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Voltage Collapse Analysis in a Graph Theoretical Framework

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***Abstract*—Voltage collapse is a type of failure with major influence in blackouts. Blackouts, although infrequent, are costly to society. For the Smart grid, integration of new technologies, new policies, and increasing power demand makes the power system becoming more disturbed, which increases the possibility of voltage collapses of all size. Hence, its complexity requires new approaches for modeling and analysis that captures the power network structure. This work presents a graph theory-based analysis of the voltage collapse problem. We present a review of voltage stability and collapse to identify its basic characteristics. We model the power system as an undirected weighted graph whose weights are voltage coupling magnitude between nodes. A steady state analysis is developed over the model to obtain properties of branches, and buses. The obtained graph properties are closely related with results obtained from Q-V modal analysis in the example. Also a dynamical system behavior is determined with the nonlinear differential equations of elements that govern the direction of network dynamics over a voltage collapse mechanism in time.**

I. INTRODUCTION

The Smart grid represents a new paradigm for electricity infrastructure including innovative changes in generation, storage and transport of energy, and information infrastructures. These new infrastructures make the power grid more dynamic, flexible, and robust [1], [2]. However, the integration of new technologies, new policies, and increasing power demand makes that power system more disturbed, which increases the possibility of blackouts of all size. Blackouts, although infrequent, are costly to society. They represent high economic losses and the possibility of failures propagation to other utility services. The influence of these events in power system stress the necessity of a deeply understanding of the phenomena and the development of methods to predict and control it.

Blackouts behavior is usually studied from a phenomenon called cascading failure. Cascading failures are strings of dependent failures of separate components that undermines the power system [3]. Combination of different kind of failures occurring during cascading failure makes this phenomenon highly complicated. Traditionally, it includes reactive power problems, voltage collapses, overloads, failures of protection equipment, software and communication malfunctions, and human operational errors. One of the most representative cause of blackout is related, according to [4], with deficiencies in voltage stability support and the supply of reactive power. Because of the influence of voltage collapse in the cascading failure patterns and blackouts, it is evident the necessity of studying this phenomena. It must be studied with analysis techniques that incorporate dynamic stability, complex systems effects, structural properties and simple models for failures as cascading overloads and voltage collapse.

Voltage collapse is a process by which a sequence of events leads to adversely low voltages in a big part of the power system [5]. It is characterized by initial progressive decline of voltage magnitude in the power system buses and a final rapid drop-out of the voltage magnitude. The analysis of the voltage collapse mechanism has been developed by the definition of the instability region where different types of equilibrium can been found. In [6], the voltage collapse mechanism is identified from a physical perspective. It presents the mechanism of interaction between synchronous machine and the system network. In [7], saddle-node bifurcation nonlinearity is used to analyze the geometrical properties of voltage collapse. In [8], influences of on-load tap changers and composition of load over the voltage collapse mechanism are studied.

Most common analysis tools had been focusing on the use of linear system approaches to identify local properties, to define time distances from collapse and voltage instability margins. More recent work adopts these techniques for the analysis of voltage stability in microgrids where the generators are controlled [9], [10]. In [11] an approach based in the relation between power network voltage stability limits and the limits of individual transmission branch is presented. Although these approaches presents a useful insight for understand voltage collapse, they does not afford with the identification of interaction patterns between elements and network topology influence. To include these restrictions in the analysis, a graph based approach must be developed.

There exist some network approaches to the analysis of power systems. In [12] a vulnerability analysis is presented. It propose a set of electrical centrality measures based in the admittance matrix. The electrical centrality measures are compared with the traditional structural centrality measures of the grid. In [13] spectral theory is used to rank substation importance in terms of relevance for the security of the power network. In [14] electrical closeness and betweenness measures are defined by using the node dependency matrix for nodes in shortest paths. With this approach an analysis of vulnerability for the power network is presented. In [15] a comparison between electrical and topological networks for the American power network is presented. That work presents a distance metric based on the phase-active power sensibility. None of these graph based approaches is related, specifically, with voltage stability and collapse.

We propose a graph theory approach for the analysis of voltage collapse of the power network. We develop two network models for the power network. First, a steady state approach with a connected weighted graph whose edge weights relates the Voltage-Reactive Power (V-Q) sensibilities and coupling of nodes. Over the graph, a structural analysis is developed; it reflects properties of the network that affects the voltage profile. A numerical example is developed over the WSCC-9 node system. A dynamical model is proposed to include the properties of the voltage collapse equilibrium point. In the second part, a model for distributed bifurcation and a function for the bifurcation parameter is proposed. A 3-nodes example is presented. It is proposed to develop dynamical analysis of the phenomena. The rest of the paper is organized as follows: Section II presents voltage instability and collapse basic properties. Section III presents a steady state analysis based on graphs. Section IV presents the model proposed for dynamical analysis in a power systems graph. Finally Section V concludes the paper.

II. Review on Voltage Instability and Collapse

A. Voltage Instability on Power Networks

Voltage stability can be defined as the system ability to control voltage magnitude following large disturbances (e.g., faults, loss of load or loss of generation) and small disturbances as gradual changes in load [5]. If the network has not enough resilience then voltage instability occurs.

The power system model for the voltage instability analysis is as follows,

$$\dot{x} = f(x, V, \mu) \tag{1}$$

$$I = g(x, V, \mu) = YV \tag{2}$$

$$(x_0, V_0) \tag{3}$$

For the model, $x \in \mathbb{R}^n$ represents the state vector, V represents the bus voltage vector, μ represents a varying parameter related with reactive power loads, I represents the current injection vector in buses, Y is the network node admittance matrix, and (x_0, V_0) are the initial condition of state and algebraic variables. According to [16] There exist two different types of voltage instability: lack of attraction through a stable post disturbance point and loss of post disturbance equilibrium point. The definitions of instability are as follows.

Lack of attraction through a stable post-disturbance equilibrium point

Let x_0 be a stable pre-disturbance equilibrium point for system in (1) at instant $t = t_{cont}^-$ where t_{cont} is contingency time. At instant t_{cont} the parameter μ changes. It produces a system disturbance and a new post-disturbance stable equilibrium point x^* appears,

$$F = \{x^* : g(x^*, V, \mu) = 0, f(x^*, V, \mu) = 0\} \tag{4}$$

and there exist around the equilibrium point a region of attraction R_A defined by,

$$R_A = \{x : f(x, V, \mu) \to x^*, t \to \infty\} \tag{5}$$

The new equilibrium point is not globally stable, if the state operation condition at instant $t = t_{cont}^+$ resides out of region of attraction defined in (5), the fixed point will be unstable.

Loss of post-disturbance equilibrium point

Because system in (1) is a Differential Algebraic Equation (DAE), trajectories will exist in the augmented state space X, V and constrained by the set U,

$$U = \{x, V, \mu \in X \times V : g(x, V, \mu) = 0\} \tag{6}$$

Every trajectory of the system model must lie within U. However U contains points where does not exist a solution or does not exist a unique local solution. These singular points forms a surface defined by,

$$S = \{(x, V, \mu) \in U : \Delta(x, V, \mu) \equiv \det(\frac{\delta g}{\delta V} = 0)\} \tag{7}$$

Trajectories that encounter S generally cannot continue, so the equilibrium is missed. For example in a power system operation point where mechanical and electrical trajectory curves do not intersects in time. This instability mechanism is related with a saddle-node bifurcation where the constrained space of solution intersects the impasse singular surface. At these points the state satisfies both singularity and equilibrium conditions and a voltage collapse occurs, i.e.,

$$f(x^*, V^*, \mu^*) = 0, \quad g(x^*, V^*, \mu^*) = 0 \quad \forall (x^*, V^*, \mu^*) \in S \tag{8}$$

Voltage collapse is the course of events by which voltage instability leads to a blackout or low voltages in a important part of the power network. Its analysis is developed for two main aspects: proximity to voltage collapse and mechanism of voltage collapse. For the proximity to voltage collapse analysis the question is how near is the system to instability. In voltage collapse mechanism the question is how and why voltage collapse occurs, what are the main influential factors in instability, and what are the weak areas. For the proximity to voltage collapse different index and techniques are defined. Usually the analysis is developed based on modal analysis and V-Q sensitivity [17], [18]. For the mechanism of voltage collapse the approach is based on bifurcation theory [7] or time domain simulations [8].

Using the system proposed in [7], an example of voltage collapse is presented here. The system is a three bus system where the nodes of the circuit are the slack bus, the generator node represented by a constant voltage, and the load bus whose dynamics are represented by the state equation of an inductor motor.

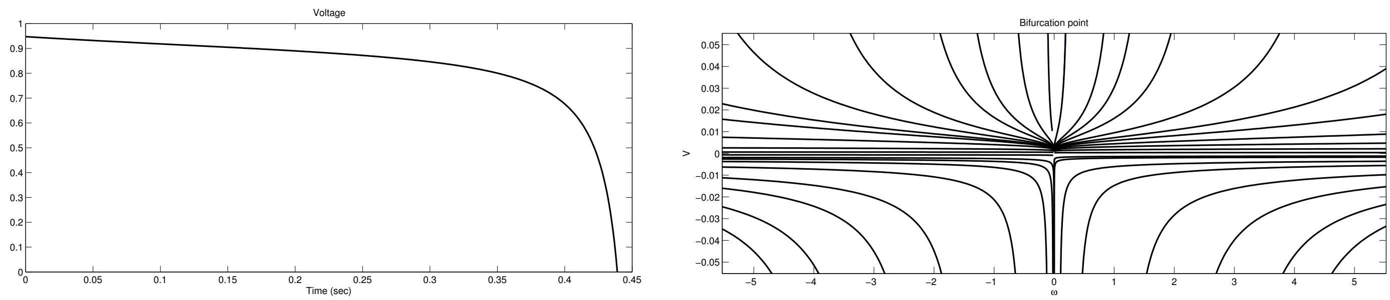

Fig. 1: *Left.* Voltage collapse for a tree bus system in bifurcation point obtained from variations in reactive load μ. *Right.* Phase plane at the point of bifurcation. Simulation parameters from model proposed in [7].

That model includes the algebraic equations into the dynamic system through the dynamic equations for the inductor motor in the load bar. The changing parameter μ represents the power constant demand in load node. The voltage collapse study is obtained through the variation of this parameter across the bifurcation point. The system has two equilibrium points before the bifurcation point, one stable and one unstable. The system is operating with initial fixed conditions in the stable equilibrium point. After the non-critical parameter perturbation the systems moves around the equilibrium point but the region of attraction maintain the trajectories near the stable equilibrium. For each load demand increase the equilibrium point is moving near the unstable equilibrium point. When the load constant demand increase to the critical point point $\mu = \mu^*$, both equilibrium collides and the fixed point for the system becomes unstable. Fig. 1 at the right shows the post disturbance equilibrium point. There exist a lack of stability in this point and the system goes to collapse. Fig. 1 left shows the bus voltage magnitude during the voltage collapse.

III. Steady State Graph Based Analysis

An appropriate model for the analysis of voltage instability and collapse, in steady state, should reflects node couplings in terms of reactive power and voltage. Consider the power flow linear system approach presented in Equation (9). The coefficient matrix is the jacobian from the nonlinear flows of active power and reactive power in each node [19]. This equation shows the incremental variations of bus voltage and angle in some node caused by the incremental variation of active or reactive power in other buses.

$$\begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{9}$$

Here, we are interested in the voltage-reactive power relation between nodes. To obtain sensitivities between reactive power injection and bus voltage, we assume $\Delta P = 0$. In this way we decoupled the flows and now we can work based on the reduced jacobian J_R,

$$J_R = \left[\frac{\partial Q}{\partial V}\right] = \left[L - MH^{-1}N\right] \tag{10}$$

Its inverse J_R^{-1} is the sensitivity matrix. It is a real and non-symmetrical matrix whose elements reflect the propagation of voltage variations following a current or reactive power injection in a given node throughout the system. Using J_R^{-1} we can define a voltage distance metric that give us a measure of voltage relations between nodes.

In [20] a useful voltage distance metric is proposed. It can be used to quantify node proximities in terms of voltage. To build this measure, the elements of each column of J_R^{-1} are divided by the diagonal terms as follows,

$$\alpha_{ij} = \left[\frac{\partial V_i}{\partial Q_j}\right]\left[\frac{\partial V_j}{\partial Q_j}\right]^{-1} \tag{11}$$

then, the voltage relation is defined by α_{ij} such that,

$$\Delta V_i = \alpha_{ij}\Delta V_j \tag{12}$$

The voltage distance between nodes is obtained by the logarithm of voltage relation α_{ij},

$$D_{ij} = D_{ji} = \log(\alpha_{ij} \cdot \alpha_{ji}) \tag{13}$$

The matrix D_{ij} has the properties of positivity, symmetry, and it is a real mathematical distance [20]. This metric is valid in cases where exist active- reactive power flow uncoupling.

A. Static Power System Model

By using the voltage distance measure defined in (13), we model a power network in steady-state as a connected, undirected and complex-weighted graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, Y)$, where $\mathcal{V} = \{1, \ldots, n\}$ is the set of nodes representing buses and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of edges. The weight of edge (i, j) is its voltage distance $d_{ij} = d_{ji} \in \mathbb{R}$. For each node $i \in \mathcal{V}$ there exist a phasor current $I_i \in \mathbb{C}$, a phasor voltage $U_i = E_i e^{j\theta_i} \in \mathbb{C}$, and a power injection $S_i = P_i + jQ_i \in \mathbb{C}$, whose real part $P_i \in \mathbb{R}$ is the active power and $Q_i \in \mathbb{R}$ is the reactive power. The power flow injection for each $i \in \mathcal{V}$ is defined by, $P_i = \sum_n^{j=1} \mathbb{I}m(Y_{ij})E_iE_j sin(\theta_i - \theta_j) + \sum_n^{j=1} \mathbb{R}e(Y_{ij})E_iE_j cos(\theta_i - \theta_j)$ and $Q_i = -\sum_n^{j=1} \mathbb{I}m(Y_{ij})E_iE_j cos(\theta_i - \theta_j) + \sum_n^{j=1} \mathbb{R}e(Y_{ij})E_iE_j sin(\theta_i - \theta_j)$.

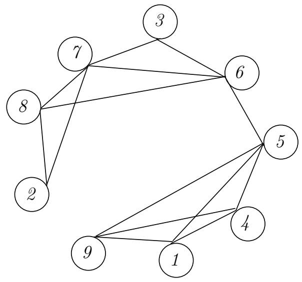

Fig. 2: V-Q distances graph for the WSCC 9-bus system.

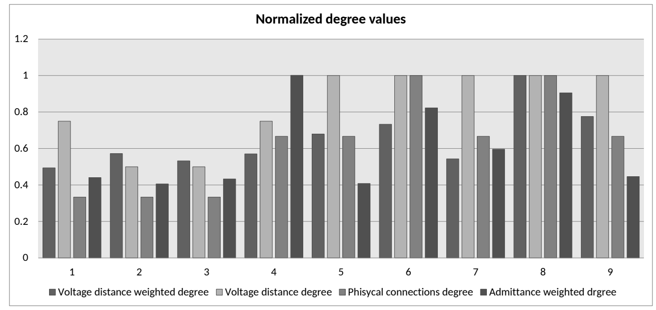

Fig. 3: Normalizes degree centrality measures for the WSCC 9 bus system.

B. Graph Based Analysis

The model described in Section III-A presents the V-Q couplings between elements. This graph based model of the power system presents information about how near is one node of another in terms of voltage variation influence. The graph includes physical connections but also it includes electrical non physical couplings that exist in the power network. This couplings are related with reactive power variations in loads and devices. The voltage stability analysis can be developed in terms of traditional network metrics as degree, closeness and betweeness. As the weights in the graph model contains information about voltage coupling for a specific operation point, the network measures give us information about the influence of nodes in these couplings. Also, if a critical voltage mode is evaluate the measures give us information about participation factors of nodes and branches in that critical mode. Also if the mode is near to a voltage collapse some measures give us information about which nodes will participate in the event and the magnitude of their collapses. Now we present a numerical example of the voltage stability analysis using the WSCC-9 nodes system.

A numerical example is developed over the WSCC-9 bus system on a stable operation point obtained from the loadflow with Matpower toolbox in MATLAB. Jacobian matrix J is builded with the loadflow results of a specific operation point in (9). Also, this results are used for the voltage distance matrix D_{ij}. Applying the model in III-A, the power network presents the set $\mathcal{V}$ numerated as the buses in WSCC system, and edges representing voltage distances between nodes obtained from (12). Fig. 2 presents the most significative voltage couplings in network. Each edge in $\mathcal{E}$ has a weight corresponding to d_{ij}. The smallest components of D_{ij} represents the strongest coupling between nodes in terms of V-Q sensibility. There exist non physical couplings between elements physically distant which have strong components in matrix D_{ij}.

Some measures are applied to the system to identify properties of their voltage stability profile. First, degree centrality is calculated in different cases. Fig. 3 presents the results of the normalized measures to compare them. The physical connection degree shows the number of physical connections for each node. Also, the admittance weighted degree is calculated for the system. In this case, the weight of the physical link is defined by the admittance matrix. The most influential node is bus 4 because it is connected with the slack bus. Two other degrees are calculated. Voltage distance degree is a measure of connections for the adjacency matrix associated with the model proposed in this work. Based on this measure, it is possible to identify that PQ buses have a strong influence in voltage coupling. Using the voltage distance weighted graph, we can identify from this measure the PQ buses with the highest influence in the voltage critical mode associated with the equilibrium point. The voltage distance weighted degree has the advantage to bring specific information about voltage instability.

Voltage stability profile for this system present a stable operation point with minimum eigenvalue of $\lambda_{min} = 3.6568$. The λ_{min} eigenvalue of J_R (been J_R the reduced uncoupled jacobian)is a measure of voltage proximity to instability in the traditional modal analysis. Associated with this mode there exist a right and left eigenvectors ξ_{ki} and η_{ik} which can be used to determine the bus participation factor p_{ki} for the λ_{min} critical mode. In this example, the results depict two nodes with high participation, node 4 and node 9. The bus participation factor shows that voltage critical mode is very localized. The factor describes these two nodes like possible candidates for reactive power compensation and voltage control. This results coincide with data obtained from two spectral centrality measures called algebraic connectivity and fielder vector. The algebraic connectivity of the network is directly related with the λ_{min} eigenvalue of modal analysis. It gives us information about the connectivity of the network in terms of voltage. If there exist the algebraic connectivity eigenvalue the graph is connected and can be divided on independent subgraphs. Also there will exist influence of every node to each other in terms of voltage stability. In this way, the approach of the voltage distance matrix model as a fully connected weighted graph is appropriate. From Fig. 2 it is possible to identify the branch with highest participation factor. Edge between node 5 and node 6 is the most vulnerable node in the system. It connects the two complete areas. Overloads, failures or maintenance of this link will result in a voltage collapse.

IV. Dynamic Graph Based Analysis

We model a power network in continuous time as a connected, undirected, and weighted graph $G(\mathcal{V}, \mathcal{E}, Y)$, of order n

TABLE I: MEASURES FOR THE WSCC-9 BUS VOLTAGE DISTANCE GRAPH.

Bus number	Type	Voltage profile	Fielder vector	$p_{ki} = \xi_{ki}\eta_{ki}$
1	PV	1.0	0.2706	-
2	PV	1.0	-0.2441	-
3	PV	1.0	-0.1554	-
4	PQ	0.987	-0.5396	0.1394
5	PQ	0.975	0.1053	0.030
6	PQ	1.003	0.3105	0.003
7	PQ	0.986	-0.3724	0.0001
8	PQ	0.996	0.5474	0.0650
9	PQ	0.958	0.0777	0.0949

where each node has associated a set of differential equations defined by

$$\dot{x} = f(x, \mu) = h(g(x, \mu)) \tag{14}$$

x is the state vector that includes voltage magnitudes and angles, μ is a vector of real varying parameters. Usually, for voltage collapse analysis, μ is considered as a vector of real and reactive load powers and $g(x, \mu)$ represents the steady state model. Solutions for equation $g(x, \mu) = 0$ will be the equilibrium points of $h(\cdot)$. To generalize the nonlinear network behavior for the analysis of a voltage collapse we use the results proposed by [7] and [21].

The power system is modeled as a particular class of distributed nonlinear network where $f_i : \mathbb{R}^n \to \mathbb{R}$ for each $i \in \{1, \ldots, n\}$ is a saddle node nonlinearity $f_i(x) = \mu_i - x_i^2$. The perturbed parameter μ is a vector defined in function of the Laplacian flow $\mathcal{L}(\mathcal{G})\xi$, where ξ is the right eigenvector associated with the i_{th} mode of $\mathcal{L}$ and the Laplacian matrix is defined by the Jacobian reduced matrix in (10),

$$\mu = \mathcal{L}(\mathcal{G})\xi \tag{15}$$

The definition of μ in (15) give us the advantage of including varying parameters related with structural perturbations of the underlying graph(e.g., variations of the laplacian), variations of voltage reactive power sensitivities due to impedance variations (e.g., weight variations of links), and influence of nodes and direction of state trajectories (e.g., weights and component signs in right eigenvector ξ). The conditions for the existence of bifurcation point in this power network model are,

Theorem 1: For the power network nonlinear model with saddle-node nonlinearity the following statements holds:

Equilibrium points:

1) For $\mu = 0$, the set of equilibrium points is the set containing all the elements in the diagonal of $\mathbb{R}^n$

$$E_c = diag(\mathbb{R}^n) \tag{16}$$

2) For $\mu > 0$, the set of equilibrium points is

$$F_b = \{x_i^* | x_i \in \{-\sqrt{\mu}, \sqrt{\mu}\}^{n-1}\} \tag{17}$$

where $n - 1$ is the rank of $\mathcal{L}(\mathcal{G})$

Bifurcation

1) For $\mu = 0$ each equilibrium point $x^* \in E_c$ is unstable
2) For $\mu < 0$ the equilibrium point for any n is missed. At this case the constrained space of solution intersects the impasse singular surface of parameter restriction.
3) For $\mu > 0$ each equilibrium point $x^* \in F_b$ is stable

According to [7], when system trajectories are near from a saddle-node point, the jacobian matrix J,(i.e., $\mathcal{L}(\mathcal{G}) = J_R$) has a unique smallest eigenvalue with a corresponding right eigenvector ξ. Right eigenvector ξ defines the direction in state space of the initial dynamics of voltage collapse. Any state variable can collapse when it moves in the direction of ξ. Also, the extent of this collapse is given by the relative magnitude of the corresponding component in this vector. Nodes where voltage magnitude falls more quickly have the largest components. Product $\mathcal{L}(\mathcal{G})\xi$ includes also this information in the model.

For example consider a three node reduced system $\mathcal{G}_1$ where each nodes has associated a dynamic equation $\dot{x} = f(x, \mu) = g(x) + \mu = g(x) + \mathcal{L}(\mathcal{G})\xi$ where $\mathcal{L}(\mathcal{G})$ is the derivative of $f(x, \mu)$ in terms of x and ξ is the right eigenvector associated with the minimum eigenvalue different from zero of $\mathcal{L}(\mathcal{G})$. For the power network $\mathcal{L}(\mathcal{G})$ can be assumed as the jacobian matrix of the power system and definitions of distance and adjacency are followed by description in Section III. Then, as the elements of J_R change in time, the equilibrium point x^* is slowly varying. So the right eigenvector ξ gives information about the variations of trajectories in time to the equilibrium point x^*. The equilibrium point slowly variations can be seen as small variations of parameter μ. When the system achieves the point of bifurcation the jacobian J_R is singular and the μ parameter is equal to zero , i.e., product $J_R\xi = 0$. At this point a voltage collapse occurs and the trajectories of state variable diverges.

For another example of this model, considers a three node undirected network. For this network we select 4 connection topologies: $\mathcal{E}_1 = \{1, 2\}, \{1, 3\}, \{2, 3\}$; $\mathcal{E}_2 = \{2, 3\}$; $\mathcal{E}_3 = \{1, 2\}, \{1, 3\}$; $\mathcal{E}_4 = \{1, 2\}, \{2, 3\}$. Fig. 4 presents results of dynamic simulation for each topology. It is easy to see that variations in network topology changes the convergence of the system. Moreover, the network topology reflected in Laplacian shows their influence over the equilibrium point. For $\mathcal{E}_1$ the system is fully connected and the stability is maintained after a perturbation. Results for $\mathcal{E}_2$ shows the case when one of the nodes is disconnected. It influence the dynamics of the other nodes collapse. Laplacian matrix $\mathcal{E}_3$ and $\mathcal{E}_4$ presents a case where the collapse is partial. Unless, the graph structure is the same, the behavior of the entire system depends on the qualities of each node.

V. CONCLUSION

Static and dynamic graph based approaches for the analysis of voltage collapse were presented in this paper. The static graph model with its structure that includes voltage couples

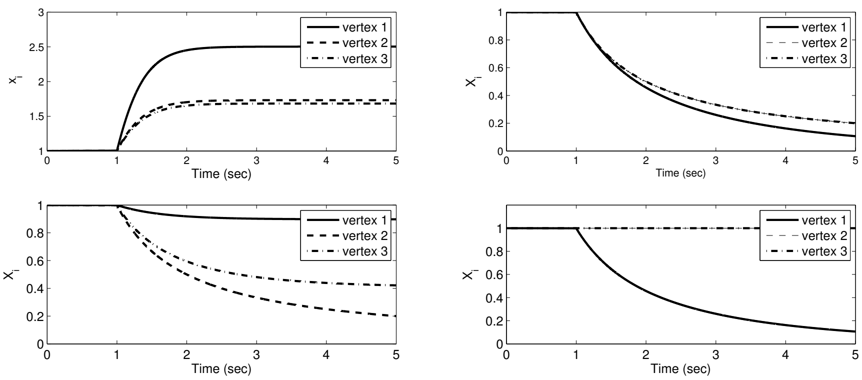

Fig. 4: V-Q distances graph for the WSCC 9-bus system.

between nodes allows us to analyze proximity to voltage collapse through graph structure measures. The dynamic graph based approach resumes the behavior of nonlinear dynamics in power system with the inclusion of saddle-node nonlinearities for each node. This model allows us to analyze disturbances related with changes in network voltage couplings and failures that generates voltage instability. As future trends, a deep understanding of network and other measures must be used to analyse the network. Also, specific conditions for the collapse point must be demonstrated in the dynamic graph.

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