A Reconfiguration Analysis Tool for Distribution Networks using Fast Decoupled Power Flow

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***Abstract*— This work proposes a tool for distribution network analysis focused on grid reconfiguration using the Fast Decoupled Newton-Raphson power flow method. The methodology uses the information of the network switching equipment status, without the need of a previous topological processing. The complex normalization approach was used to make viable the load flow calculation by the fast decoupled approach when applied to networks presenting lines with high R/X ratio, which is typical in distribution systems.**

***Index Terms*—Fast-Decoupled Load Flow, Bus Section Level, Complex Normalization, Reconfiguration, Distribution Networks.**

I. INTRODUCTION

The concept of smart grids may be, in a simple way, defined as the adaptation of the existing power systems to future challenges such as the response to the growing demand for electricity complying, as well with the needs of sustainability and security of power supply. In this context, the integration of informatics, automation, communication and measurement infrastructure is seen as an alternative to make power systems more reliable and flexible in its operation [1].

Within the electricity distribution area, and specifically regarding operational aspects, methodologies, systems and equipment are being constantly improved to perform network reconfiguration [2]. They are driven by either to accomplish emergency switching procedures, in order to restore the largest number of customers after a fault event (self-healing), or to reconfigure the grid during its normal operation, reducing technical losses and/or improving voltage regulation.

Many of the proposed methodologies [3], [4] make use of a load flow tool to determine the network states in order to verify the feasibility of a given switching operation. The load flow tools are also important for engineers who observe system operation or perform its planning, simulating scenarios to verify, for example, the best spots for switching equipment installation. However, commonly used power flow tools such as backward-forward sweep based methods do not accommodate in its original formulation the modelling of switchable branches, making necessary a previous topology processing.

This work proposes a distribution system analysis tool using Newton-Raphson power flow method that allows calculating the network states including in the formulation switches and circuit breakers states, making it a comprehensive tool for network reconfiguration studies. As an alternative to the increase computational burden dictated by the integration of switchable branches, the fast decoupled load-flow approach [5] is also considered. To make the application of this method in distribution networks viable, the complex normalization technique [6] is adopted.

The paper is organized as follows: Section II revisits the conventional Newton-Raphson power flow formulation in order to give a better background to the understanding of Sections III and IV, which, respectively, present the Extended Newton-Raphson formulation with its switchable branch modelling and the Extended Fast Decoupled Newton-Raphson. Section V resumes the complex normalization technique and Section VI presents the results of the methodology simulated in a case study. The conclusions are discussed in Section VII.

II. CONVENTIONAL NEWTON-RAPHSON FORMULATION

The original formulation of Newton's method to solve the power flow problem has been firstly presented in [7]. The problem is modeled as a set of non-linear equations and solved iteratively. This consolidated formulation is revisited in this Section aiming to ease the comprehension of the extended

978-1-4673-6605-2/15/$31.00

formulation, which contemplates the explicit modelling of switchable branches.

The active and reactive power injection equations are expressed as:

$$P_k = \sum_{m \in \Omega_k} P_{km}(V_k, V_m, \theta_k, \theta_m) \quad (1)$$

$$Q_k = Q_k^{sh}(V_k) + \sum_{m \in \Omega_k} Q_{km}(V_k, V_m, \theta_k, \theta_m) \quad (2)$$

where:

Ω_k	Set of adjacent buses to k, except the own k bus.
P_k, Q_k	Active and reactive power injections on bus k, respectively;
P_{km}, Q_{km}	Active and reactive power flowing from bus k to bus m;
θ_k, V_k	Voltage angle and module on bus k, respectively;
θ_m, V_m	Voltage angle and module on bus m respectively;
Q_k^{sh}	Reactive power injection due to a susceptance connected to bus k.

The basic power flow formulation is represented as a set of mismatches between specified and calculated power injections. This set is defined by the system buses' classification, which can be Vθ, PV or PQ [8].

Within the context of distribution networks, distributed generation is commonly modeled as a PQ bus, since for some of these sources such as photovoltaics and small wind turbines, it is not usual to supply reactive power apart from the fact that these generators also need a steady voltage reference to keep connected to the grid.

The Newton-Raphson power flow formulation is summarized as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}^r = \begin{bmatrix} H & N \\ M & L \end{bmatrix}^r \cdot \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}^r \quad (3)$$

$$\theta^{r+1} = \theta^r + \Delta\theta^r \quad (4)$$

$$V^{r+1} = V^r + \Delta V^r \quad (5)$$

where:

r	Iteration counter;
$\Delta P, \Delta Q$	Mismatches between specified and calculated active and reactive power, respectively;
$\Delta\theta, \Delta V$	Step vectors to update the voltage angle and module, respectively;
H, N, M, L	Matrices containing the corresponding derivatives: ${\partial P}/{\partial \theta}, {\partial P}/{\partial V}, {\partial Q}/{\partial \theta}, {\partial Q}/{\partial V}$;

First, equation (3) is solved. Then, the network states are updated using (4) and (5) in order to recalculate ΔP, ΔQ, H, N, M and L. The process is repeated until the mismatches vector be smaller than a preset tolerance.

It is important to note that the mismatches vector has a (2NPQ + NPV x 1) size, where NPQ and NPV are respectively the number of PQ and PV buses.

III. Extended Power Flow

The extended formulation of Newton's method to model switchable branches has its origins in [9] and [10]. In these proposals, the modelling of switchable branches was first designed for state estimation, in order to take advantage of electrical measurements available on circuit breaker instrumentation within substations. In [11] the extended power flow formulation is presented using the concept of modeling switchable branches in Newton-Raphson formulation. This new formulation allows, for example, the expansion of buses in transmission power systems into substations in different topologies, enabling the calculation of power flows inside the substation.

The extended power flow's main idea is the declaration of active and reactive power flows in switches and circuit breakers as system state variables such as the magnitude and angle of bus voltages. This approach bypasses the numerical problems of declaring zero or infinite impedances, allowing the method to converge.

$$P_k = \sum_{m \in \Omega_k} P_{km}(V_k, V_m, \theta_k, \theta_m) + \sum_{l \in \Gamma_k} t_{kl} \quad (6)$$

$$Q_k = Q_k^{sh}(V_k) + \sum_{m \in \Omega_k} Q_{km}(V_k, V_m, \theta_k, \theta_m) + \sum_{l \in \Gamma_k} u_{kl} \quad (7)$$

where:

Ω_k	Set of adjacent buses to k connected by conventional branches (non-zero impedance);
Γ_k	Set of adjacent buses to k connected by switchable branches;
t_{kl}, u_{kl}	Active and reactive power flow state variables coming from bus k to bus l. $k-l$ must be a switchable branch.

In the extended formulation, the power flows in switchable branches are added to the conventional state vector. Besides that, the system contains a new set of equations concerning the switches' status (opened or closed). The extended equation set can be expressed by [12]:

$$\begin{bmatrix} \Delta P \\ \Delta Q \\ f_\theta \\ f_V \end{bmatrix}^r = \begin{bmatrix} H & N & T & 0 \\ M & L & 0 & U \\ C & 0 & O & 0 \\ 0 & D & 0 & P \end{bmatrix}^r \cdot \begin{bmatrix} \Delta\theta \\ \Delta V \\ \Delta t \\ \Delta u \end{bmatrix}^r \quad (8)$$

$$f_{\theta-i,j}^{CL\ \ r} = \theta_i^{\ r} - \theta_j^{\ r} = 0 \quad (9)$$

$$f_{V-i,j}^{CL\ \ r} = V_i^{\ r} - V_j^{\ r} = 0 \quad (10)$$

$$f_{\theta-i,j}^{OP\ \ r} = t_{ij}^{\ r} = 0 \quad (11)$$

$$f_{V-i,j}^{OP\ \ r} = u_{ij}^{\ r} = 0 \quad (12)$$

where:

T, U, C, D, O, P Matrices containing the corresponding derivatives: $\partial P/\partial t$, $\partial Q/\partial u$, $\partial f_{\theta-i,j}/\partial \theta$, $\partial f_{V-i,j}/\partial V$, $\partial f_{\theta-i,j}/\partial t$, $\partial f_{V-i,j}/\partial u$;

In this formulation, the system of equation is solved as the conventional approach. The state variables t and u are updated similarly as given by (4) and (5).

The incorporation of the switchable branches extends the state variable vector, and consequently the Jacobian matrix and the mismatches vector. So, the new state vector has the size of (2NPQ + NPV + NSB x 1), where NSB is the number of switchable branches.

When a switchable branch is closed, (9) and (10) are used for its modelling. Likewise, in opened state, (11) and (12) are used. The new derivatives added to the Jacobian matrix, therefore, always assume constant values and may be, -1, 0 or 1, depending on the case.

IV. Fast Decoupled Extended Power Flow

The fast decoupled power flow presented in [13], proposed the decoupling of the original system of equations considering mathematical simplifications to transmission power systems which have a high X/R line impedance ratio. These simplifications omit the matrices N and M, allowing the equations system (3) be divided in two sets. Moreover, matrices H and L are replaced by matrices B' and B'', which are constant and based on admittance matrix elements.

The proposition of the extended fast decoupled power flow has been originally presented in [11]. In that work, results of the methodology application for transmission systems including the calculation of power flows along some substations internal buses are presented.

The decoupled formulation of the extended power flow is as follows:

$$\begin{bmatrix} \Delta P \\ f_\theta \end{bmatrix}^{r_P} = \begin{bmatrix} B' & T \\ C & O \end{bmatrix}^{r_P} \cdot \begin{bmatrix} \Delta\theta \\ \Delta t \end{bmatrix}^{r_P} \quad (13)$$

$$\begin{bmatrix} \Delta Q/V \\ f_V \end{bmatrix}^{r_Q} = \begin{bmatrix} B'' & U \\ D & P \end{bmatrix}^{r_Q} \cdot \begin{bmatrix} \Delta V \\ \Delta u \end{bmatrix}^{r_Q} \quad (14)$$

It is observed that the extended vectors are already decoupled due to its respective equations (9) to (12). Then, using the concept presented in [13], it is possible to decouple the equation set it two parts: the first for the active power injections and the second for the reactive power injections. Each equation subset has its own iteration counter: *rP* and *rQ*.

The main advantage of the decoupled formulation is the constant Jacobian matrix, which increases significantly the method's calculation performance.

V. Complex Per-Unit Normalization

The complex normalization approach has been first presented in [14], referred as "axis rotation", and later revisited by [6], which refers the process of axis rotation to a more comprehensive idea of a base complex power $\dot{S}_{base} = |\dot{S}_{base}|.e^{-j\Phi_{base}}$ in order to normalize the system line impedances in module and angle, modifying its original X/R ratios.

This approach becomes particularly interesting when applying the fast decoupled power flow method for distribution networks, once those systems does not usually present the method's required high X/R ratio.

The normalization of an impedance by a complex base power is given by:

$$\frac{X_{km}{}^{c.p.u}}{R_{km}{}^{c.p.u}} = tg(\zeta_{km} + \phi_{base}) \quad (15)$$

Where the "c.p.u." notation stands for "complex per unit" and ζ_{km} is the original angle of the normalized impedance.

VI. Case Study and Results

To demonstrate the methodology effectiveness, a case study, based on the distribution system presented in [15], has been modelled. This system has five tie switches connecting different sections of a distribution feeder.

The original system's loads were increased in 30% in order to magnify the effect of closing tie switches on the voltage profile. The average X/R ratio of the system's line impedances is 0.79. According to [6], each perfect line power rotation angle can be calculated as follows.

$$\phi_{base}{}^{perfect} = \frac{\pi}{2} - tan^{-1}\left(\frac{X_{km}}{R_{km}}\right) \quad (16)$$

Since all complex impedances are normalized with the same angle, the simplest way to define a suitable rotation angle is to calculate the average of perfect rotation angles. Considering that lines with zero impedance (switchable branches) have a perfect rotation angle of 0°, the average rotation angle for the whole system is 44.85°.

A. Distribution feeder in radial operation

The first simulation is a radial operation of the distribution system, which will present a voltage drop area due to the load increase.

Figure 1 presents the system topology and its voltages for radial operation. Green buses and lines represent voltages above 0.93 pu, and orange represents voltages between 0.93 and 0.90 pu. White squares represent opened switches. Figure 2 presents all modules and angles calculated using the extended fast decoupled power flow with a normalization base angle of 45º.

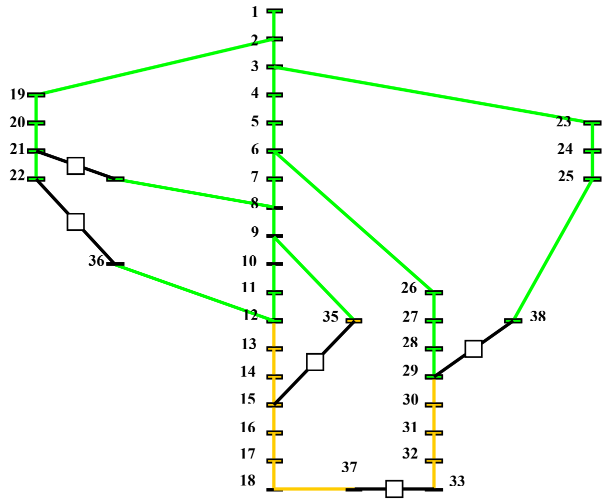

Figure 1. Test system's topology and voltages for radial operation

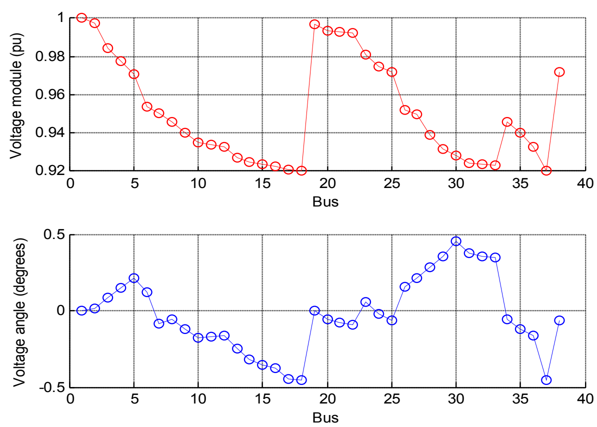

Figure 2. Voltage profile for radial operation

Although the complex normalization modifies the original system and changes line impedances and loads, the system's conventional states (magnitude and angle of the bus voltages) remain the same for any rotation angle. The conventional extended load flow converges in three iterations. The convergence analysis for the fast decoupled extended load flow is presented in Table 1.

Table 1. Convergence analysis for radial operation

Φ_{base}	Number of iterations needed	
	Pθ	*QV*
0°	Not converged	
20°	24	24
30°	15.5	15.5
45°	9.5	9.5
60°	15.5	15.5

The extended formulation demands the extension of the conventional state vector, calculating also the active and reactive power flows of switchable branches. In this particular switching scenario, where every switch is open, all calculated power flows for those branches were evaluated as zero.

B. Closed ring operation

Now, simulating a closed ring operation, closing switches 36-22 and 37-33, the system's topology and resulting voltages are presented in Figure 3.

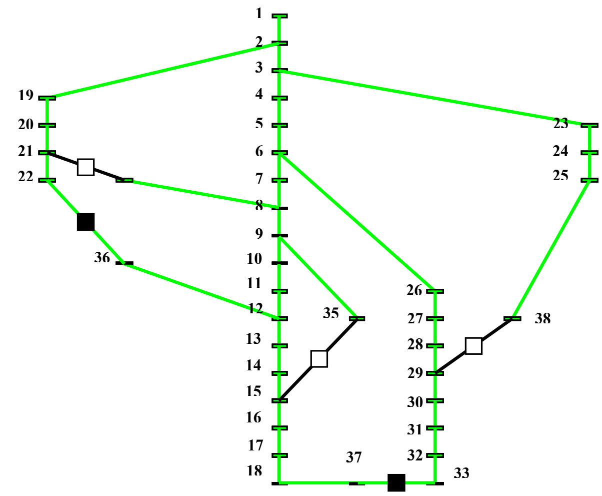

Figure 3. Test system's topology and voltages for closed ring operation

As the system operates in closed ring, it is possible to see that the voltage drop on buses 13 to 18 and 30 to 33 decreases, showing that the load flow method takes into account the parallelism between lines. Figure 4 shows the voltage profile for the closed ring operation.

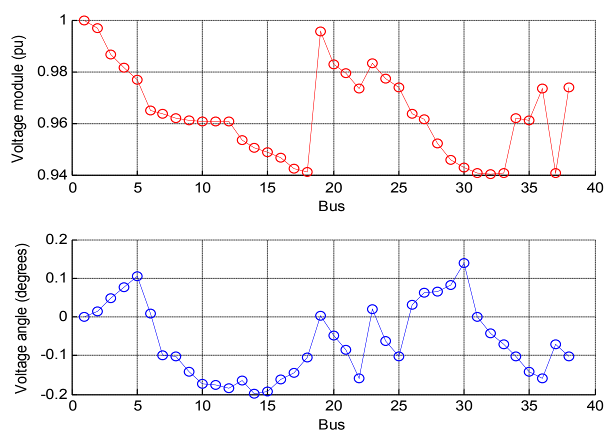

Figure 4. Voltage profile for closed ring operation

The conventional extended power flow converges in 3 iterations. As the system changes its topology, the convergence pattern is also modified. The convergence analysis for the fast decoupled version of the extended power flow for this scenario is presented in Table 2.

Table 2. Convergence analysis for closed ring operation

Φ_{base}	Number of iterations needed	
	Pθ	***QV***
0º	Not converged	
20º	Not converged	
30º	14	14
45º	8	8
60º	15	15

Once there are closed switches, the method calculates the power flowing through closed switchable branches. It is important to notice that these states are actually modified by the complex normalization. Therefore, it is necessary to apply an inverse rotation in those states, in order to have a result related to the original system.

$$S_{kl}^{c.p.u.} = |S_{kl}^{c.p.u.}|.e^{j\zeta_{kl}} \quad (17)$$

$$t_{kl}^{p.u.} = |S_{kl}^{c.p.u.}|.\cos(\zeta_{kl} - \phi_{base}) \quad (18)$$

$$u_{kl}^{p.u.} = |S_{kl}^{c.p.u.}|.\sin(\zeta_{kl} - \phi_{base}) \quad (19)$$

Table 3 presents the switchable branches active and reactive power flows.

Table 3. Switchable branches state on closed ring operation

k	l	$t_{kl}^{p.u.}$	$u_{kl}^{p.u.}$
34	21	0	0
35	15	0	0
36	22	-0.7263	-0.6786
37	33	0.0256	0.2593
38	29	0	0

C. *Closed ring operation with all switches closed*

The last scenario to be presented is a situation where all switches are closed. Even though this particular scenario would not be a possible switching in real system operation, it shows the results for the power flow states from all the switchable branches. Figure 5 shows the distribution system topology and voltages. Figure 6 shows the voltage profile with module and angle.

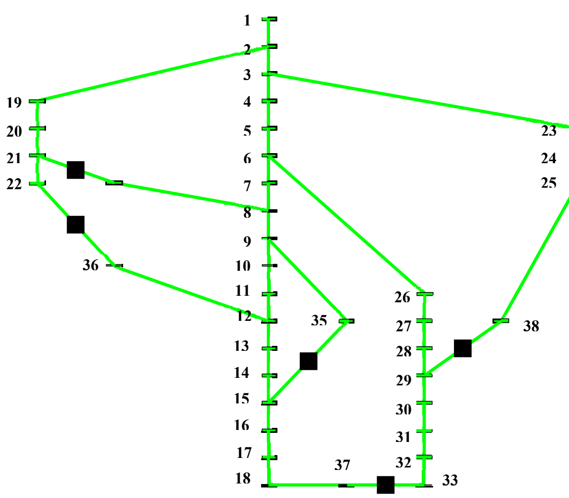

Figure 5. Test system's topology and voltages condition whenall switches are closed

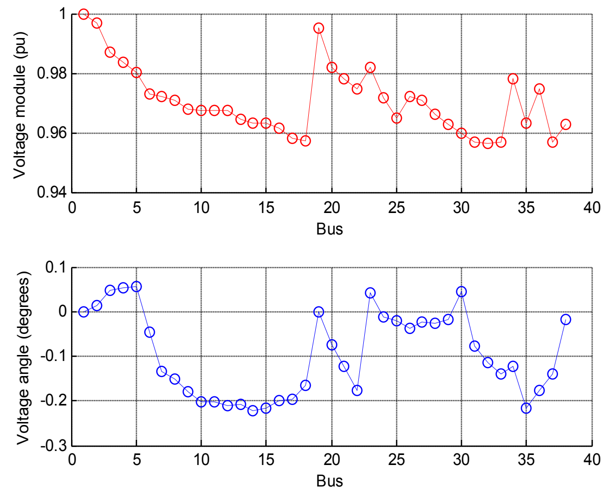

Figure 6. Voltage profile when all switches are closed

Just like the previous scenarios, the conventional extended power flow converges in three iterations. The convergence analysis for the fast decoupled version is presented in Table 4.

Table 4. Convergence analysis for all switches closed

Φ_{base}	Number of iterations needed	
	Pθ	***QV***
0º	Not converged	
20º	Not converged	
30º	13.5	13.5
45º	8	8
60º	14.5	14.5

It can be observed that in this scenario the fast decoupled method uses less iterations to solve the load flow problem, since the more meshed is the distribution system, closer the system voltages get to the flat start. Figure 7 shows the convergence pattern for the fast decoupled solving the problem with a rotation angle of 45º.

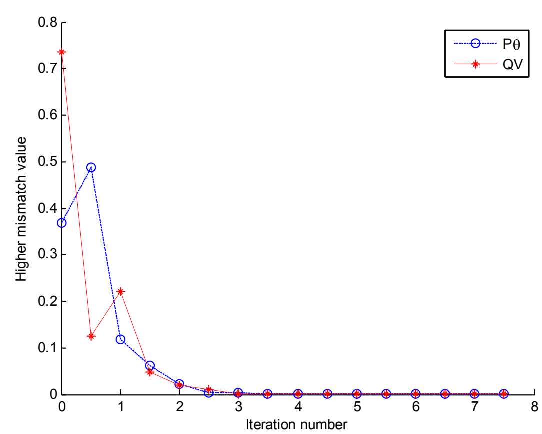

Figure 7. Convergence pattern for Φ_{base} = 45º and all switches closed

The calculated system state variables for this scenario are presented in Table 5.

Table 5. System states calculated for switchable branches when all switches are closed

k	l	$t_{kl}^{p.u.}$	$u_{kl}^{p.u.}$
34	21	-0.4128	-0.3627
35	15	0.2981	0.2276
36	22	-0.4124	-0.3533
37	33	-0.0030	0.1860
38	29	0.4933	0.5120

VII. CONCLUSION

In this paper the extended power flow formulation, using the switchable branches modelling, and the concept of complex normalization have been presented and combined in order to perform reconfiguration analysis on distribution networks.

Reconfiguration studies are relevant in distribution pre-operation and planning, providing useful information to power system engineers about losses, voltage profile and line power flows for different network topologies. This information is crucial for decision making under emergency circumstances or even to define the best switching to a given set of interconnected distribution feeders, in order to reduce technical losses and improve reliability.

A test case concerning different topological scenarios based on switching operations evidence the tool's functionality and performance. Different convergence patterns for distinct base angles are also presented, indicating the relevance of the proposed approach as an effective tool to support distribution systems reconfiguration studies.

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