Physics Problems

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2 Introduction

Ignore the numbering on the problems. This is a historical glitch.

Problem 1

The volume charge density ρ_E within a sphere of radius r_0 is distributed in accordance with the following spherical symmetric relation

$$\rho_E(r) = \rho_0 \left[1 - \frac{r^2}{r_0^2}\right]$$

where r is measured from the center of the sphere and ρ_0 is a constant. For a point P inside the sphere ($r < r_0$), determine the electrical potential V. Let $V = 0$ at infinity.

Solution

In general the work done by an $\boldsymbol{E}$ field as it act on a charge q to move it from point A to point B is defined as the electric potential difference between points A and B. That is,

$$V_B - V_A = -\frac{W}{q} = -\int_A^B \boldsymbol{E} \cdot d\boldsymbol{\ell}. \tag{2.1}$$

In particular if $B = \infty$ and $A = r$, and we assume that $V_\infty = 0$, we have, from equation 2.1

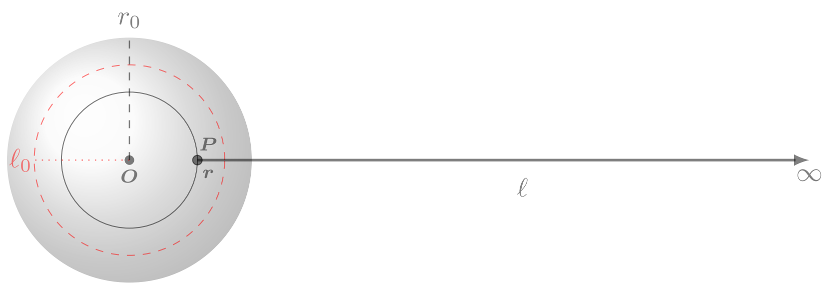
$$0 - V(r) = -\int_r^\infty \boldsymbol{E} \cdot d\boldsymbol{\ell},$$

that is

$$V(r) = \int_r^\infty \boldsymbol{E} \cdot d\boldsymbol{\ell}, \tag{2.2}$$

along the ℓ line as shown in Figure 1

Figure 1: Charged Sphere, observation point P and line of integration $\boldsymbol{\ell}$.



To find the electrical field we use Gauss law. That is, the total flow is equal to the total enclosed charge. That is, given a volume V with surface S and total charge Q, we have

$$\int_S \boldsymbol{E}\cdot\boldsymbol{n} = \frac{Q}{\epsilon_0} \tag{2.3}$$

Note that we should find the electical field everywhere as a function of the distance ℓ to the center of the sphere before integrating it. We consider two cases

(i) $\ell_0 < r_0$. Pick a sphere of radius ℓ_0. We want to evaluate the Gauss integral 2.3. At the sphere of radius ℓ_0. The field is constant along this sphere and so the integral is equal to the magnitue of $\boldsymbol{E}$, which we call E (no bold-face) times the area of the sphere. That is equation 2.3 becomes

$$4\pi\ell_0^2 E(\ell_0) = \frac{Q(\ell_0)}{\epsilon_0}$$

from which

$$E(\ell_0) = \frac{Q(\ell_0)}{4\pi\epsilon_0\ell_0^2}$$

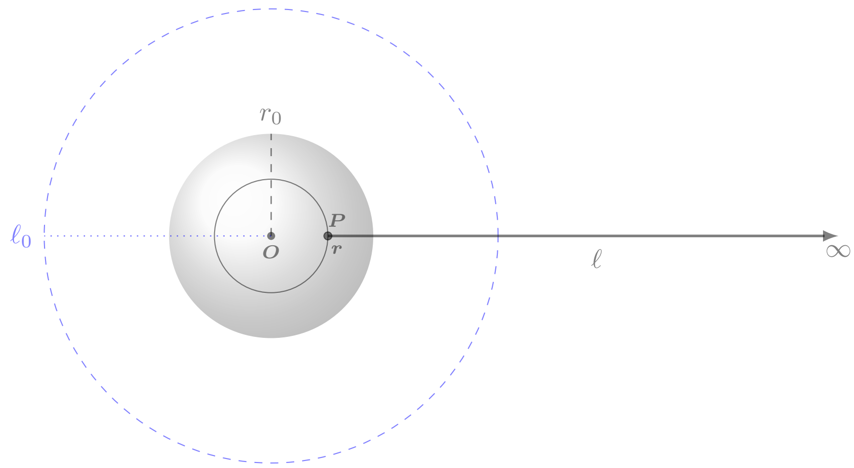
We find $Q(\ell_0)$ by integration. That is, the total charge in the sphere with radius ℓ_0 is

$$\begin{aligned} Q(\ell_0) = \int_V \rho_E(\ell)dV &= 4\pi\int_0^{\ell_0} \ell^2\rho_E(\ell)d\ell \\ &= 4\pi\int_0^{\ell_0}\rho_0\ell^2\left[1-\frac{\ell^2}{r_0^2}\right]d\ell \\ &= 4\pi\ell_0^3\rho_0\left[\frac{1}{3}-\frac{\ell_0^2}{5r_0^2}\right] \end{aligned} \tag{2.4}$$

So, we find that

$$E(\ell_0) = \frac{\cancel{4\pi}\ell_0^{\cancel{3}}\rho_0\left(\frac{1}{3}-\frac{\ell_0^2}{5r_0^2}\right)}{\cancel{4\pi\ell_0^2}\epsilon_0} = \frac{\rho_0\ell_0}{\epsilon_0}\left(\frac{1}{3}-\frac{\ell_0^2}{5r_0^2}\right)$$

Figure 2: Charged Sphere, observation point P and line of integration $\boldsymbol{\ell}$. This for the case $\ell_0 > r_0$, outside of the charged sphere, in blue



(ii) $\ell_0 \geq r_0$. Refer there to figure 2. Pick a sphere of radius ℓ_0. We want to evaluate the Gauss integral 2.3. We can think of the elecrical field as concentrated in the center of the sphere with the total charge, already computed in equation 2.4 (changing ℓ_0 by r_0,

$$Q(r_0) = 4\pi r_0^3 \rho_0 \left[\frac{1}{3} - \frac{1}{5}\right] = \frac{8\pi r_0^3 \rho_0}{15}$$

The total flow is

$$\Phi = EA = E(4\pi l_0^2) = \frac{Q(r_0)}{\epsilon_0}$$

and so the electrical field due to this concentrated charge is

$$E = \frac{Q(r_0)}{4\pi \ell_0^2} = \frac{8\pi r_0^3 \rho_0 / 15}{4\pi \ell_0^2 \epsilon_0} = \frac{8 r_0^3 \rho_0}{60 \ell_0^2 \epsilon_0}$$

In summary, we found the (magnitude) electrical field

$$E(\ell) = \begin{cases} \frac{\rho_0 \ell_0}{\epsilon_0}\left(\frac{1}{3} - \frac{\ell_0^2}{5 r_0^2}\right) & \text{if } \ell < r_0 \\ \\ \frac{2 r_0^3 \rho_0}{15 \ell^2 \epsilon_0} & \text{if } \ell \geq r_0 \end{cases}$$

Note that I changed back from ℓ_0 to ℓ because it is more convenient (it is just notation).

We now evaluate the integral 2.2. That is

$$
\begin{aligned}
V(r) &= \int_r^{\infty} \boldsymbol{E} \cdot d\ell \\
&= \int_r^{r_0} \boldsymbol{E} \cdot d\ell + \int_{r_0}^{\infty} \boldsymbol{E} \cdot d\ell \\
&= \int_r^{r_0} \frac{\rho_0 \ell}{\epsilon_0} \left(\frac{1}{3} - \frac{\ell^2}{5r_0^2} \right) d\ell + \int_{r_0}^{\infty} \frac{2r_0^3 \rho_0}{15\ell^2 \epsilon_0} d\ell \\
&= \frac{\rho_0}{\epsilon_0} \frac{\ell^2}{6} \bigg|_r^{r_0} - \frac{\rho_0}{20\epsilon_0 r_0^2} \ell^4 \bigg|_r^{r_0} + \frac{2\rho_0 r_0^3}{15\epsilon_0} \frac{\ell^{-1}}{-1} \bigg|_{r_0}^{\infty} \\
&= \frac{\rho_0}{6\epsilon_0}(r_0^2 - r^2) - \frac{\rho_0}{20\epsilon_0 r_0^2}(r_0^4 - r^4) + \frac{2\rho_0 r_0^3}{15\epsilon_0 r_0} \\
&= \frac{\rho_0}{\epsilon_0} \left(\frac{r_0^2}{6} - \frac{r_0^2}{20} + \frac{2r_0^2}{15} - \frac{r^2}{6} + \frac{r^4}{20r_0^2} \right) \\
&= \frac{\rho_0}{\epsilon_0} \left(\frac{r_0^2}{4} - \frac{r^2}{6} + \frac{r^4}{20r_0^2} \right)
\end{aligned}
$$

We want

$$
\text{answer} = \frac{\rho_0}{\epsilon_0} \left(\frac{r_0^2}{4} - \frac{r^2}{6} + \frac{r^4}{20r_0^2} \right)
$$

Problem 2

A 2.0-m-long wire carries a current of 8.2A and is immersed within a uniform magnetic field $\vec{B}$. When this wire lies along the $+x$ axis, a magnetic force $\vec{F} = (-2.5\hat{\boldsymbol{j}})$ N acts on the wire, and when it lies on the $+y$ axis, the force is $\vec{F} = (2.5\hat{\boldsymbol{i}} - 5.0\hat{\boldsymbol{k}})$ N. Find $\vec{B}$.

Solution

$$
\vec{F} = I\boldsymbol{\ell} \times \vec{B}
$$

We need to find B_x, B_y, and B_z. So we set up a system of several equations with three unknonws.

- From the x direction we see that

  $$
  -2.5\hat{\boldsymbol{j}} = I\hat{\boldsymbol{i}}\ell \times (B_x\hat{\boldsymbol{i}} + B_y\hat{\boldsymbol{j}} + B_z\hat{\boldsymbol{k}})
  $$

  and from

  $$
  \hat{\boldsymbol{i}} \times \hat{\boldsymbol{i}} = 0 \quad \hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}} \quad \hat{\boldsymbol{i}} \times \hat{\boldsymbol{k}} = -\hat{\boldsymbol{j}}
  $$

  and the distributive law of the cross product we see that this reduces to

  $$
  -2.5\hat{\boldsymbol{j}} = I\ell B_y\hat{\boldsymbol{k}} - I\ell B_z\hat{\boldsymbol{j}}
  $$

and by equating vectors, it has to happen that

$$B_y = 0 \quad B_z = \frac{2.5}{I\ell} = \frac{2.5}{(8.2A)(2m)} \approx 0.152T.$$

- From the y direction

$$2.5\hat{\boldsymbol{i}} - 5.0\hat{\boldsymbol{k}} = I\hat{\boldsymbol{j}}\ell \times (B_x\hat{\boldsymbol{i}} + B_y\hat{\boldsymbol{j}} + B_z\hat{\boldsymbol{k}})$$

from which

$$2.5\hat{\boldsymbol{i}} - 5.0\hat{\boldsymbol{k}} = I\ell(-B_x\hat{\boldsymbol{k}} + B_z\hat{\boldsymbol{i}})$$

Matching coefficients we find

$$B_z = \frac{2.5}{I\ell} \approx 0.152T \qquad B_x = \frac{5.0}{I\ell} = \frac{5.0}{(8.2A)(2.0m)} = 0.305T$$

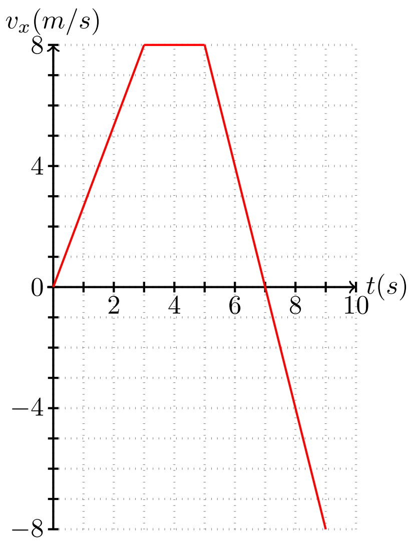


Figure 3: Problem P2.12

Problem 12

A student drives a moped along a straight road as described by the velocity versus-time Figure 3. Sketch this graph in the middle of a sheet paper.

(a) Directly above the graph, sketch a graph of the positions versus time, aligning the time coordinates of the two graphs.

Solution:

- Between 0 and 3 seconds the velocity has a constant acceleration of

$$a = \frac{8\text{m/s}}{3s} = \frac{8}{3}\ \text{m/s}^2.$$

  So the position is given by

$$x(t) = x(0) + v(0)(t - t_0) + \frac{1}{2}a(t)(t - t_0)^2 = x(0) + \frac{1}{2}\frac{8}{3}(t - t_0)^2$$

  Assuming $x(0) = 0$ (this data should be provided in the problem) and knowing that $t(0) = 0$ we find

$$x(t) = (4/3)t^2$$

  Figure 4 shows in blue the position curve (which is a parabola going through the origin)

- Between $t_0 = 3$ seconds and 5 seconds the velocity is constant of 8 m/s, and so the displacement is given by

$$x(t) = x(t_0) + v(t)(t - t_0) = x(0) + 8(t - 3),$$

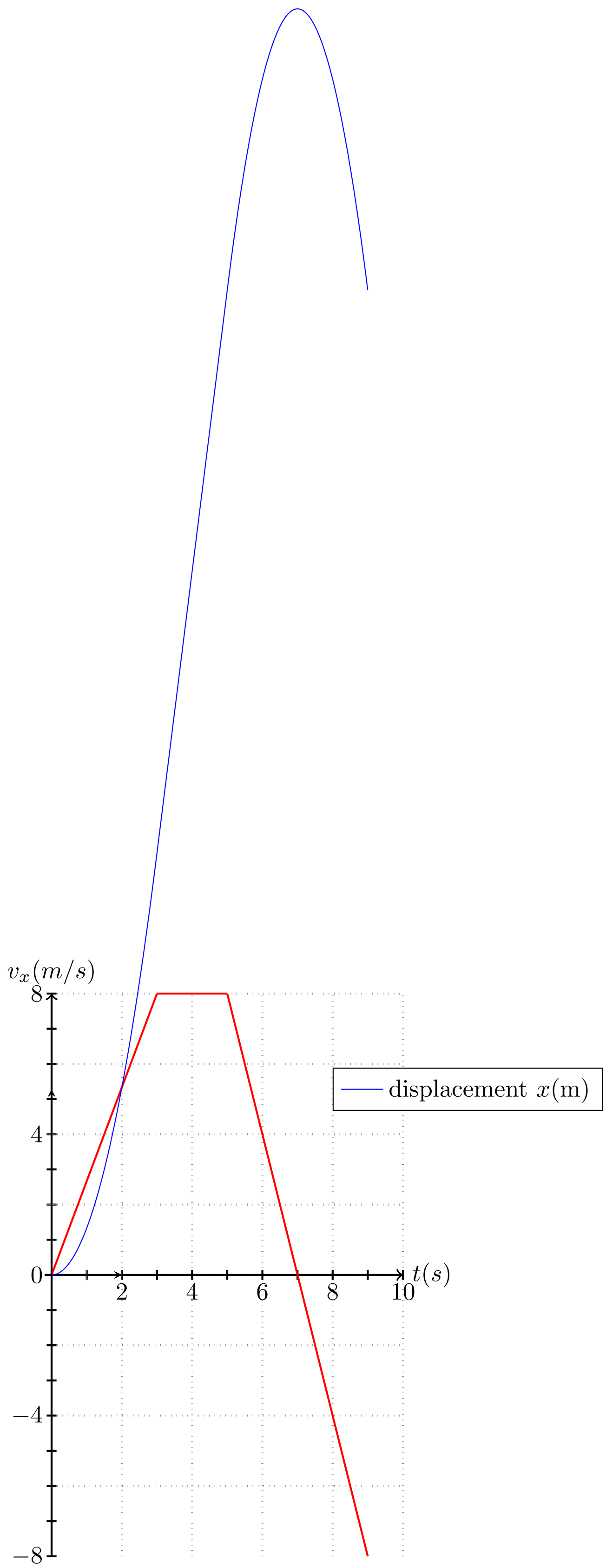


Figure 4: Problem P2.12. Velocity curve in red, displacement in blue.

now $x(t_0) = x(3) = (4)(3)(3)/3 = 12$ m. and so

$$x(t) = 12 + 8(t - t_0) = 12 + 8(t - 3),$$

Note that this displacement is out of reality. Who can mop that far? $x(5) = 28$ meters!!! even 28 feet is too much!!

- Between $t_0 = 5$ and 9 seconds we have a constant acceleration of

$$a = (-8 - 8)/4 = -4 \text{ m/s}^2$$

So the displacement is given by

$$x(t) = x(t_0) + v(t_0)(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

with $x(t_0) = 28$ m. That is

$$x(t) = 28 + 8(t - 5) - 2(t - 5)^2$$

(b) Sketch the graph of the acceleration versus time directly below the velocity-versus-time graphi, again aligning the time coordinates. On each graph, show the numerical values of x and a_x for all points of inflection.

Solution: The acceleration, as found above is given by

$$a(t) = \begin{cases} 8/3 \quad \text{m/s}^2 & 0 < t \leq 3 \\ 0 \quad \text{m/s}^2 & 3 < t \leq 5 \\ -4 \quad \text{m/s}^2 & 5 < t \leq 9 \end{cases}$$

This curve is sketeched in Figure 5

A point of inflection happens when the concavity (second derivative) changes sign. That is, at the point of inflection the second derivative is 0, while to its left has one sign and to its right another.

The velocity curve in Figure 3 does not have points of inflection. The derivative is 0 everywhere except at the corners (at $x = 0, 3, 5$) where the derivate does not exist. The displacement curve is a composed of three curves. The parabolic curve (first piece) has no inflection points, then there is a straight line (the middle piece) which does not have an inflection point and finally another parabola which does not have inflection points. The acceleration does not have inflection points. The argument is the same as that for the velocity curve.

So ... what does the problem mean by "inflection" points? It is not in the calculus sense. Perhaps they are talking about the extremes of the line which is in the middle of the two parabolas (one concave up, the other concave down). Those would be $x = 3, 5$. If that is the case, just put points there, but they are not strictly speaking inflection points.

3 Problem 13

A particle starts from rest and accelerates as shown in Figure 6. Determine

(a) The particle's speed at $t = 10.0$ s and at $t = 20.0$ s, and

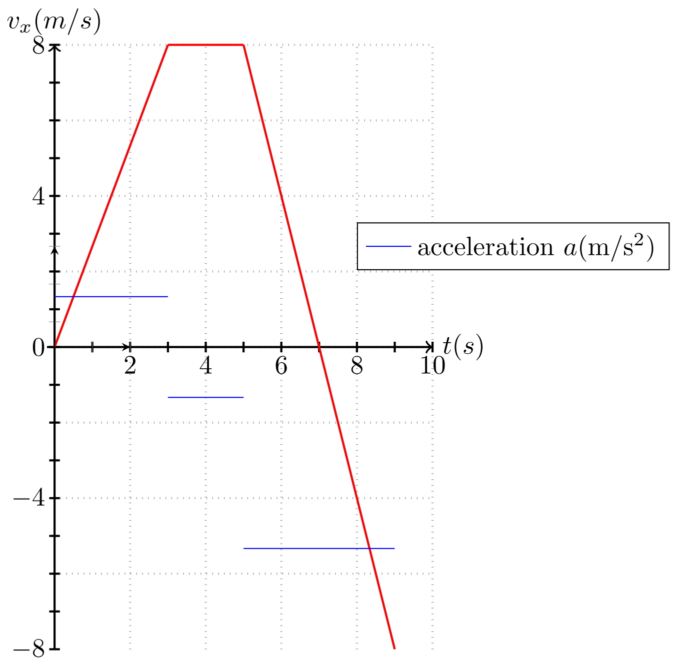


Figure 5: Problem P2.12. Velocity curve in red, displacement in blue.

Solution: We know that for constant acceleration the speed is given by

$$v(t) = v(t_0) + a(t - t_0).$$

Since the particle starts at rest, this means $v(t_0 = 0) = 0$, so

$$v(t) = at = 2t.$$

At $t = 10$ the velocity is $v(10) = 20$ m/s.

Now, from time $t_0 = 10$ s to $t = 15$ s, the acceleration is 0, that means the velocity is constant. So the velocity here would be $v(10 - 15) = 20$ m/s.

From time $t_0 = 15$ s to $t = 20$ s, the acceleration is -3 m/s^2.

so the velocity is computed as

$$v(t) = v(t_0) + a(t - t_0) = 20 \text{ m/s} - 3(t - 15) \text{ m/s}.$$

So at $t = 20$, the velocity is

$v(20) = [20 - (5)(3)]$ m/s $= 5$ m/s.

(b) The distanced traveled in the first 20.0 s.

Solution: The distanced traveled can be computed directly from the acceleration using the equation

$$x(t) = x(t_0) + v(t_0)(t - t_0) + \frac{1}{2}a(t - t_0)^2,$$

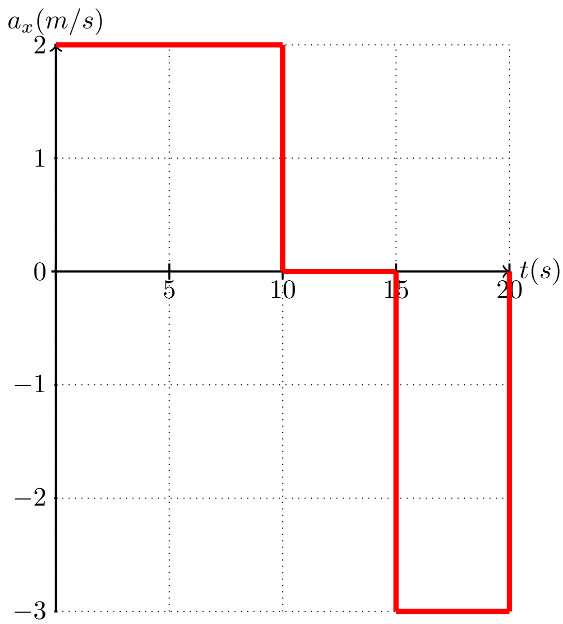


Figure 6: Problem P2.13

In three pieces. That is

(i) Between times 0 and 10 seconds, and assuming $x(0) = 0$,and since $v(0) = 0$, we find

$$x(t) = \frac{1}{2}at^2 = t^2,$$

so at $t = 10$ s, we have $x(10) = 100$ m.

(ii) Between seconds 10 and 15, the velocity is constant of $v(t) = 20$ m/s, so

$$x(t) = x(10) + v(10)(t - 10) = 100 + 20(t - 10) \text{ m/s}$$

So, at $t = 15$ we have $x(15) = 100 + 100 = 200$ m.

(iii) Finally, between times 15 and 20 seconds, we have

$$x(t) = x(15) + v(15)(t - 15) - (3/2)(t - 15)^2$$

That is

$$x(t) = 200 + v(15)(t - 15) - (3/2)(t - 15)^2 = 200 + 20(t - 15) + (3/2)(t - 15)^2 \text{ m.}$$

so

$$x(20) = 200 + 100 - (3/2)(25) = 262.5 \text{ m}$$

4 Problem 20

In Example 2.7, we investigated a jet landing on an aircraft carrier. In a later maneuver, the jet comes in for a landing on solid ground with a speed of 100 m/s, and its acceleration can have a maximum magnitude of $5.00\,\mathrm{m/s^2}$ as it comes to rest. (a) From the instant the jet touches the runway, what is the minimum time interval needed before it can come to rest? (b) Can this jet land at a small tropical island airport where the runway is 0.800 km long? (c) Explain your answer.

Solution

(a) We need to find the time to stop by using the largest negative acceleration of $a = -5.00\mathrm{m/s^2}$ and an initially speed of 100 m/s.

Integrating $a = dv/dt$ we find

$$v - v_0 = a(t - t_0).$$

From $v = 0$ (rest) and $t_0 = 0$,

$$v = at \qquad \Rightarrow \quad t = -\frac{v_0}{a} = \frac{100m/s}{5m/s^2}\,\mathrm{s} = 20\,\mathrm{s}.$$

(b) Again, we use the maximum negative acceleration of $a = -5\mathrm{m/s^2}$. The distance traveled by the aircraft is

$$x = x_0 + v_0 t + \frac{1}{2}at^2,$$

with $x_0 = 0$,

$$x = (100\mathrm{m/s})(20s) - \frac{1}{2}(5\mathrm{m/s^2})20^2\mathrm{s}^2 \;= 2000\,\mathrm{m} - 1000\,\mathrm{m} = 1000\,\mathrm{m}.$$

(c) Sorry :(Short by 200 meters. Cannot make it. $800 - 1000 = -200$

5 Problem 22

A speedboat moving at 30.0 m/s approaches a no–wake buoy marker 100 m ahead. The pilot slows the boat with a constant acceleration of -3.50 $\mathrm{m/s^2}$ by reducing the throttle. (a) How long does it take the boat the reach the buoy? (b) What is the velocity of the boat when it reaches the buoy?

Solution:

(a) Distance to cover $x = 100$ m. Acceleration $a = -3.50\mathrm{m}s^2$. From

$$x = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2.$$

With $x_0 = 0$ and $t_0 = 0$ we have

$$x = v_0 t + \frac{1}{2}at^2.$$

Replacing numbers

$$100 = 30\,t - \frac{1}{2}(3.50)t^2$$

Multiply by 4 and

$$7\,t^2 - 120\,t + 400 = 0.$$

The solution to this quadracic equation is

$$t = \frac{120 \pm \sqrt{120^2 - (4)(7)(400)}}{14}.$$

That is

$$t = \frac{20}{7}(3 \pm \sqrt{2})\,\text{s}.$$

Both times are positive.

$$t \approx 4.53082\,\text{s} \quad \text{and} \quad t \approx 12.612\,\text{s}$$

The time to reach the buoy is $t = 4.54082$s. The second time $t = 12.612$ happens because the acceleration is negative and after stopping, ahead of the buoy the speedboat will return to it about 8 seconds later.

(b) Velocity is

$$v = v_0 + at = 30.0\,\text{m/s} - 3.50\,(\text{m/s}^2)(4.54082)\,\text{s} = 14.10713\,\text{m/s}.$$

Problem 24

In Example 3.5, we fond the centripetal acceleration of the Earth as it revolves around the Sun. From information on the end papapers of this books, compute the centripetal acceleration of a point on the surface of the Earth at the equator caused by the rotation of the Earth about its axis.

Solution: The centripetal acceleration is given by the equation

$$\frac{v^2}{r}\ \text{m/s}^2.$$

We need to compute the velocity of a point in the Equator, as the earth rotates. The radius of the earth at the Equator (from Wikipedia) is

$$r = 6378137 \text{ meters}$$

We use this radius to find the speed at a point on the Equator. This is

$$v = \frac{2\pi r}{T}$$

where $T = 24$ hours or $T = (24)(3600) = 86400$ seconds. Then,

$$v = 463.831211639 \text{ m/s}.$$

Then

$$a = \frac{v^2}{r} = 0.03373075756 \text{ m/s}^2.$$

Problem 30

A point on a rotating turntable 20.0 cm from the center accelerates from rest to a final speed of 0.700 m/s in 1.75 s. At $t = 1.25$ s, find the magnitude an direction of (a) the radial acceleration, (b) the tangential acceleration, and (c) the total acceleration of the point.

Solution: I believe that the order of the answers should be change for convenience. That is, we need to find the tangent acceleration to be able to find the radial acceleration, since the radial acceleration is a function of the tangent velocity, which is a function of the tangent acceleration. That is,

- **(b) Tangent Acceleration.**

  we assume that the turning table is speeding in a uniform fashion (that is with **constant tangent acceleration** . The tangential acceleration of the point (since we assume that the point is not moving in the radial direction, otherwise the trajectory would be instead of a circle, a spiral) is given by the equation

$$a_T = \frac{v(t_f) - v(t_0)}{t_f - t_0}$$

  with $t_0 = 0$, $t_f = 1.75$, and $v(0) = 0$ (from rest) we find

$$a_T = \frac{0.7}{1.75}\ \mathrm{m/s^2} = 0.4\ \mathrm{m}s^2$$

  To find the exact direction of the tangential and radial acceleration, we should find the total rotation angle. Let us assume that the initial angle is $\theta_0 = 0$. To find the total rotated angle we first find the total tangential distance traveled by the particle. This is

$$x = x_0 + v_0(t - t_0) + \frac{1}{2}a_T(t - t_0)^2,$$

  with $x_0 = 0$ m, $v_0 = 0m/s$, $t = 1.2$ s and $a_T = 0.4\mathrm{m/s^2}$ we find that

$$x = 0.3125\ \mathrm{m}.$$

  We should convert from meters to radians. Every turn sweeps a distance of $C = 2\pi r = 1.25663706143592$ m, so the total number of turns is

$$\mathrm{turns} = \frac{x}{C} = 0.248679598581086$$

  Now we convert to radians. Since 1 turn is 2π, then

$$\theta = (turns)(2\pi) = 1.5625\mathrm{radians} \approx 90\mathrm{degrees}$$

  So the answer here is

$$a_T = \mathrm{m/s^2} = 0.4\mathrm{m/s^2}$$

  90 degrees from starting angle

- **(a) Radial Acceleration.**

  The radial acceleration produced by the rotation is outward and given by the formula

  $$a_r = \frac{v^2(t)}{r^2}$$

  We use the time $t = 1.25$, but before we find $v(t)$. Since the tangent acceleration is constant, then the tangent velocity is given by

  $$v(t) = v(t_0) + a_T(t - t_0)$$

  With $t_0 = 0$ s, $t = 1.25$ s, $v(0) = 0$ m/s, and $a_T = 0.4\text{m/s}^2$ we find

  $$v(1.25) = 0.5\,\text{m/s}$$

  From which the magnitude of the radial acceleration is (recall 20 cm = 0The.2 m)

  $$a_r(1.25) = \frac{0.5^2}{0.2}\ \text{m/s}^2 = (1/4)(5) = (5/4) = 1.25\ \text{m/s}^2$$

  The radial direction has a phase shift (direction) of 90 degrees with respect to the tangential acceleration. This means that if the table is turning counter–clockwise the inward acceleration would have a direction of 180 degrees while the outward a direction of 0 degrees. The acceleration due to the speed is outward. If the table is turning clock–wise the outward acceleration would have a direction of 180 degrees, from the initial direction of the particle.

- **Total Acceleration:** The magnitude of the total acceleration is given by

  $$a = \sqrt{a_T^2 + a_r^2} = 1.31244047484067\ \text{m/s}^2$$

  With a direction of

  $$\arctan(a_r/a_T) = 1.25109338\ \text{rad} \approx 72\ \text{degrees}$$

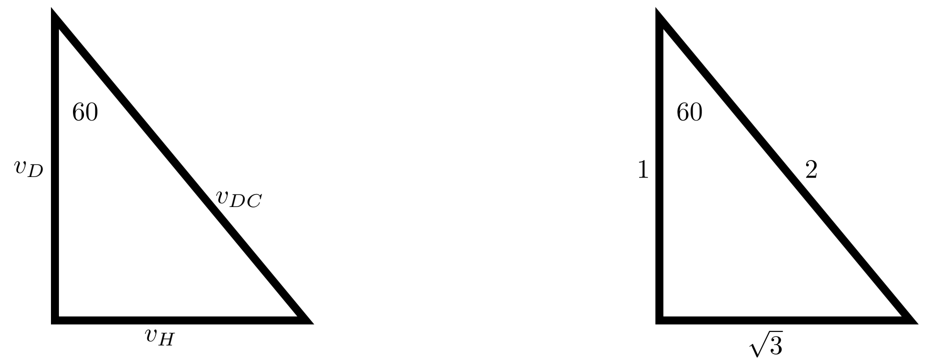
  with respect to the tangent direction.

Problem 36

A car travels due east with a speed 50.0 km/h. Raindrops are falling at a constant speed vertically with respect to the Earth. The traces of the rain on the side-windows of the car make an angle of 60.0 degrees with the vertical. Find the velocity of the rain with respect to (a) the car and (b) the Earth.

Solution: There is a triangle that we draw in the window of the car.

The vertical side has a length represents the speed of the drops with respect to the Earth V_D, the horizontal line is the speed of the car v_H, and the diagonal V_{DC} is the speed of the drop with respect to the car. The angle at the top is 60 degrees. So we have. For the problem refer to Figure 5 where the right triangle has the proportions of the triangle of the problem.



(a) The diagonal of the triangle.

$$\frac{v_H}{v_{DC}} = \frac{\sqrt{3}}{2},$$

since $v_H = 50$, we see that

$$v_{DC} = \frac{2}{\sqrt{3}} V_H \approx 57.74$$

(b) The vertical line (velocity of the drop which respect to the Earth)

$$\frac{v_H}{v_D} = \sqrt{3} \approx 1.7320$$

from which

$$v_D = \frac{v_H}{1.7320} = \frac{50}{1.7320} \approx 28.87$$

Checking:

$$v_{DC} = \sqrt{v_D^2 + v_H^2} = \sqrt{28.87^2 + 50^2} \approx 57.74 \text{ good}$$

Problem 4

Two forces, $\mathbf{F_1} = (-\mathbf{6\hat{i}} - \mathbf{4\hat{j}})$ N and $\mathbf{F_2} = (-\mathbf{3\hat{i}} + \mathbf{7\hat{j}})$ N , act on a particle of mass 2.00 kg that is initially at rest at coordinates (-2.00 m, + 4.00 m). (a) What are the components of the particles's velocity at $t = 10.0$ s? (b) In what direction is the particle moving at $t = 10.0$ s? (c) What displacement does the particle undergo during the first 10.0 s? (d) What are the coordinates of the particle at $t = 10.0$ s?

Solution:

(a) Initial coordinates and velocities are

$$\begin{aligned} x_0 &= -2\text{m} \qquad & y_0 &= 4\text{m} \\ v_{x0} &= 0\text{m} \qquad & v_{y0} &= 0\text{m} \end{aligned}$$

Total forces:

$$F_x = (-6-3)N = -9N \qquad F_y = -4+7N = 3N,$$

Accelerations (are constant)

$$a_x = \frac{F_x}{2}\text{m/s}^2 = -4.5\text{m/s}^2 \qquad a_y = \frac{F_y}{2}\text{m/s}^2 = 1.5\text{m/s}^2.$$

Velocities at $t = 10$ s are

$$v_x(10) = 0 + a_x t \text{ m/s} = -45 \text{ m/s} \qquad v_y(10) = 0 + a_y t \text{ m/s} = 15 \text{ m/s}$$

(b) At $t = 10$ s, the particle is moving in the direction (-45,15), that is, in the direction (-3,1). Three units left, one unit up. The angle is

$$\theta = \pi - \arctan\left(\frac{1}{3}\right) \text{ radians} \approx 2.82 \text{ radians} \approx 161.57 \text{ degrees.}$$

(c) The displacement along each coordiante is

$$\begin{aligned} x &= x_0 + v_{x0}t + \frac{1}{2}a_x t^2 = -(2.25)(100)m = -225m \\ y &= y_0 + v_{y0}t + \frac{1}{2}a_y t^2 = (0.75)(100)m = 75m \end{aligned}$$

The total displacement is

$$d = \sqrt{225^2 + 75^2} \text{ m} \approx 237.17 \text{ m.}$$

(d) The coordinates were found in part (c). Those are

$$(x, y) = (-225, 75) \text{ m.}$$

Problem 8

A 3.00 kg object is moving in a plane with its x and y coordinates given by $x = 5\,t^2 - 1$ and $y = 3\,t^3 + 2$, where x and y are in meters and t is in seconds. Find the magnitude of the net force acting on this object at $t = 2.00$ s.

Solution: To find the force we need to find the acceleration. We use

$$a_x = \frac{d^2x}{dt^2} \qquad a_y = \frac{d^2y}{dt^2}$$

That is

$$a_x = 10 \qquad a_y = 18t,$$

and at $t = 2.00$ s, we find

$$a_x = 10 \text{ m/s}^2 \qquad a_y = 36 \text{ m/s}^2$$

For a total acceleration of

$$a = \sqrt{a_x^2 + a_y^2} \approx 37.36 \text{ m/s}^2$$

and so the net force is

$$\|F\| = ma = (41.19)(3)N \approx 112.09 \text{ N}.$$

Problem 12

A man weights 900 N on the Earth, what would he weigh on Jupiter, where the free-fall acceleration is 25.8 m/s^2.

Solution: The mass should be the same, no matter where. The mass, based on the Earth gravity is

$$m = \frac{F}{g} = \frac{900}{9.8} \text{ Kg} = 91.83\text{Kg}.$$

The weight in Jupier would be

$$W_J = mg_J = (91.83)(25.8) \text{ N} = 8433.3673 \text{ N}$$

. . . . .he can barely walk.. too heavy :(

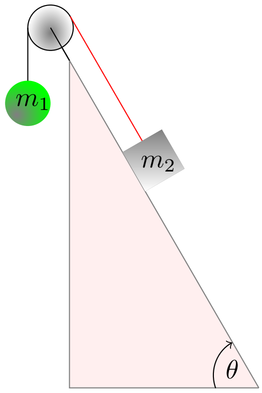


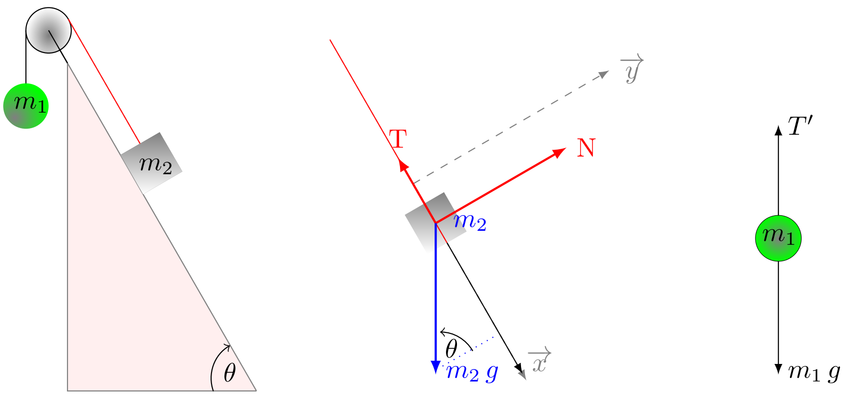
Figure 7: P4.30

Problem 30

Two objects are connected by a light string that passes over a frictionless pulley as shown in the Figure 7 . Assume the incline is frictionless and take $m_1 = 2.00$kg, $m_2 = 6.00$ kg, and $\theta = 55.0°$. (a) Draw free-body diagrams of both objects. Find (b) the magnitude of the acceleration of the objects, (c) the tension in the string, and (d) the speed of each object 2.00 s after it is released from rest.

Solution:

(a) Next is the free body diagram:



(b) Assuming that the string is not elastic so that the length is constant at all times, the two bodies are moving with the same speed and acceleration.

The hanging body has only two forces acting on it. They are the tension T' up and the gravity $m_1 g$ down. Then we have

$$T' - m_1 g = m_1 a \tag{5.5}$$

For the second body m_2 we have no component of N along the $\overrightarrow{x}$ axis. The tenstion T and the component of $m_2 g$ along the plane produces a second equation

$$m_2 g \sin\theta - T = m_2 a \tag{5.6}$$

The tension in the string is the same. That is $T = T'$, so adding equations 5.5 and 5.6 we find

$$m_2 g \sin\theta - m_1 g = (m_1 + m_2)a$$

From which the acceleration a is equal to

$$a = \frac{m_2 g \sin\theta - m_1 g}{m_1 + m_2} = \frac{g(m_2 \sin\theta - m_1)}{m_1 + m_2} \approx 3.57\,\mathrm{m/s^2}.$$

in the positive direction $\overrightarrow{x}$ for the body m_2 and upward for the body m_1. How can we know if this formula and the result make sense?

- Dimensional analysis. The units of a are $\mathrm{m/s^2}$. That is good.
- Extreme values
  - Both masses so $m_1 = m_2 = 0$. No sense, no physics here.
  - Mass $m_2 = 0$. Free fall for m_1 and $a = g$, checks good.
  - Mass $m_1 = 0$. A simple object down the ramp with acceleration $a = g \sin theta$, correct.
  - Angle $\theta = 90$. This is just a pulley and the two objects hanging. From it, here $a = g(m_2 - m_1)/(m_2 + m_1)$, Good.
  - Angle $\theta = 0$ Object going down with acceleration $a = -g m_1/(m_1 + m_2)$. Not obvious here, except that the weight m_2 is slowing it down to avoid free fall.
  - Direction of movement. Since m_2 is higher we could think that is moving down the ramp and up the hanging string, however the angle makes a difference. As in the previous step, is $\theta = 0$ it moves in the other direction. Here is an interesting problem. What is the angle θ so that $a = 0$? (hint: solve $m_2 \sin\theta - m_1 = 0$.

(c) The tension of the string is easier to compute from equation 5.5. That is,

$$T' = m_1(a + g) \approx 26.74N.$$

(d) After two seconds from released from rest, the hanging object if being pulled up at the speed of

$$v = v_0 + at = 0 + at = at \approx 7.14\,\mathrm{m/s}.$$

The speed of the object along the ramp is the same since the string is assumed to be inelastic.

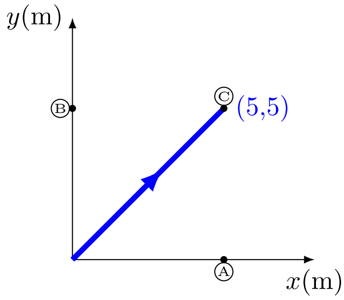


Figure 8: Figure P6.42 Problems 42 through 45.

Problem 43

A foce acting on a particle moving in the xy plane is given by $\boldsymbol{F} = (2y\hat{\boldsymbol{\imath}} + x^2\hat{\boldsymbol{\jmath}})$, where $\boldsymbol{F}$ is in Newtons and x and y are in meters. The particle moves from the origin to a final position having coordinates $x = 5.00$ m and $y = 5.00$ m as showin in Figure 8. Calculate the work done by $\boldsymbol{F}$ on the particle as it moves along the blue path.

Solution: By definition

$$W = \int_S \boldsymbol{F}.d\boldsymbol{r}$$

where S is some path, In this case we have that the path S is given by the parametric equation

$$x(s) = x$$
$$y(s) = x$$

for $x \in [0,5]$. Or in a more physics community notation the path is given by $S = x\hat{\boldsymbol{\imath}} + x\hat{\boldsymbol{\jmath}}$, $x \in [0,5]$. So the expression $d\boldsymbol{r}$ really means

$$d\boldsymbol{r} = dx(\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}) = dx(1,1).$$

Now for the force along this trajectory (the trajectory (x,x), we have $y = x$, so the force is actually given by $\boldsymbol{F} = (2x, x^2)$. and the dot product

$$\boldsymbol{F} \cdot d\boldsymbol{r} = (2x, x^2) \cdot (1,1)dx = (2x + x^2)dx.$$

So we evaluate

$$W = \int_S \boldsymbol{F}.d\boldsymbol{r} = \int_0^5 (2x + x^2)dx = x^2 + \frac{x^3}{3}\bigg|_0^5 = 66.666\cdots \text{J}$$

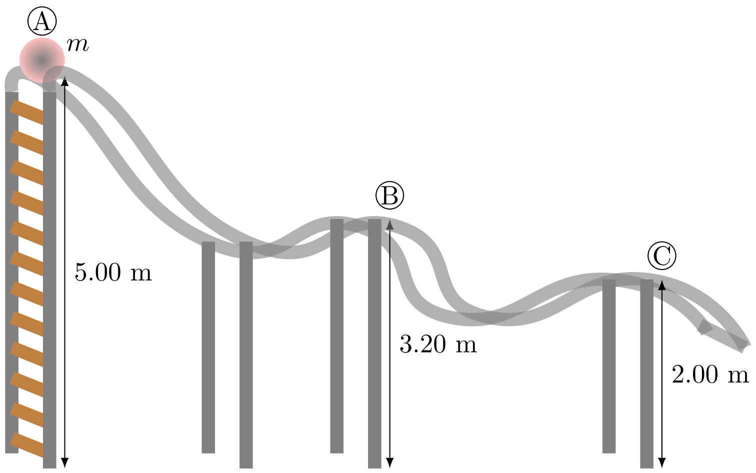


Figure 9: P7.6

Problem 6

A block of mass $m = 5.00$ kg is released from point Ⓐ and slides on the frictionless track shown in Figure 9. Determine (a) the block's speed at points Ⓑ and Ⓒ and (b) the net work done by the gravitational force on the block as it moves from point Ⓐ to Ⓒ.

Solution: The solution is done by the principle of conservation of energy. The total energy E is equal to the sum of potential plus kinetic engergy. That is

$$E = mgh + \frac{1}{2}mv^2.$$

Let us do it by parts.

(A) First, at the point Ⓐ, the speed is 0, so we have

$$E = mgh_A = (5)(9.8)(5) = 245J.$$

where we know all the terms on the right, and so the total energy is known.

(B) At point Ⓑ we have,

$$E = mgh_B + \frac{1}{2}mv_B^2,$$

but since we know E then

$$E - mgh_B = \frac{1}{2}mv_B^2$$
$$v_B = \sqrt{2(E - mgh_B)} = \sqrt{2(245 - (5)(9.8)(3.2)}\text{m/s} = 13.28\text{m/s}.$$

(C) At point Ⓒ we have

$$v_B = \sqrt{2(E - mgh_C)} = \sqrt{2(245 - (5)(9.8)(2.0)}\text{m/s} = 14.00\text{m/s}.$$

(D) The work doing by the gravitational force from points Ⓐ to Ⓒ is

$$W = mg(h_B - h_A) = (5)(-9.8)(3.2 - 5.0)\text{J} = 88.20\text{J}$$