARM-UP

Find the equivalent decimal.

1. 4 10
2. 33 100
3. 3 4

TEACHING NOTES

WARM-UP ANSWERS

1. 0.4    2. 0.33    3. 0.75

INSTRUCTION

Read Example 22 on p.182 in *Math in Focus 2A*. This problem includes the concept of percent. As you read through the problem, note the key points that need to be translated. The parts are listed and then combined in a step-by-step, logical manner. Percents can be written as decimals, as shown in the speech bubble. If this does not make sense, try substituting an actual percent for \( p \) percent to see how dividing the number by 100 results in an equivalent decimal.

It can be helpful to use a table like the following to organize the information.

<table>
<thead>
<tr>
<th>Students Who Wear Glasses</th>
<th>Number</th>
<th>Percent Who Wear Glasses</th>
<th>Number Who Wear Glasses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>40</td>
<td>0.01( p )</td>
<td>0.4( p )</td>
</tr>
<tr>
<td>Girls</td>
<td>40</td>
<td>0.01( p )</td>
<td>0.4( p )</td>
</tr>
</tbody>
</table>

Complete Guided Practice on p. 182. Use substitution to check your answers.
Textbook Answer Key

WATCH FOR THESE COMMON ERRORS

Some students may factor $80 - 0.4p - 0.4q$ incorrectly as $80 - 0.4(p - q)$. Redirect your student to the part of Example 22 that models how to rewrite and factor the expression before simplifying. Point out that the $-0.4p - 0.4q$ portion of the expression is rewritten as $(-0.4)(p) + (-0.4)(q)$. This is then factored as $(-0.4)(p + q)$.

PRACTICE

Complete problems 7–13 of Practice 3.7 on pp. 183–184 in Math in Focus 2A.

WRAP-UP

Today you learned how to solve problems using algebraic reasoning.

A middle school ordered $t$ raffle tickets for a school fundraiser. Three-fourths of the tickets were given to students and 200 tickets were given to parents. Seventy percent of the remaining tickets were given to teachers. The rest were given to sponsors. How many tickets were given to sponsors?

Number of raffle tickets remaining after giving to students and parents:

$$\frac{1}{4}t - 200$$

Number of tickets given to sponsors:

$$0.3\left(\frac{1}{4}t - 200\right) = 0.3(0.25t - 200) = 0.075t - 60$$

Rewrite $\frac{1}{4}t$ as a decimal. Then factor out $0.25$.

The number of raffle tickets given to sponsors is $0.075t - 60$.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Writing an Algebraic Expression - Part 7

Objectives
- Apply mathematical concepts and skills to solve problems.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition
- protractor
- straightedge

Assignments
- Complete Warm-up.
- Read and complete Brain @ Work in Math in Focus 2A.
- Complete the Practice activity.
- Complete the Practice Questions.

LEARN

WARM-UP

Multiply.

1. $3 \cdot \frac{3}{4}$
2. $15 \cdot \frac{4}{5}$
3. $45 \cdot \frac{5}{9}$

TEACHING NOTES

WARM-UP ANSWERS
1. 9/4 2. 12 3. 25

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Number of Sides</th>
<th>Sum of Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>180°</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>360°</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>540°</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>720°</td>
</tr>
<tr>
<td>Heptagon</td>
<td>7</td>
<td>900°</td>
</tr>
<tr>
<td>Octagon</td>
<td>8</td>
<td>1,080°</td>
</tr>
</tbody>
</table>

$(n - 2) \cdot 180°$
Read and complete Brain @ Work on p. 184 in *Math in Focus, Course 2, Volume A, Chapter 3*.

Before you begin the problem, think about what you know. The Fahrenheit and Celsius scales both measure temperature, but they use different numbers. Water freezes at 0° Celsius and at 32° Fahrenheit. Water boils at 100° Celsius and at 212° Fahrenheit. These measures are shown in the diagram in the textbook.

The following diagram shows a few key pieces of information:

- The difference between the freezing point and the boiling point is 100° in Celsius, but the difference is 180° in Fahrenheit. The scales start at different numbers (0 and 32).
- The scales rise at different rates (100 units and 180 units). You can think of this as a rate. Every 100° Celsius corresponds to 180° Fahrenheit.

Use this information to help you find the conversion formula: $C = \frac{5}{9}(F - 32)$

**TEACHING NOTES**

**Textbook Answer Key**

If your student needs assistance, encourage him to write the fractions $\frac{100}{180}$ and $\frac{180}{100}$ in simplest form. Then ask if either of the resulting fractions ($\frac{5}{9}, \frac{9}{5}$) look familiar. Your student should recognize $\frac{5}{9}$ from the formula shown on p. 184 in *Math in Focus 2A*. See if he can determine why 32 is subtracted in the formula. If he needs help, suggest that he look at the starting point of each scale.

**PRACTICE**

Write a constructed response to explain how you solved Bryan's problem on p. 184 in *Math in Focus 2A*. 
Then use reasoning to find the formula for the number of degrees in the angles of a polygon.

1. First, copy this chart on separate paper.

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Number of Sides</th>
<th>Sum of Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quadrilateral</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pentagon</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hexagon</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heptagon</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Octagon</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Use your straightedge to draw a sample of each type of polygon.

3. Use your protractor to measure the angles in each polygon and find the sum.

4. Finally, use this information to write a formula that will help you find the number of degrees in a polygon if you know the number of sides.

**TEACHING NOTES**

Your student may need help finding the relationship between the number of sides and angles of a polygon and the sum of the interior angles. Suggest he draw different polygons and divide the polygon into the least number of triangles possible. Remind him that the sum of the angles of a triangle equals 180°.

If you would like to extend your student’s learning, have him add a column to his chart and title it "Angle Measure in a Regular Polygon." Remind your student that a regular polygon has sides and angles that are all the same. Have him calculate the measure of a single interior angle in each polygon by dividing the sum of the angle measures by the number of sides. Then have him write a formula that will help find the number of degrees in a single interior angle if he knows the number of sides of the regular polygon.

**WRAP-UP**

Today you learned to apply your knowledge of algebraic expressions to find formulas.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Jonathan's age is one-third Belinda's age now. In 12 years, Belinda will be \( m \) years old.

a) Write an algebraic expression to represent Belinda's age now.

b) Write an algebraic expression to represent Jonathan's age 5 years from now.

c) If \( m = 36 \), what will Jonathan's age be 5 years from now?

d) Explain how you found the answers.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information given to answer all three questions?
- Write an algebraic expression to represent Belinda's age now?
- Write an algebraic expression to represent Jonathan's age 5 years from now?
- Answer what Jonathan's age will be 5 years from now if $m = 36$?
- Show your work?
- Explain how you found the answers to the three questions?
SHOW

Now you need to decide what you will do on your trip. Are you going to a park that will require tickets or maybe a zoo or museum that you will need tickets for? Maybe a sporting event? It is your vacation. In this part you will need to find out how much it will cost for you and your family to complete these activities. You will also make a list of all of these sources and find the total cost. In the next part, you will add this total cost to each expression from Task #1 and write the new expression in your Math Notebook.

✅ RATE YOUR PROGRESS

Please go online to view and submit this assessment.
SHOW

In the last part, you determined how much it will cost for you and your family to complete activities on your vacation. In this part, you will need to take the total cost from the last part and add this total cost to each expression from Task #1. You will also need to write the new expression in your Math Notebook.

RATE YOUR PROGRESS

Please go online to view and submit this assessment.
Solve Equations and Inequalities - Part 1

Objectives
- Solve one-step equations.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete the Practice Questions.

LEARN

WARM-UP
Simplify each expression.

1. $3y + 7y$
2. $\frac{3}{4}w - \frac{1}{2}w$
3. $6 \div \frac{2}{3}$

TEACHING NOTES

WARM-UP ANSWERS
1. 10y
2. $(1/4)w$
3. 9

INSTRUCTION

Today's part covers important ideas you need to understand before beginning the parts in this chapter. Read p. 188 in Math in Focus 2A. Think about whether you would use an algebraic equation or an algebraic inequality to represent the amount of money you would need if you wanted to go on five rides at the amusement park.

Read Solving algebraic equations by balancing on p. 189. Recall that inverse operations undo each other. Addition and subtraction are inverse operations because adding
a number and then subtracting the same number is the same as adding zero, which does not change the value of the original number. Multiplication and division are inverse operations because multiplying by a nonzero number and then dividing by that number is the same as multiplying by 1, which does not change the value of the original number.

Then read Solving algebraic equations by substitution on p. 190. Recall that to solve an algebraic equation using substitution, you must replace the variable with the given value, evaluate both sides of the equation, and check to see if both sides of the equation equal the same number. If they do, the equation is said to be true and the number you substituted for the variable is a solution to the equation.

**TEACHING NOTES**

Textbook Answer Key

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the Quick Check sections.

Point out to your student that the substitution method can be used to check equations solved algebraically. Encourage her to use the substitution method to check the examples that were solved by balancing on p. 189.

**SKILLS CHECK**

Complete the Quick Check sections on pp. 189–190 (top) in Math in Focus 2A.

**TEACHING NOTES**

After your student completes the Quick Check in the Recall Prior Knowledge lesson of this chapter, review the questions that were answered incorrectly. If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.

<table>
<thead>
<tr>
<th>Quick Check Question(s)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td>Solving Basic Equations - Part 1</td>
</tr>
<tr>
<td>3–4</td>
<td>One-Step Equations with Multiplication and Division</td>
</tr>
<tr>
<td>5–8</td>
<td>Evaluating the Expression: Constants and Variables in Formulas</td>
</tr>
</tbody>
</table>
WRAP-UP

Today you reviewed solving algebraic equations by balancing.

**Example 1**

\[
\begin{align*}
x + 3 &= 9 \\
x + 3 - 3 &= 9 - 3 \\
x &= 6
\end{align*}
\]

\[
\begin{align*}
\frac{2}{5}x &= 4 \\
\frac{2}{5}x \div \frac{2}{5} &= 4 \div \frac{2}{5} \\
x &= 10
\end{align*}
\]

Solve \(2x = 12\).

If \(x = 6\), \[2x = 2 \times 6 = 12\]

The equation \(2x = 12\) is true when \(x = 6\), so the solution of the equation is \(x = 6\).

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
**LEARN**

**WARM-UP**

Compare using <, > or =.

1. $2 \cdot 3$  \(\bigcirc\) $12 \div 2$

2. $9 + 6$  \(\bigcirc\) $22 - 8$

3. $\frac{16}{4}$  \(\bigcirc\) $\frac{15}{3}$

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. $=$  
2. $>$  
3. $<$

**INSTRUCTION**

Today's part covers important ideas you need to understand before beginning the parts in this chapter. Read *Graphing inequalities on a number line* on p. 190 in *Math in Focus 2A*.

Recall that an empty (or open) circle is used to graph inequalities that contain a *less than* symbol, $<$, or *greater than* symbol, $>$. A shaded (or closed) circle is used to graph inequalities that contain a *less than* symbol, $\leq$, or *greater than or equal to* symbol, $\geq$. A shaded circle indicates that the number is included in the solution set, while an open circle indicates that the number is not included. For example, $x > 5$ would be graphed with an open circle at 5, indicating that all numbers greater than 5 are solutions.
or equal to symbol, $\leq$, or a greater than or equal to symbol, $\geq$. Think about which direction the arrow points when graphing inequalities.

Read Writing algebraic inequalities on p. 191. Pay special attention to which inequality symbol in the right-hand column corresponds to the red highlighted words in the left-hand column. Consider what the graph of each inequality would look like and decide if the graph would have an open or closed circle.

Textbook Answer Key

After your student completes the Quick Check in the Recall Prior Knowledge lesson of this chapter, review the questions that were answered incorrectly. If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.

<table>
<thead>
<tr>
<th>Quick Check</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question(s)</td>
<td></td>
</tr>
<tr>
<td>9–18</td>
<td>Inequalities</td>
</tr>
</tbody>
</table>

PRACTICE

Complete the Quick Check sections on pp. 190–191 in Math in Focus 2A.

WRAP-UP

Today you reviewed graphing inequalities on a number line.

$x > 1.5$

$x \leq 3$

You also reviewed writing algebraic inequalities.
The length of the box, \( l \), is at least 4 inches. 
\[ l \geq 4 \]

The number of bags, \( b \), is at most 12. 
\[ b \leq 12 \]

The cost, \( c \), is less than $10. 
\[ c < 10 \]

The height of the tree, \( h \), is more than 7 feet. 
\[ h > 7 \]

The total number of people, \( p \), is not 22. 
\[ p \neq 22 \]

---

**QUICK CHECK**

Please go online to view and submit this assessment.

---

**MORE TO EXPLORE**

Remember that all solutions are plotted, which form a line. Anything shaded is a solution, and anything not shaded is not a solution. View the video, *Graphing Inequalities on a Number Line* (03:14) to learn how to graph inequalities on a number line.

Please go online to view this video ▶
Solve Equations and Inequalities - Part 3

**Objectives**
- Identify equivalent equations.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition
- algebra tiles

**Assignments**
- Complete Warm-up.
- Read and complete pages in Math in Focus A.
- Complete the Practice Questions.

**LEARN**

**WARM-UP**

Determine whether the expressions are equivalent.

1. $4x + 5$ and $3x + 5 + x$
2. $3(2x - 1)$ and $6x - 1$
3. $8x + 4$ and $4(2x + 1)$

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. equivalent  2. not equivalent  3. equivalent

**INSTRUCTION**

Read **Identify Equivalent Equations** on pp. 192–193 in *Math in Focus 2A*. Equivalent equations are equations that have the same solution. Use algebra tiles and follow along with the book to model the equation $x - 1 = 7$. Then use the tiles to model subtracting 2 from both sides of the equation. You can use substitution to verify that the solution to both the original equation and the resulting equation is 8.

Next, model the equation $x - 3 = 5$ using the algebra tiles, and then model adding 3 to both sides. The solution to both equations is the same, 8. Now model multiplying the equation $x = 8$ by 2. Think about whether the solution will change and use the algebra tiles to verify your prediction. Finally, model dividing the equation $2x = 16$ by 4. Again, notice that the solution is the same.

Consider the processes you just modeled and what conclusions you can make about performing the same operation on both sides of an equation, including whether or not you obtain an equivalent equation.
Review **Example 1** on p. 194. Complete **Guided Practice** on p. 195. The problems guide you through two ways of determining whether equations are equivalent. For problem 1, can the first equation be rewritten to match the second equation? If it can, then the equations must have the same solution and therefore are equivalent equations. If it cannot, then the equations must have different solutions and therefore are not equivalent equations.

For problems 2–4, is the solution to one equation also the solution to the other equation? Remember that you can use substitution to check whether a given value is a solution to an equation.

Remember to apply the operation to every single term on both sides to keep the equation balanced. Discuss the **Think Math** question with your Learning Guide.

---

### TEACHING NOTES

**Textbook Answer Key**

If your student has difficulty identifying equivalent equations, she may find it helpful to create and use a balance model. Explain that the model is balanced if the two sides contain equivalent equations. If a scale is balanced, and one side has several smaller objects, she can arrange them differently without affecting the balance. This shows why she can combine the terms on one side of an equation without having to change the value of the other side to keep it equal. Your student may find it helpful to see equations that are not equivalent using a balance model. After your student gains an understanding of equivalent equations, help her to transition from using a balance model to using algebra tiles.

---

### PRACTICE

Complete **Practice 4.1** on p. 196 in *Math in Focus 2A.*
Today you learned how to identify equivalent equations.

Example 1

\[ x - 7 + 5x = 11 \] and \[ 6x = 18 \]

\[ x - 7 + 5x = 11 \]

\[ 6x - 7 = 11 \]

\[ 6x - 7 + 7 = 11 + 7 \]

\[ 6x = 18 \]

\[ x - 7 + 5x = 11 \] can be rewritten as \[ 6x = 18 \], so the equations have the same solution and are equivalent equations.

Example 2

\[ 1.5x = 15 \] and \[ x + 4 = 10 \]

\[ x + 4 = 10 \]

\[ x + 4 - 4 = 10 - 4 \]

\[ x = 6 \]

If \[ x = 6 \], then \[ 1.5x = 1.5 \cdot 6 \]

\[ = 9 (\neq 15) \]

The equations have different solutions, so they are not equivalent equations.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
LEARN

WARM-UP

Solve each equation.

1. \(x + 4 = 7\)
2. \(3x = 12\)
3. \(x/5 = 9\)

WARM-UP ANSWERS

1. \(x = 3\)  
2. \(x = 4\)  
3. \(x = 45\)

INSTRUCTION

Read pp. 197−199 in Math in Focus 2A. Before writing an equation for a verbal description, first decide what the variable in the equation will represent. For word problems, the variable is usually the amount you are trying to find.

Method 1 uses substitution to solve. Think about the pros and cons of using this method. (This process is the same as guess-and-check.)

Method 2 on pp. 198−199 uses balancing to solve the equation. Remember that you must perform the same operation to both sides of an equation to keep it balanced.

Use algebra tiles to model \(2x + 6 = 9\) as shown on p. 198. Think about what operation you can perform to isolate the variable term, \(2x\). Use the algebra tiles to model each step of this method. Remember that...
solving an equation means finding the value of the variable that makes the equation true. Continue performing operations on both sides of the equation until it is in the form $x = a$ number as shown on pp. 198–199. Decide what operation you can perform to get it to this form.

The last step to solving an equation is to check your answer. Substitute the value you found for $x$ into the original equation, $2x + 6 = 9$. If you get a true equation, then you solved for $x$ correctly. If you do not get a true equation, go back and check your work. Finally, state what the solution represents in the context of the problem.

Review Example 2 on pp. 200–201. In part b, $\frac{1}{6}y$ can be written as $y \cdot \frac{1}{6}$ because $\frac{1}{6}y = \frac{1}{6} \cdot y \cdot 1 = y \cdot \frac{1}{6}$. Method 2 illustrates how you can first multiply both sides of an equation by the least common denominator (LCD) to obtain an equivalent equation that does not contain fractions. If you use this method, be sure to multiply each term on both sides of the equation by the LCD.

Complete Guided Practice on p. 202. Problem 3 has two variable terms on the same side of the equation. Think about how to combine these like terms. For problem 4, decide whether you prefer to work with the fractions or to multiply each term on both sides of the equation by the LCD to get rid of the fractions.

### TEACHING NOTES

**Textbook Answer Key**

Point out to your student that it is typically easiest to solve an equation by applying the order of operations in reverse. First undo addition or subtraction and then undo multiplication or division. For example, to solve $2x + 6 = 9$, you can first use subtraction to undo the addition, and then you can use division to undo the multiplication.

If she chooses to first divide both sides of the equation by 2 and then subtract 6 from both sides, she must be sure to divide each term on both sides of the equation by 2.

\[
\begin{align*}
2x + 6 &= 9 \\
\frac{2x}{2} + \frac{6}{2} &= \frac{9}{2} \\
x + 3 &= 4.5 \\
x &= 4.5 - 3 \\
x &= 1.5
\end{align*}
\]
INTERACTIVE ACTIVITY

Use these virtual algebra tiles as you work through the lesson to help you solve the equations.

PRACTICE

Complete problems 1–13 of Practice 4.2 on p. 209 in Math in Focus 2A.

WRAP-UP

Today you learned how to solve algebraic equations with variables on the same side of the equation.

\[ 4x - 3 = 17 \]

\[ 4x - 3 + 3 = 17 + 3 \]

\[ 4x = 20 \]

\[ \frac{4x}{4} = \frac{20}{4} \]

\[ x = 5 \]

You also learned that it is important to check your solution using substitution.

\[ 4x - 3 = 4(5) - 3 \]

\[ = 20 - 3 \]

\[ = 17 \]

When \( x = 5 \), the original equation is true, so \( x = 5 \) is the solution.
Quick Check

Please go online to view and submit this assessment.

More to Explore

Remember to check your answers by substituting the value into the equation to see if it makes a true statement.
## Objectives
- Solve equations with variables on both sides.

## Books & Materials
- Math in Focus 2A
- *Math in Focus - Teacher Edition*
- algebra tiles

## Assignments
- Complete Warm-up.
- Read and complete pages in *Math in Focus 2A*.
- Complete the Practice Questions.

### LEARN

### WARM-UP

Solve each equation.

1. \(3x + 1 = 7\)
2. \(1.5y - 9 = 12\)
3. \(\frac{x}{3} - 6 = 4\)

### TEACHING NOTES

#### WARM-UP ANSWERS

1. \(x = 2\)  
2. \(y = 14\)  
3. \(x = 30\)

### INSTRUCTION

Read the instructional section on pp. 202–203 in *Math in Focus 2A*. The first step to solving an equation with a variable on both sides is to use addition or subtraction to isolate the variable terms on a single side of the equation. It does not matter which side of the equation you choose to isolate the variable terms on.

Use algebra tiles to model the equation \(4x + 7 = x + 10\) (shown on p. 202) and then isolate the variable terms on the left side of the equation. Continue to use the algebra tiles to model solving the equation. Check your solution by substituting it for \(x\) into the original equation.

Model the equation \(4x + 7 = x + 10\) again. This time, use the algebra tiles to model isolating the variable terms on the right side of the equation. Then continue to use the algebra tiles to model solving the equation.

Compare the two methods. Think about which method you prefer and why. Remember that the equal sign is not directional; it represents that both sides of the equation are equal in value.
Review Example 3 on pp. 203–205. In Method 1 of part a, the variable is isolated on the left side of the equation, and in Method 2, it is isolated on the right side.

Notice that in part b, another way to solve is to first multiply all of the terms on both sides of the equation by 10 to get rid of the decimal coefficients.

Complete Guided Practice on p. 205.

Textbook Answer Key

Point out to your student that isolating the variable on one side of the equation involves adding or subtracting a variable term from both sides and then simplifying the equation by combining like terms. Review that like terms have the same variable raised to the same exponent.

In some cases, your student may prefer to isolate the variable on the right side of the equation, but then prefer to rewrite the equation with the variable on the left side so the solution is in the form $x = a$ \textit{number} instead of $a$ \textit{number} $= x$. Point out that flipping the left and right side of the equation is an application of the Symmetric Property of Equality.

Complete problems 14–26 of Practice 4.2 on pp. 209–210 in Math in Focus 2A.

Instructional Video: Solving Equations with Variables on Both Sides

Today you learned how to solve equations with variables on both sides. The variable may be isolated on either side of the equation; however, sometimes fewer steps are needed when isolating on one side as opposed to the other.
**Method 1: Isolate on the left.**

\[2x - 7 = 3x - 12\]

\[2x - 7 - 3x = 3x - 12 - 3x\]

\[-x - 7 = -12\]

\[-x - 7 + 7 = -12 + 7\]

\[-x = -5\]

\[-\frac{x}{-1} = \frac{-5}{-1}\]

\[x = 5\]

**Left side:**

\[2x - 7 = 2(5) - 7\]

\[= 10 - 7\]

\[= 3\]

**Right side:**

\[3x - 12 = 3(5) - 12\]

\[= 15 - 12\]

\[= 3\]

When \(x = 5\), the left side equals the right side, so \(x = 5\) is the solution.

---

**✓ PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
**Solve Equations and Inequalities - Part 6**

**Objectives**
- Solve equations in factored form.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition
- algebra tiles

**Assignments**
- Complete Warm-up.
- Read and complete pages in *Math in Focus 2A*.
- Complete the Practice Questions.

---

**LEARN**

**WARM-UP**

Solve each equation.

1. $4x + 1 = 7 + 2x$
2. $1.2y - 3.4 = 2.6 - 0.8y$
3. $\frac{x}{5} + 2 = 2x - 7$

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. $x = 3$  
2. $y = 3$  
3. $x = 5$

---

**INSTRUCTION**

Read *Solve Algebraic Equations in Factored Form* on p. 206 in *Math in Focus 2A*. For Method 1, remember that to expand using the Distributive Property, you multiply each term inside the parentheses by the factor outside the parentheses.

Use algebra tiles to model the equation $2(3x + 1) = 11$ (shown on p. 206). Use the algebra tiles to model distributing the factor of 2 on the left side of the equation by creating 2 groups of $(3x + 1)$. Then continue to use the algebra tiles to model solving the equation. Check your solution by substituting it into the original equation.

Use algebra tiles to model $2(3x + 1) = 11$ again. This time, use the algebra tiles to first model dividing both sides of the equation by 2. Then continue to use the tiles to model solving the equation.

Compare the two methods. Think about which method you prefer and why.
Review Example 4 on pp. 207–208. In Method 1 of part a, the Distributive Property is applied first, whereas in Method 2, the equation is solved using inverse operations. Notice that both methods yield the same solution to the equation.

Read and discuss the Think Math section on p. 207 with your Learning Guide. Remember that inverse operations are used to solve equations. Complete Guided Practice on p. 208.

Textbook Answer Key

Discuss with your student that there can be many different ways to solve an equation, but one method can be more convenient than another method. Explain to your student that one method is not more correct than another; they are merely different ways to work, and that as she becomes more comfortable with them, she will be able to more easily determine the most convenient method for her.

PRACTICE

Complete problems 27–42 of Practice 4.2 on p. 210 in Math in Focus 2A.

WRAP-UP

Today you learned how to solve equations in factored form.

\[
5(x + 3) = 50
\]

\[
5x + 15 = 50
\]

\[
5x + 15 - 15 = 50 - 15
\]

\[
5x = 35
\]

\[
\frac{5x}{5} = \frac{35}{5}
\]

\[
x = 7
\]
Check your solution using substitution.

\[ 5(x + 3) = 5(7 + 3) \]
\[ = 5(10) \]
\[ = 50 \]

When \( x = 7 \), the equation is true, so \( x = 7 \) is the solution.

✔️ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
### WARM-UP

Write an algebraic expression for each situation.

1. A box contains 8 pencils. How many pencils are in \( x \) boxes?

2. Suppose your brother is twice as old as you. If you are \( a \) years old, how old is your brother?

### TEACHING NOTES

#### WARM-UP ANSWERS

1. \( 8x \)  
2. \( 2a \)

#### INSTRUCTION

Read page \( p. 211 \) in *Math in Focus 2A*. To solve a real-world problem, it can help to draw a model and then write an equation that can be used to solve. For example, the model on page 211 represents the drawing Shirley wants to frame. Labeling all parts of the drawing makes it easier to write an equation to find the width of the frame. Think about what the solution represents in the context of the problem.

Review Example 5 on page \( p. 212 \). In Method 1, a bar model is used to solve the problem, and in Method 2, an equation is used. Both methods yield the same solution.

Complete Guided Practice on page \( p. 213 \). The beginning of a bar model and an algebraic approach are both provided. Choose the method you prefer. Be sure to check your solution in the original riddle.

Review Example 6 on page \( p. 214 \). In part \( a \), you are asked to write an expression, not an equation. Remember that expressions do not contain equal signs, whereas equations do. Expressions can be evaluated by substituting different values for the variable; equations are solved for a variable. Think about whether you prefer using a table or algebraic reasoning to think through the problem.
Complete **Guided Practice** on p. 215. Use either a table or algebraic reasoning to organize the information given in the problem.

Review **Example 7** on p. 215. When solving real-world problems, it is important to write your solution as a statement in the context of the problem. Notice that the value of the variable \(x\) is not the solution to this real-world problem. Always reread the original problem to make sure you are answering the question that was asked.

Complete **Guided Practice** on p. 216. Be sure to find the number of cards James has, as well as the number of cards Fay has.

Read and discuss the **Think Math** section on p. 216 with your Learning Guide. Remember that there is often more than one way to solve a problem.

---

**TEACHING NOTES**

**Textbook Answer Key**

Encourage your student to try different methods to solve the same problem, such as organizing information in a table, drawing a picture, using a bar model, and using algebraic reasoning. Discuss which method she prefers and why. She may prefer one method for some problem types and another method for other problem types. Remind her that while the method she uses is not important, it is important to reread the original problem after solving to ensure she has answered all parts, as well as to check her answers in the original problem.

---

**PRACTICE**

Complete **Practice 4.3** on pp. 217–219 in *Math in Focus 2A*.

---

**SUPPLEMENTAL**

**Instructional Video:** *Writing Equations for Word Problems*
WRAP-UP

Today you learned how to solve real-world problems algebraically.

The length of a rectangle is four times greater than its width. The perimeter of the rectangle is 40 inches. Find the length and width of the rectangle.

Let the width of the rectangle be $x$.

Then the length of the rectangle is $4x$.

$$2(l + w) = P$$

$2(4x + x) = 40$

$2(5x) = 40$

$10x = 40$

$$\frac{10x}{10} = \frac{40}{10}$$

$x = 4$

$4x = 16$

The width of the rectangle is 4 inches, and the length is 16 inches.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
The numbers 1 through 8 are to be placed in the circles in the diagram shown such that the sum of the numbers on each side of the figure is 15. Find the values of \( x \), \( y \), and \( z \). Then copy the diagram on a piece of paper and fill in each circle with the appropriate number. Be sure to show all your work. Upload the finished problem below.

Note: \( x \) satisfies \(-4(x - 2) = -4\),

\[
y \text{ satisfies } 9 + 2(y - 3) = 3(y - 2) + 1,
\]

and \( z \) satisfies \( \frac{z}{3} + \frac{1}{2} = \frac{5}{2} \).
Did you:

- Use the information given to solve for $x, y,$ and $z$?
- Only use the numbers 1 through 8 one time each to complete the circles in the diagram?
- Check that the sum of the numbers on each side of the diagram is 15?
- Copy the diagram on a piece of paper and upload the completed diagram when finished?
- Show your work?
You have a budget of $3000 for your family's hotel stay and activities. Write inequalities to show your expressions based on the provided budget.

Please go online to view and submit this assessment.
Solve Inequalities - Part 1

Objectives
- Solve and graph inequalities with addition and subtraction.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition
- algebra tiles

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete the Practice Questions.

LEARN

WARM-UP
Solve each equation.

1. \( x + 6 = 18 \)
2. \( 0.2x - 4 + 0.8x = 7 \)

WARM-UP ANSWERS
1. \( x = 12 \)  2. \( x = 11 \)

TEACHING NOTES

INSTRUCTION
Read Solve Algebraic Inequalities Using Addition and Subtraction on pp. 220–211 in Math in Focus 2A. The solution to an inequality is called a solution set because it includes more than one value. For example, the solution to the inequality \( x < 3 \) includes all values that are less than 3.

In Method 1, different values of the variable are substituted into the inequality to determine if the inequality is true for that value. Method 2 uses inverse operations to solve the inequality. This method isolates the variable.

Read Graph the Solution Set of an Inequality on a Number Line on pp. 221–222. Notice that if you swap the expressions on either side of an inequality symbol, you must reverse the inequality symbol. Otherwise, the inequalities will not be equivalent. For example, to rewrite \( 0 < x \) with \( x \) on the left side of the inequality symbol and 0 on the right side, you must reverse the direction of the inequality symbol. Therefore, \( 0 < x \) is equivalent to \( x > 0 \).
Review **Example 8** on pp. 223–224. Remember to use an empty circle to graph inequalities that use the < or > symbol and a shaded circle to graph inequalities that use the ≤ or ≥ symbol.

Complete **Guided Practice** on p. 224. Just like when solving equations, always perform the same operation on both sides of the inequality. You should also remember to check your solution set in the original inequality.

Review **Example 9** on p. 225. Read and discuss the **Think Math** section with your Learning Guide. Complete **Guided Practice** on p. 226.

---

**TEACHING NOTES**

**Textbook Answer Key**

Point out to your student that solving inequalities is similar to solving equations. Just as you must perform the same operation to both sides of an equation to produce an equivalent equation, you also must perform the same operation to both sides of an inequality to produce an equivalent inequality. However, remind your student that if she swaps the expressions on either side of an inequality symbol, she must also reverse the inequality symbol.

Explain to your student that while she can check the solution to an equation by substituting the value into the original equation, it is not possible to check all solutions to an inequality. Still, she should check some values from the solution set to make sure her answer is reasonable. Explain that it is also helpful to check that a value outside of the solution set makes the original inequality false.

---

**PRACTICE**

Complete problems 1–14 of **Practice 4.4** on p. 233 in **Math in Focus 2A**.

---

**WRAP-UP**

Today you learned how to solve inequalities using addition and subtraction.

\[
6x + 5 - 5x > 6
\]

\[
x + 5 > 6
\]

\[
x + 5 - 5 > 6 - 5
\]

\[
x > 1
\]

![Graph of the solution set for the inequality 6x + 5 - 5x > 6]
PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Solve Inequalities - Part 2

Follow these instructions for the activity shown below.

Click here to view the activity in a new window.

In this activity, you will explore inequalities and find their solutions. In the Gizmo, select $x + a > b$. Set $a$ to 3, $b$ to 8, and select $<$. (To set the value of a slider quickly, type the number into the text box to the right of the slider and press Enter.) The inequality $x + 3 < 8$ should now be showing in the red box. Be sure Test different values for $x$ is selected. Drag the purple dot on the number line to 2. Look at the test shown in the right pane. The inequality is true when $x = 2$. Notice that the purple dot is on the red part of the number line, which is the graph of the inequality.

Now move the purple dot to 8. Is 8 a solution for this inequality? Where is it located on the number line in relation to the graph of the inequality? Then place the purple dot at 5. Is 5 a solution for this inequality? Where is it located on the number line in relation to the red graph? Can you come up with a general description of all the numbers that are solutions for this inequality? Share your observations with your Learning Guide.

Use algebra to find the solution set for each of the following inequalities. Graph the solution set in your Math Notebook. Then use the Gizmo to check your work.

1. $x - 4 \leq 2$
2. $x + 1 > -5$
3. $x - 7 \geq -8$
The number 8 is not a solution to the inequality. It is not on the red line of the graph.

The number 5 is not a solution to the inequality. It is in the dot at the end of the graph, which means it is not included in the solution set.

Any number less than 5 is a solution for this inequality.

1

\[
\begin{align*}
&\quad x - 4 \leq 2 \\
&x - 4 + 4 \leq 2 + 4 \\
&\quad x \leq 6
\end{align*}
\]

\[\text{Graph with red line for } x \leq 6\]

2

\[
\begin{align*}
&\quad x + 1 > -5 \\
&x + 1 - 1 > -5 - 1 \\
&\quad x > -6
\end{align*}
\]

\[\text{Graph with open dot at } x = -6\]

3

\[
\begin{align*}
&\quad x - 7 \geq -8 \\
&x - 7 + 7 \geq -8 + 7 \\
&\quad x \geq -1
\end{align*}
\]

\[\text{Graph with red line for } x \geq -1\]

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

✅ RATE YOUR UNDERSTANDING

Please go online to view and submit this assessment.
Solve Inequalities - Part 3

Objectives

• Solve and graph inequalities with multiplication and division.

Books & Materials

• Math in Focus 2A
• Math in Focus - Teacher Edition

Assignments

• Complete Warm-up.
• Read and complete pages in Math in Focus 2A.
• Complete the Quick Check.

LEARN

WARM-UP

Solve each equation.

1. \(-3x = 18\)
2. \(-0.2x = 10\)
3. \((-1/4)x = 8\)

WARM-UP ANSWERS

1. \(x = -6\)  2. \(x = -50\)  3. \(x = -32\)

TEACHING NOTES

INSTRUCTION

Read and complete Hands-On Activity on pp. 226–227 in Math in Focus 2A. Pay special attention to the sign of the answer. Remember that the product or quotient of two integers with the same sign is positive, and the product or quotient of two integers with different signs is negative. You may find it helpful to use a number line to compare the numbers.

Read Solve Algebraic Inequalities Using Multiplication and Division on pp. 227–228. Solving inequalities using multiplication and division is similar to solving equations using multiplication and division with one very important difference. Be sure to apply the rule you learned in Hands-On Activity: If you multiply or divide both sides of an inequality by a negative number, you must reverse the direction of the inequality symbol. If you do not reverse the direction of the inequality symbol, the resulting inequality is not equivalent to the original inequality.
Review **Example 10** on pp. 228–229. Remember to use an empty circle to graph inequalities that use the < or > symbol and a shaded circle to graph inequalities that use the ≤ or ≥ symbol. Also, check the solution set for each part of the example.

Complete **Guided Practice** on p. 229.

---

### ♦ TEACHING NOTES

**Textbook Answer Key**

Your student may sometimes forget to reverse the direction of the inequality symbol when she multiplies or divides by a negative number. However, emphasize that if she always checks her answer, then she will catch this mistake and be able to correct her error.

---

### PRACTICE

Complete problems 15–22 of **Practice 4.4** on p. 233 in *Math in Focus 2A*.

---

### WRAP-UP

Today you learned how to solve inequalities using multiplication and division. When you multiply or divide both sides of an inequality by a negative number, you must reverse the sign of the inequality.

\[-7x > 21\]

\[-7x \div (-7) < \frac{21}{-7}\]

\[x < -3\]

---

### ✅ QUICK CHECK

Please go online to view and submit this assessment.
MORE TO EXPLORE

If you struggled with this question, you can check to see if a number is in the solution set by substituting it into the original inequality to see if it gives a true statement. Then revisit the material in this lesson.
Solve Inequalities - Part 4

Objectives
- Solve and graph inequalities with two or more steps.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete the Quick Check.

LEARN

WARM-UP

Solve each equation.

1. \(2x + 4 = 16\)
2. \(5x - 4 + 2x = 24\)

WARM-UP ANSWERS

1. \(x = 6\)  
2. \(x = 4\)

TEACHING NOTES

WARM-UP ANSWERS

1. \(x = 6\)  
2. \(x = 4\)

INSTRUCTION

Read Solve Multi-Step Algebraic Inequalities on p. 229 in Math in Focus 2A. Notice that solving a multistep inequality is very similar to solving a multistep equation. The only difference is that you must change the direction of the inequality symbol if you multiply or divide by a negative number in the solution process.

Review Example 11 on pp. 230–232. In part c, think about why the direction of the inequality symbol was reversed in Method 1, versus why it was reversed in Method 2. Both methods produce the same solution set. Check the solution set in the original inequality.

In part d, Method 1 uses the Distributive Property and then inverse operations to solve the inequality, while Method 2 uses only inverse operations. Both methods involve dividing by a negative number, so the direction of the inequality symbol is reversed.

Complete Guided Practice on p. 232. Remember to reverse the direction of the inequality symbol if you multiply or divide by a negative number and to check each solution set in the original inequality.
Point out to your student that there are several different ways to solve multistep inequalities, including some techniques for simplifying before solving, like she learned when solving equations. For example, in part a on p. 230, each term of the inequality could be multiplied by the LCD of the fractions, 5, in order to eliminate the fractions. Similarly, in part b, each term could be multiplied by 10 to remove the decimals. Regardless of the method she chooses, remind her to reverse the inequality symbol whenever she multiplies (or divides) both sides of the inequality by a negative number.

Complete problems 23–49 of Practice 4.4 on p. 234 in Math in Focus 2A.

Today you learned how to solve inequalities with two or more steps.

Please go online to view and submit this assessment.

Always reverse the sign when multiplying or dividing both sides by a negative. Use substitution to check answers to equations and inequalities.
Solve Inequalities - Part 5

Books & Materials

- Math in Focus - Teacher Edition

Assignments

- Complete Interactive Activity.
- Complete Rate Your Enthusiasm.

LEARN

INTERACTIVE ACTIVITY

In this activity, Solve Inequalities, you will practice your skills. You can choose the level of difficulty and check your answers.

RATE YOUR ENTHUSIASM

Please go online to view and submit this assessment.
Solve Inequalities - Part 6

Objectives
- Use inequalities to solve real-world problems.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete the Practice Questions.

LEARN

WARM-UP

Solve each inequality.

1. $2x + 8 < 12$
2. $5x - 1 - 7x \leq -9$

TEACHING NOTES

WARM-UP ANSWERS

1. $x < 2$  
2. $x \geq 4$

INSTRUCTION

Read the first instructional section on p. 235 in Math in Focus 2A.

Review Example 12 on p. 235. Remember that to find the average of a list of numbers, divide the sum of the numbers by the number of numbers. Identify the verbal phrase that indicates which inequality symbol to use.

Complete Guided Practice on p. 236. Remember to begin by defining the variable. Any letter may be used to represent the unknown fourth number.
Solve each inequality.

1. $2x + 8 < 12$
2. $5x - 1 - 7x \leq -9$

WARM-UP ANSWERS
1. $x < 2$
2. $x \geq 4$

Read the first instructional section on p. 235 in Math in Focus 2A.

Review Example 12 on p. 235. Remember that to find the average of a list of numbers, divide the sum of the numbers by the number of numbers.

Identify the verbal phrase that indicates which inequality symbol to use.

Complete Guided Practice on p. 236. Remember to begin by defining the variable. Any letter may be used to represent the unknown fourth number.

Review Example 13 on p. 236. Be sure to check that the solution set is reasonable based on the context of the given situation. Also, notice that if you substitute 26 into the original inequality, you get $24.8 \leq 25$. Think about what this means in terms of the amount of money Kelly will have left if she goes on 26 rides.

Complete Guided Practice on p. 237. Remember to interpret the solution set based on the context of the given situation. Think about whether decimals, fractions, or negative numbers should be included in the solution set.

Review Example 14 on p. 237. Then complete Guided Practice on p. 238. Point out that when solving real-world inequalities, your student must check to make sure the solution set makes sense in the context of the problem. For instance, in Example 13 on p. 236, a decimal does not make sense since Kelly cannot go on a portion of a ride. The solution set is stated using a whole number. You can expand on this by explaining that only whole numbers in the solution set make sense in terms of the context of the problem, so the graph of the solution is not represented using a line, but rather points at each whole number less than or equal to 26.

Complete problems 1–14 of Practice 4.5 on pp. 239–240 in Math in Focus 2A.

Today you learned how to solve real-world problems involving inequalities.
John earned test scores of 77, 92, and 84. What score must John earn on his next test to have a test average of at least 80?

Let $x$ be the fourth test score.

\[
\frac{77 + 92 + 84 + x}{4} \geq 80
\]

\[
\frac{253 + x}{4} \geq 80
\]

\[
253 + x \geq 320
\]

\[
x \geq 67
\]

John must earn a score of at least 67 on his next test.

✅ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
## Solve Inequalities - Part 7

### Objectives
- Apply mathematical concepts and skills to solve problems.
- Design a project to apply skills with solving algebraic inequalities.

### Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition

### Assignments
- Complete Warm-up.
- Read and complete Brain @ Work in Math in Focus 2A.
- Complete the Practice activity.
- Complete the Practice Questions.

## LEARN

### WARM-UP
Write an algebraic expression for each.

1. twice the sum of $x$ and 4
2. five less than a number

### TEACHING NOTES

#### WARM-UP ANSWERS
1. $2(x + 4)$
2. $x - 5$

### INSTRUCTION
Read and complete Brain @ Work on p. 240 in Math in Focus 2A.

Begin by defining a variable. Use the variable to write an algebraic equation for the problem. Then interpret your solution to the equation in the context of the problem. Does the solution to the equation answer the problem? If not, think about how you can use the solution to the equation to answer the problem.

Remember to check your answer. If your answer does not work, then first reread the problem and check that the algebraic expressions and equation you wrote match the descriptions given. Then check that you solved the equation correctly by substituting your answer into the original equation you wrote. You may need to look back at whose age you defined to be the variable and adjust your solution accordingly.
If your student struggles with how to begin to solve this problem, remind her that the first step to solving real-world equations and inequalities is to define the variable. Guide her to recognize that it is most helpful to define the daughter’s age as the variable, since she is the youngest. For example, let \( d \) be the daughter’s age. Help your student use the daughter’s age, \( d \), to write algebraic expressions for the ages of the son, wife, and father.

Son: \( 5d \)

Wife: \( 5(5d) = 25d \)

Father: \( 2(25d) = 50d \)

Remind your student that the grandmother’s age is given, 81, and that she is as old as the sum of everyone else. Your student can now write an algebraic equation and solve for \( d \). After solving the equation, if your student gives the daughter’s age as the answer, remind her that the original problem asks for the age of the son. Stress the importance of rereading the original problem and interpreting the answer in its context.

It can also be helpful to write an answer sentence before writing the equation:

The man’s son is ________ years old.

Then, when the equation is solved for \( d \), your student will only have to fill in the blank. Seeing the answer sentence will remind her that she is looking for the son’s age, \( 5d \), not the daughter’s age, \( d \).

**PRACTICE**

Write a constructed response to explain each step in solving the family’s problem on p. 240 in *Math in Focus 2A*. Remember that a constructed response answers the question *how do you know?* and can be in the form of a word sentence, a drawing, a chart or table, an equation, or any combination of these.

Then complete the following problem.

There are three kinds of fruit in a basket: apples, oranges, and pears. The number of apples is 1 less than the number of oranges. The number of pears is 2 times the number of apples. There are 17 fruits in the basket. How many of each kind are there?
If your student struggles with how to begin to solve this problem, remind her that the first step to solving real-world equations and inequalities is to define the variable. Guide her to recognize that it is most helpful to define the daughter's age as the variable, since she is the youngest. For example, let \( d \) be the daughter's age. Help your student use the daughter's age, \( d \), to write algebraic expressions for the ages of the son, wife, and father.

Son: \( 5 + d \)

Wife: \( 5(5 + d) = 25 + 5d \)

Father: \( 2(25 + d) = 50 + 2d \)

Remind your student that the grandmother's age is given, 81, and that she is as old as the sum of everyone else. Your student can now write an algebraic equation and solve for \( d \). After solving the equation, if your student gives the daughter's age as the answer, remind her that the original problem asks for the age of the son. Stress the importance of rereading the original problem and interpreting the answer in its context.

It can also be helpful to write an answer sentence before writing the equation:

\[ \text{The man's son is } \_\_\_\_\_\_\_\_\text{ years old.} \]

Then, when the equation is solved for \( d \), your student will only have to fill in the blank. Seeing the answer sentence will remind her that she is looking for the son's age, \( 5 + d \), not the daughter's age, \( d \).

**TEACHING NOTES**

**Practice Answers:**

4 apples, 5 oranges, 8 pears

To solve the Practice problem, guide your student to define a variable. For example, she may define the number of oranges to be \( x \). Then lead her to write an algebraic expression for each type of fruit in the basket.

**WRAP-UP**

Today you learned to apply mathematical concepts and skills to solve problems.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Joyce rented a booth at a carnival at a cost of $95 to sell handmade beaded necklaces. The cost of making and packaging each beaded necklace was $15. If Joyce sells the beaded necklaces at $35 each, how many beaded necklaces must she sell to make a profit of at least $500?

Upload your work to show how you got to the answer.
Joyce rented a booth at a carnival at a cost of $95 to sell handmade beaded necklaces. The cost of making and packaging each beaded necklace was $15. If Joyce sells the beaded necklaces at $35 each, how many beaded necklaces must she sell to make a profit of at least $500?

Did you:

- Use the information given to show how many beaded necklaces Joyce must sell to make a profit of at least $500?
- Solve the problem using inequalities?
- Show the correct steps to solve for an inequality?
- Label your answer to give the numerical value meaning?
- Organize your work neatly?
- Show logical steps in your problem solving?
- Upload your work showing how you found your answers?
Solve Inequalities - Part 9

Books & Materials
- Math in Focus - Teacher Edition

SHOW

Use your inequalities from the previous task to solve for the maximum number of nights you can stay at each hotel. Record your information in your Math Notebook.

RATE YOUR PROGRESS

Please go online to view and submit this assessment.
Solve Proportions - Part 1

Objectives
- Write a ratio to show a comparison.
- Find unit rates.
- Plot points on a coordinate grid.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete the Practice Questions.

LEARN

WARM-UP
1. Solve: $4x - 2 = 26$
2. Solve and graph the solution set: $-3.5x + 12 > 19$

=WARM-UP ANSWERS
1. 6
2. $x < -2$

INSTRUCTION

Today's part covers important ideas you need to understand before beginning the parts in this chapter. Read p. 244 in Math in Focus 2A.

Read both instructional sections on p. 245. Finding equivalent ratios and writing ratios in simplest form is similar to the procedures you use to find equivalent fractions and to write fractions in simplest form.

Read the first two instructional sections on pp. 246–247. Always start from the origin when identifying or plotting points on the coordinate plane. The first number tells how many units to move horizontally. The second number tells how many units to move vertically.
Textbook Answer Key

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the Quick Check sections.

Write three different ratios using each of the forms shown on p. 245 in Math in Focus 2A. Ask your student to find an equivalent ratio for each one, writing it in the same form.

Point out that a unit price is a type of unit rate. A unit price is often shown as price per pound or price per ounce.

SKILLS CHECK

Complete the Quick Check sections on pp. 245–247 (top) in Math in Focus 2A.

Quick Check

<table>
<thead>
<tr>
<th>Question(s)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–8</td>
<td>Ratios</td>
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<tr>
<td>9–11</td>
<td>Rates</td>
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<tr>
<td>12</td>
<td>Rectangular Coordinate System for Problems</td>
</tr>
<tr>
<td>13–14</td>
<td>Percents</td>
</tr>
</tbody>
</table>
WRAP-UP

Today you reviewed writing ratios in simplest form and writing equivalent ratios.

6:12 = 1:6

Divide both terms by 6 to simplify.

You also reviewed finding and comparing unit rates.

Brand A: $15.80 for 20 oz of shampoo $15.80 ÷ 20 = $0.79 per ounce
Brand B: $20.70 for 30 oz of shampoo $20.70 ÷ 30 = $0.69 per ounce

Brand B costs $0.10 less per ounce, so it is the better buy.

You reviewed identifying and plotting coordinates on a graph.

The coordinates of point J are (2, 6).

✅ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Solve Proportions - Part 2

Objectives
- Use bar models to solve percent problems.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition
- Using Bar Models to Find Percents Worksheet

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete Using Bar Models to Find Percents Worksheet.
- Complete the Quick Check.

LEARN

WARM-UP
Express each percent as a fraction in simplest form.

1. 73%
2. 55%
3. 20%

WARM-UP ANSWERS
1. 73/100  2. 11/20  3. 1/5

TEACHING NOTES

INSTRUCTION
Read Solving percent problems on p. 247 in Math in Focus 2A. The bar model problem shows how to find the missing whole given a quantity, or part, and its percent. In this problem, the given quantity is 600 people, and the given percent is 80%. From the model, you can see that 20% of the people equals 600 people. Divide 600 by 20 to find the value of 1%. Multiply the value of 1% by 100 to find the whole.
You can use bar models to find a missing part.

What is 30% of 150 units?

The model shows the following:

\[
100\% \Rightarrow 150 \\
1\% \Rightarrow \frac{150}{100} = 1.5 \\
30\% \Rightarrow 30 \cdot 1.5 = 45
\]

30% of 150 units is 45 units.

You can also use bar models to find a missing percent.

Mark has completed 15 of 50 homework problems. What percent of his homework has Mark completed?

The model shows the following:

\[
50 \text{ problems (100\%)} \\
15 \text{ problems (30\%)}
\]

\[
50 \text{ problems } \Rightarrow 100\% \\
1 \text{ problem } \Rightarrow \frac{100}{50} \% \\
15 \text{ problems } \Rightarrow 15 \cdot \frac{100}{50} \% = 30\%
\]

Mark has completed 30% of his homework.

**TEACHING NOTES**

**Textbook Answer Key**

Point out to your student that he can use bar models to represent three types of percent problems: find the missing whole, find the missing part, or find the missing percent.
SKILLS CHECK

Complete Quick Check at the bottom of p. 247 in Math in Focus 2A. Then complete the Using Bar Models to Find Percents Worksheet.

TEACHING NOTES

For problems 13 and 14 on p. 247 in Math in Focus 2A, ask your student what he is being asked to find. Make sure he understands that he is finding the missing whole. He should use the given part and percent to find the whole.

WORKSHEET ANSWERS

Your Turn Answers:

Method 1: 1 360 2 360/100 = 3.6 3 3.6 \cdot 30 = 108 4 108

Method 2: 1 30; 108 2 108

Your Turn Answers:

1 35/20 = 1.75 2 100 \cdot 1.75 = 175 3 175 4 60; 21, 60, 35; 35 5 21/62; 21/60, 100; 35; 35

Your Turn Answers:

Method 1: 1 60 2 21; 60; 35 3 35

Method 2: 1 21/60 2 21/60; 100; 35 3 35

Practice Answers:
Practice Answers:

1. \[ \frac{220}{100} = 2.2 \]
2. \[ \frac{540}{100} = 5.4 \]
3. \[ \frac{180}{100} = 1.8 \]
4. \[ \frac{250}{100} = 2.5 \]
5. \[ \frac{60}{100} = 0.6 \]
6. \[ \frac{300}{100} = 3.0 \]
7. \[ \frac{160}{100} = 1.6 \]
8. \[ \frac{625}{100} = 6.25 \]
9. \[ \frac{75}{100} = 0.75 \]
WRAP-UP

Today you learned how to represent and solve three types of percent problems. You learned to find a missing whole, a missing part, or a missing percent.

QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

Use reasoning to estimate. Remember that 40% is a little less than half. Half of 200 is 100, so the answer should be a little less than 100. Think of how you could set up a bar model to represent the problem.
**LEARN**

**WARM-UP**

1. What is 20% of 450?

2. Avina thinks of a number. 30% of her number is 75. What is Avina’s number?

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. 90  
2. 250

**INSTRUCTION**

Review Solving percent problems on p. 247 in Math in Focus 2A. You can use percent to solve various kinds of problems.

You can use percent to find the sales tax on the purchase of an item.

Carlos purchases a television for $250.00. Sales tax of 7% is added to the cost. What is the total cost?

Sales tax = 7% of $250.00

\[
\text{Sales tax} = \frac{7}{100} \cdot \$250.00 = \$17.50
\]

\[
\$250.00 + \$17.50 = \$267.50
\]

The total cost of the television is $267.50.

You can use percent to solve problems involving commission.
Mrs. Schwartz earns a 4% commission on all vacation packages she sells. If she receives $1,250 in commission, what is the dollar amount of her sales?

4% ⇒ $1,250
1% ⇒ $1,250 ÷ 4 = $312.50
100% ⇒ 100 • $312.50 = $31,250

The dollar amount of Mrs. Schwartz’s sales is $31,250.

You can use percent to solve problems involving interest rate.

A company has $40,000 in a bank account with an interest rate of 3% per year. How much interest will it earn at the end of a year?

Interest = 3% of $40,000
= \frac{3}{100} \cdot $40,000
= $1,200

The company will earn $1,200 in interest.

You can use percent to compute a markup in price.

Mr. Meyer sells cheese at a 65% markup. He pays $4.20 per pound for gouda. At what price per pound will he sell it?

65% of $4.20 = \frac{65}{100} \cdot $4.20
= $2.73

The price is marked up by $2.73.

$4.20 + $2.73 = $6.93

Mr. Meyer will sell the gouda at $6.93 per pound.

Point out to your student that a markup is a percent increase. Most merchants sell items for more than what they paid for them in order to make a profit. This is a price markup.

Complete the Solving Percent Problems with Bar Models Worksheet.
Mrs. Schwartz earns a 4% commission on all vacation packages she sells. If she receives $1,250 in commission, what is the dollar amount of her sales?

You can use percent to solve problems involving interest rate.

A company has $40,000 in a bank account with an interest rate of 3% per year. How much interest will it earn at the end of a year?

The company will earn $1,200 in interest.

You can use percent to compute a markup in price.

Mr. Meyer sells cheese at a 65% markup. He pays $4.20 per pound for gouda. At what price per pound will he sell it?

Mr. Meyer will sell the gouda at $6.93 per pound.

Point out to your student that a markup is a percent increase. Most merchants sell items for more than what they paid for them in order to make a profit. This is a price markup.

Complete the Solving Percent Problems with Bar Models Worksheet.

Worksheet Answers:

Your Turn Answers:
Method 1: 1 7; 7/100; 38.50 2 38.50; 588.50 3 588.50
Method 2: 1 550 2 550/100 = 5.50 3 7; 5.50; 38.50 4 38.50; 588.50 5 588.50

Your Turn Answers:
1 3; 3,240 2 3,240; 1; 1,080 3 1,080; 108,000 4 108,000

Your Turn Answers:
1 3; 10,000; 3/100; 10,000; 300 2 300

Your Turn Answers:
Method 1: 1 40/100; 50; 20 2 20 3 20; 70 4 70
Method 2: 1 50/100; 0.5 2 40; 0.5; 20 3 20 4 20; 70 5 70

Your Turn Answers:
1 75; 45; 30 2 30 3 75; 100/75; 30; 30; 100/75 = 40 4 40

Practice Answers:
Today you learned how to represent and solve different types of percent problems involving sales tax, commission, interest rate, and markup.

Please go online to view and submit this assessment.
MORE TO EXPLORE

View the video Discounts to practice with another application of percents.
Solve Proportions - Part 4

**Objectives**
- Identify a direct proportion from a table.
- Identify the constant of proportionality.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition
- yardstick
- ruler
- cardboard tube
- tape

**Assignments**
- Complete Warm-up.
- Read and complete pages in Math in Focus A.
- Complete problems 1–4 and 9–11, Math in Focus A.
- Complete the Practice Questions.

---

**LEARN**

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**WARM-UP**

Write equivalent ratios.

1. 5:30
2. 9:27

**WARM-UP ANSWERS**

1. Answers will vary. Sample answer: 10:60
2. Answers will vary. Sample answer: 1:3

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. Answers will vary. Sample answer: 10:60
2. Answers will vary. Sample answer: 1:3

---

**INSTRUCTION**

Read pp. 248–249 in *Math in Focus 2A*. The price per pound of strawberries is an example of a unit price. Other common unit prices you may be familiar with are price per pound of meat or price per gallon of gas.

Reread the text in the shaded box on p. 249 that explains the constant of proportionality. Note that the value of $k$ depends on the situation. For example, if the price per pound of strawberries was $3 instead of $2, then the value of $k$ would be 3.

Read and complete Hands-On Activity on p. 250. Tape the yardstick to the wall at eye level so you can clearly see it through the tube. If you only have one yardstick, use repeated measurements with a ruler to determine distance from the wall.
Then read Example 1 on p. 251. For part a, the constant of proportionality 32 does not mean there is 1 whole fish and 1 half fish in every gallon of water. Rather, it is a unit rate. It is used to find the number of fish in a given number of gallons of water.

Part b illustrates why you must check all pairs of values. Though the first two pairs of values have the same proportion, the third pair does not. Therefore, d and t are not in direct proportion in this example.

Complete Guided Practice on p. 252. Remember to check all the rates or ratios in a table. For y to be directly proportional to x, all pairs of values must have the same proportion.

HELPFUL ONLINE RESOURCES

Instructional Video: Introduction to Proportions

UTOR NOTES

Textbook Answer Key

For Hands-On Activity on p. 250, make sure your student understands that he is finding the ratio $H:L$. Ask your student what happens to the values of $H$ and $H:L$ as $L$ increases. The value of $H:L$ should be constant.

PRACTICE

Complete problems 1–4 and 9–11 of Practice 5.1 on p. 257 in Math in Focus 2A.

UTOR NOTES

Textbook Answer Key

WRAP-UP

Today you learned how to find a direct proportion from a table and to identify the constant of proportionality.

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>6</td>
<td>12</td>
<td>18</td>
</tr>
</tbody>
</table>

$6 \div 2 = 12 \div 4 = 18 \div 6 = 3$

$y$ is directly proportional to $x$. The constant of proportionality is 3.

The direct proportion equation is $y = 3x$. 
The direct proportion equation is \( y = 3x \).

Please go online to view and submit this assessment.
LEARN

WARM-UP

Tell whether \( y \) is directly proportional to \( x \).

1. 

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y )</td>
<td>5</td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

2. 

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y )</td>
<td>10</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

WARM-UP ANSWERS

1. no  2. yes

TEACHING NOTES

INSTRUCTION

Read the instructional section at the bottom of p. 252 in *Math in Focus 2A*. Look at the Think Math box. Since \( x \) represents pounds of strawberries, substitute \( x = 10 \) in the direct proportion equation \( y = 2x \).

Then read Example 2 on p. 253. Notice the difference between the rewritten equations in parts a and b: \( y = 6x \) and \( y = 5x + 2 \). The first equation has two terms, and it matches the form \( y = kx \). The second equation includes a third term (+ 2), so it does not match the form \( y = kx \). Therefore, it is not a direct proportion.
Complete **Guided Practice** on pp. 253–254. The constant $k$ does not have to be a whole number. It can also be a decimal or fractional value. To determine whether a given equation represents a direct proportion, you need to find whether it is possible to rewrite the equation in the form $y = kx$ for some value, $k$.

Read the instructional section on p. 254. When two quantities are in direct proportion, the constant of proportionality is a unit rate.

Read **Example 3** on p. 254. Then complete **Guided Practice** at the top of p. 255. Note that the constant of proportionality in this situation is a unit rate showing the number of baseballs produced each day.

---

**TEACHING NOTES**

**Textbook Answer Key**

Observe your student as he performs algebraic steps to rewrite a given equation. An error in performing the steps can lead to the wrong conclusion as to whether an equation represents a direct proportion.

---

**PRACTICE**

Complete problems 5–8 and 12–13 of **Practice 5.1** on pp. 257–258 in *Math in Focus 2A*.

---

**WRAP-UP**

Today you learned how to identify from an equation whether quantities are in direct proportion. If the equation can be written in the form $y = kx$, the equation represents a direct proportion.

$y = 3x$

If the equation cannot be written in this form, it does not represent a direct proportion.

$y = 4x - 3$

---

**QUICK CHECK**

Please go online to view and submit this assessment.
MORE TO EXPLORE

If you answered incorrectly, view the Instructional Video, *Solving Proportions*. 
Solve Proportions - Part 6

Objectives
- Determine the constant of proportionality.

Books & Materials
- Math in Focus 2A
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete the Practice Questions.

LEARN

WARM-UP

1. In the equation $y = kx$, which letter represents the constant of proportionality?
2. Which equation represents a direct proportion?

$3y = 6x$ \hspace{1cm} y + 4 = 8x$

TEACHING NOTES

WARM-UP ANSWERS

1. $k$ \hspace{1cm} 2. $3y = 6x$

INSTRUCTION

Read Example 4 on p. 255 in Math in Focus 2A.

In verbal descriptions like this one, the phrase is *directly proportional* to is a clue to identifying the quantities to represent with a variable. In this example, the cost of $8 is a constant of proportionality. This means that no matter how many caps are purchased, the cost is always $8 per cap.

Suppose, however, the cost of the cap decreases if a large quantity of caps are purchased. For example, if the cost were $8 each for up to 10 caps and $6 each for 11 or more caps, the unit cost would not be constant. In this case, the relationship between the number of caps purchased and the total cost would not be directly proportional.
Complete **Guided Practice** on the bottom of p. 255. Although many examples so far have used the variables $x$ and $y$, you can use other variables. For example, you could use $a$ for the amount paid and $n$ for the number of sandwiches.

Read **Example 5** on p. 256. Note that the problem states that $y$ is directly proportional to $x$. Then complete **Guided Practice** on p. 256.

---

**TEACHING NOTES**

**Textbook Answer Key**

You may wish to tell your student that he has learned that a direct proportion can be represented in tables, equations, or verbal descriptions. Two quantities that are in direct proportion are related by a constant of proportionality. Two quantities are in direct proportion if one increases or decreases by the same factor as the other quantity.

---

**INTERACTIVE ACTIVITY**

Ask your Learning Guide to help you access this activity. Then use it to explore proportions on a double number line.
Follow these steps to access the activity:

1. Go to the Sixth Grade Number Sense and Proportional Reasoning section of the activity list.
2. Click on Proportional Reasoning on a Double Number Line.
3. Click on View Teacher Tool or View Teacher Tool in Spanish.
4. Read and agree to the Terms of Use. Click Continue.

Complete problems 14–20 of Practice 5.1 on p. 258 in Math in Focus 2A.

Today you learned how to identify a constant of proportionality in a verbal description.

Mrs. Restivo is printing study guides. Each study guide contains 12 pages. The total number of pages printed is directly proportional to the number of study guides she prints. Let \( s \) be the number of study guides Mrs. Restivo prints. Let \( t \) be the total number of pages printed.

Number of pages per study guide: 12 pages

The direct proportion equation is \( t = 12s \).

You also learned how to identify the constant of proportionality in an equation.

\[ d \text{ is directly proportional to } c, \text{ and } d = 4 \text{ when } c = 16. \]

\[ d \text{ is directly proportional to } c, \text{ and } d = 4 \text{ when } c = 16. \]

Constant of proportionality: \( \frac{d}{c} = \frac{4}{16} = \frac{1}{4} \)

The constant of proportionality is \( \frac{1}{4} \)

The direct proportion equation is \( d = \frac{1}{4}c \).
Please go online to view and submit this assessment.
Solve Proportions - Part 7

**Objectives**
- Interpret a direct proportion from a graph.

**Books & Materials**
- Math in Focus 2A
- Math in Focus - Teacher Edition

**Assignments**
- Complete Warm-up.
- Read and complete pages in Math in Focus 2A.
- Complete the Quick Check.

---

**LEARN**

---

**WARM-UP**

1. Look at the table. Is $y$ directly proportional to $x$?

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>4</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

2. Which graph shows a straight line passing through the origin?

[Images of three graphs labeled A, B, and C]

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. no  
2. C

---

**INSTRUCTION**

Read p. 259 in Math in Focus 2A. The table represents a direct proportion because the ratios are constant. Since the graph was plotted from points given in the table, it also represents a direct proportion:

Look at the Think Math box. For the point $(1, y)$, $x = 1$. The constant of proportionality is $k = y 1 = y$. This means that at the point $(1, y)$, $k = y$. 

---

Grade 7 Calvert Math in Focus 246 Unit 2
Then read **Example 6** on pp. 260–261. A graph must meet three conditions to represent a direct proportion:

- The graph is a straight line.
- The line passes through the origin.
- The line does not lie along the x- or y-axis.

Note that in part a, you could obtain the same constant of proportionality using any point except (0, 0). For (2, 20), \( k = \frac{20}{2} = 10 \); for (3, 30), \( k = \frac{30}{3} = 10 \).

Complete **Guided Practice** on p. 261. Remember to check for all three conditions to determine whether the graph represents a direct proportion. For **Think Math**, consider whether a line that passes through (3, 0) could meet all three conditions. Draw a sketch of such a line to help you.

Then read **Example 7** on p. 262. Note that the letters a, c, d, and e on the graph are not labels for points. They are references to help you answer each of the corresponding questions in the problem. For **Think Math**, recall that each point on the graph \((t, w)\) represents the time in hours \((t)\) that Jonathan works to earn \(w\) dollars. Find the point that shows how many hours Jonathan must work to earn $65.

Complete **Guided Practice** on p. 263. Remember to check the labels on the axes to see which quantity each axis represents.

---

**TEACHING NOTES**

**Textbook Answer Key**

If your student struggles with interpreting a graph, ask him what real-world quantity is represented by each axis. Guide him to find information on the graph. For example, in **Guided Practice** part 4d on p. 263, have your student locate the \(t\)-coordinate 3 on the \(t\)-axis. He can trace up with his finger to find the corresponding point on the line. Then he can trace across with his finger to find the corresponding \(y\)-coordinate, 150.

---

**PRACTICE**

Complete **Practice 5.2** on pp. 264–265 in **Math in Focus 2A**.

---

**TEACHING NOTES**

For problem 7 on p. 265, point out to your student that exchange rates change from day to day. The exchange rate shown in the graph is the rate for one particular day.
WRAP-UP

Today you learned how to identify a direct proportion from a graph.

The graph is a straight line through the origin and does not lie along the x- or y-axis. It represents a direct proportion.

The line passes through (1, 5), so the constant of proportionality is 5. The direct proportion equation is \( y = 5x \).

QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

View the video, Proportional vs. Non-Proportional (Relationships on Graphs) (07:18), to review how to tell proportional relationships by drawing graphs.

Please go online to view this video ▶
Solve Proportions - Part 8

**Objectives**
- Use proportions to solve real-world problems involving ratios.

**Books & Materials**
- Math in Focus 2A
  - Math in Focus - Teacher Edition

**Assignments**
- Complete Warm-up.
- Read and complete pages in *Math in Focus 2A*.
- Complete the Practice Questions.

---

**LEARN**

**WARM-UP**

When y is directly proportional to x, find the constant of proportionality.

1. \( \frac{y}{x} = \frac{25}{5} \)
2. \( y = 36, x = 4 \)

**WARM-UP ANSWERS**

1. \( 5 \)  
2. \( 9 \)

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. \( 5 \)  
2. \( 9 \)

---

**INSTRUCTION**

Read the instructional section on pp. 266–267 in *Math in Focus 2A*. The order that you write the cross products does not change their value. For example, even if you wrote the cross products of \( 2 \cdot 5 = 4 \cdot 10 \) as \( 4 \cdot 5 = 2 \cdot 10 \), the cross products remain equal.

When using Method 1, the proportion can be written in other ways. You can compare T-shirts to dollars, or dollars to dollars and T-shirts to T-shirts.

\[
\frac{8 \text{ T-shirts}}{40 \text{ dollars}} = \frac{5 \text{ T-shirts}}{y \text{ dollars}} \quad \text{or} \quad \frac{y \text{ dollars}}{40 \text{ dollars}} = \frac{5 \text{ T-shirts}}{8 \text{ T-shirts}}
\]

In each case, the cross products are equal. To correctly write a proportion, the key is to be sure both ratios in the proportion compare the quantities in the same order.

---

**CAREER CONNECTION**

Chefs use proportions and cross products to adjust recipes to serve different numbers of people.
Notice the Check at the bottom of p. 267. The unitary method means to first identify the value of a single unit. In the example, the cost of 8 T-shirts is used to find the cost of 1 T-shirt. The unit cost is then multiplied by 5 to find the cost of 5 T-shirts.

Then read Example 8 on p. 268. You can use either method to solve the problem. You can use the unitary method to check the solution as follows:

\[
\begin{align*}
18 \text{ hours} & \Rightarrow 432 \\
1 \text{ hour} & \Rightarrow \frac{432}{18} = 24 \\
21 \text{ hours} & \Rightarrow 24 \cdot 21 = 504
\end{align*}
\]

Complete Guided Practice on p. 269. Use the unitary method to check your solutions.

HELPFUL ONLINE RESOURCE
Instructional Video: Solving Proportions

Textbook Answer Key

Ensure that your student understands the different proportions that can be written to represent a given situation. For instance, in Example 8 on p. 268, the following proportions could also be used to find Belle’s pay for 21 hours of work:

\[
\begin{align*}
\frac{18 \text{ h}}{432 \text{ dollars}} &= \frac{21 \text{ h}}{y \text{ dollars}} \\
\frac{18 \text{ h}}{21 \text{ h}} &= \frac{432 \text{ dollars}}{y \text{ dollars}}
\end{align*}
\]

The following proportion, however, is incorrect, as it does not compare the given quantities in the same order.

\[
\begin{align*}
\frac{18 \text{ h}}{21 \text{ h}} &= \frac{y \text{ dollars}}{432 \text{ dollars}}
\end{align*}
\]

PRACTICE

Complete problems 1–2 and 5–14 of Practice 5.3 on pp. 272–273 in Math in Focus 2A.
WRAP-UP

Today you learned how to solve real-world problems using proportions.

Christopher is cooking a meal for his family. He has a recipe in which the amount of pasta is directly proportional to the number of people it feeds. The recipe calls for 8 ounces of pasta and feeds 4 people. Use a proportion to find how much pasta Christopher needs to feed 10 people.

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use a proportion.</td>
<td>Use a direct proportion equation.</td>
</tr>
<tr>
<td>Let ( y ) be the amount of pasta needed.</td>
<td>Let ( x ) be the number of people.</td>
</tr>
</tbody>
</table>
| \[
\frac{8 \text{ oz}}{4 \text{ people}} = \frac{y \text{ oz}}{10 \text{ people}}
\] | Let \( y \) be the amount of pasta. |
| \[
\frac{8}{4} = \frac{y}{10}
\] | Constant of proportionality: |
| \[
y \cdot 4 = 10 \cdot 8
\] | \[
\frac{y}{x} = \frac{8}{4} = 2
\] |
| \[
4y = 80
\] | Direct proportion equation: |
| \[
\frac{4y}{4} = \frac{80}{4}
\] | \[
y = 2x
\] |
| \[
y = 20
\] | When \( x = 10 \) and \( y = 2x \), |
| Christopher will need 20 oz of pasta to feed 10 people. | \[
y = 2 \cdot 10 = 20.
\] |

✔ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
LEARN

WARM-UP

Find the unknown value.

1. \( \frac{2}{70} = \frac{y}{910} \)
2. \( \frac{8}{120} = 38 \times \)

TEACHING NOTES

WARM-UP ANSWERS

1. \( y = 26 \)  
   2. \( x = 570 \)

INSTRUCTION

Read Example 9 on p. 270 in Math in Focus 2A. Note that part a shows how to find the constant of proportionality. In this case it is a unit rate, or the number of peaches per crate.

Complete Guided Practice on p. 270.

Then read Example 10 on p. 271. In the bar model, notice that the entire bar represents the regular price of the phone, $228 (100%). The unshaded portion represents the discount, $45.60 (? percent). Recall there is more than one way to write equivalent proportions. In this example, Method 1 uses one equivalent proportion, and Method 2 uses another equivalent proportion.

In Think Math, consider whether the ratios in the proportion are written in the correct order.

Complete Guided Practice on p. 271.

HELPFUL ONLINE RESOURCES

Instructional Video: Using Proportions to Find Percents
Textbook Answer Key

If your student struggles with setting up the ratios in a proportion in the proper order, use Example 9 on p. 270 to show how he can use a table to organize his thinking. Each ratio in the proportion corresponds to a column of the table.

\[
\frac{15 \text{ crates}}{600 \text{ peaches}} = \frac{? \text{ crates}}{1,000 \text{ peaches}}
\]

Complete problems 3–4 and 15–22 of Practice 5.3 on pp. 272–274 in Math in Focus 2A.

Teaching Notes

Practice

WRAP-UP

Today you learned how to solve a direct proportion problem from a table.

The number of pencils for sale at a store, \(P\), is directly proportional to the number of boxes the pencils are packed in, \(B\). The table shows the relationship between the total number of pencils for sale and the number of boxes.

<table>
<thead>
<tr>
<th>Number of Pencils ((P))</th>
<th>240</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Boxes ((B))</td>
<td>20</td>
<td>45</td>
</tr>
</tbody>
</table>

Write a direct proportion equation that relates \(P\) and \(B\).

Number of pencils per box: 240/20 = 12

The direct proportion equation is \(P = 12B\).

Find the missing value in the table.

When \(B = 45\) and \(P = 12\cdot 45\)

\[P = 540\]

There are 540 pencils in 45 boxes.

You also learned how to solve a direct proportion problem involving percent.
The regular price of a shirt is $27. Today the price is marked down by $4.05. Use a proportion to find the percent discount.

\[
\frac{100\%}{27} = \frac{x\%}{4.05}
\]

\[x \cdot 27 = 100 \cdot 4.05\]

\[27x = 405\]

\[x = 15\]

The percent discount is 15%.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Solve Proportions - Part 10

**Books & Materials**
- Math in Focus - Teacher Edition

**USE**

**USE FOR MASTERY**

The volume of paint used, $V$ liters, is directly proportional to the area, $A$ square feet, that the paint can cover. 5 liters of paint can cover a wall with an area of 75 square feet.

a) What is the constant of proportionality? Tell how you found it.

b) Write an equation relating $V$ and $A$.

c) How much paint would be needed to cover an area of 180 square feet? Show your work.
Did you:

- Use the information given to answer parts A–D?
- Explain all your answers thoroughly?
- Use labels to show what the numbers represent?
- Upload your work showing how you got your answers?

The volume of paint used, \( V \) liters, is directly proportional to the area, \( A \) square feet, that the paint can cover. 5 liters of paint can cover a wall with an area of 75 square feet.

a) What is the constant of proportionality? Tell how you found it.

b) Write an equation relating \( V \) and \( A \).

c) How much paint would be needed to cover an area of 180 square feet? Show your work.
Now it is time to review all of your calculations and decide on a hotel. Make a poster or a PowerPoint presentation to represent and share your trip idea. Include the following information:

1. Which hotel did you choose?
2. What made you reach your decision?
3. How many nights will you be staying?
4. What will you be doing on your trip?
5. What will be the total cost of your hotel and activities?

Remember to attach all of your notes, calculations, and sources.

The Project Rubric will help you understand how your project will be scored. Your goal should be to earn all possible points for each part.

Upload your answers below.
COLLABORATION

How did your budget impact your visit? If you were given $6000, how would your vacation be different? Would you stay longer, stay in a different hotel, or complete different activities? Respond to two of your peers.
Unit Quiz: Algebraic Expressions and Equations

Books & Materials

- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

UNIT QUIZ

Please go online to view and submit this assessment.
Unit 3 - Plane and Solid Figures
In this project, you are going to make a beautiful design out of tiles. Perhaps you have visited the Magic Kingdom or have seen it on television and are familiar with Cinderella's Castle in the center. Many people do not realize that when you walk through the center of the castle, there is a beautiful tile design that tells the story of Cinderella.

If you would like to learn more about these designs or see them up close, you can click on the link to [Allears.net: The World According to Jack: Cinderella Castle Mosaic Murals](https://www.allears.net/). Although these designs are created from a variety of different shapes and patterns, we often see tile designs that form a pattern. These are called tessellations. To read a bit more about tessellations, visit [Math is Fun: Tessellations](https://www.mathsisfun.com/geometry/tessellations.html). Focus on “semi-regular tessellations,” as that is what you will be doing for this project.

In this project, you will need to:

- Create an original tessellation using the following shapes:
  - A parallelogram
  - An equilateral triangle
  - A square
  - A right triangle
- Use a straightedge to draw all shapes.
- Use a protractor to measure and draw your angles.
- Color your images.
- Upload your design.
- Answer questions about your shape relationships.
You will submit your project in Scale Drawings - Part 7.

**PROJECT RUBRIC**

The [Project Rubric](#) will help you understand how your project will be scored. Your goal should be to earn all possible points for each part.

---

**COLLABORATION**

Search the Internet to look for different tessellations and tile designs. Share one of your favorites with your peers. What do you like about it? What shapes are used to create the design?

---

**RATE YOUR EXCITEMENT**

Please go online to view and submit this assessment.
Angles - Part 1

**Objectives**
- Classify angles as acute, obtuse, right, or straight.
- Identify parallel and perpendicular lines.

**Books & Materials**
- Math in Focus 2B
- Math in Focus - Teacher Edition

**Assignments**
- Complete Warm-up.
- Read and complete pages in *Math in Focus 2B*.
- Complete the Practice Questions.

**LEARN**

**WARM-UP**
Tell what kind of angle is described: *acute, obtuse, or right*.

1. an angle that measures exactly 90°
2. an angle that measures less than 90°
3. an angle that measures greater than 90° but less than 180°

**TEACHING NOTES**

**WARM-UP ANSWERS**
1. right angle  2. acute angle  3. obtuse angle

**INSTRUCTION**

Today’s part covers important ideas you need to understand before beginning the lessons in this chapter.

Read pp. 2–4 in *Math in Focus 2B*. Recall the differences between lines, rays, and line segments.

- A *line* is a set of points that extends without end in opposite directions.
- A *ray* is part of a line, having one endpoint and extending in one direction without end.
- A *line segment* is part of a line or ray, consisting of two endpoints and all points between those endpoints.
The angles, lines, and figures you will learn about in this chapter are all made from lines, rays, or line segments.

Read **Classifying angles** on p. 3. Notice that two rays that meet at a common endpoint form an angle, and angle measures are based on the 360° of a circle.

Then read **Identifying parallel lines and perpendicular lines** on p. 4. Notice the symbols used to indicate parallel lines and the right angle symbol used to indicate perpendicular lines. Also notice the small line symbol above the letters $AB$. This symbol indicates that $AB$ is a line and is read as “line $AB$.”

**TEACHING NOTES**

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the **Quick Check** sections.

**SKILLS CHECK**

Complete the **Quick Check** sections on pp. 3–4 in *Math in Focus 2B*.

**TEACHING NOTES**

**Textbook Answer Key**

Review your student’s answers to **Quick Check**, noting the problems that she answered incorrectly. Click on the link to access the appropriate **Reteach** activity that your student should complete for the remainder of this part.

**Reteach**

After your student completes the **Quick Check** in the **Recall Prior Knowledge** lesson of this chapter, review the questions that were answered incorrectly. If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.

<table>
<thead>
<tr>
<th>Quick Check Question(s)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–6</td>
<td><strong>Angles</strong></td>
</tr>
<tr>
<td>7–11</td>
<td><strong>Parallel and Perpendicular Lines</strong></td>
</tr>
</tbody>
</table>
Today you reviewed that angles are classified according to their measure.

An acute angle measures less than $90^\circ$.

A right angle measures exactly $90^\circ$.

An obtuse angle measures greater than $90^\circ$ but less than $180^\circ$.

A straight angle measures exactly $180^\circ$.

You also reviewed how to identify parallel and perpendicular lines.

Two lines that remain the same distance apart and never intersect are parallel to each other.

Two lines that intersect to form a $90^\circ$ angle are perpendicular to each other.

Please go online to view and submit this assessment.
**WARM-UP**

Complete the following problems.

1. How many endpoints are shared by two rays that form an angle?  
2. Can you tell whether an angle is right, acute, or obtuse without measuring? Explain.

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. 1 endpoint

2. Yes. A right angle forms a square corner; an acute angle is narrower than a right angle; an obtuse angle is wider than a right angle.

**INSTRUCTION**

Read *Explore the Properties of Complementary Angles* on p. 5 in *Math in Focus 2B*. In the first sentence, the term $m \angle ABC = 30^\circ$ is spoken as, “the measure of angle $ABC$ is equal to 30 degrees.” The letter $m$ stands for “measure.” The symbol $\angle$ is used to represent an angle, and the symbol $^\circ$ is used to represent degree.

For angles to be complementary, the sum of their measures must equal exactly $90^\circ$.

Then read *Technology Activity* on p. 6. If you have geometry software, perform the steps shown in the activity. For step 1, draw line segment $BC$ perpendicular to line segment $AB$. For
step 5, you should find that regardless of the location of point \(D\), the sum of the measures of the angles is always 90°, so the angles are complementary.

Study Example 1 on p. 7. Practice interpreting the symbols correctly by reading the sentence out loud. For example, the first line of the solution is read as:

“The measure of angle \(ABC\) is equal to 66 degrees, and the measure of angle \(DEF\) is equal to 24 degrees.”

Notice that angles are described by giving three points, with the middle point naming the vertex of the angle.

Complete Guided Practice on p. 7. When the sum of the measures of two angles equals exactly 90°, the angles are complementary.

Read Example 2 on p. 8. Sometimes an angle is described only by its vertex. When angles are described this way, you will only see one letter instead of three. Because angles \(K\) and \(P\) are complementary, you can subtract \(m\angle K\) from 90° to find \(m\angle P\).

Complete Guided Practice on p. 8. Since the angles are complementary, subtract the given angle measure from 90° to find the unknown angle measure.

### TEACHING NOTES

**Textbook Answer Key**

An angle can be identified by the angle symbol \(\angle\), followed by three points that define the angle. The middle letter must be the vertex. The other two points lie on the legs. The angle below could be named \(\angle DEF\) or \(\angle FED\). The angle can also be named by just the vertex, \(\angle E\). In this part, angles are named both ways.

### WATCH FOR THESE COMMON ERRORS

Your student may confuse the math term *complementary* with the everyday word *complimentary*. Both the spellings and meanings of the words differ. The sentence “Complementary angles complete a right angle” can help your student remember the correct spelling.
Complete the following problems.

1. How many endpoints are shared by two rays that form an angle?
2. Can you tell whether an angle is right, acute, or obtuse without measuring? Explain.

WARM-UP ANSWERS
1. 1 endpoint
2. Yes. A right angle forms a square corner; an acute angle is narrower than a right angle; an obtuse angle is wider than a right angle.

Read Explore the Properties of Complementary Angles on p. 5 in Math in Focus 2B. In the first sentence, the term \( \text{m} \angle ABC = 30^\circ \) is spoken as, “the measure of angle \( \angle ABC \) is equal to 30 degrees.” The letter \( \text{m} \) stands for “measure.” The symbol \( \angle \) is used to represent an angle, and the symbol ° is used to represent degree.

For angles to be complementary, the sum of their measures must equal exactly 90°.

Then read Technology Activity on p. 6. If you have geometry software, perform the steps shown in the activity. For step 1, draw line segment BC\( \overline{\text{C}} \) perpendicular to line segment A\( \overline{\text{B}} \). For step 5, you should...

Objectives
Identify complementary angles.

Books & Materials
Math in Focus 2B
Math in Focus - Teacher Edition
geometry drawing software (Optional)

Assignments
Complete Warm-up.
Read and complete pages in Math in Focus 2B.
Complete the Practice Questions.

WARM-UP

INSTRUCTION

WATCH FOR THESE COMMON ERRORS

Your student may confuse the math term complementary with the everyday word complimentary. Both the spellings and meanings of the words differ. The sentence "Complementary angles complete a right angle" can help your student remember the correct spelling.

Complete problems 1–4, 9–12, and 21 of Practice 6.1 on p. 17 in Math in Focus 2B.

Today you learned how to identify complementary angles.

\[ \text{m} \angle FGH + \text{m} \angle JKL = 25^\circ + 65^\circ \]
\[ = 90^\circ \]

\( \angle FGH \) and \( \angle JKL \) are complementary angles.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Complete the following problems.

1. If two angles are complementary angles, what can you say about the sum of their measures?
2. Angles $A$ and $D$ are complementary, and $m\angle A = 44^\circ$. Find $m\angle D$.

**WARM-UP ANSWERS**

1. The sum of their measures is $90^\circ$.
2. $m\angle D = 46^\circ$

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. The sum of their measures is $90^\circ$.
2. $m\angle D = 46^\circ$

**INSTRUCTION**

Read **Explore the Properties of Supplementary Angles** at the bottom of p. 8 in *Math in Focus 2B*. In the previous part, you learned that two angles with measures that add to $90^\circ$ are complementary angles. In this part, you learn that two angles with measures that add to $180^\circ$ are called **supplementary** angles.

Read the **Technology Activity** on p. 9. If you have geometry software, perform the steps shown in the activity. For step 1, point $Q$ can be located at any position along $PR$. For step 5, you should find that regardless of the location of point $S$, the sum of the measures of the angles is always $180^\circ$. Although the measures of $\angle SQP$ and $\angle SQR$ may change, the sum of their measures remains $180^\circ$, so the angles are supplementary.

Read **Example 3** on p. 10. A pair of supplementary angles can be made up of one acute angle and one obtuse angle, or it can be made up of two right angles. A pair of supplementary angles cannot be made up of two acute angles or two obtuse angles.
Complete **Guided Practice** on p. **10**.

Then read **Example 4** on p. **11**. You can check your answer for reasonableness by recalling that if one angle is acute, the other should be obtuse. In part **a**, since \( m\angle K = 22^\circ \), it is an acute angle. That means \( \angle P \) will be an obtuse angle.

Complete **Guided Practice** on p. **11**. Find the difference between \( 180^\circ \) and the measure of the given angle to find the unknown angle measure.

**TEACHING NOTES**

**Textbook Answer Key**

<table>
<thead>
<tr>
<th>Alphabetically,</th>
<th>Numerically,</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>complementary</strong></td>
<td>( \Rightarrow 90 )</td>
</tr>
<tr>
<td>comes before</td>
<td>comes before</td>
</tr>
<tr>
<td><strong>supplementary</strong></td>
<td>( \Rightarrow 180 )</td>
</tr>
</tbody>
</table>

**PRACTICE**


**WRAP-UP**

Today you learned how to identify supplementary angles.

\[
m\angle FGH + m\angle JKL = 123^\circ + 57^\circ = 180^\circ
\]

\( \angle FGH \) and \( \angle JKL \) are supplementary angles.
Please go online to view and submit this assessment.

Remember that supplementary angles always have a sum of 180°. You can view the Instructional Video, *Types of Angles*, to review the difference between complementary and supplementary angles.
Angles - Part 4

Books & Materials
- Math in Focus - Teacher Edition

Assignments
- Complete Interactive Activity.
- Complete Rate Your Understanding.

LEARN

INTERACTIVE ACTIVITY

Click on this activity to explore different types of angles.

RATE YOUR UNDERSTANDING

Please go online to view and submit this assessment.
Angles - Part 5

Objectives
- Identify adjacent angles.

Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2B.
- Complete the Practice Questions.

LEARN

WARM-UP

Complete the following problems.

1. In $\angle ABC$, which point is the vertex?

2. Must two angles be situated right next to each other in order to be complementary or supplementary angles? Explain.

WARM-UP ANSWERS

1. Point $B$

2. No. Two angles are complementary if the sum of their measures equals 90°; two angles are supplementary if the sum of their measures equals 180°. In either case, the two angles do not need to be right next to each other.

TEACHING NOTES

INSTRUCTION

Read Explore the Properties of Adjacent Angles at the bottom of p. 11 in Math in Focus 2B. Notice that the angles in each pair share the same point in the middle of their names. In the examples shown, the common point is $O$. This point is the vertex of each angle. Each pair of angles also shares another common point along one ray.
Note that two angles must meet three criteria to be classified as *adjacent angles*.

- They share a common vertex.
- They share a common side.
- They have no common interior points. They do not overlap.

Then read **Example 5** on *p. 12*. The key word perpendicular lets you know that $\angle PQR$ is a right angle; it measures $90^\circ$, so $\angle PQS$ and $\angle SQR$ are adjacent complementary angles. Notice the abbreviation for complementary angles given in the speech bubble.

Read and discuss the **Think Math** question with your Learning Guide. Review the definition of adjacent angles and look carefully at $\angle PQR$ and $\angle SQR$. Although they share a common vertex and a common side, the angles contain common interior points. Since the angles overlap, they are not adjacent.

Complete **Guided Practice** on *p. 12*. $\angle ABD$ and $\angle DBC$ are adjacent complementary angles. Subtract the given angle measure from $90^\circ$ to find the value of $x$.

---

**TEACHING NOTES**

**Textbook Answer Key**

Problems 25 and 27–28 on *p. 18* in *Math in Focus 2B* include more than two angles. You may need to remind your student that only a pair of angles can be complementary, but she can still use the fact that $m \angle ABC = 90^\circ$ and the known adjacent angles to find the measure of the unknown angle. In addition, problems 26–28 include both a variable and a coefficient, such as $2x$. Point out that the value of $x$, the variable, is not an angle measure, so its solution has no degree symbol. In the case of $2x$, your student needs to find the value of the unknown angle measure and divide by 2 to find the value of $x$.

**PRACTICE**

**WRAP-UP**

Today you learned how to identify adjacent angles.

Look at the angles in the diagram.

∠POQ and ∠QOR are adjacent angles; ∠QOR and ∠ROS are also adjacent angles. ∠POQ and ∠ROS are not adjacent angles; although they share a common vertex and do not overlap, they do not share a common side.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Angles - Part 6

Objectives
- Apply knowledge about angles to find unknown angle measures.

Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus B.
- Complete problems 29–34, Math in Focus B.
- Complete the Quick Check.

LEARN

WARM-UP

Complete the following problems.

1. The measure of a straight angle is exactly __________.
2. True or false: Adjacent angles can overlap.
3. What are the three necessary conditions for angles to be adjacent?

WARM-UP ANSWERS

1. 180°  2. false  3. a common vertex, a common side, and no overlap

TEACHING NOTES

INSTRUCTION

Read the instructional section at the top of p. 13 in Math in Focus 2B. There can be two or more nonoverlapping angles on a straight line, and the sum of their measures will be 180°. Read the Caution note. Recall that supplementary angles are pairs of angles with measures that total 180°.

Then read Example 6 on pp. 13–14. For part a, since PR is part of a straight line, the measure of ∠PQR is 180°. Notice the abbreviation in the speech bubble on p. 13. For part b, you can use a similar method to find the unknown angles. Review the steps that show how to write an equation to represent the situation and then solve for x. Notice that the variable x in the solution has no degree symbol because it is not the measure of an angle; it is just a number.
The diagram shows that $\angle DOB = 2x^\circ$. Since $x = 18$, find the measure of the angle by multiplying the value of $x$ by 2.

$2 \cdot 18^\circ = 36^\circ$

$m\angle DOB = 36^\circ$

Check the sum of the measures of the three angles: $126^\circ + 18^\circ + 36^\circ = 180^\circ$

Complete Guided Practice on pp. 14–15. Write an equation for the adjacent angles on a straight line and solve for $y$. After finding the value for $y$, check that the sum of the angle measures is $180^\circ$.

Then read Example 7 on pp. 15–16. You can use a bar model or algebra to find angle measures involving ratios. Method 1 demonstrates using a bar model and the unitary method. These steps have been used in previous lessons to solve problems involving ratios. Method 2 provides a way to solve the problem using a variable to represent the measure of the angle.

Complete Guided Practice on p. 16. Solve the problem using the method you are most comfortable with. Check your answer using the other method.

Textbook Answer Key

This part shows that the measures of any number of adjacent angles along one side of a straight line have a sum of $180^\circ$. This information can be used to draw a bar model or to write an equation and solve for the unknown variable to find the measures of unknown angles along a straight line.

Complete problems 29–34 of Practice 6.1 on pp. 18–19 in Math in Focus 2B.
Today you learned how to find unknown angle measures along one side of a straight line.

In the diagram, $\angle FOJ$, $\angle JOH$, and $\angle HOG$ are angles on a straight line. Find the value of $x$.

\[
\begin{align*}
m\angle FOJ + m\angle JOH + m\angle HOG &= 180^\circ \\
108^\circ + x^\circ + 2x^\circ &= 180^\circ \\
108^\circ + 3x^\circ &= 180^\circ \\
108^\circ + 3x^\circ - 108^\circ &= 180^\circ - 108^\circ \\
3x &= 72 \\
\frac{3x}{3} &= \frac{72}{3} \\
x &= 24
\end{align*}
\]

Please go online to view and submit this assessment.

If you answered incorrectly, consider how you found the answer. Remember that a straight line is 180 degrees, so you need to add all three angles to equal 180 degrees. If you are still struggling with the concept, review the material in this lesson.
Angles - Part 7

Objectives

- Find measures of angles at a point.

Books & Materials

- Math in Focus 2B
- Math in Focus - Teacher Edition

Assignments

- Complete Warm-up.
- Read and complete pages in Math in Focus 2B.
- Complete the Quick Check.

WARM-UP

Complete the following problems.

1. Draw an angle and mark its vertex.
2. Draw two angles that share a vertex.

LEARN

WARM-UP ANSWERS

Sample answers are shown.

![Diagram of two angles sharing a vertex.]

TEACHING NOTES

WARM-UP ANSWERS

Sample answers are shown.

![Diagram of two angles sharing a vertex.]
INSTRUCTION

Read the instructional section on p. 20 in Math in Focus 2B. All angles at a point must share a common vertex. Because there are 360° in a circle, a full turn around a point is equal to 360°.

Then read Example 8 on pp. 20–21. For part a, since \( x = 34.5 \), you can find \( m\angle AOC \). In the diagram, the unknown angle measure is \( 4x^\circ \). Multiply: \( 4 \cdot 34.5 = 138 \). So, \( m\angle AOC = 138^\circ \). Check the answer to make sure all angle measures add to 360°.

\[ 138^\circ + 138^\circ + 84^\circ = 360^\circ \]

Read the Caution note at the bottom of p. 21. After you have solved for \( x \), you still have to substitute the value of \( x \) into the equation to find each angle measure in degrees.

Complete Guided Practice on p. 22. To find the value of \( p \), first identify whether all the angles are angles around a common point. Next, write an equation that relates all the measures of the angles at point \( O \). Then solve the algebraic equation. Finally, reread the question to determine whether you are asked to find the value of \( p \) or to find the measure of the angle. Check your answer by finding the sum of the measures of the angles at point \( O \).

Then study Example 9 on p. 22. The numbers 1, 2, 3, and 4 within the arcs are labels for the angles. This is another way to identify angles.

You can also use the Property of Angles at a Point to find the value of \( c \) in another way after you find the value of \( b \).

Complete Guided Practice on p. 23. Remember that you can use any pair of adjacent angles on a straight line to write an algebraic equation.

Study Example 10 on p. 24. This example shows two methods you can use to solve the problem: a bar model and algebra. Note that you use the Property of Angles at a Point to solve the problem, because all angles share a common vertex, and together make a 360° turn around one point. After finding the value of \( a \), you must continue to find the values of \( b \) and \( c \).

Complete Guided Practice on p. 25. The method used here is the algebraic method. You could also use a bar model to solve the problem.
This part shows how to find the measures of angles around a point. You may wish to go over problems such as Example 9 on p. 22 in Math in Focus 2B with your student to discuss strategies for solving these kinds of problems. Look at the information given. Find whether any of the angles in the diagram measure 90º or 180º. If so, look for adjacent angles within and write an equation that can be used to find an unknown angle measure. Also, notice whether the same variable appears in more than one angle, as \( b \) does in Example 9.

**PRACTICE**

Complete problems 1–4, 11, 13, 17, and 21–23 of Practice 6.2 on pp. 28–30 in Math in Focus 2B.

**WRAP-UP**

Today you learned how to apply the Properties of Angles at a Point to solve problems.

Find the value of \( x \) in the diagram.

\[
\text{m} \angle AOB + \text{m} \angle AOC + \text{m} \angle BOC = 360º \\
xº + 2xº + 2xº = 360º \\
5xº = 360º \\
\frac{5x}{5} = \frac{360}{5} \\
x = 72
\]

Always check your answer to make sure the values make the equation true.

\[
xº + 2xº + 2xº = 360º \\
72º + 144º + 144º = 360º
\]

**QUICK CHECK**

Please go online to view and submit this assessment.
MORE TO EXPLORE

If you answered incorrectly, try drawing a bar model to help you understand the problem and operations involved.
Angles - Part 8

Objectives
- Identify congruent angles.
- Identify vertical angles.
- Apply knowledge about vertical angles to find unknown angle measures.

Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition
- geometry drawing software (Optional)

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2B.
- Complete the Practice Questions.

WARM-UP
Use the diagram to answer the questions.

1. What kinds of angles do you see in the diagram?
2. How many pairs of supplementary angles do you see? Name them.

LEARN

TEACHING NOTES

WARM-UP ANSWERS
1. possible responses: adjacent angles, supplementary angles, acute angles, obtuse angles
2. 4 pairs: $\angle 1$ and $\angle 2$, $\angle 2$ and $\angle 4$, $\angle 3$ and $\angle 4$, $\angle 3$ and $\angle 1$
**INSTRUCTION**

Read the instructional section at the bottom of p. 25 in *Math in Focus 2B*. Two straight intersecting lines form vertical angles. The sample on p. 25 uses the Vertical Angle Property to show that $\angle 1$ and $\angle 3$ are **congruent**. You can also show this in another way.

\[
\begin{align*}
m\angle 1 &= 180^\circ - m\angle 2 \\
m\angle 3 &= 180^\circ - m\angle 2 \\
m\angle 1 &= m\angle 3
\end{align*}
\]

Look at the Caution note on p. 25. Because the angles are not formed by intersecting lines, they are not vertical. They are angles around a point.

Then read Technology Activity on p. 26. If you have geometry drawing software, perform the steps shown in the activity. You should find that as the measure of $\angle AOD$ increases, the measure of $\angle AOC$ decreases.

Read Example 11 on p. 26. Since the diagram is made of two intersecting straight lines, this means the angle labeled $2x^\circ$ and the angle labeled $60^\circ$ are congruent because they are vertical angles. Therefore $2x^\circ = 60^\circ$.

Complete Guided Practice at the top of p. 27. Use the method shown in Example 11 to find the value of $y$. Note that you can use supplementary angles to find the measure of the other two angles formed by the intersection of lines $AB^\|$ and $CD^\|$. ($60^\circ$)

Then study Example 12 on p. 27. The order in which you find the angle measures is not important. For example, you could use the Property of Vertical Angles first to find the value of $b$. In problems such as this, there is often more than one way to find unknown angle measures.

Complete Guided Practice at the bottom of p. 27. You can use the order shown in the problem, or you can use a different order to find the values of the variables. For example, since you know $m\angle 1$, you can find $q$ first because $m\angle 1 = m\angle 3 = 114^\circ$.

**TEACHING NOTES**

Textbook Answer Key

*Math in Focus* does not use the congruent symbol, $\cong$. However, you may want to introduce the symbol to your student. For the diagram at the bottom of p. 25, your student could write $\angle 1 \cong \angle 3$. 
Complete problems 5–10, 12, 14–16, 18–20, and 24–33 of Practice 6.2 on pp. 28–31 in Math in Focus 2B.

Today you learned how to identify vertical angles and apply knowledge about vertical angles to find unknown angle measures.

AB and CD are straight lines. Find the value of $a$.

\[
2a^\circ = 74^\circ \\
\frac{2a}{2} = \frac{74}{2} \\
a = 37
\]

Please go online to view and submit this assessment.
Angles - Part 9

Books & Materials
- Math in Focus - Teacher Edition

LEARN

INTERACTIVE ACTIVITY

Ask your Learning Guide to help you access this activity. Use the tool to explore the angles created by intersecting lines.

TEACHING NOTES

Follow these steps to access the activity:

1. Go to the Seventh Grade Geometry section of the activity list.
2. Click on Supplementary and Vertical Angles.
3. Click on View Teacher Tool or View Teacher Tool in Spanish.
4. Read and agree to the Terms of Use. Click Continue.

RATE YOUR ENTHUSIASM

Please go online to view and submit this assessment.
Use an equation to find the value of each indicated variable.

\( \overrightarrow{AB} \) is a straight line.
Find the values of \( y \) and \( z \).

Upload your work to show how you got to the answer.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use an equation to find the value of each variable?
- Upload your work to show how you found the answer?
Constructions - Part 1

**Objectives**
- Identify types of triangles (acute, obtuse, right, scalene, isosceles).
- Identify types of quadrilaterals (trapezoid, parallelogram, rectangle, rhombus, square).

**Books & Materials**
- Math in Focus 2B
- Math in Focus - Teacher Edition

**Assignments**
- Complete Warm-up.
- Read and complete pages in Math in Focus 2B.
- Complete the Practice Questions.

---

**LEARN**

**WARM-UP**

Identify each angle as acute, obtuse, or right.

1. 

2. 

3. 

---

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. acute 2. right 3. obtuse
Today's part covers important ideas you need to understand before beginning the lessons in this chapter. Read p. 62 in *Math in Focus 2B*. Think about other real-world situations that use patterns and geometric shapes. Name some shapes that you see often in everyday life.

Read *Classifying triangles* on p. 63. Notice how angles of equal measure and sides of equal length are marked in the drawings.

Then read *Naming quadrilaterals* on pp. 64–65. Notice that a square can also be described as a quadrilateral, parallelogram, rectangle, or rhombus. However, *square* is the most precise name. Notice the symbols that are used to mark angles of equal measure, side lengths of equal measure, parallel sides, and right angles. These marks are useful to include when drawing a picture of a geometric shape.

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the Quick Check sections.

Complete the *Quick Check* sections on pp. 64–65 in *Math in Focus 2B*.

Review your student's answers to the *Quick Check*, noting the problems that he answered incorrectly. Click on the link to access the appropriate *Reteach* activity that your student should complete for the remainder of this part.

After your student completes the *Quick Check* in the *Recall Prior Knowledge* part of this chapter, review the questions that were answered incorrectly.
If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.

<table>
<thead>
<tr>
<th>Quick Check Question(s)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Triangles</td>
</tr>
<tr>
<td>2</td>
<td>Quadrilaterals</td>
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<tr>
<td>3–9</td>
<td>Measuring Angles</td>
</tr>
<tr>
<td>10–12</td>
<td>Perpendicular Lines</td>
</tr>
</tbody>
</table>

**WRAP-UP**

Today you reviewed classifying triangles.

The triangle shown is an equilateral triangle because all 3 sides are of equal length. It is also an acute triangle because all 3 angles are acute.

![Equilateral Triangle](image)

You also reviewed how to name quadrilaterals.

The following quadrilateral is a square. All sides are of equal length; its opposite sides are parallel; all its interior angles are right angles; and its diagonals are equal in length.

![Square](image)

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Constructions - Part 2

**Objectives**
- Use a protractor to measure angles.
- Use a protractor to draw angles.

**Books & Materials**
- Math in Focus 2B
- Math in Focus - Teacher Edition
- protractor

**Assignments**
- Complete Warm-up.
- Read and complete pages in Math in Focus 2B.
- Complete the Quick Check.

### LEARN

**WARM-UP**
Sketch an example of each kind of angle.

1. acute
2. right
3. obtuse

### TEACHING NOTES

**WARM-UP ANSWERS**
Sample answers are shown.

1. 
   ![Example](image1)
2. 
   ![Example](image2)
3. 
   ![Example](image3)
**INSTRUCTION**

Today's part covers important ideas you need to understand before beginning the lessons in this chapter. Read p. 66 in *Math in Focus 2B*.

Notice that the point where two rays intersect to form an angle is called a *vertex*. To measure an angle using a protractor, align the vertex with the center of the base line. It is very important to read the angle measure from the correct scale. If the ray passes through zero on the inner scale, read the angle measure on the inner scale. If the ray passes through 0 on the outer scale, read the angle measure on the outer scale.

Then read p. 67. Notice that the steps for drawing an angle are similar to using a protractor to measure an angle. Once again, pay special attention to whether you should use the inner or outer scale.

Finally, read p. 68. Recall that perpendicular line segments intersect at a 90° angle. Think about why 90° is only labeled once on the protractor.

**HELPFUL ONLINE RESOURCES**

Instructional Video: *Measuring Angles*

---

**TEACHING NOTES**

Emphasize to your student the importance of using the correct scale on a protractor. Point out that he should always check the reasonableness of his answer. For example, if the angle appears to be acute and he measures an angle greater than 90°, then he should check that he measured using the correct scale.

---

**PRACTICE**

Complete the Quick Check sections on pp. 66–68 in *Math in Focus 2B*.

---

**TEACHING NOTES**

*Textbook Answer Key*
WRAP-UP

Today you reviewed how to use a protractor to measure and draw angles.

Angle ABC measures 40°.

You also reviewed how to use a protractor to draw perpendicular line segments.

Quick Check

Please go online to view and submit this assessment.

More to Explore

If you struggled with this question, use a protractor to measure angles using the outer scale and the inner scale and find the difference. You may also want to use estimation knowing that angles that are not as wide as right angles will have measures less than 90 degrees. If you have difficulty reading the protractor, revisit the material in this lesson.
Constructions - Part 3

Objectives
- Construct a triangle with given measures.

Books & Materials
- Math in Focus 2B
- *Math in Focus - Teacher Edition*
- protractor
- compass
- ruler

Assignments
- Complete Warm-up.
- Read and complete pages in *Math in Focus 2B*.
- Complete the Practice Questions.

LEARN

WARM-UP

Two measures of angles in a triangle are given. Find the measure of the third angle.

1. 80° and 76°
2. 14° and 95°

TEACHING NOTES

WARM-UP ANSWERS

1. 24° 2. 71°

INSTRUCTION

Read p. 85 in *Math in Focus 2B*. While you do not use a protractor to construct angle bisectors or perpendicular bisectors, you will use a protractor to construct a triangle with given measures.

A *unique triangle* is a triangle that can be created when specific line and/or angle measurements are given. If more than one triangle can be created with the information given, the triangle is not unique. You can construct a unique triangle if you know the lengths of two sides and the measure of the *included angle*, or the angle that lies between the two sides. You can also construct a unique triangle if you know the measures of two angles and the length of the *included side*, or the side that is shared between the two known angles.
Review Example 6 on p. 86. Begin by drawing a rough sketch to help guide you as you work. Notice that you can measure and draw any of the sides first. When you have completed the triangle, compare the construction to your sketch. Complete Guided Practice on p. 86. Remember to begin by drawing a rough sketch.

Review Example 7 on p. 87. In this case, you are given the measure of two angles and the length of the included side. Complete Guided Practice on p. 87. After sketching the triangle, identify the included side.

Review Example 8 on p. 88. In this example, you are given a different set of measurements: the length of two sides and the measure of the included angle. Read and discuss the Think Math question with your Learning Guide. Remember that knowing the length of two sides and the measure of the included angle of a triangle defines a unique triangle. Complete Guided Practice on p. 88.

Review Example 9 on p. 89. The first side drawn is one of the sides that forms the given angle measure. Read and discuss the Think Math question with your Learning Guide. Notice that in step 4, continuing the arc to the right would make it intersect XY at a second point. This would result in a completed different-shaped triangle. Complete Guided Practice on p. 89.

**TEACHING NOTES**

**Textbook Answer Key**

Discuss the meanings of included side and included angle with your student. You may want to draw several triangles with different combinations of given measurements and ask him to identify the included sides and included angles.

Emphasize the differences between the examples. Stress the importance of drawing a sketch first to understand the types of measurements given as well as to compare the sketch to the final construction.

**PRACTICE**

Complete problems 1–7 of Practice 7.3 on p. 93 in Math in Focus 2B. For problem 7, check your answer using a protractor.
Today you learned how to construct a triangle given different groups of measures. Below is a step-by-step guide for constructing a triangle given two sides and the included angle.

Construct triangle $ABC$ such that $AB = 2$ centimeters, $BC = 5$ centimeters, and $m\angle ACB = 50^\circ$.

**Step 1:** Draw a rough sketch.

**Step 2:** Use a ruler to draw $BC$.

**Step 3:** Use a protractor draw an angle of $50^\circ$ at vertex $B$.

**Step 4:** Set the compass to the length of $AB$, 2 centimeters, and draw an arc centered at $B$ that crosses the ray you drew in Step 3. Label the intersection point $A$.

**Step 5:** Connect $A$ and $C$.

![Diagram](image)

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Constructions - Part 4

Books & Materials
- Math in Focus - Teacher Edition

Assignments
- Complete Interactive Activity.
- Complete Rate Your Understanding.

LEARN

INTERACTIVE ACTIVITY
You will use this app Triangle Maker to see if a triangle is formed with given constraints. You will adjust the lengths with a slider and adjust angles by moving the endpoints.

RATE YOUR UNDERSTANDING
Please go online to view and submit this assessment.
Follow these instructions for the activity shown below.

Click [here](#) to view the activity in a new window.

First practice using the ruler and angle measure tools. Click on the red “Show ruler” tool and place one open circle on point $A$ and the other on point $B$. You now see that side $AB$ is 20 units long. Now click on the blue “Show ruler” tool and measure side $AC$. The tool shows that it is 26.93 units long.

Click off the ruler tools and click on the green “Show angle measure tool.” Place the circle at one end on point $A$, the middle circle on point $B$, and the last circle on point $C$. You have now measured angle $ABC$ and found it to measure 68.2°. Use the other Show angle measure tool to measure one of the other two angles in the triangle. This triangle appears to be isosceles. How can you find out? Use the tools and tell your Learning Guide if the triangle is isosceles or not.

Now try this activity. Set the measure of angle $ABC$ as close to 30° as you can. Set the measure of angle $ACB$ to 60°. Put a ruler tool over side $BC$ and stretch it to measure 42 units. What is the measure of angle $CAB$, rounded to the closest whole number? What is the length of side $AC$, round to the closest whole number?

Use the Gizmo to check your work on p. 93 of Math in Focus 2B.
Answers:

The triangle is isosceles because sides $AC$ and $BC$ both are 26.93 units long.

Angle $CAB$ measures approximately 90°; line segment $AC$ is about 21 units long.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

Quick Check

Please go online to view and submit this assessment.

More to Explore

If you answered incorrectly, try constructing examples using each of the criteria. You may also want to revisit the material in this lesson.
Constructions - Part 6

Objectives
- Identify when given conditions determine a unique triangle, more than one triangle, or no triangle.

Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2B.
- Complete the Practice Questions.

LEARN

WARM-UP

1. Describe the shortest distance between a point and a line segment.
2. What does it mean for a triangle to be unique?

WARM-UP ANSWERS

1. The shortest distance between a point and a line segment is the length of the perpendicular line segment that joins them.

2. It means that there is only one triangle that has that specific shape and size.

TEACHING NOTES

INSTRUCTION

Read p. 90 in Math in Focus 2B. In part a, two side lengths and a nonincluded angle measure are given. While this is generally enough information to draw a triangle, it is impossible to draw this particular triangle because the length given for line segment YZ is shorter than the minimum length needed to connect the sides. In order to connect the sides, the measure of \( \angle X \) would need to be adjusted, but that is not possible since \( \angle X \) is given as 40°.

In part b, two side lengths and a nonincluded angle measure are also given. With these particular measurements, it is possible to draw two different triangles. You should draw an arc wide enough to identify the two possible locations for point Z.

Study the diagrams in both parts a and b. The minimum distance for side YZ is the length of the perpendicular line segment that joins Y and Z.
Read and complete Hands-On Activity on p. 91. You may find it helpful to use the Triangle Inequality Theorem, which says that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. After you complete steps 1–5, record whether you could draw exactly one triangle, more than one triangle, or no triangles using the given measures.

Review Example 10 on pp. 91–92. Notice that three side lengths are given in part a. Start by using the Triangle Inequality Theorem to determine if it is possible to draw a triangle with the given side lengths. If it is, then construct the triangle.

Complete Guided Practice on p. 92.

**TEACHING NOTES**

**Textbook Answer Key**

While reviewing part b on p. 90, emphasize to your student that he must draw an arc wide enough to identify the two points where the arc intersects side XZ.

Guide your student to understand that if only one triangle can be drawn using given measures, then the triangle is unique. If there is more than one point where two sides could intersect, then more than one triangle can be drawn. If it is not possible to draw a triangle with the given measures, then a triangle with those measures does not exist.

**PRACTICE**

Complete problems 8–10 of Practice 7.3 on p. 93 in Math in Focus 2B.

**WRAP-UP**

Today you learned to identify the conditions that determine a unique triangle.

![Diagram of triangle ABC with side lengths given: AB = 4 centimeters, BC = 5 centimeters, AC = 6 centimeters. The text states that one unique triangle can be constructed.]
Please go online to view and submit this assessment.
# Constructions - Part 7

## Objectives
- Construct squares and rectangles.

## Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition
- ruler
- compass
- protractor

## Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus B.
- Complete problems 1–2 and 7 in Math in Focus B.
- Complete the Practice Questions.

## LEARN

### WARM-UP

1. Describe the properties of a square.

2. Describe the properties of a rectangle.

### TEACHING NOTES

#### WARM-UP ANSWERS

1. Opposite sides are parallel; all side lengths are equal; it has 4 right angles; diagonals intersect at a right angle; diagonals are congruent; diagonals bisect each other.

2. Opposite sides are parallel and congruent; it has 4 right angles; diagonals are congruent; diagonals bisect each other.

### INSTRUCTION

Read Use Properties of Quadrilaterals on p. 94 in Math in Focus 2B. Just as you can construct triangles, you can also construct quadrilaterals. As with triangles, it is helpful to begin with a rough sketch.

Review Example 11 on pp. 94–95. Recall that opposite sides of a rectangle are parallel and congruent, and that each angle of a rectangle measures 90°. Draw a sketch to help visualize the rectangle. Include labels in the sketch.

Notice that step 3 uses the fact that a rectangle has 4 right angles, and steps 4 and 5 use the fact that opposite sides are parallel and congruent.
Because a square is a rectangle that has 4 sides of equal length, you can construct a square using a similar process. Follow the steps in Example 11 to construct a square with a side length of 5 centimeters.

Construct the rectangle in Example 11 again, but this time do not use a protractor. Begin with a line segment longer than 5 cm and use your compass to construct perpendicular line segments to this line segment.

Complete Guided Practice on p. 96. You will need a ruler, protractor, and compass to complete the construction. Remember to begin by sketching the rectangle and labeling it with all the information you are given.

Textbook Answer Key

Point out that there are other quadrilaterals beside squares and rectangles. Have your student draw some examples, such as a trapezoid or concave quadrilateral. Help him make language connections: quad means four and lateral means side.

Guide your student to understand that because a square is a special type of rectangle, the process to construct a square is similar to the process to construct a rectangle.

Help your student construct the rectangle from Example 11 without the use of a protractor. He should draw a line segment longer than 5 cm and then mark the two endpoints 5 cm apart. He should set his compass to a relatively small setting, then mark two arcs that intersect the line segment with his compass on one endpoint. Next, he will construct the perpendicular bisector of the segment formed by the arcs he just drew. Have him increase the compass setting and place the compass at the intersection of an arc and the line segment and draw an arc above the line segment. He should repeat this for the second arc. He will draw a line segment from the intersection of the two arcs above the line segment to the endpoint. He has now constructed a line segment perpendicular to the original line segment at one endpoint. He should repeat for the second endpoint and then complete the construction.

Practice

Complete problems 1–2 and 7 of Practice 7.4 on p. 99 in Math in Focus 2B.
WRAP-UP
Today you learned how to construct squares and rectangles.

Construct rectangle $ABCD$ with a length of 6 centimeters and a width of 4 centimeters.

✔️ PRACTICE QUESTIONS
Please go online to view and submit this assessment.
Constructions - Part 8

Objectives
- Construct parallelograms and rhombuses.
- Apply knowledge of constructions to composite shapes.

Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition
- ruler
- compass
- protractor

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2B.
- Complete the Practice Questions.

LEARN

WARM-UP
1. Describe the properties of a parallelogram.
2. Describe the properties of a rhombus.

TEACHING NOTES

WARM-UP ANSWERS
1. Opposite sides are parallel and congruent; opposite angles are congruent; consecutive angle are supplementary; diagonals bisect each other.

2. Opposite sides are parallel; all four sides are congruent; opposite angles are congruent; consecutive angle are supplementary; diagonals bisect each other at right angles.

INSTRUCTION

Review Example 12 on pp. 96–97 in Math in Focus 2B. Recall that opposite sides of a parallelogram are parallel and congruent, and opposite angles are congruent. These properties can help you construct a parallelogram. Draw a rough sketch to help visualize the parallelogram. Include labels in your sketch.

Compare the steps for constructing a parallelogram with the steps for constructing a rectangle.

Read and discuss the Think Math section on p. 97 with your Learning Guide. The properties of a parallelogram can help you construct a parallelogram with given measures.
Complete Guided Practice on p. 97. You will need a ruler, protractor, and compass to complete the construction. Remember to begin by sketching the parallelogram and labeling it with all the information you know.

Then review Example 13 on p. 98. Recall that a rhombus is a special type of parallelogram with all sides of equal length and with diagonals that are perpendicular bisectors of each other.

Read and discuss the Think Math question on p. 98 with your Learning Guide. Remember that a perpendicular bisector divides a line segment into two equal pieces at a right angle.

Complete Guided Practice on p. 98. Begin by drawing a rough sketch.

**TEACHING NOTES**

**Textbook Answer Key**

Guide your student to understand that because the diagonals of a rhombus are perpendicular bisectors of each other and all sides of a rhombus are congruent, the construction steps are very similar to those for constructing a perpendicular bisector, if the length of a diagonal and a side are given.

Have your student construct a parallelogram and rhombus with angle measures that he can create without the use of a protractor. For example, he can create a parallelogram with 45° angles by constructing a perpendicular bisector to the first side that is half the length of that side. A line segment connecting the endpoint of the side with the endpoint of the perpendicular bisector will create a 45° angle.

Have him apply the principles of construction to composite shapes, such as a triangle on top of a rectangle or a semicircle on top of a square. Do not allow the use of a protractor.

**PRACTICE**

Complete problems 3–6 and 8–16 of Practice 7.4 on pp. 99–100 in Math in Focus 2B.
WRAP-UP

Today you learned how to construct parallelograms and rhombuses.

The diagonal $AC$ of rhombus $ABCD$ measures 8 centimeters and side $AB$ measures 5 centimeters. Construct rhombus $ABCD$.

![Diagram of rhombus with dimensions](image)

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
How many triangles can be constructed with the following measures?

\[ AB = 7.6 \text{ cm}, \ AC = 5.4 \text{ cm}, \text{ and } m\angle ABC = 50^\circ. \]

Explain how you got this answer.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the measurements given to construct triangles?
- Tell how many triangles can be constructed with the measurements shown?
- Explain how you found your answer?
- Show your work?

How many triangles can be constructed with the following measures:

AB = 7.6 cm, AC = 5.4 cm, and \( \angle C = 50° \).

Explain how you got this answer.
Constructions - Part 10

Books & Materials
- Math in Focus - Teacher Edition

SHOW

In the project introduction, you learned about tessellation. Now it is time to start making your shape. Here are a few more examples for you to look at:

A

Notice that Pattern A involves parallelograms, squares, and triangles.

B

Pattern B uses parallelograms, squares, equilateral triangles, and hexagons.
Pattern C uses squares, equilateral triangles, irregular hexagons, and regular hexagons.

You will need to make sure your shape includes:

- a parallelogram
- an equilateral triangle
- a square
- a right triangle

You may also use:

- a rectangle
- an obtuse angle
- a trapezoid

Also, to allow you to be a bit more creative, you may also use up to two other polygons that are not listed.

Since fitting these patterns together is not as simple as it may seem, you may want to begin by cutting shapes out of cardboard to try fitting them together in different ways. Try tracing them in different ways to decide on a pattern. Make sure to use a straightedge to assure that all of your shapes are correctly drawn to scale. As you repeat the shapes, they must all be the same size and shape.

Although there is some fun online software that you can use to draw these (and you are welcome to use one when trying to decide on a pattern), the purpose of this activity is to practice drawing and measuring these shapes, so you do need to do these with a paper, pencil, protractor, and ruler.

For the first task, decide on the pattern that you will be creating. Once you decide on the pattern, make sure that you have a piece of paper or poster board larger enough to repeat it multiple times. Try a few different designs to see what you like the best.
RATE YOUR PROGRESS

Please go online to view and submit this assessment.
SHOW

Now it is time to create your tessellation. In the previous task, you practiced putting different shapes together to decide on a pattern. In this task, you will now draw your tessellation. Make sure that your repeating shapes match exactly. All of your angles and side lengths must be precise. You do not need to color them in yet. For this activity, you should just use a regular pencil, as that it is easier to erase if you make an error as you work.

RATE YOUR PROGRESS

Please go online to view and submit this assessment.
LEARN

WARM-UP

1. Name an object that is too large for you to draw its full-size version.
2. Name an object that is too small for you to draw its full-size version.
3. Write the ratio 3:4 two other ways.

WARM-UP ANSWERS

1. Answers will vary.  2. Answers will vary.  3. 3 to 4, 3:4

INSTRUCTION

Read p. 101 in Math in Focus 2B. Scale drawings are a useful tool when it is not reasonable to make a drawing of an object true to size. For example, a scale drawing of an airplane shows the airplane at a smaller size, usually one that will fit on a piece of paper. A scale drawing of an atom shows the atom at a larger size, so the details can be shown.

Read Understand Scale and Scale Factor on p. 102. Notice the difference between the terms scale and scale factor. While both compare a length in the drawing to the same length of the real-life object, a scale factor compares the lengths using the same units.
Review Example 14 on p. 102. When calculating a scale factor as a fraction, it is important to write the measurement of the scale drawing in the numerator and the corresponding measurement of the actual object in the denominator. Notice that a measurement of the actual object times the scale factor equals the corresponding measurement of the scale drawing.

Complete Guided Practice on p. 103. Remember to write the scaled length in the numerator and the original length in the denominator.

Read and complete Hands-On Activity on p. 103. Before redrawing the figures, predict whether the drawing will be an enlargement or a reduction of the original figure, and think about what this means regarding the value of the scale factor. Check the scale factors you find by measuring more than one side length in the scale drawing and original figure.

HELPFUL ONLINE RESOURCE
Instructional Video: Dilations

Textbook Answer Key

Be sure your student understands the difference between a scale and a scale factor. Emphasize that a scale factor uses the same units; whereas a scale is used to convert between different units. You may want to review common conversion factors of units with your student.

Guide your student to understand that a scale drawing that is a reduction has a scale factor less than 1, and a scale drawing that is an enlargement has a scale factor greater than 1. Ask your student what a scale factor of 1 means.

PRACTICE
Complete problems 1–6 of Practice 7.5 on p. 111 in Math in Focus 2B.
WRAP-UP

Today you learned about scale drawings and how to find a scale factor.

In the diagram, square $A$ is a reduction of square $B$. The scale factor is 23.

![Diagram of squares A and B with measurements 3.6 cm and 5.4 cm]

QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

If you answered incorrectly, remember that when writing a scale factor, you need to use the same units and write it in simplest form. In this example, you would need to convert both measurements to either feet or inches, so you would rename 6 feet to 72 inches. You would then write the fraction as $3/72$ and finally simplify by dividing both the numerator and denominator by 3.
Follow these instructions for the activity shown below.

Click here to view the activity in a new window.

Click the triangle button. Set Scale factor to 1.0 and Rotation, in degrees to 0. (To set the value of a slider, drag the slider or select the number in the text field, type in a new value, and press Enter.) The pink and green triangles appear to be the same size and shape. Because they have a scale factor of 1, these triangles are congruent. This means that the corresponding side lengths and angle measures of the two triangles are the same. Select Show lengths and then Show angle measures to check.

Now drag the Scale factor slider. Notice that the size of ΔEFG (the image) changes, but ΔABC (the preimage) stays the same. How do the image and preimage compare when the scale factor is greater than one? How do the image and preimage compare when the scale factor is less than one? Share your observations with your Learning Guide.

Set the Scale factor slider to 3. Similar figures have pairs of corresponding angles and pairs of corresponding sides, just like congruent figures. Name the part of ΔEFG that corresponds to each of the following parts of ΔABC.

∠ABC and ____  ∠BCA and ____  ∠CAB and ____
AB and ____  BC and ____  CA and ____

Click on “Show lengths.” What do you notice about the measures of corresponding sides? Now click “Show angle measures.” Tell your Learning Guide how the side and angle measures compare in the two triangles.

Select the trapezoid button to view two similar trapezoids. With Show lengths selected, drag the vertices of trapezoid ABCD so that AB = 16, BC = 12, CD = 20, and DA = 12. A. If EF = 8, what is the scale factor? (Hint: Move the Scale factor slider until EF = 8.) If you know the measures of the angles of trapezoid ABCD, how do you find the measures of the angles of trapezoid EFGH? Select Show angle measures and check your answer in the Gizmo.

If you would like more practice with the Gizmo, you can click on Lesson Info and download the Student Exploration Sheet.
When the scale factor is greater than 1, the image is larger than the preimage. When the scale factor is less than 1, the image is smaller than the preimage.

<table>
<thead>
<tr>
<th>∠ABC and ∠EFG</th>
<th>∠BCA and ∠FGE</th>
<th>∠CAB and ∠GEF</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB and EF</td>
<td>BC and FG</td>
<td>CA and GE</td>
</tr>
</tbody>
</table>

The side lengths of the image can be found by multiplying the side lengths of the preimage by the scale factor. The angle measures of the image and preimage are the same.

If you know the measures of the angles of trapezoid ABCD, the measures of the angles of trapezoid EFGH will be the same because they are similar figures.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

**RATE YOUR UNDERSTANDING**

Please go online to view and submit this assessment.
Scale Drawings - Part 3

Objectives
- Find distances on a map using the map scale.

Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition
- ruler

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2B.
- Complete the Quick Check.

LEARN

WARM-UP

1. Explain the meaning of scale.
2. Explain the meaning of scale factor.
3. Solve \( \frac{25}{20} = x \).

TEACHING NOTES

WARM-UP ANSWERS

1. A scale is a comparison of a length in a scale drawing to the corresponding length of the original object.

2. A scale factor is the ratio of a length in a scale drawing to the corresponding length of the original object using the same units. \( 3x = 8 \)

INSTRUCTION

Read Use Map Scales on p. 104 in Math in Focus 2B. If the scale of the map is written without units, then the units of both quantities are the same. Scales on maps enable us to be able to calculate actual distances.

Review Example 15 on p. 104. Recall that a proportion is an equation that equates two ratios. Since both the map scale and the known and unknown distances are equivalent ratios, a proportion is used to solve for the unknown distance. You can check your answer by dividing it by 25. The quotient should be the length of the road on the map.

Complete Guided Practice on p. 104. When writing the proportion, be sure to write the map distances in the numerators and the actual distances in the denominators. Include units in your answer.
Review Example 16 on p. 105. Notice that this time you are given an actual distance and asked to find the distance on the map. Before completing Guided Practice on p. 105, think about what the map scale means and what units your answer will have.

Review Example 17 on p. 106. Just as you can use a proportion to find an actual distance from a map, you can use a proportion to find an actual length from a scale drawing. The scale does not include units, so both values have units of inches. Also note that you are asked to find the missing lengths in feet. Since all the measurements are given in inches, you must convert the missing lengths to feet. Use the fact that 12 inches equal 1 foot.

Complete Guided Practice on p. 106. Be sure that your answer is in feet.

HELPFUL ONLINE RESOURCE
Instructional Video: Scale Drawings

Textbook Answer Key

Your student will use proportions and solve equations with fractions to find distances using map scales. You may want to review these topics with your student.

Guide your student to understand that units can be cancelled because it is like multiplying (or dividing) by 1, which does not change the value. You may want to use a numerical example to illustrate this.

Point out to your student that maps are a type of scale drawing.

PRACTICE
Complete problems 7–12, 15–16, and 18–19 of Practice 7.5 on pp. 111–113 in Math in Focus 2B.

WRAP-UP
Today you learned how to find distances on a map given a map scale.

The scale of a map is 1 inch : 20 miles. If the distance on the map between the library and the grocery store is 0.4 inch, what is the actual distance in miles?

Let x miles be the actual distance.

1 inch : 20 miles = 0.4 inch : x miles

\[
\frac{1\text{ inch}}{20\text{ miles}} = \frac{0.4\text{ inch}}{x}\n\]

1 \cdot x = 0.4 \cdot 20

x = 8

The actual distance between the library and the grocery store is 8 miles.
Please go online to view and submit this assessment.

If you answered incorrectly, remember to set up a proportion and label the units. If you are still having difficulty, revisit the material from this lesson.
LEARN

1. A map has a scale of 1 inch to 20 miles. Explain how to find the actual length of a highway that is 5 inches long on the map.
2. What does it mean to square a number?
3. What units are used to measure area?

WARM-UP ANSWERS

1. Set up a proportion with the map distances in the numerators and the actual distances in the denominators. Then solve the proportion.
2. It means to multiply the number by itself.
3. Square units

INSTRUCTION

Read and complete Hands-On Activity on pp. 107–108 in Math in Focus 2B. In step 4, compare the area of each enlarged square to the area of the original square. Notice that the area of each enlarged square is the area of the original square, 1 square centimeter, times the square of the scale factor. It is important to understand this also extends to other two-dimensional figures, such as triangles, rectangles, trapezoids, etc.

Read Interpret Areas in Scale Drawings on p. 109. Recall that a map scale gives the ratio of a length on the map to the corresponding actual length. You can use the map scale to write a scale for an area on the map by squaring the scale factor.

Review Example 18 on p. 109. The first step is to write the scale of the area on the map. Then, just as you used a proportion to find actual distances with a map scale, you can also use a proportion to find the actual area. Remember to write the areas on the map in the numerators and the actual areas in the denominators. Label your answer with the appropriate unit of measure.
Complete **Guided Practice** on p. 109. Remember that the ratio of the areas is equal to the square of the ratio of the lengths.

Review **Example 19** on p. 110. In this example, you are given an actual area and asked to find the area in the scale drawing. The first step is the same—use the scale of the lengths to write the scale of the areas. When writing the proportion, write the map areas in the numerators and the actual areas in the denominators.

Complete **Guided Practice** on p. 110. Think about what information you are given and what you are asked to find. Remember to label your answer with the correct units.

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**TEACHING NOTES**

**Textbook Answer Key**

Guide your student to understand that the area of a scaled figure is equal to the area of the original figure times the square of the scale factor. Be sure your student understands that this applies to all two-dimensional figures. You may choose to repeat the activity using a different two-dimensional figure to illustrate this.

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**PRACTICE**

Complete problems 13–14 and 17 of **Practice 7.5** on pp. 112–113 in *Math in Focus 2B*.

---

**WRAP-UP**

Today you learned how to use scale factors to compare areas.

The scale on a map is 1 inch : 5 miles. The area of a park on the map is 30 square inches. What is the actual area of the park?

Map length : Actual length = 1 in. : 5 mi

Map area : Actual area = 1 in.² : 25 mi²

\[
\frac{\text{Area of park on map}}{\text{Actual area of park}} = \frac{30}{25}
\]

\[
\frac{30}{x} = \frac{1}{25}
\]

\[
x = 30 \cdot 25
\]

\[
x = 750
\]

The actual area of the park is 750 square miles.
PRACTICE QUESTIONS

Please go online to view and submit this assessment.
LEARN

WARM-UP

1. What is an angle bisector?
2. What is a perpendicular bisector?
3. What is the measure of each angle in an equilateral triangle?

TEACHING NOTES

WARM-UP ANSWERS

1. An angle bisector divides an angle into two congruent angles.
2. A perpendicular bisector intersects a line segment at a right angle and divides the line segment into two congruent segments.
3. 60°

INSTRUCTION

Read and complete Brain @ Work on p. 114 in Math in Focus 2B. To trisect an angle means to divide it into three congruent angles. In problem 1, you are asked to trisect a right angle. Think about what each of the three angles will measure after a right angle is trisected. Begin by constructing a 60° angle. Do not use a protractor. Instead, since each angle of an equilateral triangle measures 60°, construct an equilateral triangle.

To solve problem 2, follow the steps the math teacher suggests in the problem.
LEARN
1. What is an angle bisector?
2. What is a perpendicular bisector?
3. What is the measure of each angle in an equilateral triangle?

WARM-UP ANSWERS
1. An angle bisector divides an angle into two congruent angles.
2. A perpendicular bisector intersects a line segment at a right angle and divides the line segment into two congruent segments.
3. 60°

Read and complete Brain @ Work on p. 114 in Math in Focus 2B. To trisect an angle means to divide it into three congruent angles. In problem 1, you are asked to trisect a right angle. Think about what each of the three angles will measure after a right angle is trisected. Begin by constructing a 60° angle. Do not use a protractor. Instead, since each angle of an equilateral triangle measures 60°, construct an equilateral triangle.

To solve problem 2, follow the steps the math teacher suggests in the problem.

PRACTICE
Complete the following problems without the use of a protractor.

Draw a triangle. Then, draw the angle bisector of each angle all the way across the triangle.

1. Classify the triangle you drew in terms of both its side lengths and its angle measures.
2. What is the sum of the angle measures around the center?
3. What is the sum of the three half-angles?
4. Are your answers true for all possible triangles?

PRACTICE ANSWERS
1. Answers may vary. 2 360° 3 90° 4 yes

Guide your student to construct the three angle bisectors of the triangle he drew. Then lead him to answer the questions. If he has difficulty determining the sum of the angle measures around the center, remind him that the sum of the measures of angles at a point is 360°.

Guide your student to understand that since the sum of the angles of a triangle is 180°, then the sum of the half-angles of the triangle is 90°.
WRAP-UP

Today you applied mathematical concepts and skills to solve problems.

✓ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
The scale of a map is 1 inch : 9.5 miles.

a) The actual distance between Joseph’s house and the airport is 24 miles. How far apart are Joseph’s house and the airport on the map?

b) Joseph traveled from his house to the airport. He then traveled another 16 miles past the airport to a restaurant. How many inches on the map represent this distance?

c) Upload your work to show how you got to the answer.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information given to tell how far apart Joseph's house and the airport are on the map?
- Tell how many inches on the map the distance from his house to the airport plus the distance from the airport represents?
- Upload your work to show how you found the answer?
Now it is time to have some fun. You need to color in your tessellation. Choose a color for each shape and color each one in. You can use more than one color for each shape, but the repeating pattern must be the same color. For example:

This tessellation has 4 different colored trapezoids, but the colors follow a pattern.

You can use crayons, colored pencils, or markers. Remember that as your shapes repeat, so should your colors.
For your project, you have examined different tessellations and created your own. You included a parallelogram, an equilateral triangle, a square, a right triangle, and you could choose two other polygons to incorporate into your tessellation. You also colored your tessellation.

In this part, you will submit your tessellation and answer the three reflection questions listed below. Upload your answers with your tessellation. Remember to number your answers and write in complete sentences.

1. What is the difference between adjacent and supplementary angles? Why is it important to consider angles when making a tessellation?
2. What did you find the most challenging about constructing your tessellation? Be sure that your answer includes a relationship between different shapes.
3. Why is it important to measure angles accurately?

You will be graded according to the following rubric.

**FINAL SHOW**

Upload your tessellation and reflection question answers below.

Supported file formats: PDF, JPG, GIF, PNG, CSV, TXT, XPS, Word, Excel, Powerpoint, Publisher
In this part, your student will submit the tessellation and answer the following reflection questions.

Your student should answer each reflection question with the following answers:

1. What is the difference between adjacent and supplementary angles? Why is it important to consider angles when making a tessellation? Two angles are adjacent when they have a common side and a common vertex. It is important to consider angles when making a tessellation because the sum of the angles that share a common vertex need to equal 360 degrees.

2. What did you find the most challenging about constructing your tessellation? Be sure that your answer includes a relationship between different shapes. Answers will vary. Your student may mention that it was difficult to draw shapes that fit together, and that measurement and angles are important in constructing a tessellation.

3. Why is it important to measure angles accurately? Answers will vary. Your student may answer that it is important to measure angles accurately so that the shapes in the tessellation fit together without any gaps between shapes.

COLLABORATION

Share your tessellation with your peers. Review two of your peer’s tessellations and respond with the different shapes used and how color enhanced the look of the tessellation.
### Plane and Solid Figures - Part 1

#### Objectives
- Find the area of a circle.
- Find the circumference of a circle.

#### Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition
- string (Optional)
- round object such as hula hoop or plate (Optional)

#### Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2B.
- Complete the Interactive Activity.
- Complete the Practice Questions.

### LEARN

#### WARM-UP
Evaluate mentally.

1. \(3 \cdot 5^2\)
2. \(2 \cdot 4^2\)
3. \(2 \cdot 6^2\)

#### TEACHING NOTES

#### WARM-UP ANSWERS
1. 75  2. 32  3. 72

### INSTRUCTION
In this chapter, you will study solids that have faces or cross sections that are circles. Read Finding the area and circumference of a circle on p. 122 in Math in Focus 2B.

Recall that the *diameter* of a circle is a line segment that passes through the center of the circle and connects two points on the circle. The diameter is twice the *radius*, which is a line segment that connects the center of a circle to a point on the circle.
The circumference of a circle is the distance around the circle, or the perimeter of a circle. 

\[ C = \pi d, \] where \( d \) is the diameter of the circle

Recall the meaning of \( \pi \), the Greek letter \( pi \) (pronounced pie). \( Pi \) is the ratio of the circumference of a circle to its diameter. This ratio is constant for every circle. If you were to take a string that is the length of the circumference of a circle and use that string to measure the diameter of the circle, you would find that the string is a little longer than 3 diameters.

\( Pi \) is an irrational number. For ease of computation, we sometimes approximate it as 3.14 (written as a decimal) or 227 (written as a fraction).

HELPFUL ONLINE RESOURCE

Instructional Video: Finding Circumference

If your student needs additional support, you can use items such as a string and a hula hoop to demonstrate the concept of the ratio of \( pi \). Have her place the string around the perimeter of the hoop and mark where the string connects to itself. Then have her take that length of string and extend it lengthwise across the diameter of the hoop. She should see it crosses back and forth a little over three times. Explain that this ratio is the case with every circle, and mathematicians have named this ratio \( pi \).

PRACTICE

Complete problems 4–5 of Quick Check on p. 122 in Math in Focus 2B.

Textbook Answer Key
The circumference of a circle is the distance around the circle, or the perimeter of a circle. \[ C = \pi d \], where \( d \) is the diameter of the circle.

Recall the meaning of \( \pi \), the Greek letter \( \pi \) (pronounced pie). \( \pi \) is the ratio of the circumference of a circle to its diameter. This ratio is constant for every circle. If you were to take a string that is the length of the circumference of a circle and use that string to measure the diameter of the circle, you would find that the string is a little longer than 3 diameters. \( \pi \) is an irrational number. For ease of computation, we sometimes approximate it as 3.14 (written as a decimal) or \( \frac{22}{7} \) (written as a fraction).

HELPFUL ONLINE RESOURCE

Instructional Video: Finding Circumference

If your student needs additional support, you can use items such as a string and a hula hoop to demonstrate the concept of the ratio of \( \pi \). Have her place the string around the perimeter of the hoop and mark where the string connects to itself. Then have her take that length of string and extend it lengthwise across the diameter of the hoop. She should see it crosses back and forth a little over three times. Explain that this ratio is the case with every circle, and mathematicians have named this ratio \( \pi \).

Complete problems 4–5 of Quick Check on p. 122 in Math in Focus 2B.

Textbook Answer Key

Go to the online Circumference and Area of Circles activity. Click [here](#) if you would like to view the activity in a new window.

Download and print the Student Exploration Sheet. Complete the Prior Knowledge Questions. Then use the resource to complete the rest of the worksheet.

INTERACTIVE ACTIVITY

Go to the online Circumference and Area of Circles activity. Click [here](#) if you would like to view the activity in a new window.

Download and print the Student Exploration Sheet. Complete the Prior Knowledge Questions. Then use the resource to complete the rest of the worksheet.

TEACHING NOTES

Before beginning the online activity, download and print the Exploration Sheet Answer Key. Notice that, further down on the web page, you can access the Student Exploration Sheet in Spanish or French. Assist your student as needed through the activity.

WRAP-UP

Today you reviewed how to find the area and circumference of a circle.

\[ A = \pi r^2 \], where \( r \) is the radius of the circle

\[ C = \pi d \], where \( d \) is the diameter of the circle

PRACTICE QUESTIONS

Please go online to view and submit this assessment.


**Plane and Solid Figures - Part 2**

**Objectives**
- Find the surface area of a rectangular pyramid.
- Identify the nets of prisms and pyramids.

**Books & Materials**
- Math in Focus 2B
- Math in Focus - Teacher Edition
- nets of solid figures
- scissors

**Assignments**
- Complete Warm-up.
- Read and complete pages in *Math in Focus 2B*.
- Complete the Quick Check.

---

**LEARN**

**WARM-UP**
Evaluate mentally.

1. 12·60·50
2. 12·40·20
3. 30·20·40

**TEACHING NOTES**

**WARM-UP ANSWERS**
1. 1,500 2. 400 3. 24,000

**INSTRUCTION**

Read *Finding the surface area of a square pyramid* on p. 121 in *Math in Focus 2B*.
Pyramids are named by the shape of their base—the base of a square pyramid is a square. The base of a rectangular pyramid is a rectangle.

Surface area is the total area of the faces, including all bases, of a solid. The faces of a pyramid are triangles. Recall that you can use the formula $A=12bh$ to find the area of any triangle, where $b$ is the length of the base of the triangle and $h$ is the height of the triangle. Remember, the height is the perpendicular distance from the base of the triangle to the vertex of the angle opposite that base.
Read **Identifying nets of prisms and pyramids** on p. 123. Remember that a *net* is a plane figure that can be folded to make a solid. Think of a net as a flattened three-dimensional figure. Fold your cut-out nets to create three-dimensional shapes. Then unfold them. Look carefully at their flat shapes and see if you can visualize the folded shapes.

Review the difference between a prism and a pyramid.

**HELPFUL ONLINE RESOURCE**

**Instructional Video:** *Surface Area of Prisms and Pyramids*

---

**TEACHING NOTES**

Have your student brainstorm other prism shapes, such as triangular prisms, hexagonal prisms, and/or octagonal prisms.

Note that a *prism* is a solid with two identical ends. All sides are flat. The cross section of a prism is the same anywhere along the prism. Pyramids are not prisms. Pyramids have triangular sides which meet in a point at the top.

---

**PRACTICE**

Complete problems 3 and 6 for the **Quick Check** sections on pp. 122–123 in *Math in Focus 2B*.

---

**TEACHING NOTES**

**Textbook Answer Key**

Review your student’s answers to the Quick Check sections, noting the problems that she answered incorrectly. Click on the link to access the appropriate **Reteach** activity that she should complete for the remainder of this part.

**Reteach**

After your student completes the Quick Check in the **Recall Prior Knowledge** part of this chapter, review the questions that were answered incorrectly.

If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.

Note this chapter opener spans two parts.
Quick Check

<table>
<thead>
<tr>
<th>Question(s)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1−3</td>
<td>Surface Area and Volume</td>
</tr>
<tr>
<td>4−5</td>
<td>Area of a Circle</td>
</tr>
<tr>
<td>6</td>
<td>Nets of 3D Shapes</td>
</tr>
</tbody>
</table>

WRAP-UP

Today you reviewed how to find the surface area of a square pyramid, as well as the definition of a prism and pyramid. In a pyramid, the faces meet at a vertex.

You also reviewed how to name pyramids. Remember, pyramids are named according to the shapes of their bases.

You reviewed how to find the surface area of a pyramid. To find the surface area of a solid, calculate the area of each face and each base, and then add the areas together. Since the faces of a pyramid are triangles, you can find their areas by using $A=\frac{1}{2}bh$, where $b$ is the length of the base of the triangle and $h$ is the height of the triangle. The height, $h$, is the perpendicular distance from the base of the triangle to the vertex of the angle opposite that base.

Finally, you reviewed what a net is and how to recognize the net of a cube, rectangular prism, triangular prism, and square pyramid.

QUICK CHECK

Please go online to view and submit this assessment.

MORE TO EXPLORE

If you had difficulty with this question, try writing down the shape of each side and the number of each of those shapes in the pyramid. You may also want to review the material in this lesson.
LEARN

WARM-UP

Name the shape of the base of the following solids.

1. square pyramid
2. rectangular prism
3. cone
4. cylinder

WARM-UP ANSWERS

1 square 2 rectangle 3 circle 4 circle

TEACHING NOTES

INSTRUCTION

Read Identify Cross Sections of Solids on p. 126 in Math in Focus 2B. A cross section of a solid is formed when a plane slices through the three-dimensional figure, producing a two-dimensional figure.

Note that a plane that intersects only on a point on the exterior of a solid is called a tangent. A plane that is tangent to a solid does not yield a cross section.

Complete Hands-On Activity on p. 127. Notice that the shape of the cross section depends on how you make your cut. The angle at which you make your cut is the angle of the plane.
Read **Identify Cross Sections of a Square Pyramid** on p. 128. For a pyramid, if the cut is made parallel to the base, then the cross section will have the same shape as the base. However, the cross section will not be the same size because a pyramid gets narrower as you get closer to its vertex. Try it with clay models to see for yourself.

Review **Example 1** on pp. 128–129. Look for patterns as you examine the cross sections of solids. Identify the similarities and differences between the cross sections of pyramids and cones. Identify the similarities and differences between the cross sections of cylinders and prisms.

Discuss the **Think Math** questions on pp. 128–129 with your Learning Guide. Think about both the shape and the size of the cross sections.

Complete **Guided Practice** on p. 129. Identify each solid before naming the shape of the cross section.

---

**TEACHING NOTES**

**Textbook Answer Key**

Encourage your student to use modeling clay to make various solids and experiment with string to form cross sections. Slice through the clay to demonstrate what a cross section is and make sure your student discovers that any cross section of a sphere is a circle. Encourage your student to look for and find patterns in the cross sections.

Make sure your student understands the characteristics of the cross sections of a pyramid. Encourage your student to apply what she discovers about pyramids to cones. Have her use modeling clay to discover patterns.

**Looking Ahead**: Your student will need uncooked rice or beans for the next lesson part.

---

**PRACTICE**

Complete problems 11 and 14–16 of **Practice 8.1** on pp. 131–132 in *Math in Focus 2B*.

---

**WRAP-UP**

Today you learned about cross sections of spheres, square pyramids, and other solids. The shape of the cross section depends on the shape of the object and how you make the cut through the solid.

You learned that for a cylinder or prism, if a cross section is cut parallel to the base of the solid, the cross section formed is exactly the same size and shape as the base of the solid.
The cross section of a pyramid or cone can be different shapes. If the cut is made parallel to the base of the solid, then the cross section will have the same shape as the base, but it will be smaller than the base because pyramids and cones get narrower as you get closer to the vertex.

✅ QUICK CHECK

Please go online to view and submit this assessment.

معايير مراجعة السريعة

View the video Cross Sections to learn how solid shapes can be sliced.

Please go online to view this video ▶
Plane and Solid Figures - Part 4

Books & Materials
- Math in Focus - Teacher Edition

Assignments
- Complete Interactive Activity.
- Complete Rate Your Enthusiasm.

LEARN

INTERACTIVE ACTIVITY

Play this game, Geometry Slicer, to make and identify cross-sections of different solid shapes.

RATE YOUR ENTHUSIASM

Please go online to view and submit this assessment.
LEARN

WARM-UP

Name the figures with the following characteristics.

1. solid figure with identical bases and flat sides
2. solid figure with identical circular bases and curved sides
3. solid figure with a circular base and a vertex at the top

TEACHING NOTES

WARM-UP ANSWERS

1. prism  2. cylinder  3. cone

INSTRUCTION

Complete Hands-On Activity on pp. 140–141 in Math in Focus 2B. In this activity, you explore the volume relationships between a pyramid and a prism and between a cone and a cylinder. In step 4, you are using the pyramid as a measuring cup. Fill it with rice so it is level across the top. Then pour it into the prism.

Look for patterns as you explore the shapes and their volumes. As you make comparisons, keep in mind that the solids in each pair have the same height and base area.

Recall that the volume of a prism is \( V = Bh \), where \( B \) is the area of the base of the prism and \( h \) is the height of the prism. Recall that the volume of a cylinder is \( V = Bh \), where \( B \) is the area of the base of the cylinder and \( h \) is the height of the cylinder. You will use these volume formulas and the results of your experiment to write your own formula for the volume of a pyramid and the volume of a cone.
Textbook Answer Key

Make sure your student realizes that the height of the solids she is comparing is the same, as is the area of the bases of those solids.

Guide your student to see the relationship between the volume of a prism and the volume of a pyramid. The prism holds three times the amount of rice (or beans) as the pyramid with the same height and base. Also, make sure your student discovers a similar relationship between the volume of a cylinder and the volume of a cone. The cylinder holds three times the amount of rice (or beans) as the cone with the same height and base.

Complete both Math Journal problems on p. 141 in Math in Focus 2B.

Today you compared the volume of a pyramid to the volume of a prism with the same height and base. You also compared the volume of a cone to the volume of a cylinder with the same height and base.

Please go online to view and submit this assessment.
LEARN

WARM-UP

Name the shape formed by each cross section.

1. a slice through the center of a sphere
2. a slice through a cylinder that is parallel to the base of the cylinder
3. a slice through a triangular prism that is parallel to the base of the prism
4. a vertical slice through the vertex of a rectangular pyramid that is perpendicular to the base of the pyramid

TEACHING NOTES

WARM-UP ANSWERS

1. circle  2. circle  3. triangle  4. triangle

INSTRUCTION

Read and complete Brain @ Work on p. 168 in Math in Focus 2B. First, use modeling clay to make a cube. Next, use string to cut cross sections. You may use a computer drawing program or paper and pencil to draw your solutions.

TEACHING NOTES

Textbook Answer Key

Review the meaning of an isosceles triangle and regular hexagon with your student.
PRACTICE
Write a constructed response to explain how you found the answer to Brain @ Work on p. 168 in Math in Focus 2B. Be sure to explain the steps you used.

WRAP-UP
Today you found cross sections of a cube that formed different shapes—an isosceles triangle and a regular hexagon. Drawing three-dimensional pictures can be challenging, but it helps to communicate information about our three-dimensional world.

✅ PRACTICE QUESTIONS
Please go online to view and submit this assessment.
The diagram below shows a pyramid glued to the top of a cube. Given that the slant height of the pyramid is 5.9 centimeters,

Find the total surface area of the solid rounded to the nearest square centimeter.
The diagram below shows a pyramid glued to the top of a cube. Given that the slant height of the pyramid is 5.9 centimeters, find the total surface area of the solid rounded to the nearest square centimeter.

Did you:

- Use the information in the diagram to find the total surface area of the solid?
- Round your answer to the nearest square centimeter?
- Show your work?

USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information in the diagram to find the total surface area of the solid?
- Round your answer to the nearest square centimeter?
- Show your work?
Unit Quiz: Plane and Solid Figures

Books & Materials
- Math in Focus - Teacher Edition A
- Math in Focus - Teacher Edition B

☑️ UNIT QUIZ

Please go online to view and submit this assessment.
Today’s lesson covers important ideas you need to understand before beginning the lessons in this chapter. Read pp. 178–180 (middle) in *Math in Focus 2B*. You will review how to find the mean and median of a set of data.

The *mean* is also known as the average, although any measure of central tendency can be considered an average. To find the mean of a data set, find the sum of the data and then divide by the number of data values.
The median is the middle number or the mean of the middle numbers of an ordered data set. If the number of data is odd, the median is the middle number. If the number of data is even, the median is the mean of the two middle numbers. The mode is the number that occurs most frequently in a data set.

The mode is the number that occurs most frequently in a data set.

Remember that you can use the properties of addition to reorder and group the addends when finding the sum of the data. Because finding the mean requires dividing, the quotient may result in a repeating decimal. In this case, you may want to round the quotient to a specific place.

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the Quick Check sections.

Complete the Quick Check sections on pp. 179–180 in Math in Focus 2B.

Textbook Answer Key
Review your student’s answers to Quick Check, noting the problems that he answered incorrectly. Click on the link to access the appropriate Reteach activity that your student should complete for the remainder of this lesson.

Reteach
After your student completes the Quick Check in the Recall Prior Knowledge lesson of this chapter, review the questions that were answered incorrectly.

If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.

<table>
<thead>
<tr>
<th>Quick Check Question(s)</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–10</td>
<td>Mean, Median, Mode, and Range</td>
</tr>
<tr>
<td>11–12</td>
<td>Dot Plots and Frequency Tables</td>
</tr>
</tbody>
</table>
Today you reviewed finding the mean and median of a data set.

The number of people who attended the town meeting each month for 6 months is shown.

236, 188, 242, 282, 196, 158

Find the mean.

Step 1: Add the data.

\[ 236 + 188 + 242 + 282 + 196 + 158 = 1,302 \]

Step 2: Divide by the number of data.

\[ 1,302 \div 6 = 217 \]

A mean of 217 people attended the monthly meeting.

Find the median.

Step 1: Order the data from least to greatest.

158, 188, 196, 236, 242, 282

Step 2: Determine if the number of data is odd or even.

There are 6 data values, which is an even number.

Step 3: Find the mean of the two middle numbers.

\[ 196 + 236 = 432 \]

\[ 432 \div 2 = 216 \]

The median number of people that attended the monthly meeting is 216.

Please go online to view and submit this assessment.
Measures of Variation - Part 2

INTERACTIVE ACTIVITY

Follow these instructions for the activity shown below.

Click here to view the activity in a new window.

To begin, create a data set by dragging the gray dots to the line plot below. Drag dots to 1, 1, 2, 3, and 8. Check that Find mean is selected. Drag the balance point slider until the frogs are balanced. The value of the mean is 3. Click on “Calculate mean” to see how this can be found by calculating. All the values are added and then divided by the total number of values.

Now select Find median. Click the arrow buttons until the pans are balanced. The middle value of this data set is 2. Then select Find mode. Drag the bar down until it hits the highest stack of frogs. Since the highest stack of frogs is at 1, 1 is the mode of the data.

In your Math Notebook, find the mean, median, and mode of each data set. Use the Gizmo to check your work.

1. 1, 1, 2, 7, 9, 10
2. 1, 3, 6, 6, 8, 9, 9
3. 3, 3, 3, 4, 4, 4, 5, 5, 5

TEACHING NOTES

Answers:

1 mean: 5; median: 4.5; mode: 1  2 mean: 6; median: 6; mode: 6 and 9  3 mean: 4; median: 4; mode: none

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.
Quick Check

Please go online to view and submit this assessment.

More to Explore

View the video What Is the Range of a Set of Data? (02:36) to review range.

Please go online to view this video ▶
LEARN

WARM-UP

Subtract.

1. 267 − 188
2. 342 − 173
3. 58.7 − 36.9
4. 6.45 − 2.78

WARM-UP ANSWERS

1. 79
2. 169
3. 21.8
4. 3.67

INSTRUCTION

Read the instructional section Measures of Variation on p. 182 in Math in Focus 2B. Describe how different the extreme values in a data set are. The reason Team B has the greater variation is that the difference between the tallest and shortest players is 12 inches, while the difference between the tallest and shortest players on Team A is only 5 inches.

Read Understand Range on p. 183. Range is the difference between the two extremes of a data set, or the least value subtracted from the greatest value. To find the range of a data set, identify the greatest and least values and then subtract.

Review Example 1 on p. 183. Then complete Guided Practice on p. 184.
Some students may want to order all 12 numbers in Guided Practice. It is not necessary to order all of them, but if your student has trouble keeping track of the numbers, have him write them down and cross out the intermediate numbers until he is left with the most extreme values. Remind your student to align the decimal points when comparing decimals. Then instruct him to compare from left to right as he would with whole numbers.

Complete problems 1–4 of Practice 9.1 on p. 190 in Math in Focus 2B.

Measures of variation describe the distribution of data within a set. One measure of variation is the range. The range of a data set measures the difference between the greatest and least numbers.
The following table shows the normal monthly precipitation in New Orleans.

<table>
<thead>
<tr>
<th>Month</th>
<th>Precipitation (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>5.2</td>
</tr>
<tr>
<td>February</td>
<td>5.3</td>
</tr>
<tr>
<td>March</td>
<td>4.6</td>
</tr>
<tr>
<td>April</td>
<td>4.6</td>
</tr>
<tr>
<td>May</td>
<td>4.6</td>
</tr>
<tr>
<td>June</td>
<td>8.1</td>
</tr>
<tr>
<td>July</td>
<td>5.9</td>
</tr>
<tr>
<td>August</td>
<td>6.0</td>
</tr>
<tr>
<td>September</td>
<td>5.1</td>
</tr>
<tr>
<td>October</td>
<td>3.6</td>
</tr>
<tr>
<td>November</td>
<td>4.5</td>
</tr>
<tr>
<td>December</td>
<td>5.3</td>
</tr>
</tbody>
</table>

To find the range of the monthly precipitation, follow these steps.

Step 1: Identify the greatest value.

The greatest value is 8.1.

Step 2: Identify the least value.

The least value is 3.6.

Step 3: Subtract the least value from the greatest value.

\[ 8.1 - 3.6 = 4.5 \]

The range of the monthly precipitation in New Orleans is 4.5 inches. In other words, the wettest month had 4.5 inches more precipitation than the driest month.

Please go online to view and submit this assessment.
LEARN

WARM-UP

Find the median of each data set.

1. 35, 67, 42, 58, 41, 60, 72
2. 27, 19, 36, 28, 31, 42, 40, 30
3. 88, 78, 76, 58, 64, 70, 80, 92

TEACHING NOTES

WARM-UP ANSWERS

1. 58
2. 30.5
3. 77

INSTRUCTION

Read Understand Quartiles on pp. 184–185 in Math in Focus 2B. Before you can find the quartiles of a data set, you need to first find the median. Recall that the median is the middle number, or the mean of the middle numbers, in an ordered data set.

The quartiles are known as the first (or lower) quartile, the second quartile (the median), and the third (or upper) quartile. To find the quartiles in an ordered, odd-numbered data set, you do not count the median when considering the quartiles. For example, if there are 7 data points, the lower quartile is the median of the first 3 data points and the upper quartile is the median of the last 3 data points.

Note that sometimes the word quartile refers to a single data point, but other times it refers to the range of data within a certain quartile.
Review **Example 2** on p. 186. In this case, the median (132 lb) is not one of the data points. No one at the health screening weighed exactly 132 lb. However, half the people weighed less than 132 lb, and the other half weighed more than 132 lb.

Complete **Guided Practice** on p. 186. To determine the middle values of an even-numbered data set, add 1 to the number of data points and divide by 2. This will give you a decimal that ends in .5. The whole numbers on either side of the decimal are the middle data points. For example, if there are 16 data points, add 1 to get 17. Then divide by 2 to get 8.5. The two middle numbers are the eighth and ninth values.

---

**TEACHING NOTES**

**Textbook Answer Key**

A helpful analogy may be to think of the first quartile as the end of the first quarter of a football or basketball game, the median as halftime, and the third quartile as the end of the third quarter. The terms *first quartile* and *lower quartile* are interchangeable, as are *third quartile* and *upper quartile*. Your student should be consistent when choosing terms to use, to avoid using *first* and *upper* or *lower* and *third* together.

---

**PRACTICE**

Complete problems 5–8 of **Practice 9.1** on p. 190 in *Math in Focus 2B*.

---

**WRAP-UP**

Today you learned how to find the first and third quartiles in a data set. If there is an odd number of data points, exclude the median when determining the quartiles. If there is an even number of data points, divide the data in half. The *first quartile* is the median of the lower half of the data, and the *third quartile* is the median of the upper half of the data.

Find the first quartile, median, and third quartile of the data set.

87, 68, 64, 58, 72, 60, 60, 42

**Step 1**: Order the data from least to greatest.

42, 58, 60, 60, 64, 68, 72, 87
Step 2: Determine if the number of data is odd or even.

There are 8 data points, an even number. (Note that 60 occurs twice. That counts as 2 data points.)

Step 3: Find the median.

The median is the mean of the fourth and fifth values.

60 + 64 = 124, and 124 ÷ 2 = 62

The median is 62.

Step 4: Find the first quartile.

The first quartile is the median of the first 4 numbers: 42, 58, 60, 60

58 + 60 = 118 and 118 ÷ 2 = 59

The first quartile is 59.

Step 5: Find the third quartile.

The third quartile is the median of the last 4 numbers: 64, 68, 72, 87

68 + 72 = 140 and 140 ÷ 2 = 70

The third quartile is 70.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
LEARN

WARM-UP

Use the following data set to complete each problem.

80, 64, 96, 88, 76, 92, 90, 52

1. Find Q₂.
2. Find Q₁.
3. Find Q₃.
4. Find the range.

TEACHING NOTES

WARM-UP ANSWERS

1. 84
2. 70
3. 91
4. 44

INSTRUCTION

Read Understand Interquartile Range on p. 187 in Math in Focus 2B. The interquartile range is the difference between the third and first quartiles. The interquartile range (often abbreviated as IQR) describes the variation in the middle half of a data set. Recall that the range measures the variation of the entire data set.

Review Example 3 on p. 187. Remember that in an ordered data set with an odd number of values, the median is excluded from the values when finding the quartiles. In Example 3, there are 15 data points.
The first quartile is the median of the first 7 values, and the third quartile is the median of the last 7 values. The eighth value is the median, or $Q_2$.

Complete **Guided Practice** on p. 188. One way to order the data is to break the numbers into smaller groups. For example, for problem 3, order the digits with 1 in the tens place first, then move on to the numbers with 2 in the tens place, and so on.

Review **Example 4** on p. 188. While there are only 4 ticket prices, there are 22 data points because there were 22 tickets sold.

Complete **Guided Practice** on p. 189. Remember that the interquartile range is the third quartile minus the first quartile, or

\[ IQR = Q_3 - Q_1 \]

Complete **Technology Activity** on p. 189. For steps 1 and 5, you can choose to use the names of 10 people or places of your choice. For example, you can use the names of 10 family members, states, presidents, or entertainers. If you do not have access to spreadsheet software, you can perform the calculations by hand or by using a calculator.

### TEACHING NOTES

When solving problems involving quartiles, your student should be in the habit of first determining whether the median is being asked for or not. Finding the median will allow him to break the data into halves, making it more likely that the quartiles are correctly represented.

### PRACTICE

Complete problems 9–23 of **Practice 9.1** on pp. 190–192 in **Math in Focus 2B**.

### WRAP-UP

Today you learned how to find the interquartile range (IQR) of a data set.

\[ \text{Interquartile range} = \text{third quartile} - \text{first quartile} \]

To find the interquartile range, it is first necessary to find the quartiles.

74, 68, 50, 79, 84, 92, 85, 96, 78, 86, 92
Step 1: Order the data from least to greatest.

50, 68, 74, 78, 79, 84, 85, 86, 92, 92, 96

Step 2: Determine if the number of data is odd or even.

There are 11 data points, an odd number.

Step 3: Find the median.

The median is the sixth value, which is 84. (There are 5 data points above 84 and 5 below 84.)

Step 4: Find the first quartile.

The median of the first 5 numbers is the third value, 74.

The first quartile is 74.

Step 5: Find the third quartile.

The median of the last 5 numbers is the third value, 92.

The third quartile is 92.

Step 6: Find the interquartile range.

92 − 74 = 18; the interquartile range is 18.

Quick Check
Please go online to view and submit this assessment.

More to explore
If you struggled with this question, view the Instructional Video, *Measures of Variation.*
LEARN

WARM-UP

Subtract.

1. 48 − 65
2. 81 − 94
3. 57.4 − 68.2
4. 62.5 − 71.7

WARM-UP ANSWERS

1. −17
2. −13
3. −10.8
4. −9.2

TEACHING NOTES

INSTRUCTION

Read Understand Mean Absolute Deviation on pp. 205–206 in Math in Focus 2B. Mean absolute deviation (MAD) is the mean of the difference between the mean and each value in a data set. To find the mean absolute deviation, first determine the mean of the data set. Then for the values that are less than the mean, subtract them from the mean. For the values that are greater than the mean, subtract the mean from them. Find the sum of the differences and divide by the number of data values. If a value is equal to the mean, the deviation is 0.

Discuss the Think Math question on p. 206 with your Learning Guide. Because mean absolute deviation is based on the mean's distance from each data value, you do not have to order the data set to find it.
Review Example 11 on pp. 207–208. Then complete Guided Practice on p. 208. In problem 3, there are 10 data points, so the mean absolute deviation value will result in a whole number or a terminating decimal. If there are different numbers of data points, the mean absolute value can result in a repeating decimal. Should the decimal repeat, you may want to round to a specific place such as tenths or hundredths.

In Example 11 and Guided Practice, the data sets do not have outliers. Ask your student to explain what would happen to the mean absolute deviation if there were an outlier. He should be able to tell you that the MAD would increase, no matter whether the outlier was the minimum or maximum value.

Complete problems 9–15 of Practice 9.3 on pp. 210–211 in Math in Focus 2B.

Today you learned how to find the mean absolute deviation of a data set. **Mean absolute deviation** is the mean of the differences between the mean and each data value in a set. Mean absolute deviation is always a positive number.

Look at the following data set. Find the mean absolute deviation.

72, 64, 58, 81, 75, 62, 80, 76, 59, 67

**Step 1:** Find the mean of the data.

\[
72 + 64 + 58 + 81 + 75 + 62 + 80 + 76 + 59 + 67 = 694 \\
694 ÷ 10 = 69.4
\]

The mean is 69.4.
Step 2: Subtract the mean from each data value. Find the absolute value of the differences.

<table>
<thead>
<tr>
<th>Data Item</th>
<th>Subtract</th>
<th>Absolute Value of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>$72 - 69.4 = 2.6$</td>
<td>2.6</td>
</tr>
<tr>
<td>64</td>
<td>$64 - 69.4 = (-5.4)$</td>
<td>5.4</td>
</tr>
<tr>
<td>58</td>
<td>$58 - 69.4 = (-11.4)$</td>
<td>11.4</td>
</tr>
<tr>
<td>81</td>
<td>$81 - 69.4 = 11.6$</td>
<td>11.6</td>
</tr>
<tr>
<td>75</td>
<td>$75 - 69.4 = 5.6$</td>
<td>5.6</td>
</tr>
<tr>
<td>62</td>
<td>$62 - 69.4 = (-7.4)$</td>
<td>7.4</td>
</tr>
<tr>
<td>80</td>
<td>$80 - 69.4 = 10.6$</td>
<td>10.6</td>
</tr>
<tr>
<td>76</td>
<td>$76 - 69.4 = 6.6$</td>
<td>6.6</td>
</tr>
<tr>
<td>59</td>
<td>$59 - 69.4 = (-10.4)$</td>
<td>10.4</td>
</tr>
<tr>
<td>67</td>
<td>$67 - 69.4 = (-2.4)$</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Step 3: Find the sum of the absolute values.

$$2.6 + 5.4 + 11.4 + 11.6 + 5.6 + 7.4 + 10.6 + 6.6 + 0.4 + 2.4 = 74$$

Step 4: Divide the sum of the absolute values by the number of data values.

$$74 ÷ 10 = 7.4$$

The mean absolute deviation is 7.4.

Quick Check:

Please go online to view and submit this assessment.

More to Explore:

If you answered incorrectly, make sure that you are following these steps: 1. Find the mean. 2. Find the distance from each value to the mean by using subtraction. 3. Find the mean of those deviations. If you are still having difficulty, revisit the material in this lesson.
LEARN

WARM-UP

Find the mean absolute deviation for each data set.

1. 36, 52, 48, 60, 44
2. 58, 75, 67, 92, 81, 70, 54, 63
3. 1.7, 4.1, 3.5, 2.8, 2.4
4. 2.8, 4, 3.6, 3.2, 2.4, 4.2, 3, 2.4

TEACHING NOTES

WARM-UP ANSWERS

1. 6.4
2. 9.5
3. 0.72
4. 0.55

INSTRUCTION

Complete Technology Activity on p. 209 in Math in Focus 2B. For most spreadsheet software, the formula to enter in step 3 is =AVERAGE(firstcell:lastcell), or in the example on p. 209, =AVERAGE(A1:J1). Note that the equal sign is part of the formula you enter. That is what tells the spreadsheet to perform a calculation.

In step 5, the formula is usually =AVEDEV(firstcell:lastcell), or in this example, =AVEDEV(A1:J1).

If you do not have spreadsheet software, follow these steps instead of what is on p. 209. Use your calculator. (Some calculators have functions that allow you to find an average. You can look in the manual to see if yours has that capability.)
Step 1: Pick 10 whole numbers as your data.

Step 2: Find the mean of your data.

Step 3: Find the absolute value of the difference between the mean and each data value.

Step 4: Find the mean absolute deviation of the data values.

Step 5: Explain what the MAD tells you about the data.

Step 6: Pick 10 different whole numbers and repeat steps 1 to 5.

TEACHING NOTES

Textbook Answer Key

Your student will have a greater mean absolute value if the data values are spread out. If the data values are clustered, the mean absolute value will be less.

PRACTICE

Complete problems 16–23 of Practice 9.3 on p. 211 in Math in Focus 2B.

WRAP-UP

Today you used technology to help you compare the mean absolute deviations for two sets of data.

Look at the following two data sets.

Data Set 1: 72, 65, 60, 57, 86

Data Set 2: 81, 39, 54, 76, 90

Both data sets have a mean of 68. Identify the data set that has the greater MAD. By how much do the MADs differ?
**Step 1:** Subtract the mean from each data value. Find the absolute value of the differences. (Do this for both sets; only the calculations for Data Set 1 are shown in here.)

**Data Set 1**

<table>
<thead>
<tr>
<th>Data Item</th>
<th>Subtract</th>
<th>Absolute Value of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>72 − 68 = 4</td>
<td>4</td>
</tr>
<tr>
<td>65</td>
<td>65 − 68 = (-3)</td>
<td>3</td>
</tr>
<tr>
<td>60</td>
<td>60 − 68 = (-8)</td>
<td>8</td>
</tr>
<tr>
<td>57</td>
<td>57 − 68 = (-11)</td>
<td>11</td>
</tr>
<tr>
<td>86</td>
<td>86 − 68 = 18</td>
<td>18</td>
</tr>
</tbody>
</table>

**Step 2:** Find the mean absolute deviations for both sets.

- **Data Set 1:** \(4 + 3 + 8 + 11 + 18 = 44\) and \(44 ÷ 5 = 8.8\)
- **Data Set 2:** \(13 + 29 + 14 + 8 + 22 = 86\) and \(86 ÷ 5 = 17.2\)

**Step 3:** Find the difference of the MADs.

\[17.2 − 8.8 = 8.4\]

Data Set 2 has a greater MAD by a margin of 8.4.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
The tables show Finn's history test scores and science test scores.

**History Test Scores**

| 78 | 65 | 72 | 85 |
| 68 | 70 | 80 | 82 |

**Science Test Scores**

| 78 | 45 | 80 | 88 |
| 80 | 78 | 62 | x |

**A)** Calculate the mean and the mean absolute deviation of Finn's history test scores. Give each answer correct to 2 decimal places.

**B)** The score $x$ represents the score of the science test Finn has yet to take. If Finn wants to have the same mean score in science as in history, what score must he achieve on his last science test?

**C)** Including the score of the last science test from part **B)**, calculate the mean absolute deviation of the science test scores and compare it with the mean absolute deviation of the history test scores. What conclusion can be drawn about Finn's test scores?
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information in the tables shown to answer ALL the questions?
- Calculate the mean and the mean absolute deviation of Finn's history test scores, correct to 2 decimal places?
- Solve for $x$ to show what score Finn must achieve on his last science test if he wants to have the same mean score in science as in history?
- Calculate the mean absolute deviation of the science test scores, including the value of $x$ from part B?
- Compare the mean absolute deviation of the science test scores with the mean absolute deviation of the history test scores?
- Write a conclusion that could be drawn about Finn's test scores?
- Show your work?
### Random Samples - Part 1

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Books &amp; Materials</th>
<th>Assignments</th>
</tr>
</thead>
</table>
| • Describe the purpose and processes of random sampling. | • Math in Focus 2B  
• Math in Focus - Teacher Edition B  
• container  
• random number table (Optional) | • Complete Warm-up.  
• Read and complete Math in Focus 2B.  
• Complete Practice Questions. |

## LEARN

**WARM-UP**

Use the following data set to complete each the problem.

89, 95, 71, 67, 98, 78

1. Order the numbers from least to greatest.
2. Find the mean.
3. Find the mean absolute deviation.

### WARM-UP ANSWERS

1. 67, 71, 78, 89, 95, 98
2. 83
3. 11

### TEACHING NOTES

**WARM-UP ANSWERS**

1. 67, 71, 78, 89, 95, 98
2. 83
3. 11

### INSTRUCTION

Read **Understand the Concepts of a Population and Sample** on p. 212 in *Math in Focus B*. A population is a group about which information is desired. Because collecting data from every member of a population is not always practical, you can collect it from part of the population, or a sample. The number of members in a sample is called a sample size.

Read **Understand the Purpose of Random Sampling** on p. 212–213. When doing research, you need to make sure you collect a random sample. This will provide an unbiased sample that is representative of the population. A biased sample will skew your results. For instance, if you asked people in an ice cream parlor what their favorite dessert is, more of them are likely to say ice cream is than if you asked people in a bakery.
Read Understand Simple Random Sampling on p. 213–214. In each case, the sample is unbiased because each person has the same chance of being picked.

Discuss the Think Math question on p. 213 with your Learning Guide. Be sure to consider the requirements that in a random sample, every member of the population should have an equal chance of being selected, and that sample members must be selected independently of each other.

Complete Hands-On Activity on p. 215 with modifications from your Learning Guide.

HELPFUL ONLINE RESOURCES

Random Integer Generator

Today you learned how to collect data from a sample and the difference between biased and unbiased, or random, samples.

PRACTICE

Complete problems 1–4 of Practice 9.4 on p. 220 in Math in Focus 2B.

WRAP-UP

Today you learned how to collect data from a sample and the difference between biased and unbiased, or random, samples.
A random sample gives every member of the population an equal chance of being selected. Three types of simple random samples are as follows.

- A sampling frame uses a database from which you can randomly pick names from the population.
- A lottery method assigns the members of a population numbers that are randomly selected.
- A random number table assigns numbers to members of a population. The difference between a random number table and a lottery method is the numbers in a random number table are preassigned. Because the numbers are in no particular order, they are random. (A computer can also be used to assign random numbers.)

✅ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Random Samples - Part 2

**Objectives**
- Describe the purpose and processes of stratified random sampling and systematic random sampling.
- Determine the appropriate random sampling method for a given situation.

**Books & Materials**
- Math in Focus 2B
- Math in Focus - Teacher Edition B

**Assignments**
- Complete Warm-up.
- Read and complete Math in Focus 2B.
- Complete Quick Check.

---

**LEARN**

**WARM-UP**

Answer the following questions.

1. Robert interviewed 5 friends who own dogs to determine the most popular pet in his neighborhood. Did Robert use a random sample?
2. Without looking, Jessica removed 15 marbles out of a jar of 100 marbles to determine the color that appears the most. Is this a random sample?

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. no
2. yes

**INSTRUCTION**

Read Understand Stratified Random Sampling on pp. 216–217 in Math in Focus 2B. To make a stratified random sample, divide a population into nonoverlapping groups based on common characteristics. For example, three nonoverlapping age groups are infants, toddlers, and teenagers.

Read Understand Systematic Random Sampling on p. 217. To make a systematic random sample, randomly select a member from a population and then choose subsequent members at regular intervals. Make sure your intervals are consistent throughout the population.

Review Example 12 on p. 218. It is important to understand the three random sampling methods you have learned and why one method may be more appropriate for a given situation than the others.
Complete **Guided Practice** on p. 219. In some situations, it is possible that a combination of methods may be used to obtain a better representative sample.

You may complete the optional **Hands-On Activity** on p. 219. This activity will help you understand which sampling method is appropriate for different types of situations.

---

**TEACHING NOTES**

**Textbook Answer Key**

If your student struggles with the difference between stratified random sampling and systematic random sampling, remind him to look closely at their names: stratified has the root word stratum, which your student may be familiar with from science courses, meaning “layers.” Stratified random sampling uses layers of random sampling from subsets of a population. Systematic has the word system, which means “order.” Systematic random sampling involves counting and taking samples at regular, orderly intervals.

---

**PRACTICE**

Complete problems 5–18 of **Practice 9.4** on pp. 220–221 in *Math in Focus 2B*.

---

**WRAP-UP**

Today you learned about different types of sampling methods and why one method may be more appropriate in a given situation than another.

*Stratified random sampling* is a sampling method in which the population is divided into nonoverlapping groups from which members are randomly selected. The following is an example.

An ice-cream company owner with 4 stores in the state wants to survey his customers. He divides the customers by the 4 stores they patronize, and then he randomly selects 25 customers from each store. He surveys the customers about their favorite flavors.

*Systematic random sampling* is a sampling method in which the first member is randomly selected, and then the other members are chosen at regular intervals. The following is an example.

At a town meeting, the mayor randomly picked a resident and then selected every ninth resident thereafter.
Please go online to view and submit this assessment.

If you answered incorrectly, make a list of the methods with an example of each to review. This will allow you to have a resource to understand each method. You may also want to revisit Random Samples, Part 2.
Random Samples - Part 3

Objectives
- Make inferences about a population based on a random sample.

Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition B

Assignments
- Complete Warm-up.
- Read and complete Math in Focus 2B.
- Complete Practice Questions.

LEARN

WARM-UP
Find the mean.

1. 87, 65, 48, 57, 72
2. 97, 112, 62, 73, 81, 96, 68, 83

WARM-UP ANSWERS

1. 65.8
2. 84

INSTRUCTION

Have you ever wondered how news outlets can predict a winner in an election before the voting has been completed? It is because they have made inferences about a population.

Read p. 222 in Math in Focus 2B. An inference is an educated prediction. Remember that a random sample is one in which every member of the population has an equal chance to be selected.

Read p. 223. Because the population is often too large to study, samples are used. Good random samples of a population will have approximately the same mean. The mean of the sample means should be close to the actual mean of a population.

Review Example 13 on pp. 224–225. Remember that the mean absolute deviation (MAD) is the mean of the distances that each data value is from the mean. Because distance is only measured with positive numbers, you can subtract one number from the other and use the absolute value of the differences.
Complete **Guided Practice** on p. 225. Your dot plot should use equal intervals on the number line. Look at the range of the data before choosing your intervals. In problem 1, the range is 13, which means you could use 1 or 2 as the intervals for the dot plot.

Complete **Hands-On Activity** on p. 226.

---

**TEACHING NOTES**

In **Hands-On Activity**, make sure your student uses a random sample. One way to do this is to decide, before collecting, how the data will be collected. Among the possibilities for random samples are taking the first word from 20 pages (and repeating to obtain five random samples) or picking 20 consecutive words from the first full paragraph from 5 randomly selected pages.

---

**PRACTICE**

Complete problems 1–5 and 12–15 of **Practice 9.5** on pp. 232–233 in **Math in Focus 2B**.

---

**WRAP-UP**

Today you used random samples of a population and made inferences about that population using mean absolute deviation and the ratio of mean absolute deviation to the mean. The greater the ratio of mean absolute deviation to the mean (MAD/mean), the more the data is spread out.

A population consists of 50 scores. Ten scores have been randomly collected from the population.

15, 9, 12, 17, 16, 10, 12, 15, 14, 16

**Step 1**: Find the mean.

\[
15 + 9 + 12 + 17 + 16 + 10 + 12 + 15 + 14 + 16 = 136
\]

\[
136 \div 10 = 13.6
\]

The mean is 13.6.
**Step 2:** Find the MAD. First find the distances from 13.6 to each data value. Then use the absolute value of each difference to find the sum, and then divide to find the MAD.

\[
1.4 + 4.6 + 1.6 + 3.4 + 2.4 + 3.6 + 1.6 + 1.4 + 0.4 + 2.4 = 22.8
\]

\[
22.8 \div 10 = 2.28
\]

The mean absolute deviation is 2.28.

**Step 3:** Find MAD/mean. Round to the nearest percent.

\[
2.28 \div 13.6 \approx 0.168
\]

The MAD/mean is about 17%.

✅ **PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Random Samples - Part 4

Objectives
- Make comparative inferences about two populations based on random samples.

Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition B

Assignments
- Complete Warm-up.
- Read and complete Math in Focus 2B.
- Complete Quick Check.

WARM-UP
Find the mean absolute deviation. Round your answers to the nearest hundredth.

1. 128, 237, 172, 186, 138
2. 238, 227, 242, 218, 232, 246, 229, 250

WARM-UP ANSWERS
1. 31.44
2. 8.75

TEACHING NOTES

INSTRUCTION
Read Make Comparative Inferences About Two Populations on p. 227 in Math in Focus B. Recall that measures of variation include range and interquartile range.

Review Example 14 on p. 227. From left to right, a box plot shows the minimum value, first quartile, median, third quartile, and maximum value. Then complete Guided Practice on p. 228. The interquartile range can be thought of as the length of the box. To find the difference of the quartiles, subtract the first quartile from the median and the median from the third quartile.

Review Example 15 on pp. 228–229. A box plot clearly shows the range and the middle 50% of a data set. The range of a box plot is the difference between the extremes. The middle 50% of a data set is the data between the first and third quartiles.

Complete Guided Practice on p. 229. When you are asked about how a data set is spread out, you are being asked about the range.
Review Example 16 on pp. 229–231. There are many ways to calculate equivalent means. As you can see, one data set is fairly clustered and the other has a greater distribution.

Complete Guided Practice on p. 231. Your dot plots should use number lines with the same intervals.

HELPFUL ONLINE RESOURCE
Instructional Video: *Box and Whisker Plots*

---

**TEACHING NOTES**

*Textbook Answer Key*

A data set with outliers will have a greater MAD than a data set with the same mean that does not have outliers.

---

**PRACTICE**


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**WRAP-UP**

Today you used box plots to compare two populations. To find the difference between quartiles, subtract the first quartile from the median and subtract the median from the third quartile.

A box plot can also be used to compare the ranges and interquartile ranges of two data sets.

![Box Plots](image)

Sample 1 has a range of 45 and Sample 2 has a range of 55.

The interquartile range of both samples is 20.

The difference of the median and first quartile in each sample is 10.

The difference of the third quartile and median in each sample is 10.
Two data sets can have the same mean, but different mean absolute deviations.

**Sample 3**: 6, 8, 10, 12, 14
The mean of Sample 3 is 10.
The MAD of Sample 3 is 2.4.

**Sample 4**: 8, 9, 9, 11, 13
The mean of Sample 4 is 10.
The MAD of Sample 4 is 1.6.

The dot plot shows that Sample 3 has a greater distribution than Sample 4.

---

**QUICK CHECK**

Please go online to view and submit this assessment.

---

**MORE TO EXPLORE**

View the video, *Quartiles, Boxplots, and Outliers*, to review this material.

Please go online to view this video ▶
Use the following data set to complete each problem.

93, 74, 68, 52, 76, 84

1. Find the mean.
2. Find the median.
3. If you were to make a stem-and-leaf plot, how many stems would there be?

WARM-UP ANSWERS

1. 74.5
2. 75
3. 5

A double stem-and-leaf plot is used to compare two sets of related data.

Review Example 7 on p. 197 in Math in Focus 2B. As you can see, the leaves in the left side of a double stem-and-leaf plot are organized from greatest to least. The key shows how the double stem-and-leaf plot reads for both halves of the plot. It might help to think of it like a mirror image, or a tree reflected in the water.

Complete Guided Practice on p. 198. Remember that the leaves on the left side of a double stem-and-leaf plot are ordered from greatest to least.
Review Example 8 on pp. 198–199. This stem-and-leaf plot contains a large gap. Like a dot plot, the data will be skewed. Because the outliers are the maximum values, the mean will be greater than the median. An outlier greatly affects the range and may or may not greatly affect the mean, depending on the number of data points. An outlier will not affect the median or the interquartile range any more than any other value.

Complete Guided Practice on p. 199. This stem-and-leaf plot has a large gap, but this time the outlier is the minimum value. This means that the mean will be less than the median.

TEACHING NOTES

Textbook Answer Key

If your student struggles with using a double stem-and-leaf plot, he can make two stem-and-leaf plots. The leaves of the left side of the double stem-and-leaf plot would have to be reversed.

PRACTICE

Complete problems 15–18 of Practice 9.2 on p. 201 in Math in Focus 2B.

WRAP-UP

A double stem-and-leaf plot is used to represent two sets of related data. The left side of a double stem-and-leaf plot arranges the leaves from greatest to least. The key of a double stem-and-leaf plot shows how each side arranges its numbers.

The following double stem-and-leaf plot represents the number of points that a football team scored and allowed in its games this season.

<table>
<thead>
<tr>
<th>Points Scored and Allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scored</strong></td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>7 7 4</td>
</tr>
<tr>
<td>8 7 4 1</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>9 2</td>
</tr>
</tbody>
</table>

Scored: 4 1 represents 14
Allowed: 1 0 represents 10
A double stem-and-leaf plot can be used to find and compare measures of central tendency and measures of variation.

Stem-and-leaf plots and double stem-and-leaf plots are useful displays for identifying outliers. An outlier is a number or numbers that are greatly different from the majority of the data. When the outlier or outliers are the minimum values, the mean will be less than the median. When the outlier or outliers are the maximum values, the mean will be greater than the median.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6 8</td>
</tr>
<tr>
<td>3</td>
<td>2 7 7 7</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1 4</td>
</tr>
</tbody>
</table>

2|6 represents 26

In this stem-and-leaf plot, the outliers are 61 and 64. In this case, the mean will be greater than the median.

✔️ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Objectives
- Make comparative inferences about two populations based on random samples.

Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition B

Assignments
- Complete Warm-up.
- Read and complete Math in Focus 2B.
- Complete the Practice activity.
- Complete Practice Questions.

LEARN

WARM-UP
Find the MAD/mean ratio to the nearest percent.

1. 24, 28, 32, 35, 48
2. 36, 41, 45, 46, 50
3. 45, 68, 72, 76, 80
4. 38, 52, 61, 78, 94

WARM-UP ANSWERS
1. 19%
2. 9%
3. 14%
4. 27%

TEACHING NOTES

INSTRUCTION
Read Brain @ Work on p. 234 in Math in Focus 2B.

For part a, remember that the mean is the sum of the data values divided by the number of data values.

For part b, the mean absolute deviation is found by finding the sum of the absolute values of the differences between the mean and each data value and dividing by the number of data values.

For part c, look closely at the bar graph before deciding whether you agree or disagree with Alex's statement. Then write a constructed response to explain why Alex is correct or incorrect.
The bar graph shown on p. 234 is an example of a misleading graph. The broken scale in the bar graph makes the differences in the data appear greater than they really are. In this bar graph, it appears as if the May rainfall was 6 times greater than the July rainfall, when the reality is that May had only about 6.3% more rainfall than July.

If your student calculated correctly, he found that the mean absolute deviation is 2.8 and the MAD/mean is about 1.7%, indicating that the monthly rainfall is fairly consistent.

You can discuss with your student how graphs can be constructed to be misleading at a glance; however, a closer inspection will reveal reality.

Complete the following problems using the bar graph.

The bar graph shows the average monthly temperatures from April to September in Chicago.

1. Find the mean monthly temperature for the six months shown.
2. Calculate the mean absolute deviation.
3. Calculate MAD/mean. Round to the nearest percent.
4. Explain the spread of the data.
Your student should be able to complete problems 1–3 from his work in the chapter.

You can discuss with him that the average temperatures include both daytime and nighttime temperatures. These temperatures are not average high temperatures. You can extend this concept by asking him what he thinks would happen to the mean, the mean absolute deviation, and the MAD/mean if temperatures for all 12 months were included.

**WRAP-UP**

Today you learned how to solve multistep problems involving mean absolute deviation and MAD/mean.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Mr. O'Lear is conducting an experiment that involves 10 plants. He initially determined that the plants had a mean height of 72 centimeters and a mean absolute deviation of 8.5 centimeters. He later discovered that he made two errors in the height measurements of two plants. The table shows the incorrect and the correct measurements of the two plants.

<table>
<thead>
<tr>
<th>Incorrect Height</th>
<th>Correct Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>65 cm</td>
<td>70 cm</td>
</tr>
<tr>
<td>78 cm</td>
<td>73 cm</td>
</tr>
</tbody>
</table>

A) What is the correct mean height of the 10 plants?

B) What is the correct mean absolute deviation of the 10 plants?

C) Upload your work to show how you got the answer.
Did you:

- Use the information given to calculate the correct mean height of the 10 plants?
- Use the information given to calculate the correct mean absolute deviation of the 10 plants?
- Upload your work to show how you found the answer?
Probability - Part 1

LEARN

WARM-UP

Write each fraction in simplest form.

1. \( \frac{12}{18} \)
2. \( \frac{15}{21} \)
3. \( \frac{9}{24} \)

WARM-UP ANSWERS

1. \( \frac{2}{3} \)  
2. \( \frac{5}{7} \)  
3. \( \frac{3}{8} \)

TEACHING NOTES

INSTRUCTION

Today's lesson covers important ideas you need to understand before beginning the lessons in this chapter. Read pp. 240–243 in Math in Focus 2B. You will convert among fractions, percents, and decimals.

Percent means per 100. If you rewrite a percent as a fraction, the denominator is 100.

9% means 9 per 100, which is 9/100 in fraction form.
When converting a fraction to a percent, if the denominator is not a factor of 100, you can always divide the numerator by the denominator to find the percent.

When the fraction leads to a repeating decimal or percent, round to a specific place. To round, look at the digit to the right of the place that you are rounding. If the digit is 5 or greater, round up. Otherwise, round down.

To convert a percent to a decimal, divide the percent by 100 and remove the percent sign. For example, 9% = 0.09. This is equivalent to the fraction form; they are both nine hundredths.

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the Quick Check sections.

Complete the **Quick Check** sections on pp. 241–243 in *Math in Focus 2B*.

Review your student’s answers to the **Quick Check** sections, noting the problems that she answered incorrectly. Click on the link to access the appropriate **Reteach** activity that your student should complete for the remainder of this lesson.

**RETEACH**

After your student completes the **Quick Check** in the **Recall Prior Knowledge** lesson of this chapter, review the questions that were answered incorrectly.

If, after the review, you feel that your student needs additional exposure to any of these skills, click on the title below that corresponds with the number of the incorrectly answered question(s) and have him complete the activity.
Note that this chapter opener spans two lessons.

<table>
<thead>
<tr>
<th>Quick Check</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question(s)</td>
<td></td>
</tr>
<tr>
<td>1–3</td>
<td>There is no review activity for this skill because it is based on the Singapore Math Method and will be taught in this chapter.</td>
</tr>
<tr>
<td>4–12</td>
<td>Converting Fractions and Decimals to Percents</td>
</tr>
<tr>
<td>15–16</td>
<td>There is no review activity for this skill because it is based upon the Singapore Math Method and will be taught in this chapter.</td>
</tr>
</tbody>
</table>

**WRAP-UP**

Today you reviewed converting fractions to percents.

Write $\frac{18}{30}$ as a percent. Write $\frac{5}{6}$ as a percent. Round to 2 places if necessary.

\[
\frac{18}{30} \div \frac{6}{6} = \frac{3}{5}
\]

\[
\frac{5}{6} = \frac{5}{6} \cdot 100\%
\]

\[
\frac{3}{5} \cdot \frac{20}{20} = \frac{60}{100} = 60\%
\]

\[
\frac{500}{6} \approx 83.33\%
\]

You also reviewed converting percents to decimals and to fractions in simplest form.

Write 83% as a decimal. Write 72% as a fraction in simplest form.

\[
83% = \frac{83}{100} = 0.83
\]

\[
72% = \frac{72}{100} = \frac{72 \div 4}{100 \div 4} = \frac{18}{25}
\]

Finally, you reviewed converting part-to-whole ratios to percents.

The ratio of consonants to vowels in a word is 5 to 3. What percent of the letters are vowels?

\[
\frac{3}{5 + 3} = \frac{3}{8}
\]

\[
= \frac{\frac{3}{8} \cdot 125}{125} = \frac{375}{1000} = 37.5\%
\]

Of the letters, 37.5% are vowels.

**PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Probability - Part 2

LEARN

WARM-UP

There are eight 11-year-olds, six 12-year-olds, and ten 13-year-olds on the soccer team. Write the ratio of each of the following.

1. 11-year-olds to 13-year-olds
2. 12-year-olds to all students
3. All students to 11-year-olds
4. 12-year-olds to 13-year-olds

WARM-UP ANSWERS

1. 4:5
2. 1:4
3. 3:1
4. 3:5

INSTRUCTION

Read p. 245 in Math in Focus B. In a one-stage event, the number of possible outcomes is the sample space.

Complete Hands-On Activity on p. 246. If you want to save paper and time, simply record all of the three-digit numbers that you can, using the numbers 1, 2, 3, 4. Do not use the same number more than once. One way to organize the list of numbers is to put the numbers in ascending order. For example, start with 123, 124, and continue.
There are eight 11-year-olds, six 12-year-olds, and ten 13-year-olds on the soccer team. Write the ratio of each of the following.

1. 11-year-olds to 13-year-olds
2. 12-year-olds to all students
3. All students to 11-year-olds
4. 12-year-olds to 13-year-olds

WARM-UP ANSWERS
1. 4:5
2. 1:4
3. 3:1
4. 3:5

Read p. 245 in Math in Focus B. In a one-stage event, the number of possible outcomes is the sample space.

Complete Hands-On Activity on p. 246. If you want to save paper and time, simply record all of the three-digit numbers that you can, using the numbers 1, 2, 3, 4. Do not use the same number more than once. One way to organize the list of numbers is to put the numbers in ascending order. For example, start with 123, 124, and continue.

Review Example 1 on p. 246. In part b, there are two green cards. When counting the number of possible outcomes, count green twice. If there were ten green cards, you would count them as ten different possible outcomes.

Complete Guided Practice on p. 247. For problem 2, one word you can form is cab. Make an organized list to find the possible outcomes of three letters. The letter d is used twice, so you can use it twice in your three-letter words. You can use a dictionary to determine if the words that you make are actually words.

Textbook Answer Key
For problem 2 of Guided Practice, you may want to have your student find the different combinations of three letters that can be made. For this problem, the order of the letters matters, so a-b-c is different from c-b-a.

PRACTICE
Complete problems 1 and 5–7 of Practice 10.1 on p. 249 in Math in Focus 2B.

WRAP-UP
Today you learned about possible outcomes and sample spaces.

The possible outcomes are A, B, C, D, and E.
There are 5 possible outcomes.
If a possible outcome occurs more than once, count it as many times as it occurs.

The possible outcomes are A, B, B, C, C, and C.

There are 6 possible outcomes.

You also learned how to find possible outcomes when finding multiple numbers or letters. How many different two-digit numbers can be made from these cards?

Make an organized list showing the numbers in ascending order.

<table>
<thead>
<tr>
<th>45</th>
<th>47</th>
<th>56</th>
<th>64</th>
<th>67</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>46</td>
<td>54</td>
<td>57</td>
<td>65</td>
<td>74</td>
<td>76</td>
</tr>
</tbody>
</table>

There are 12 two-digit numbers that can be made from the cards.

Quick Check

Please go online to view and submit this assessment.

More to Explore

View the Instructional Video, Introduction to Probability, to review the material from this lesson.
### Probability - Part 3

#### Objectives
- Identify outcomes favorable to an event.

#### Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition B

#### Assignments
- Complete Warm-up.
- Read and complete Math in Focus 2B.
- Complete Practice Questions.

### LEARN

#### WARM-UP

Write the number of possible outcomes for each experiment.

1. tossing a coin
2. tossing a number cube
3. spinning a spinner divided into fifths

#### WARM-UP ANSWERS

1. 2 possible outcomes
2. 6 possible outcomes
3. 5 possible outcomes

### TEACHING NOTES

#### WARM-UP ANSWERS

1. 2 possible outcomes
2. 6 possible outcomes
3. 5 possible outcomes

### INSTRUCTION

Read List Outcomes Favorable to an Event on p. 247 in Math in Focus 2B. Favorable outcomes are those possible outcomes about which you are concerned. For example, if you are tossing a coin and you want to know the number of times it lands on heads, the possible outcomes are heads and tails, but the favorable outcome is heads. You can represent the event $A$ of the coin landing on heads as $A = \{\text{heads}\}$.
Review Example 2 on p. 248. You can see that the words inside the braces are those that are favorable outcomes. The unfavorable outcomes are not listed inside the braces. The number of possible outcomes in part a is the number of breakfast choices and in part b the number of words.

Complete Guided Practice on p. 248. For problem 3, \( Y \) is equal to the set of prime numbers less than 20. Remember when determining the prime numbers that even numbers greater than 2 are all composite (not prime), as are numbers ending in 5 or 0 that are greater than 5. For the rest of the numbers, try to divide by 3. For problem 4, be aware that polygons have the same number of sides as angles.

Textbook Answer Key

Make sure that your student is only including favorable outcomes in the braces. Also make sure that she understands that favorable outcomes are the outcomes she is concerned with.

Complete problems 2–4 and 8–11 of Practice 10.1 on pp. 249–250 in Math in Focus 2B.

Today you learned how to list favorable outcomes to an event.
Suppose you are to spin the following spinner once.

Let $G$ be the event that the spinner lands on a composite number.
Let $H$ be the event that the spinner lands on a number less than 4.
Let $J$ be the event that the spinner lands on a number greater than 6.

Find the favorable outcomes that comprise event $G$.

$G = \{4, 6, 8\}$

There are 3 favorable outcomes.

Find the favorable outcomes that comprise event $H$.

$H = \{1, 2, 3\}$

There are 3 favorable outcomes.

Find the favorable outcomes that comprise event $J$.

$J = \{7, 8\}$

✅ **PRACTICE QUESTIONS**

Please go online to view and submit this assessment.
Probability - Part 4

**LEARN**

Follow these instructions for the activity shown below.

Click [here](#) to view the activity in a new window.

In this activity, you will explore the probability of a dart hitting a target based on its size. To begin, check that the **Side length for red square** is set to 20 and the **Side length for blue square** is set to 6. A. Out of 10 darts, about how many do you expect to hit the blue square? Share your prediction with your Learning Guide and explain how you came to that conclusion. Now click “Throw 10.” How many darts hit the blue square? Was this close to your prediction?

Study the ratio at the bottom of the screen. It shows you the ratio of darts that hit the target compared to the number of darts thrown. This represents the *experimental probability* of your throws. Now click on “Show area information” on the left. This ratio compares the area of the target to the area of the red square, which is 9%. This represents the *theoretical probability* of this event. You would you expect the ratio of the areas to be about the same as the ratio of the darts thrown at the target. You can click on “Throw 10” several more times to see what might happen in different trials. Are the results what you would expect?

Now set **Side length for red square** to 12 and **Side length for blue square** to 10. (To set the value of a slider quickly, type the value in the text box to the right of the slider and press Enter.) How will this affect how many darts hit the blue square? Explain your new prediction to your Learning Guide. Click “Throw 10.” How many darts hit the blue square? How closely did it match your prediction? How closely do the results (experimental probability) match the area information (theoretical probability)?

Suppose you were to click on the “Throw 100” button. How would you expect this to affect the experimental and theoretical probabilities? Share your ideas with your Learning Guide. Click on the button a few times and then click on the **Table** tab at the top left of the screen. All your attempts for this target are recorded. Use the data to tell whether your ideas were correct.

---

**Books & Materials**
- Computer

**Assignments**
- Complete Interactive Activity.
- Complete Rate Your Understanding.

---

**INTERACTIVE ACTIVITY**

Follow these instructions for the activity shown below.

Click [here](#) to view the activity in a new window.

In this activity, you will explore the probability of a dart hitting a target based on its size. To begin, check that the **Side length for red square** is set to 20 and the **Side length for blue square** is set to 6. A. Out of 10 darts, about how many do you expect to hit the blue square? Share your prediction with your Learning Guide and explain how you came to that conclusion. Now click “Throw 10.” How many darts hit the blue square? Was this close to your prediction?

Study the ratio at the bottom of the screen. It shows you the ratio of darts that hit the target compared to the number of darts thrown. This represents the *experimental probability* of your throws. Now click on “Show area information” on the left. This ratio compares the area of the target to the area of the red square, which is 9%. This represents the *theoretical probability* of this event. You would you expect the ratio of the areas to be about the same as the ratio of the darts thrown at the target. You can click on “Throw 10” several more times to see what might happen in different trials. Are the results what you would expect?

Now set **Side length for red square** to 12 and **Side length for blue square** to 10. (To set the value of a slider quickly, type the value in the text box to the right of the slider and press Enter.) How will this affect how many darts hit the blue square? Explain your new prediction to your Learning Guide. Click “Throw 10.” How many darts hit the blue square? How closely did it match your prediction? How closely do the results (experimental probability) match the area information (theoretical probability)?

Suppose you were to click on the “Throw 100” button. How would you expect this to affect the experimental and theoretical probabilities? Share your ideas with your Learning Guide. Click on the button a few times and then click on the **Table** tab at the top left of the screen. All your attempts for this target are recorded. Use the data to tell whether your ideas were correct.
In the first exercise, the area of the blue target appears to be about 1/10 of the red area, so your student should reasonably expect 1 or 2 darts to hit it. In the second exercise, the area of the blue square seems to be about 90% of the area of the red square, so your student’s prediction should be closer to 9 or 10.

When the trial is set to Throw 100, the actual percentage is closer to area ratio. If the Gizmo were to allow trials of 1,000 or more, the experimental and theoretical probabilities would probably be even closer.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

RATE YOUR UNDERSTANDING

Please go online to view and submit this assessment.
Probability - Part 5

LEARN

WARM-UP

Use a number cube with faces labeled 1–6 to write the number of favorable outcomes for each event.

1. M is the event that the number is odd.
2. N is the event that the number can be evenly divided by 3.
3. O is the event that the number is greater than 1.

WARM-UP ANSWERS

1. 3 favorable outcomes
2. 2 favorable outcomes
3. 5 favorable outcomes

TEACHING NOTES

WARM-UP ANSWERS

1. 3 favorable outcomes
2. 2 favorable outcomes
3. 5 favorable outcomes

INSTRUCTION

Read p. 251 in Math in Focus B. The sum of the probability of an event happening and not happening is 100%. An event that has a probability of 100% is certain. An event that has a probability of 0% is impossible. A probability of 50% means the event is equally as likely to happen as not to happen.

Review Example 3 on p. 252. Discuss the Think Math question with your Learning Guide. Sometimes one outcome is more likely to occur than another, such as when a coin is heavier on one side. That is why the word fair is used to describe a coin (If the coin is not fair, it is called biased.). For you to be able to write a probability as a fraction, each possible outcome must be equally probable.

Complete Guided Practice on p. 252. For problem 1a, write the probability as a fraction. The numerator is the number of favorable outcomes. The denominator is the number of possible outcomes.
Review **Example 4** on p. 253. When writing probability as a fraction, the numerator cannot be greater than the denominator because an outcome cannot have a probability greater than 100%. The formula for probability as a fraction indicates that the numerator is the number of favorable outcomes and the denominator is the number of possible outcomes.

Complete **Guided Practice** on p. 254. To find the probability of an event as a fraction, find the number of possible outcomes, which is the denominator, and then the number of favorable outcomes, which is the numerator. Remember that your probability cannot be less than 0% or greater than 100%.

**TEACHING NOTES**

**Textbook Answer Key**

For probability, your student might find the ratio of favorable outcomes to unfavorable outcomes. That is the ratio for odds, not probability. Make sure your student understands that when given a ratio as in problem 4 on p. 254, the percent is written as the ratio of favorable outcomes to possible outcomes (not unfavorable outcomes).

**PRACTICE**

Complete problems 1–5 of **Practice 10.2** on p. 263 in *Math in Focus 2B*.

**WRAP-UP**

*Probability* is the measure of the likelihood of an event occurring. It is measured as a number from 0 to 1, or 0% to 100%.

- An event that is *certain* has a probability of 1, or 100%.
- An event that is *impossible* has a probability of 0, or 0%.
- An event that is *equally likely* has a probability of $\frac{1}{2}$, or 50%.

Other probabilities can be found using the following formula:

\[
\text{Probability} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}
\]

To be able to write a probability as a fraction, each possible outcome must have an equal chance of occurring. The probability of event $A$ can be expressed as $P(A)$.

A shelf holds 8 fiction and 4 nonfiction books. One book is randomly chosen. Let $N$ be the event of picking a nonfiction book. What is $P(N)$?
Step 1: Write the number of possible outcomes.

The number of possible outcomes is the total number of books.

\[ 8 + 4 = 12 \]

There are 12 possible outcomes.

Step 2: Write the number of favorable outcomes.

The favorable outcomes are the nonfiction books.

There are 4 favorable outcomes.

Step 3: Write the probability as a fraction in simplest form.

\[ P(\text{N}) = \frac{4}{12} = \frac{1}{3} \]

Quick Check

Please go online to view and submit this assessment.

More to Explore

View the video, Basic Probability (11:23) to learn about probability.

Please go online to view this video ▶
Probability - Part 6

Books & Materials
- Computer

Assignments
- Complete Interactive Activity.
- Complete Rate Your Enthusiasm.

LEARN

INTERACTIVE ACTIVITY

Follow these instructions for the activity shown below.

Click here to view the activity in a new window.

This activity models an experiment in which colored marbles are drawn from a bag. To begin, drag two blue marbles and one green marble into the bag. Set Replace marbles after each draw? to No and set Number of draws to 2. This means the marbles will not be replaced after each draw.

Look at the THEORETICAL tab on the right side of the Gizmo. Notice the notation $B$ is used for drawing a blue marble and $G$ for a green marble. You can see that the probability of each outcome is $\frac{2}{6}$, or $\frac{1}{3}$, except for drawing 2 green marbles. This outcome is impossible because there is only 1 green marble in the bag. Copy this chart in your Math Notebook.

Now click the EXPERIMENTAL tab. Click “Run 1 trial” 10 times. How many times did BG (blue first, then green) occur? How do these results compare with the theoretical probability for each event? Click “Clear,” and then click on “Run 1,000 trials.” How do the experimental and theoretical probabilities compare now? Share your observations with your Learning Guide.

This activity models a dependent event. The second outcome is dependent on the first outcome. This is why the probability of drawing a green marble twice is 0 (impossible). Now look at an independent event in the Gizmo. Again, drag two blue marbles and one green marble into the bag. Make sure Replace marbles after each draw? is set to Yes. Now set Number of draws to 2 and click on the THEORETICAL tab. Compare the probabilities of each event this time with the probabilities you wrote in your Math Notebook for the dependent event. Explain to your Learning Guide why the calculations are different.

Now click on Run 1 trial 10 times and compare the probabilities. Do the same with Run 1,000 trials. Are the results what you expect? Try other combinations of marbles and experiment with the probabilities.
TEACHING NOTES

Go to Lesson Info to access the Exploration Sheet Answer Key. You may also wish to explore the other teaching resources in the activity.

RATE YOUR ENTHUSIASM

Please go online to view and submit this assessment.
Probability - Part 7

Objectives
- Identify an event as mutually exclusive or nonmutually exclusive.

Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition B

Assignments
- Complete Warm-up.
- Read and complete Math in Focus 2B.
- Complete Practice Questions.

LEARN

WARM-UP

There are 4 red marbles, 6 blue marbles, and 8 green marbles in a jar. One marble is randomly chosen from the jar.

1. What is the probability of picking a red marble?
2. What is the probability of picking a blue marble?
3. What is the probability of picking a green marble?

WARM-UP ANSWERS
1. 2/9
2. 1/31
3. 4/9

INSTRUCTION

Read Use Venn Diagrams to Show Relationships for Events on pp. 254–255 in Math in Focus 2B. Think of mutually exclusive as two related events that cannot occur together. A Venn diagram is often shown as overlapping circles that can be used to show the events that belong to Group A, Group B, and both Groups A and B. For mutually exclusive events, the Venn diagram will not overlap. If any of the possible outcomes are not mutually exclusive, the Venn diagram will overlap.

Review Example 5 on pp. 255–256. In a Venn diagram, outcomes that appear outside the circles are not favorable to either group. In Example 5, the numbers 16, 17, 19, 22, and 23 are not favorable to either group; they are neither multiples of 3 or multiples of 5.
Complete Guided Practice on p. 256. By completing part a, you should be able to answer parts c and d. The Venn diagram will help you visualize the problem.

Review Example 6 on p. 257. Then complete Guided Practice.

TEACHING NOTES

Textbook Answer Key

One way for your student to determine if two events are mutually exclusive is to list the favorable outcomes for each event. If any of the favorable outcomes are the same for both events, the events are not mutually exclusive. Your student should understand that some of the possible outcomes could be unfavorable to both events. In that case, those outcomes would fall outside of the circles in the Venn diagram.

PRACTICE

Complete problems 7–8 of Practice 10.2 on p. 264 in Math in Focus 2B.

WRAP-UP

Mutually exclusive events are two events that cannot happen at the same time.

A Venn diagram is an organizing tool that shows which data belong to Group A, to Group B, or to both. For mutually exclusive events, the circles do not overlap.

Ten numbers from 40 to 49 are on separate cards. Let A be the event of picking an odd number. Let B be the event of picking a number divisible by 6. Are the events mutually exclusive?

Step 1: Write the odd numbers from 40 to 49.

A = {41, 43, 45, 47, 49}

Step 2: Write the numbers divisible by 6.

B = {42, 48}
Step 3: Make a Venn diagram.

Events $A$ and $B$ are mutually exclusive.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
The numbers 30 through 50 are on separate cards. One card will be picked randomly. Determine if the following events are mutually exclusive or not.

1. $A$ is the event of picking a prime number and $B$ is the event of picking an odd number.
2. $C$ is the event of picking a number divisible by 8 and $D$ is the event of picking a number divisible by 9.

**WARM-UP ANSWERS**

1. no
2. yes

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. no
2. yes

**INSTRUCTION**

Read **Define Complementary Events** on p. 258 in *Math in Focus 2B*. Complementary events are mutually exclusive and always have a combined probability of 1.

Review **Example 7** on pp. 258–259. The circles in the Venn diagram of complementary events will never overlap. To find the probability of the complement of an event, you can find the probability of the event and subtract it from 1.

Complete **Guided Practice** on p. 259. Make a Venn diagram with the months that have the letter *a* in their names inside the circle, and the months that do not have the letter *a* in their names outside the circle.
Review Example 8 on p. 260. Complementary events always have a sum of 100%. The Venn diagram on p. 260 shows that one group, band members who play trombone, is part of a larger group, band members who play a brass instrument.

Complete Guided Practice on p. 261. For part b, the events of apples being rotten and not being rotten are complementary. You know the percentage of red apples that are rotten, so you can determine the percentage of red apples that are not rotten. Because red apples that are not rotten is a subgroup of red apples, the probability of picking an apple that is not rotten will be less than that of picking a red apple.

Review Example 9 on pp. 261–262. Then complete Guided Practice on p. 262. In addition to using a Venn diagram, you may want to make a table to help you organize the data.

### TEACHING NOTES

Textbook Answer Key

For events that are not mutually exclusive, the circles in a Venn diagram overlap. For mutually exclusive events, including complementary events, there are no overlaps. For a probability problem that includes a subgroup, that subgroup is shown inside one of the circles.

### PRACTICE

Complete problems 6 and 9–15 of Practice 10.2 on pp. 263–265 in Math in Focus 2B.

### WRAP-UP

Today you learned about complementary events. If $E$ is an event of something happening, the complement $E'$ is the event that $E$ does not happen. The sum of the probability of complementary events is 1; the event either happens or it does not.

You learned how to find probability involving percents and ratios. These problems often involve groups and subgroups. To find the probability of the subgroup occurring, multiply the probability of the subgroup by the probability of the group.

Out of 80 teenagers surveyed, 48 play sports for at least one team. Of those who play sports, 2 out of 3 also play a musical instrument. What is the probability that if you pick one teenager, she will play sports and an instrument?

**Step 1:** Find the percentage of teenagers who play sports.

$$\frac{48}{60} = 60\%$$
Step 2: Find the percentage of teenagers who play sports and a musical instrument.

\[
2 \frac{3}{3} \times 60\% = 40\%
\]

The probability of picking a teenager who plays sports on at least one team and plays a musical instrument is 40%.

✅ QUICK CHECK

Please go online to view and submit this assessment.

👌 MORE TO EXPLORE

Remember that \( \frac{5}{12} \) and the answer should add up to a sum of 1. Practice subtracting fractions mentally by determining how many numbers would need to be added to the numerator to equal the denominator. What is \( 1 - \frac{3}{5} \)? (2/5) What is \( 1 - \frac{8}{11} \)? (3/11)
LEARN

WARM-UP

There are 9 pens in a cup. Six are blue, and the rest are black. One pen is chosen randomly from the cup.

1. What is $P(\text{blue})$?
2. What is $P(\text{not blue})$?

TEACHING NOTES

WARM-UP ANSWERS

1. $\frac{2}{3}$
2. $\frac{1}{3}$

INSTRUCTION

Compound probability is the probability of two or more simple events that occur either together or one after the other. These kinds of events are called compound events. For example, tossing a coin or tossing a number cube are both simple events. Tossing both a coin and a number cube together is a compound event.

Review parts A and B in the Compound Probability Worksheet. Unless you are told otherwise, use the possibility diagram that you prefer to organize the possible outcomes. When possible, use numerical or alphabetical order to help you organize the possible outcomes. For example, for finding the two-digit numbers that can be made from the digits 1, 2, and 3, start with all of the possible two-digit numbers with 1 in the tens place, followed by 2 in the tens place, and then 3 in the tens place.
The tree diagram is a popular possibility diagram. If you imagine using a dime and a quarter, you might be able to visualize better that there are four different possible outcomes. There are two possible HT outcomes; the quarter could be heads and the dime tails, or vice versa. This is true even if there are two quarters; they are still two distinctly different objects.

Review parts C and D. These examples show how possibility diagrams can be used to find the probabilities of compound events. The table works well for the number cube sums, while the tree diagram works well for the weather chart. A tree diagram can become very bulky for large numbers of possible outcomes.

Review part E. Compound probabilities can be found by multiplying the probabilities of each simple event. Expect the probability of a compound event to be less than the probability of either event individually.

Complete the Technology Activity. Simulations are used to generate data for experiments that could be time consuming or difficult to complete. For example, using random numbers in the spreadsheet means that thousands of simulations can be done nearly instantly.

Complete the Your Turn section.

To find the number of possible outcomes in a compound event, your student can use the Fundamental Counting Principle, which states that the number of possible outcomes of a compound event is the product of the number of possible outcomes for each simple event. For example, to find the number of possible outcomes of tossing a number cube and a coin, multiply $6 \cdot 2 = 12$.

YOUR TURN ANSWERS

<table>
<thead>
<tr>
<th>Flavor</th>
<th>V</th>
<th>V</th>
<th>C</th>
<th>C</th>
<th>S</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Container</td>
<td>Cone</td>
<td>Cup</td>
<td>Cone</td>
<td>Cup</td>
<td>Cone</td>
<td>Cup</td>
</tr>
<tr>
<td>Outcome</td>
<td>V-Cone</td>
<td>V-Cup</td>
<td>C-Cone</td>
<td>C-Cup</td>
<td>S-Cone</td>
<td>S-Cup</td>
</tr>
</tbody>
</table>

2 3 6 3 17 4 17/36
PRACTICE ANSWERS

1 false 2 true

3a Possibility diagrams will vary, but all should include the following outcomes. Sample diagram is shown.

<table>
<thead>
<tr>
<th>Flavor</th>
<th>V</th>
<th>V</th>
<th>C</th>
<th>C</th>
<th>S</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Container</td>
<td>Cone</td>
<td>Cup</td>
<td>Cone</td>
<td>Cup</td>
<td>Cone</td>
<td>Cup</td>
</tr>
<tr>
<td>Outcome</td>
<td>V-Cone</td>
<td>V-Cup</td>
<td>C-Cone</td>
<td>C-Cup</td>
<td>S-Cone</td>
<td>S-Cup</td>
</tr>
</tbody>
</table>

b 6

4a Possibility diagrams will vary. Sample diagram is shown.

<table>
<thead>
<tr>
<th>Spinner 1</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1, 1)</td>
<td>(2, 1)</td>
<td>(3, 1)</td>
<td>(4, 1)</td>
</tr>
<tr>
<td>2</td>
<td>(1, 2)</td>
<td>(2, 2)</td>
<td>(3, 2)</td>
<td>(4, 2)</td>
</tr>
<tr>
<td>3</td>
<td>(1, 3)</td>
<td>(2, 3)</td>
<td>(3, 3)</td>
<td>(4, 3)</td>
</tr>
</tbody>
</table>

b 12

5 Possibility diagrams will vary. Sample diagram is shown.

<table>
<thead>
<tr>
<th>Number Cube</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>3</th>
<th>3</th>
<th>4</th>
<th>4</th>
<th>5</th>
<th>5</th>
<th>6</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coin</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td>T</td>
</tr>
<tr>
<td>Outcome</td>
<td>H</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td>H</td>
<td>T</td>
<td>T</td>
<td>H</td>
<td>T</td>
</tr>
</tbody>
</table>

6a 1/12 b 1/4 c 1/6 d 5/12

7 Possibility diagrams will vary, but all should include the following outcomes. Sample diagram is shown.

8a 1/8 b 3/8 c 3/8 d 1/8

9a 1 b These are all possible outcomes, so the probability is 1. Answers will vary for percentages. The theoretical probability is given after each problem.

10 1/18 11 1/9 12 1/6 13 1/12
WRAP-UP

Today you learned about compound probability. A compound event consists of two or more simple events occurring together or one after another. To find the probability of a compound event, you can use a possibility diagram or multiply the probabilities of each simple event.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
In a school, 40% of the students play tennis, 24% of the students play baseball, and 58% of the students play neither tennis nor baseball. If you pick a student at random, what is the probability that the student plays both tennis and baseball?

Upload your work to show how you got to the answer.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information given to solve the problem?
- Find the probability that a student plays both tennis and baseball if you pick one student at random?
- Upload your work to show how you found the answer?
Probability Models - Part 1

Objectives
- Identify and calculate relative frequency.

Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition B
- coin

Assignments
- Complete Warm-up.
- Read and complete Math in Focus 2B.
- Complete Practice Questions.

LEARN

WARM-UP

Two number cubes labeled 1–6 are tossed.

1. What is $P(\text{sum of 8})$?
2. What is $P(\text{sum of at least 8})$?

TEACHING NOTES

WARM-UP ANSWERS

1. $5/36$
2. $5/12$

INSTRUCTION

Read Find the Relative Frequency in a Chance Process on p. 266 in Math in Focus 2B.

Complete Hands-On Activity on pp. 266–267. Relative frequency can also be written as a percent. The sum of the relative frequencies of an event is 1, or 100%, because the sum accounts for all of the possible outcomes.

Review Example 10 on pp. 267–268. Because there were 600 people in the park, the sum of the number of men, women, and children is equal to 600.

Complete Guided Practice on p. 268. To find the number of monitors for each size, multiply the total number of monitors by the relative frequency of each size. If it helps, you can convert the decimals to fractions and then simplify common factors before multiplying.
Today you learned about relative frequency. Relative frequency is the ratio of the observed frequency of a data value to the total number of observations. Relative frequency can be written as a fraction, decimal, or percent. The sum of the relative frequencies of a data set is always 1, or 100%.

The table shows the relative frequencies of children, adults, and seniors who went to a movie theater yesterday.

<table>
<thead>
<tr>
<th>People</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children</td>
<td>0.25</td>
</tr>
<tr>
<td>Adults</td>
<td>0.45</td>
</tr>
<tr>
<td>Seniors</td>
<td>0.3</td>
</tr>
</tbody>
</table>

A total of 840 people attended the movies at that theater. How many more adults than children attended?

**Step 1:** Subtract the relative frequency of children from the relative frequency of adults.

\[0.45 - 0.25 = 0.2\]

**Step 2:** Multiply the difference in relative frequency by the total number of people.

\[0.2 \times 840 = 168\]

Yesterday, 168 more adults than children attended the movie theater.
Please go online to view and submit this assessment.
Probability Models - Part 2

**Objectives**
- Interpret relative frequency from a histogram.

**Books & Materials**
- Math in Focus 2B
- Math in Focus - Teacher Edition B

**Assignments**
- Complete Warm-up.
- Read and complete Math in Focus 2B.
- Complete Quick Check.

**LEARN**

**WARM-UP**

A spinner is spun 40 times. It lands on A 16 times, on B 14 times, and on C 10 times. Write each of the following relative frequencies as a decimal.

1. What is the relative frequency of A?
2. What is the relative frequency of B?
3. What is the relative frequency of C?

**WARM-UP ANSWERS**

1. 0.4
2. 0.35
3. 0.25

**TEACHING NOTES**

**TEACHING NOTES**

**INSTRUCTION**

Read the instructional section at the top of p. 269 in *Math in Focus 2B*.

Review Example 11 on pp. 269–270. The histograms that you have used and made give a range of numbers on the horizontal axis. Any GPA greater than or equal to 2 and less than 2.5 is included in the first bar. Any GPA greater than or equal to 2.5 and less than 3 is in the second bar. Any GPA greater than or equal to 3 and less than 3.5 is in the third bar, and any GPA greater than or equal to 3.5 and less than 4 is in the fourth bar. Based on this histogram, no student had a GPA of 4.

Complete Guided Practice on p. 271. To find the relative frequency for each interval, divide the number of fish with a mass in that interval by the total number of fish. Remember that the sum of the relative frequencies when written as percents must equal 100%.
In the histograms on pp. 269–270, if a GPA of 4 were included, the intervals of the bars would not be equal. What the intervals would be are $2–2.49$, $2.5–2.9$, $3–3.49$, and $3.5–4$, which are unequal. In general, for each interval in the frequency histogram, the lower value for each bar is included in the interval, and the upper value is not.

Complete problem 4 of Practice 10.3 on p. 277 in Math in Focus 2B.

Problem 4 involves a bar graph and not a histogram. The difference between a bar graph and a histogram is that a bar graph shows specific values, while a histogram shows ranges of values. Also, the bars of a histogram are always vertical, while the bars of a bar graph may be either vertical or horizontal.

Today you learned how to use a histogram showing relative frequency. The histogram for relative frequency may not necessarily show the range of numbers for each bar. Instead, the minimum value for the bar will be the number at the left and the maximum number will be any number that is less than the number at right.

The histogram shows the frequency of batting averages for the players on a team. The relative frequency for each bar is given above the bar.
Please go online to view and submit this assessment.

View the Discovery video *Modeling Probability: Rainy Weather* to view how to construct a probability model from observations.
LEARN

Probability Models - Part 3

Objectives
- Differentiate between experimental probability and theoretical probability.
- Use experimental probability to make a prediction.

Books & Materials
- Math in Focus 2B
- Math in Focus - Teacher Edition B
- 10 counters
- paper bag or similar container

Assignments
- Complete Warm-up.
- Read and complete pages in Math in Focus 2B.
- Complete problems 5–8, pp. 277–278, Math in Focus B.
- Complete Quick Check.

WARM-UP

Two number cubes labeled 1–6 are tossed. Use this information to write each probability as a fraction in simplest form.

1. P(sum of 9)
2. P(product of 4)

WARM-UP ANSWERS
1. 1/9
2. 1/12

TEACHING NOTES

WARM-UP ANSWERS
1. 1/9
2. 1/12

INSTRUCTION

Read p. 272 in Math in Focus B. Experimental probability measures what has happened, while theoretical probability measures what is expected to happen. Depending on the number of trials, the experimental probability may either be close to the theoretical probability or very different. The greater the number of trials, the closer the experimental probability should be to the theoretical probability. Relative frequency is an example of experimental probability.

Review Example 12 on pp. 273–274. Then complete Guided Practice on p. 274. The color that has the greatest observed frequency will also have the greatest relative frequency, which also means that it will be the color most likely to be hit.

Read the instructional section at the top of p. 275. An example is tossing a coin. If you toss a fair coin 2 times, it could land heads up both times, giving an experimental probability of 100% for heads (which is
0% for tails). If you toss a fair coin 100 times, it is likely that the experimental probability of it landing on heads will be near its theoretical probability of 50%.

Complete **Hands-On Activity** on p. 275.

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### TEACHING NOTES

#### Textbook Answer Key

Many students confuse experimental probability and theoretical probability. Your student can think of batting averages or free throw percentages as experimental probability. It varies depending upon the skill of the athlete. A theoretical probability is the same for everyone.

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### PRACTICE

Complete problems 5–8 of **Practice 10.3** on pp. 277–278 in *Math in Focus 2B*.

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### WRAP-UP

Today you compared experimental probability with theoretical probability. *Experimental probability* is based on what has happened and *theoretical probability* is based on what should happen. Both probabilities can be used to make a prediction. You also learned that relative frequency is a form of experimental probability.

More than likely, the greater the number of trials or observations, the closer the experimental probability will be to the theoretical probability.

There are 8 cards in a bag. 4 are labeled E, 3 are labeled F, and 1 is labeled G. A card was randomly picked from a bag, the letter was recorded, and then the card was placed back in the bag.

The observed frequency is shown in the following table.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Observed Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>55</td>
</tr>
<tr>
<td>F</td>
<td>48</td>
</tr>
<tr>
<td>G</td>
<td>17</td>
</tr>
</tbody>
</table>
The relative frequency (experimental probability) is compared with the theoretical probability in the following table.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Observed Frequency</th>
<th>Relative Frequency (Experimental Probability)</th>
<th>Theoretical Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>55</td>
<td>45.83%</td>
<td>50%</td>
</tr>
<tr>
<td>F</td>
<td>48</td>
<td>40%</td>
<td>37.5%</td>
</tr>
<tr>
<td>G</td>
<td>17</td>
<td>14.17%</td>
<td>12.5%</td>
</tr>
</tbody>
</table>

Please go online to view and submit this assessment.

Remember that theoretical probability does not depend on experimental data. To find the theoretical probability, you would find the probability of the event based on the number of what you are looking for (in this case, children) out of the total number of events (in this case, total number of index cards). Here is a video reviewing theoretical vs. experimental probability: *Theoretical vs. Experimental Probability (03:59).*

Please go online to view this video ▶
Follow these instructions for the activity shown below.

Click [here](#) to view the activity in a new window.

In this activity, you will be working with spinners in probability model. To begin, check that the **Number of spinners** is 1, **Sections** is 6, **Number** is 2, and the sign is chosen. In this game, a win (a favorable outcome) occurs if the spinner lands on 2. Based on what you have learned about probability, how likely do you think it is that a player will win the game? Explain your ideas to your Learning Guide.

On the EXPERIMENTAL tab, click “Run 1 trial.” What was the outcome? 3. Click “Clear.” Then, click “Run 10 trials.” How many trials were favorable? Click “Run 10 trials” 5 more times so there are a total of 60 trials. How many favorable outcomes did you get out of 60 trials?

Go to the THEORETICAL TAB. The tab shows a table of outcomes. The red numbers show the possible numbers on the spinner, and the blue number represents the selected number. In the table, Y represents a favorable outcome, while N represents an unfavorable outcome. Now click on “Show theoretical probabilities.” The table shows the number of favorable and unfavorable outcomes and the theoretical probabilities as fractions and percentages. Which experimental probability is closer to the theoretical probability: the result after 10 trials or the result after 60? What is the sum of the favorable and unfavorable probabilities? Share your answers with your Learning Guide.

Now try this activity:

1. Turn off **Show theoretical probabilities**. Change the **Sections in the spinner** to 7, the **Number** to 4, and the sign to ≥. In this game, what are the favorable outcomes? How many possible outcomes are there? What is the theoretical probability of a favorable outcome? Give your answer as a fraction and as a percentage. Turn on “Show theoretical probabilities“ to check your work.

2. On the EXPERIMENTAL tab, run 100 trials. How many favorable outcomes occurred? What is experimental probability of a favorable outcome? How did the experimental probability compare to the theoretical probability you calculated?
Sample Answers: Spinning a 2 is one outcome in a sample space of 6 possible outcomes, so the theoretical probability is $\frac{1}{6}$, or about 16.7%. More trials with the spinner will give an experimental probability that is closer to the theoretical probability. The sum of favorable and unfavorable probabilities is always 100%.

Activity Answers: 1 The favorable outcomes are 4, 5, 6, and 7. There are 7 possible outcomes. The theoretical probability of a favorable outcome is $\frac{4}{7}$, or approximately 57%. 2 Answers will vary.

If you would like, you can click on Lesson Info and download the Student Exploration Sheet and Exploration Sheet Answer Key to have your student try some other activities with the Gizmo.

RATE YOUR ENTHUSIASM

Please go online to view and submit this assessment.
A soccer team won 8 games, lost 6 games, and tied 2 games.

1. What is the experimental probability that the team will win its next game?
2. What is the experimental probability that the team will lose its next game?

WARM-UP ANSWERS

1. 1/2
2. 3/8

INSTRUCTION

Read Understand a Probability Model on p. 279 in Math in Focus 2B. When conducting a probability experiment, random means that the outcome is unknown before it happens.

Read Develop a Uniform Probability Model on p. 279. Prior to this lesson, all the probability experiments you have done have used uniform probability models.

Review Example 13 on p. 280. The bar graph shows the probability that each color will be chosen. It shows that the probability model is uniform.

Complete Guided Practice on p. 281. Your bar graph should show the probability of each letter occurring.
Read the instructional section in the middle of p. 281. The fraction \( \frac{n}{1} \) means that each outcome has the same chance of occurring.

Review Example 14 on p. 281. Then complete Guided Practice on p. 282. To find the denominator of the fraction that describes probability, count the number of numbers.

**TEACHING NOTES**

Textbook Answer Key

Discuss with your student how viewing probabilities in a bar graph strengthens the concept of a uniform probability distribution: the heights of the bars are uniform in height.

**PRACTICE**

Complete problems 1–4 of Practice 10.4 on p. 289 in Math in Focus 2B.

**WRAP-UP**

Today you learned that a uniform probability model is one in which each possible outcome has an equal chance of occurring. In a probability experiment, randomness is essential. Randomness means that the outcome of the experiment is unknown before it happens.

A letter is chosen at random from the list: A, C, E, G, I, K, M, O, Q, S.

The following is true:

- The letters form a uniform probability model because each letter has an equal opportunity of being chosen.
- The probability of choosing any one letter is \( \frac{1}{10} \).
- The probability of choosing a consonant is \( \frac{6}{10} = \frac{3}{5} \).
- The probability of choosing a vowel is \( \frac{4}{10} = \frac{2}{5} \).

**QUICK CHECK**

Please go online to view and submit this assessment.
MORE TO EXPLORE

To find the probability, make a list of the possible outcomes and circle the favorable outcomes. Then, write a fraction to represent the probability. You may also want to revisit the material in this lesson.
Probability Models - Part 6

**Objectives**
- Determine and graph events in a nonuniform probability model.
- Develop an experimental probability model.

**Books & Materials**
- Math in Focus 2B
- Math in Focus - Teacher Edition B
- spreadsheet software (Optional)

**Assignments**
- Complete Warm-up.
- Read and complete Math in Focus 2B.
- Complete Practice Questions.

**LEARN**

**WARM-UP**
The letters $A–H$ are on tiles that are placed inside a bag. One tile will be chosen randomly from the bag.

1. What is $P($the letter $A$)?
2. What is $P($consonant$)?$  

**TEACHING NOTES**

**WARM-UP ANSWERS**

1. 1/8  
2. 3/4

**INSTRUCTION**

Read Develop a Nonuniform Probability Model on p. 282 in Math in Focus B.

Review Example 15 on p. 282. The 15 pencils each have a probability of 115 of being chosen, which is uniform. If the model involves the colors of the pencils, then that is nonuniform because there are different numbers of each of the four colors of pencils. The bar graph of a uniform probability model showed all equal bars. For the nonuniform probability model, at least 2 of the bars have different heights.

Complete Guided Practice on p. 284. Your bar graph should show the probability of each tea bag being selected. Remember, the sum of the probabilities is 1.

Review Example 16 on pp. 284–285. This is an example of an unfair number cube. Although there are 6 numbers, the cube was made such that one number will be rolled more frequently than the others. Complete Guided Practice on p. 285.
Review Example 17 on pp. 285–286. Remember that in this histogram, the minimum value of each bar is given on the left and the maximum value is any number less than the number to the right. For example, a value of 75 would have needed a new bar.

Complete Guided Practice on p. 287. You can use a stem-and-leaf plot to organize the data for your histogram.

---

**TEACHING NOTES**

**Textbook Answer Key**

Whether a probability model is uniform or nonuniform depends on the question being asked. For example, the probability of picking any letter in FITZGERALD is equal. That is a uniform probability model. Asking about the probability of picking a consonant or a vowel makes it a nonuniform probability model because there are 7 consonants and 3 vowels in FITZGERALD.

If your student has access to spreadsheet software, she can complete the optional Hands-On Activity on pp. 287–288. In cell A1, have your student write the formula = INT(RAND) (*9 + 0) to provide random numbers. To model 50 tosses, select cells A2 to A51 and choose **Fill Down** from the Edit menu.

---

**PRACTICE**

Complete problems 5–8 of Practice 10.4 on pp. 290–291 in Math in Focus 2B.

---

**WRAP-UP**

Today you learned that a *nonuniform probability model* is one in which each possible outcome does not have an equal chance of occurring. Each individual possible outcome may have an equal chance of occurring, but when the items are grouped, such as prime vs. composite numbers or various colors of an item, the probability of choosing an item from one group is different from the probability of choosing one from a different group. The model becomes nonuniform.

The bar graph of a nonuniform probability model will show bars of different lengths.

There are 5 cherry ice pops, 4 grape ice pops, and 3 orange ice pops in a freezer. One ice pop is chosen randomly.
The following is true:

- The flavors of ice pops form a nonuniform probability model because the flavors are in unequal numbers.
- The probability of choosing any one ice pop is \( \frac{1}{12} \).
- The probability of choosing a cherry ice pop is \( \frac{5}{12} \).
- The probability of choosing a grape ice pop is \( \frac{4}{12} = \frac{1}{3} \).
- The probability of choosing an orange ice pop is \( \frac{3}{12} = \frac{1}{4} \).

✅ PRACTICE QUESTIONS

Please go online to view and submit this assessment.
**Probability Models - Part 7**

**Objectives**
- Apply mathematical concepts and skills to solve problems.

**Books & Materials**
- Math in Focus 2B
- Math in Focus - Teacher Edition B

**Assignments**
- Complete Warm-up.
- Read and complete Math in Focus 2B.
- Complete the Practice activity.
- Complete Practice Questions.

---

**LEARN**

**WARM-UP**
Find the probability of randomly picking the following from the word MISSISSIPPI.

1. $P(M)$
2. $P(I)$
3. $P(\text{consonant})$

**WARM-UP ANSWERS**
1. $\frac{1}{11}$
2. $\frac{4}{11}$
3. $\frac{7}{11}$

---

**TEACHING NOTES**

**INSTRUCTION**
Read and complete Brain @ Work on p. 291 in Math in Focus 2B. For parts a and b, you can make a list of each possible outcome. The difference between each pair of numbers should be written as the absolute value, as it does not matter in which order the numbers are subtracted.

From your organized list, you can find the number of times you would win and then calculate the probability.

---

**TEACHING NOTES**

Textbook Answer Key
Today you learned how to solve multi-step problems involving rational numbers and probability.

WRAP-UP

Today you learned how to solve multi-step problems involving rational numbers and probability.

PRACTICE

Write a constructed response for your solution to part c in Brain @ Work.

Then complete the following problems.

1. A quiz has 5 true-false questions. What is the probability of guessing and passing the quiz with four or five correct answers? Explain your answer.
2. A quiz has 10 true-false questions. What is the probability of guessing all 10 questions correctly? Explain your answer.

TEACHING NOTES

Encourage your student to use the Fundamental Counting Principle to find the number of possible outcomes. Each answer can only be correct (C) or wrong (W). If she knows how to use exponents, she may use $2^5$ for problem 1 and $2^{10}$ for problem 2.

For problem 1, instead of writing all 32 possible outcomes, encourage her to write only those that will provide 4 or 5 correct answers. For 4 correct answers, only 1 answer can be incorrect, so it can happen in only one of the first, second, third, fourth, or fifth questions.

PRACTICE ANSWERS

1. $\frac{3}{16}$; There are 32 possible outcomes because $2 \times 2 \times 2 \times 2 \times 2 = 32$. There are 5 ways to get 4 correct answers: C-C-C-C-W, C-C-C-W-C, C-C-W-C-C, C-W-C-C-C, and W-C-C-C-C. There is 1 way to get all 5 correct. The probability is $\frac{6}{32} = \frac{3}{16}$.
2. $\frac{1}{1,024}$; There are 1,024 possible outcomes because $32 \times 32 = 1,024$. There is only 1 way to get 10 correct answers. The probability is $\frac{1}{1,024}$.

PRACTICE QUESTIONS

Please go online to view and submit this assessment.
Probability Models - Part 8

Books & Materials
- Math in Focus - Teacher Edition B

USE

USE FOR MASTERY

A fair cube has a red face, a blue face, a green face, and a white face. During an experiment, the cube is tossed and the color that it lands on is recorded.

The cube lands on red 4 times as often as it lands on blue. The cube lands on blue twice as often as it lands on green, and it lands on green as often as it lands on white.

A) Create and complete a probability distribution table of the four colors.

B) Draw a bar graph to represent the probability distribution.

Upload both answers here.
USE FOR MASTERY GUIDELINES & RUBRIC

Did you:

- Use the information given to create and complete a probability distribution table of the four colors?
- Draw a bar graph to represent the probability distribution?
- Show the probability in its lowest form?
- Upload your work to show how you found your answers?
Unit Quiz: Statistics and Probability

Please go online to view and submit this assessment.
Appendix
This form is to be used when completing Use for Mastery assessments or Projects offline. Your assessment can then be scanned and uploaded into the correct lesson online.

Please Fill In This Form Completely

Student's Name  

Course Name  

Lesson Title  

Provide your answer in the space below.
### Planning a Trip

**Student Facing Project Rubric**

Read the chart below to understand how your project will be scored. Your goal should be to earn all 4 points for each part.

<table>
<thead>
<tr>
<th>CRITERIA</th>
<th>4 POINTS</th>
<th>3 POINTS</th>
<th>2 POINTS</th>
<th>1 POINT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expressions</td>
<td>You correctly wrote three expressions to represent your total cost with your total number of days at three different hotels. The number of days is represented as $x$, which is multiplied by the cost of the hotel each day. The activity costs have been added to this amount.</td>
<td>You correctly wrote two expressions to represent your total cost with your total number of days at three different hotels. The number of days is represented as $x$, which is multiplied by the cost of the hotel each day. The activity costs have been added to this amount.</td>
<td>You correctly wrote one expression to represent your total cost with your total number of days at three different hotels. The number of days is represented as $x$, which is multiplied by the cost of the hotel each day. The activity costs have been added to this amount.</td>
<td>You wrote three expressions to represent your total cost based on your number of days. There is at least one error in each expression.</td>
</tr>
<tr>
<td>Inequalities</td>
<td>You correctly wrote three inequalities representing your total cost as being less than or equal to 3000.</td>
<td>You correctly wrote two inequalities representing your total cost as being less than or equal to 3000.</td>
<td>You correctly wrote one inequality representing your total cost as being less than or equal to 3000.</td>
<td>You wrote three inequalities representing your total and its relationship to 3000, but they were not correct.</td>
</tr>
<tr>
<td>Solving for and Interpreting $x$</td>
<td>You correctly solved for the value of $x$ as the maximum number of days you can stay at your hotel for each inequality. You also correctly answered if you had to stay $x$ nights. This answer also included a justification with mathematical reasoning.</td>
<td>You correctly solved for the value of $x$ as the maximum number of days you can stay at your hotel for two of the three inequalities. You also correctly answered if you had to stay $x$ nights. This answer also included a justification with mathematical reasoning.</td>
<td>You correctly solved for the value of $x$ as the maximum number of days you can stay at your hotel for one of the three inequalities. You also correctly answered if you had to stay $x$ nights. This answer also included a justification with mathematical reasoning.</td>
<td>You correctly solved for the value of $x$ as the maximum number of days you can stay at your hotel for each inequality. You also correctly answered if you had to stay $x$ nights, but you did not include your reasoning.</td>
</tr>
<tr>
<td>Presentation</td>
<td>Your presentation includes all of the information needed in the project. Your presentation is well organized and does not contain any spelling or grammatical errors.</td>
<td>Your presentation includes all of the information needed in the project. Your presentation is well organized and contains 2 to 3 spelling or grammatical errors.</td>
<td>Your presentation is missing 1 piece of information needed in the project. Your presentation has some flaws with organization. It contains more than 3 spelling or grammatical errors.</td>
<td>Your presentation is missing 2 or more pieces of information needed in the project. Your presentation has several flaws with organization. It contains more than 3 spelling or grammatical errors.</td>
</tr>
<tr>
<td>Total Final Cost</td>
<td>You created a poster or PowerPoint showing your destination. You decided on a hotel and explained your reasoning. You have shown all your work and sources. All of your calculations are correct.</td>
<td>You created a poster or PowerPoint showing your destination. You decided on a hotel and explained your reasoning. You have shown all your work and sources. Most of your calculations are correct.</td>
<td>You created a poster or PowerPoint showing your destination. You decided on a hotel and explained your reasoning. You did not show all your work and sources. Some of your calculations are incorrect.</td>
<td>You did not create a poster or PowerPoint showing your destination. You did not show all your work and sources. Some of your calculations are incorrect. You reached a decision regarding the hotel where you would stay and told why.</td>
</tr>
</tbody>
</table>

**Total Possible Points: 20**
### Drawing a Tessellation

**Student Facing Project Rubric**

Read the chart below to understand how your project will be scored. Your goal should be to earn all 4 points for each part.

<table>
<thead>
<tr>
<th>CRITERIA</th>
<th>1 POINT</th>
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<tr>
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<td>Requirements</td>
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<td>Your tessellation included at least a parallelogram, an equilateral triangle, a right triangle, and a square.</td>
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<td>Your tessellation included three of the following: a parallelogram, an equilateral triangle, a right triangle, and a square.</td>
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<td>Your tessellation included two of the following: a parallelogram, an equilateral triangle, a right triangle, and a square.</td>
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**Total Possible Points: 20**
A. Using a Possibility Diagram or an Organized List

Suppose you design a game such that winning means getting a 5 on a toss of a fair number cube \textit{and} heads on a toss of a fair coin. Represent the possible outcomes of this compound event using a possibility diagram.

One way to represent the possible outcomes is a two-way grid. List the outcomes for one simple event on the horizontal axis and the outcomes for the other simple event on the vertical axis. The intersection of grid lines represents possible outcomes of the compound event.

There are 12 possible outcomes when tossing a number cube and a coin.

Another way to represent the outcomes of the compound event is an organized list. Write all the ways a number cube can land and combine that with heads.

1-H, 2-H, 3-H, 4-H, 5-H, 6-H

Then write all the ways a number cube can land and combine that with tails.

1-T, 2-T, 3-T, 4-T, 5-T, 6-T

Add the possible outcomes.

$6 + 6 = 12$

Just as you found using the two-way grid, there are 12 possible outcomes when tossing a number cube and a coin.
B. Using a Tree Diagram

A tree diagram is a possibility diagram used to represent a compound event. The following tree diagram represents tossing a fair coin. Each possible outcome is shown by a branch.

Follow these guidelines to create a tree diagram.

- The number of branches indicates the number of outcomes the event has.
- The outcome for the event is written at the end of the branch.
- The probabilities of the branches from each node must add up to 1.

Now suppose you toss two coins. Then the tree diagram looks like this.

There are 4 equally likely possible outcomes.
C. Using a Possibility Diagram to Find Probability of a Compound Event

Two fair number cubes labeled 1–6 are tossed. Find the probability that the sum of the two numbers is a composite number.

**Step 1:** Make a diagram to represent each possible outcome and its sum. Then circle the sums that are composite numbers.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

**Step 2:** Count the number of possible outcomes.

There are 36 possible outcomes.

**Step 3:** Count the number of favorable outcomes, which are the circled numbers.

There are 21 favorable outcomes.

**Step 4:** Write the probability as a fraction in simplest form.

\[
\frac{21}{36} \div \frac{3}{3} = \frac{7}{12}
\]

The probability of tossing a sum that is a composite number is \(\frac{7}{12}\), which is slightly greater than one-half, or 50%. 
D. Using a Tree Diagram to Find Probability of Compound Events

Suppose that it is equally likely to snow or not snow on any given day. Draw a tree diagram and use it to find the probability that it snows on one of two consecutive days.

**Step 1:** Make a tree diagram.

```
   1st Day      2nd Day      Outcome
    S          N            (S, S)
       / \         /  \        /
      S   N       S   N        (S, N)
       /     \     /     \      /
      S     N     S     N        (N, S)
       /         /     \         /
      N         N       N        (N, N)
```

**Step 2:** The favorable outcome is snow on one of two consecutive days. Look for the outcome that gives \((S, N)\) or \((N, S)\), where \(S\) represents snowing and \(N\) represents not snowing.

**Step 3:** Find the probability.

\[
P(\text{snow on exactly one day}) = \frac{2}{4} = \frac{1}{2}
\]

The probability of snow on one of two consecutive days is \(\frac{1}{2}\), or 50%.

E. Using Multiplication to Find Probability of Compound Events

Two number cubes labeled 1–6 are tossed. What is the probability that the first number cube lands with a number less than 2 and the second lands with a number greater than 4?

**Step 1:** Find the probability that a number cube lands with a number less than 2.

There are 6 possible outcomes. There is 1 favorable outcome—the number 1.

The probability is \(\frac{1}{6}\).

**Step 2:** Find the probability that a number cube lands with a number greater than 4.

There are 6 possible outcomes. There are 2 favorable outcomes—the numbers 5 and 6.

The probability is \(\frac{2}{6} = \frac{1}{3}\).

**Step 3:** Multiply the probabilities to find the probability of the compound event.

\[
\frac{1}{6} \cdot \frac{1}{3} = \frac{1}{18}
\]

The probability that the first toss is less than 2 and the second toss is greater than 4 is \(\frac{1}{18}\).
Be careful using multiplication to find the probability of compound events. When there are multiple ways to achieve a favorable outcome (such as 2 heads and 2 tails when flipping 4 coins), it is best to use a probability diagram or tree.

**Technology Activity: Using a Simulation with Random Numbers and Technology**

**Materials:** spreadsheet software or two number cubes

Two number cubes labeled 1–6 are tossed 100 times. Use a spreadsheet to generate data to determine how many times each sum from 2 to 12 occurs. (Alternatively, you can toss two number cubes 50 times by hand and record the sums using a dot plot or a table with tick marks. You learned about both of these data displays in Chapter 9.)

If you are using the spreadsheet, follow these instructions.

1. On your spreadsheet, label cell A1 as *Cube A*, cell B1 as *Cube B*, and cell C1 as *Sum*.

2. In cell A2, enter the formula \( = \text{INT}(\text{RAND}() \times 6 + 1) \) to simulate tossing a number cube. A random number from 1–6 should appear in the cell.

3. To model 100 tosses, select cells A2 to A101 and choose **Fill Down** from the Edit menu. If this option is unavailable, copy and paste the formula from A2 to cells A3 through A101. To copy and paste, you can copy cell A2, and then highlight cells A3 to A101 all at once before hitting paste. This will copy the formula 99 times.

4. Repeat steps 2–3 for cells B2 to B101, using appropriate cell references when you copy and paste.

5. In cell C2, enter the formula \( = \text{A2} + \text{B2} \). Select cells C2 to C101 and choose **Fill Down** from the Edit menu. If this option is unavailable, copy and paste the formula from C2 to cells C3 through C101. This column will give the sum of the two number cubes.

6. To see how many times a sum of 2 occurs, in cell E1, type the number 2 and in cell E2, enter the formula \( = \text{COUNTIF}(\text{C2:C101},2) \). To see how many times a sum of 3 occurs, in cell F1, type the number 3 and in cell F2, enter the formula \( = \text{COUNTIF}(\text{C2:C101},3) \).

7. To find the number of times the sums 4–12 occur, repeat step 6 for cells G1–G2, H1–H2, I1–I2, J1–J2, K1–K2, L1–L2, M1–M2, N1–N2, and O1–O2, increasing the number in the top row and at the end of the parentheses in the formula by 1 each time.

Your simulation is complete. Save your simulation for the Practice section.
Your Turn

Two fair number cubes labeled 1–6 are tossed. Find the probability that the two numbers will have a product of at least 12.

1. Make a possibility diagram to record all of the possibilities.

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2. There are ________ possible outcomes.

3. There are ________ favorable outcomes (outcomes with a product of at least 12).

4. The probability of tossing two numbers that have a product of at least 12 is ________.

Practice

Tell whether each statement is True or False.

1. Selecting the letter A from the word PROBABILITY is a compound event.

__________

2. Selecting the letter B from both of the words BASEBALL and ABLE is a compound event.

__________
Solve.

3. A deli offers vanilla, chocolate, and strawberry ice cream. The ice cream can go in a cone or a cup.
   a. Display the possible outcomes in a possibility diagram.
   
   b. There are _______ possible outcomes.

4. The following two spinners are spun.

   a. Display the possible outcomes in a possibility diagram.

   b. There are _______ possible outcomes.
A coin is tossed and a number cube labeled 1–6 is tossed. Use this information to complete problems 5–6.

5. Display the possible outcomes in a possibility diagram.

6. Find the probability for each outcome. Write each probability as a fraction in simplest form.

   a. P(heads, 6) ________
   b. P(heads, even number) ________
   c. P(tails, composite number) ________
   d. P(tails, number greater than 1) ________

Three fair coins are tossed. Use this information to complete problems 7–9.

7. Write the possible outcomes in a tree diagram.

8. Write each probability as a fraction in simplest form.

   a. P(3 heads) ________
   b. P(2 heads and 1 tails) ________
   c. P(1 heads and 2 tails) ________
   d. P(3 tails) ________

9. a. P(3 heads) + P(2 heads and 1 tails) + P(1 heads and 2 tails) + P(3 tails) = ________
   
   b. Explain what your answer to part a means.
Use your simulation from Technology Activity to answer problems 10–13.

10. Find your percentage of times that the sum was 3. Compare your percentage to the probability that the sum will be 3.

11. Find your percentage of times that the sum was 5. Compare the percentage to the probability that the sum is 5.

12. Find your percentage of times that the sum was 7. Compare the percentage to the probability that the sum is 7.

13. Find your percentage of times that the sum was 10. Compare the percentage to the probability that the sum is 10.
Problems Involving Taxes

You can use a bar model to help you solve different kinds of percent problems. One kind of problem you can solve is finding the amount of sales tax added to the purchase price of an item.

Maya purchases flowers for her mother. The total cost of the flowers is $28.50, and a 6% sales tax is added to the bill. What is the total bill?

![Bar Model]

Method 1
The model shows the following:
Sales tax = 6% of $28.50
\[ = \frac{6}{100} \times 28.50 \]
\[ = 1.71 \]
$28.50 + $1.71 = $30.21
The total cost of the flowers is $30.21.

Method 2
Sales tax:
100% ⇒ $28.50
1% ⇒ \[ \frac{28.50}{100} = 0.285 \]
6% ⇒ 6 \times 0.285 = $1.71
$28.50 + $1.71 = $30.21
The total cost of the flowers is $30.21.

Your Turn
Solve using both methods.

A tablet computer in a store costs $550. A sales tax of 7% will be added to the price. What is the total cost of the tablet?

Method 1
1. Sales tax = _______% of $550
\[ = \frac{7}{100} \times 550 \]
\[ = 38.50 \]
2. $550 + $38.50 = $______
3. The total cost of the tablet is $______.

Method 2
Sales tax:
1. 100% ⇒ $______
2. 1% ⇒ \[ \$______ \]
3. 7% ⇒ ______ \times ______ = $______
4. $550 + ______ = $______
5. The total cost of the tablet is $______.
Problems Involving Commission

A commission is a fee a salesperson earns when he or she makes a sale. It is a percent of the sale price.

Robyn earns a 4% commission on all the clothing she sells. She receives $250 for her sales this week. What is the dollar amount of her sales this week?

4% ⇒ $250
1% ⇒ $250 ÷ 4 = $62.50
100% ⇒ 100 • $62.50 = $6,250

The dollar amount of Robyn's sales is $6,250.

Your Turn

Solve the problem.

Mr. Boyd earns a 3% commission on all the cars he sells. If he receives $3,240 in commission, what is the dollar amount of his sales?

1. _______ % ⇒ $_______
2. 1% ⇒ $_______ ÷ _______ = $_______
3. 100% ⇒ 100 • $_______ = $_______
4. The dollar amount of Mr. Boyd's sales is $______.

Problems Involving Interest

When you deposit money in a savings account, the bank pays you interest on that money. The interest rate is the rate at which your money earns interest in a given amount of time.

Monish deposits $750 in a savings account. The bank will pay him 4% interest at the end of a year. How much interest will Monish receive?
Interest for the year = 4% of $750
\[= \frac{4}{100} \cdot 750\]
\[= 30\]

Monish will receive $30 in interest.

**Your Turn**

Solve the problem.

A robotics club has $10,000 in a bank account at the beginning of the year. Interest will be paid at a rate of 3% at the end of the year. How much interest will the robotics club receive for the year?

1. Interest for the year = \[\_\_\_/\ \] % of \[\_\_/\ \]
\[= \_\_\_/\ \cdot \_\_\_/\ \]
\[= \_\_/\ \]

2. The robotics club will receive \[\_\_/\ \] in interest for the year.

**Problems Involving Percent Increase/Markup**

A markup is a kind of percent increase. A price is marked up when a merchant sells an item for more than he paid for it.

A furniture store sells furniture at a 75% markup. The store pays $200 for a sofa. At what price will the store sell the sofa?

**Method 1**

\[
\text{Markup} = 75\% \text{ of } 200 \\
= \frac{75}{100} \cdot 200 \\
= 150
\]

The price is marked up by $150.

\[200 + 150 = 350\]

The sofa will sell for $350.

**Method 2**

\[
100\% \Rightarrow 200 \\
1\% \Rightarrow \frac{200}{100} = 2 \\
75\% \Rightarrow 75 \cdot 2 = 150
\]

The price is marked up by $150.

\[200 + 150 = 350\]

The sofa will sell for $350.
Your Turn

Solve the problem using both methods.

The weight of grain in a storage bin is 50 pounds. After another bag of grain is added to the bin, the weight of the grain increases by 40%. Find the weight after the bag of grain is added.

Method 1

1. $40\%$ of 50 = \[ \_ \cdot \_ = \_ \]
2. The weight of the grain increases by _______ pounds.
3. $50 + \_ = \_ $
4. The weight of the grain after the bag of grain is added is _______ pounds.

Method 2

1. $100\% \Rightarrow 50$ lb
2. $40\% \Rightarrow \_ \cdot \_ = \_ $ lb
3. The weight of the grain increases by _______ pounds.
4. $50 + \_ = \_ $
5. The weight of the grain after the bag of grain is added is _______ pounds.

Problems Involving Percent Decrease/Discount

A discount is a kind of percent decrease. A price is discounted when an item is on sale for less than its regular price.

The regular price for a backpack is $45. During a sale, its price is reduced to $36. Find the percent discount.

\[ \frac{\text{Regular price} - \text{Sale price}}{\text{Regular price}} \times 100\% \]

\[ \frac{45 - 36}{45} \times 100\% = 20\% \]

The percent discount is 20%.
Your Turn

Solve the problem.

The regular price for a jacket is $75. During a sale, its price is reduced to $45. Find the percent discount.

1. $\square - $\square = $\square$

2. The discount is $\square$.

3. $\square \Rightarrow 100\%$

\[
\begin{array}{c}
\$1 \Rightarrow \square \\
\% \\
\end{array}
\]

\[
\begin{array}{c}
\$\square \Rightarrow \square \cdot \square = \square \% \\
\end{array}
\]

4. The percent discount is _______%.

Practice

Solve. Show your work. Use bar models to help you.

1. Mrs. Price buys a printer that costs $350. How much does she pay for the printer if the sales tax rate is 7%?

2. A business deposits $30,000 into a savings account at the beginning of the year. Interest is paid at a rate of 4% at the end of the year. How much interest will the business receive for the year?
3. A bookstore buys paperback books at $1.50 each and sells them at an 80% markup. At what price does the bookstore sell each book?

4. A gallery owner receives a 5% commission on artwork sold in his gallery. If he receives $25,000 in commission, what is the dollar amount of the artwork sold?

5. Emily earned $500 last summer walking dogs. This summer she earns 25% more. How much does she earn this summer?

6. The original price of a camera is $850. During a sale, the selling price of the camera is $680. Find the percent discount.
7. Shayna buys a pair of shoes for $36. She also pays sales tax of 6%. How much does she pay for the shoes?

8. Alex had $600 in his savings account. After he withdraws money, he has $450 left in his account. By what percent did the money in his savings account decrease?

9. Debra earns a 4% commission on each piece of furniture she sells. If she receives $800 in commission, what is the dollar amount of the furniture she sold?

10. A gift shop sells snow globes at a 70% markup. The store pays $2.50 for each snow globe. At what price does the store sell each snow globe?
Finding the Quantity Represented By a Percent

You can use a bar model to help you solve different kinds of percent problems. One kind of problem you can solve is to find the quantity represented by a percent when given the whole quantity. In this case, you are finding a missing part.

15% represents 15 units for every 100 units.

So 15% of a whole quantity refers to the part of the quantity that represents 15%.

Find 15% of 180.

**Method 1**

The model shows the following:

100% ⇒ 180
1% ⇒ \(\frac{180}{100} = 1.8\)
15% ⇒ 15 \cdot 1.8 = 27
15% of 180 is 27.

**Method 2**

15% of 180 = \(\frac{15}{100} \cdot 180\)
= \(\frac{3}{20} \cdot 180\)
= \(\frac{540}{20}\)
= 27

15% of 180 is 27.

The phrase 15% of 180 means 15% times 180. The word of means multiply by in problems like this.
Your Turn

What is 30% of 360 meters?

Method 1

\[
\begin{align*}
\text{m (100\%)} & \quad \text{m (1\%)}
\end{align*}
\]

The model shows the following:
1. 100% ⇒ ______ m
2. 1% ⇒ ______ = ______ m
3. 30% ⇒ ______ \cdot ______ = ______ m
4. 30% of 360 meters is ______ meters.

Method 2

1. 30\% of 360 = \frac{30}{100} \cdot 360 = ______ m
2. 30\% of 360 meters is ______ meters.

Finding the Whole Given a Quantity and its Percent

You can use a bar model to help you find the whole quantity when given a part and its percent.

12\% of the children at a science fair received prizes. There are 63 children who received prizes. How many children were at the science fair?

\[
\begin{align*}
\text{? children (100\%)}
\end{align*}
\]

The model shows the following:
12\% ⇒ 63 children
1\% ⇒ \frac{63}{12} = 5.25 children
100\% ⇒ 100 \cdot 5.25 = 525 children

There were 525 children at the science fair.
Your Turn

Solve using the bar model.

Of Noah's book collection, 20% are nonfiction books. He has 35 nonfiction books. How many books are in Noah's book collection?

\[ \text{35 books (20\%)} \]

The model shows the following:

1. \( \text{1\%} \Rightarrow \text{books} \)
2. \( \text{100\%} \Rightarrow 100 \cdot \text{books} \)
3. There are \text{books} in Noah's collection.

Finding the Percent Represented by a Quantity

You can use a bar model to help you find the percent when given a part and the whole.

There are 102 apples at a fruit stand. There are a total of 340 pieces of fruit at the fruit stand. What percent of the fruits are apples?

**Method 1**

\[ \text{340 fruits (100\%)} \]

The model shows the following:

340 fruits ⇒ 100%
1 fruit ⇒ \( \frac{100}{340} \)
102 fruits ⇒ \( 102 \cdot \frac{100}{340} = 30\% \)
30% of the fruits are apples.

**Method 2**

\[ \frac{\text{apples}}{\text{all fruit}} = \frac{102}{340} = \frac{3}{10} \]

\( \frac{3}{10} \cdot 100\% = 30\% \)
30% of the fruits are apples.
Your Turn

Solve using both methods.

Jenna is using 60 beads to make a necklace. She uses 21 red beads and the rest are gold beads. What percent of the beads are red?

**Method 1**

- **60 beads (100%)**
- **21 red beads ( ? %)**

The model shows the following:

1. 60 beads ⇒ 100%
2. 1 bead ⇒ \( \frac{100}{1} = \) %
3. 21 beads ⇒ \( \frac{21}{1} \times \frac{100}{1} = \) %

**Method 2**

1. Fraction of red beads in necklace = \( \frac{21}{60} \)  
2. Percent of red beads in necklace = \( \frac{21}{60} \times \) %  
3. ______% of the beads are red.

**Practice**

Draw a bar model to help solve each problem.

1. Find 20% of 220.
2. Find 15% of 540.
3. Find 5% of 180.

4. 18% of the marbles in a jar are blue. There are 45 blue marbles in the jar. How many marbles are there in the jar?

5. Martin has a collection of stamps. He has 15 international stamps, which is 25% of his collection. How many stamps does Martin have in his collection?

6. An apple orchard has 24 trees that produce golden delicious apples. The golden delicious trees make up 8% of the orchard. How many trees are in the apple orchard?
7. In a neighborhood of 160 houses, 48 of the houses are made of bricks. What percent of the houses are brick houses?

8. The distance from Camila's house to her grandmother's house is 625 miles. Camila has traveled 175 miles so far on the way to her grandmother's house. What percent of the total distance has she traveled?

9. Mr. Norris is selling 75 items at a garage sale. Nine of the items he is selling are chairs. What percent of the items are chairs?