A Complex Network Analysis of the Brazilian Power Test System

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***Abstract*—This paper presents a complex network analysis of the Brazilian Power Test System (BPTS), which is based on the real Brazilian electric power system. The aim is to model the BPTS cases via graphs, characterize them according to existing complex network models, compute several invariants from graph and complex networks, and analyze them with respect to these invariants, in order to obtain useful information for the application. The analysis of these metrics applied to the electrical power system can help us to identify and solve problems such as network overload, catastrophic failures and blackouts. Each invariant leads to different critical nodes. Thus, we could look for the best combination of invariants and weights to analyze power system networks.**

***Index Terms*—Complex Networks, Smart Grids, Centrality Measures, Resilience Analysis.**

I. INTRODUCTION

Nowadays the high electrical energy consumption due to the increasing of world population and to the technology advances is a challenging problem to the scientific community, since this demand is leading the power grid to collapse.

For instance, several large blackouts have occurred in several countries in recent years, such as a blackout occurred in Northeastern America in 2003, when 50 million people were disconnected [1], and a blackout occurred in Brazil in 2009, that affected all regions of the country (South, Southeast, Midwest, North and Northeast) and interrupted 24,436 MW, that corresponds to 40% of the loads of the Brazilian National Interconnected System [2].

In this context, the Smart Grids appear as a great candidate solution, which have been concentrating multidisciplinary researches on tools and technologies aiming to improve quality and reliability of the power system [3].

The current power system is one of the most complex man-made infrastructure, with large numbers of diverse components connected through a vast and geographically extensive network.

Indeed, power systems can be seen as complex networks [4], [5], where nodes represent generators, substations and load bus, and links represent the transmission lines interconnecting them. This approach help us to explore the structure and the behavior of power systems.

Complex network theory has been growing rapidly in recent years, and it provides tools for efficient network vulnerability analysis. The identification of vulnerable points is important to prevent network failures and avoid blackouts, ensuring system reliability.

A survey on analyses of power grid systems using complex network theory is provided in [6], considering real power grids and synthetic topologies, such as the bus models of IEEE. The complex network analysis comprises topological aspects (e.g., node degree distribution, path length analysis, and betweenness distribution), disruption behavior, and small-world investigation. Almost all power grids analyzed there are from United States, Europe and China, and no one is from South America.

As an approach to understand the Brazilian power grid as a complex network, this paper considers a set of five test systems with 9, 16, 33, 65 and 107 AC buses. This set is called the Brazilian Power Test System (BPTS), and all the BPTS cases are based on the real Brazilian electric power grid [7], therefore taking into account its particular characteristics.

This paper aims applying the Complex Network Theory tool in the Brazilian Power Test System for investigating reliability and resilience. Some results on complex network analysis of the BPTS cases are presented, including topological and electrical aspects, and disruption behavior.

The rest of the paper is organized as follows. Section II brings important concepts from complex network theory. Section III describes the BPTS under investigation, and our proposed approach. Results are presented and discussed in Section IV, where the power systems are analyzed in the context of complex networks. Finally, Section V presents conclusions and future work.

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II. COMPLEX NETWORK THEORY

This section provides some basic graph and complex network concepts and metrics that are used throughout this paper. The books [8] and [9] are the main references used here.

A. Basic Graphs Concepts

A power grid can be represented as a connected graph G with a set $V(G)$ of N nodes (or vertices) representing the substations, generators and loads, and a set $E(G)$ of L links (or edges) representing the high voltage transmission lines. A graph G is connected if there is at least a path between every pair u, v of nodes in G.

Two nodes u and v are adjacent if there is a link $(u, v) \in E$ interconnecting them. The degree of a node v, denoted as $deg(v)$, is the number of nodes adjacent to v in G. The degree of a node is an important property that indicates the number of connections it has with other nodes. The average node degree is given by $\langle k \rangle = 2L/N$.

A vertex $v \in V(G)$ is a cut-vertex in G if $G \setminus \{v\}$ is a disconnected graph, i.e., if the removal of v from G disconnects G. A link $e \in E(G)$ is a bridge in G if $G \setminus \{e\}$ is a disconnected graph, i.e., if the removal of e from G disconnects G. Thus, the set of cut-vertices and bridges is important to identify critical nodes and links in the power grid.

The distance between two nodes $u, v \in V(G)$ is the number of links in any of the shortest paths between u and v. If no such path exists, then the distance is set equal to infinite. The diameter of G is the length of the longest shortest path between any two nodes u and v in G. A disconnected graph has infinite diameter. The average distance of G, denoted as $\langle d \rangle$, is also known as characteristic path length, and it is given by the half-sum of the distances between every pair of nodes, over the number of node-pairs.

B. Centrality Measures

Centrality measures for the nodes and links in a graph G are used to identify the importance of each node or link in a complex network, regarding some criterion of interest.

The degree centrality of a node k indicates its connectivity, resource and access to information in the network. It is defined as:

$$C_D(k) = \frac{deg(k)}{N-1}. \tag{1}$$

The electrical degree centrality indicates the connectivity, taking into account the power flow as the link weights. It is defined as:

$$C_D^E(k) = \frac{\sum_{k \sim t} P_{kt}}{N-1}, \tag{2}$$

where P_{kt} is the power flow between k and t.

The betweenness centrality is one of the most used centrality measures. It is based upon the frequency with which a node falls between pairs of other nodes on the shortest paths connecting them. It indicates how a node is communicable with the rest of the network.

The edge betweenness centrality for an edge e is the sum of the fraction of all-pairs shortest paths that pass through e, indicating how an edge is communicable with the rest of network. Mathematically:

$$C_B(e) = \sum_{s,t \in V(G)} \frac{\sigma(s,t|e)}{\sigma(s,t)}, \tag{3}$$

where $V(G)$ is the set of nodes, $\sigma(s,t)$ is the number of shortest (s,t) paths, and $\sigma(s,t|e)$ is the number of those paths passing through edge e.

The electrical betweenness centrality has a slightly different definition, which takes into account the power flow as the link weights [10]. Mathematically:

$$C_B^E(k) = \sum_{s=1}^{N} \sum_{t=1}^{N} \frac{P_{st}(k)}{P_{st}}, s \neq t \neq k \in V(G), \tag{4}$$

where P_{st} is the maximum power flow in electrical shortest paths between nodes s and t, and $P_{st}(k)$ is the maximum inflow and outflow in node k with electrical shortest paths between s and t.

C. Classical Complex Network Models

The complex network theory is based on graph theory and aims to bring models for real-world networks, which can not be described by usual analyses, i.e., complex networks display heterogeneous structures that result from different mechanisms of evolution. Random networks, small-word networks, and scale-free networks are among the most studied complex network models.

The random network model or ER model was presented in late 1950's by Paul Erdos and Alfred Renyi. In this model, for a given number of nodes, and some given probability p, the network is built by randomly interconnecting the node pairs, according to p. Thus, the node degree distribution follows a binomial probability distribution.

The small-word model generates networks whose characteristics are between regular graphs and random graphs. In small-world networks, the diameter and the average distance depend logarithmically on the number of nodes, i.e., $diameter \sim logN/log\langle k \rangle$ and $\langle d \rangle \sim logN/log\langle k \rangle$ [8].

The scale-free model generates networks for which the vast majority of nodes have small degree, but some nodes (called hubs) have very large degree. In this model, the node degree distribution follows a power-law probability $P(k) \sim ke^{-\gamma}$, where $2 < \gamma < 3$ [8].

III. PROPOSED APPROACH

The Brazilian Power Test System (BPTS) is a set of five test systems with 9, 16, 33, 65 and 107 buses, considering only AC transmission. All BPTS cases are based on the real Brazilian electric power grid [7], therefore taking into account its particular characteristics.

For instance, Fig. 1 ilustrates the one line diagram for BPTS 107 bus case, that is available in [7]. Our analyses will focus on this case.

The BPTS 107 bus case is divided in three subsystems, called as Sul, Sudeste and Mato Grosso, with 22 GW total capacity of generation and 12.7 GW total load. Almost 56% of total generation capacity is on Sudeste subsystem. Energy interchange is often made in the Brazilian power system, and the BTPS cases take into accont this aspect.

From this BTPS 107 bus case, a graph topology was built, where the high voltage transmission lines are modeled by links, parallel lines are considered, and all substations, generators and loads are modeled as nodes. The labels of the nodes in the resulting graph correspond to the original labels shown in Fig. 1.

More generally, our proposed approach is: i) to build a graph for each BTPS case; ii) to analyze each BTPS case with respect to topological aspects and disruption behavior; and iii) to check whether the power grid fits the classic models of complex networks (random, small-world, and scale-free). The following global parameters are considered: number of nodes and links, minimum, maximum and average degree, average path length, and diameter. The resilience analysis includes diameter inspection after the removal of each node or link. Moreover, the behavior of node and link centrality measures is investigated for unweighted and weighted systems. In the latter case, the links are weighted by the power flow. The next section presents results focusing on the BTPS 107 bus case.

IV. RESULTS AND DISCUSSION

For each BPTS case, a graph was built and analyzed, as described in the previous section. Table I shows the main topological parameters for all BPTS cases, i.e., the number of nodes N, the number of links L, the minimum degree k_{min}, the maximum degree k_{max}, the average degree $\langle k\rangle$, the average path length $\langle d\rangle$, and the diameter.

Table I also presents parameters for analyzing disruption behavior, namely, the percentage of bridges and cut-vertices for all BPTS cases, i.e., the percentage of links and nodes for which an individual failure brings the systems disconnected. The last column of Table I shows a logarithmic relation of nodes and average degree, that is useful for checking whether these systems have the small-world property.

The classification of a power grid in the context of complex network theory can be verified using its node degree distribution. Figure 2 shows the probability distribution function (PDF) of the node degree distribution for the BPTS 107 bus case, where the horizontal axis shows the degrees of the nodes and the vertical axis shows the probability of the occurrence of this node degree.

Firstly, the BPTS 107 bus case is not a random network, since its probability distribution does not follow a binomial distribution.

Notice also in Fig. 2 that the probability of having nodes with small degree is greater than the probability of having nodes with high degree. It suggests that BPTS 107 bus case could fit the scale-free model. However, the power-law fitting for this network degree distribution resuts in $\gamma = 3.5$. Another evidence supporting that this system does not follow the scale-free model is the absence of hubs. According to [8], the degree of a hub should be 100 to 1000 times greater than the minimum degree, which is not the case for this system.

In order to check if the BPTS 107 bus case fits the small-world model, we have to compare the ratio $r = logN/log\langle k\rangle$ with the diameter and the average distance $\langle d\rangle$ [8]. For this system, we have $r = 5.26$, whereas $\langle d\rangle = 8.25$ and the diameter is equal to 21. Since the diameter is rather greater than r, one could conclude that the system does not fit the small-world model. On the other hand, it could be considered as a small-world model since $\langle d\rangle$ is not so greater than r.

It is important to know the network model (random, small-word or scale-free) which best fits power grid networks, since it helps to design more robust and resilient networks, contributing to power grid expansion projects.

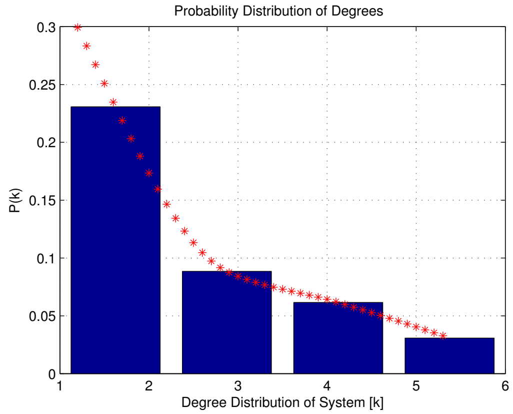


Fig. 2. Network degree distribution for the BPTS 107 bus case.

The network vulnerability can be analyzed from the identification of critical points that can be affected by any kind of failures (overcharge, short circuit, natural

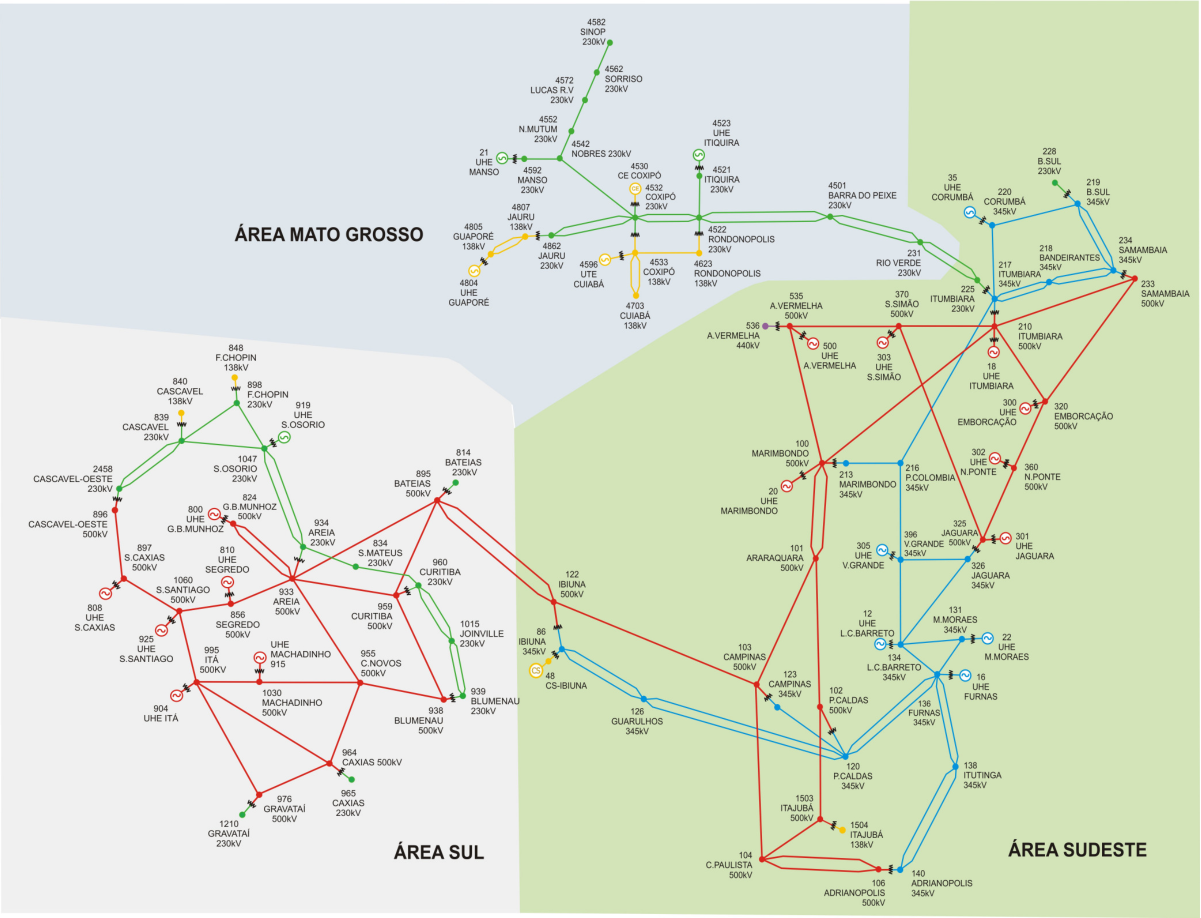


Fig. 1. One line diagram for the BPTS with 107 AC buses [7].

TABLE I
MAIN TOPOLOGICAL PARAMETERS OF ALL BPTS CASES.

System case	N	L	k_{min}	k_{max}	$\langle k \rangle$	$\langle d \rangle$	diameter	% bridges	% cut-vertices	log N/log $\langle k \rangle$
BPTS 9	9	10	1	4	2.22	2.13	5	32.16	33.33	2.75
BPTS 16	16	20	1	6	2.00	3.45	7	32.16	31.25	4.00
BPTS 33	33	41	1	6	2.27	4.03	10	32.16	45.45	4.25
BPTS 65	65	96	1	6	2.36	6.18	13	32.16	40.00	4.85
BPTS 107	107	171	1	6	2.43	8.25	21	32.20	42.00	5.26

causes such as rain and storms, equipments failures, etc). A single failure in a critical point can induce cascade failures, either for not support the power flow redistribution, or for slow dynamic for failure detection.

For the BPTS 107 bus case, the critical points with respect to the diameter can be identified in Fig. 3, that shows the diameter of the network when each node is removed. Notice that, whereas the network diameter is 21 (see Table I), the diameter ranges from 20 to 25 when a node is removed, for the cases where the system remains connected. However, 42% of the nodes are cut-vertices, i.e., when removing each of them, the system becomes disconnected. A failure in any of these nodes can affect system components such as transformers, that can overload transmission lines, that can lead to cascade failures. Moreover, 32.2% of the lines are bridges, i.e., when removing each of them, the system becomes disconnected. A failure in any of these lines can affect other lines and the power flow, that can overload transmission lines, that can also lead to cascade failures.

An intense monitoring is required for vulnerable points, i.e, they need a robust and reliable technology to avoid failures, such as the Phasor Mesurements Units (PMUs), that allows real time network monitoring. The vulnerable points identified are the first candidates to be changed in network restructuring projects.

Network projects should consider more robust networks, but respecting the cost limits. Having in mind that first projects in the beginning of the evolution towards Smart Grids are looking for enhance the existing networks with new technologys, the detection of critical points is very important.

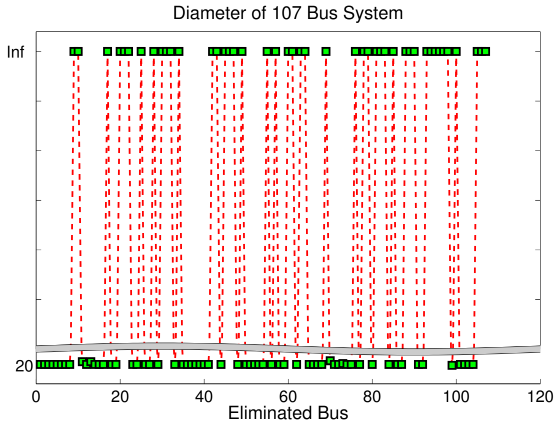


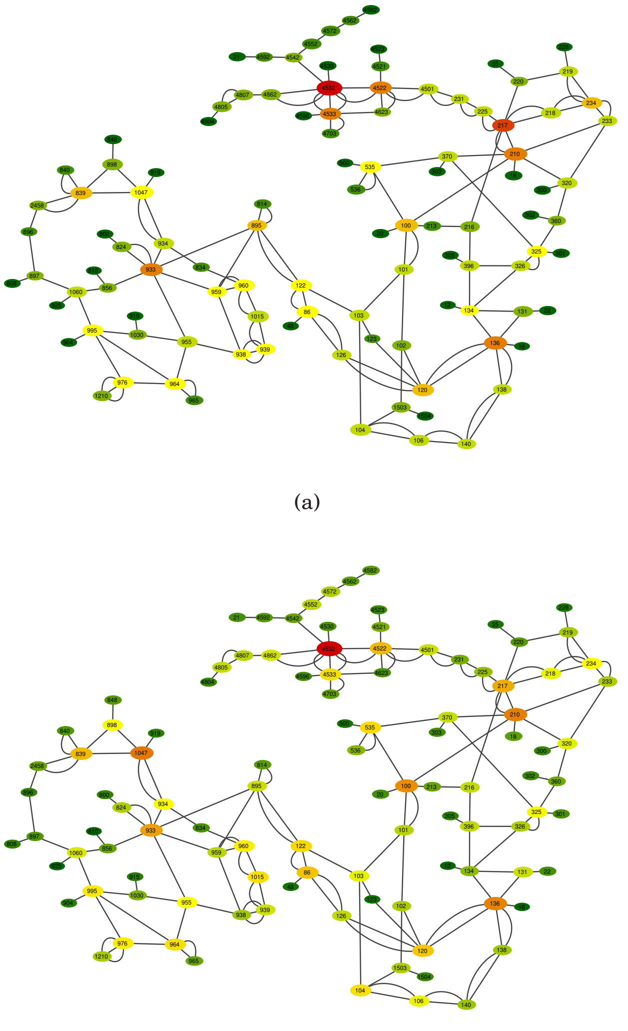
Fig. 3. Diameter of the BPTS 107 bus case after node removal.

The centrality measures also help us to identify critical points of power grid systems, with respect to some criterion of interest. Here we analyze the behavior of the BTPS 107 bus case regarding both the degree centrality and the betweenness centrality, considering the unweighted system, and the system weighted by the power flow. For these analyses, we have used the Cytoscape software [11].

Figure 4 considers the graph corresponding to the BTPS 107 bus case, where nodes represent the substations, loads and generators, and the different node colors represent the different values of degree centrality of the nodes. For the unweighted system, the degree centrality can be seen at Fig. 4(a), where the node colors represent the node degrees. For the weighted system, shown in Fig. 4(b), the colors of the nodes indicate the power flow of the links adjacent to them.

Figure 5 also shows the graph corresponding to the BTPS 107 bus case, but the different node and link colors represent the different values of betweenness centrality of the nodes and links. The betweenness centrality values for the unweighted system are shown in Fig. 5(a), and the corresponding results for the system weighted by the power flow are shown in Figure 5(b).

In Fig. 5, the colors follow the betweenness centrality values. From the highest to the smallest betweenness centrality, the node and link colors vary from red to green. This figure shows the most important nodes and lines of the network, which represent the backbone of the system. The same figure can also help us to identify the most important lines, that correspond to critical lines and may lead to cascading failures or blackouts in the occurrence of a short circuit, for instance. Notice that the weighted and the unweighted betweenness centrality indicate different critical points.

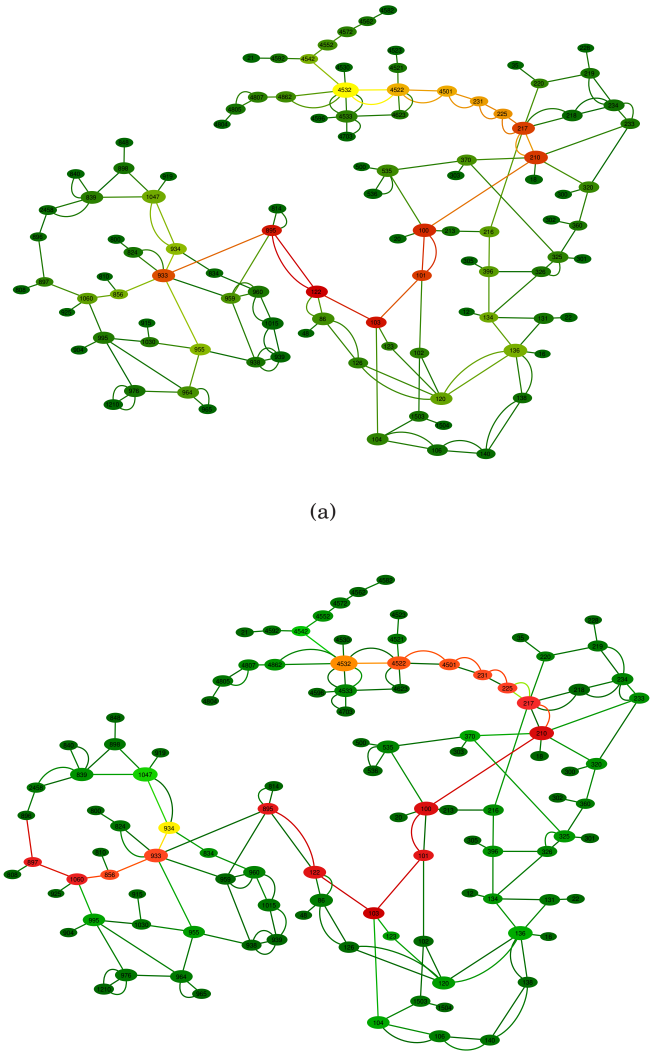

(a)

(b)

Fig. 4. Degree centrality for the BTPS 107 bus case: 4(a) unweighted system, and 4(b) system weighted by the power flow.

The electrical power grids are rulled by the Kirchhoff laws, where the modeling shows the network parameters and its couplings [12]. As presented in [13], there is evidence that topological power grid analyses can not model all required parameters. Metrics such as betweenness centrality, degree centrality, distance and average distance should be rolled out with the insertion of electrical parameters such as line impedance, power angle, and power flow.

Since centrality measures potentially contribute for identifying critical points of power grid systems, and they are sensitive to the different node or link weights considered, it is interesting to extend the presented analyses for other centrality measures such as eigenvector centrality, closeness centrality, and harmonic centrality, and for other electrical parameters such as line impedance, and power angle.



(a)

(b)

Fig. 5. Node and link betweenness centrality for the BTPS 107 bus case: 5(a) unweighted system, and 5(b) system weighted by the power flow.

V. CONCLUSION AND FUTURE WORK

This paper presents results on complex network analysis of the Brazilian Power Test System (BPTS), including topological and electrical aspects, and disruption behavior. The analysis of the BPTS cases using graphs and metrics from complex networks might help us to identify and solve problems such as network overload, catastrophic failures and blackouts. However, more studies and analyses with different cases are necessary. Ongoing research includes other centrality measure analysis considering different node and link weights, for identifying critical points of the BPTS cases. After that, the best combination of invariants and weights could be used to analyze the real Brazilian power system.

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