A SuperCapacitor Agent for Providing Real-Time Power Services to the Grid

Lorenzo Reyes-Chamorro
Mario Paolone
Distributed Electrical Systems Laboratory
École Polytechnique Fédérale de Lausanne
Switzerland
{lorenzo.reyes, mario.paolone}@epfl.ch

Andrey Bernstein
Jean-Yves Le Boudec
Laboratory for Communications and Applications 2
École Polytechnique Fédérale de Lausanne
Switzerland
{andrey.bernstein, jean-yves.leboudec}@epfl.ch

Abstract—**Supercapacitors-based storage systems are expected to play a key role in microgrids in view of their capability to compensate high-power imbalances. We define an agent for the control of supercapacitor arrays within the context of the novel control framework Commelec, proposed by the Authors as a composable method for real-time control of active distribution networks with explicit power setpoints. An important function of such an agent is to advertise the real-time power capabilities and operational preferences of the supercapacitor array based on local information. Given the small energy capacity of such a device, its internal state can largely vary from one setpoint implementation to the next one. For this reason, the use of an accurate model is crucial in the agent definition. We show that it is possible to infer the real-time power capabilities of the device by using simple measurements on the supercapacitor array suitably coupled with an accurate representation of the cells composing the array. Results show that the agent is able to speak for the resource, thus allowing its use from an external controller.**

Index Terms—**Supercapacitors, Storage System, Explicit Power Control, Agent-based Control.**

I. Introduction

The large penetration of renewable energy resources in the distribution level of power systems introduces new complexities for ensuring quality-of-service and reliability of the entire electrical infrastructure due to the stochasticity of these resources. As generation is getting disperse, new technical solutions have emerged. This is the case, for example, of microgrids where generation, storage and controllable demand interact in a low voltage network that can operate both islanded or connected to the main grid as a virtual power plant. The multiplicity of resources with different power flexibility, operation willingness and degrees of uncertainty, added to the fact that they are interdependent through the power network, makes the problem of quality of supply difficult to solve. Note that the elements that play the most important role for the control of the system are the energy storage devices [1]. The classic way for addressing this problem is the droop control method, which has been discussed in the literature extensively [2]–[4]. This solution is based on locally setting the controller parameters on each resource; it can only react to local measurements, and cannot take into account some global aspects of the system state, such as the state of storage in other resources. Also, it cannot expose the actual internal willingness of the resource and can produce stability issues [5].

It this respect, a composable framework for the real-time control of distribution networks by using explicit power setpoints has been introduced in [6]. In this framework the resources can advertise their current internal needs and power availability by simple messages in order to allow a grid controller to take wise and informed decisions and maintain the state of the system within secure limits. The framework, called Commelec, is designed to be robust (i.e. it avoids the problems inherently posed by software controllers) and scalable (i.e. it easily adapts to grids of any size and complexity). It is based on software agents, which are responsible for resources/subsystems (Resource Agents) or entire grids (Grid Agents) and communicate using a simple, yet powerful protocol with a refresh rate of around 100 ms. See Section II for more details on this framework.

Within this context, in this paper we show how to abstract a supercapacitors-based storage by using the Commelec framework. We provide an accurate mathematical representation of a supercapacitors array that allows for the analytic computation of the power limits and the update of its internal state in real-time. We also show the definition of the associated software agent, whose goal is to respect the internal constraints of the device and expose its flexibility to an external controller. The main challenge is then to accurately abstract the storage capabilities in the time-frame of Commelec. This is achieved by considering a very detailed cell model but simplifying its representation for minimizing the computational effort when computing its real-time capabilities. It is important to note that the proposed problem formulation fits all typical supercapacitors models used in literature. Indeed, several applications of supercapacitors-based storage systems with constant power demand can be found in literature. In particular, applications such as wind power, electrical vehicles and elevators are the typical ones (e.g. [7]–[9]). In general, the power capabilities of the supercapacitors are considered as a linear function of the terminal voltage given by the well-known capacitor's

equation. In [10] the sizing of a supercapacitors array is done by taking into account the minimum and maximum allowable instantaneous powers using a simplified static model. This approach may not be suitable for following time-varying setpoints with dynamics comparable with the supercapacitors ones while the redistribution phenomenon is taking place [11]. This will require the assessment of the supercapacitors predictive behaviour to feed an optimal controller.

The paper is structured as follows. In Section II we present in detail the requirements defined by the Commelec framework in order to abstract the resource internal state. We present then the design of the agent in Section III. Section refsec:adv is dedicated to show all the needed information that the agent has to compute. In Section V simulation results using a detailed model are presented. Finally Section VI outlines the conclusions of the work.

II. THE COMMELEC FRAMEWORK

A. Agents Advertisements

In the Commelec framework, a software *agent* is associated with a resource (henceforth called "Resource Agent", RA), or an entire system including a grid and/or a number of devices (henceforth called "Grid Agent", GA). There is a well-defined relationship between the agents, which follows from the tree structure of the distribution networks. An example of agents relationship is shown in Figure 1 where GA is in charge of controlling RAs $A_1, ..., A_N$, who are responsible for subsystems $S_1, ..., S_N$.
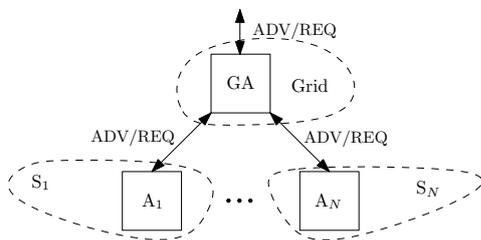

Fig. 1.   A general scheme for showing agents interaction.

Each resource agent advertises its internal state to its grid agent using the following three elements. (1) The *PQ profile* $\mathcal{A}$ is the region in the (P, Q)-plane (for active and reactive power) that the subsystem under the control of this resource agent can deploy. (2) The *virtual cost* C is a function, defined for every (P, Q) in the PQ profile, which returns a number $C(P, Q)$ interpreted as the willingness of this subsystem to apply a requested power setpoint (P, Q). (3) The *belief function* BF returns the set of all possible (actual) setpoints that this subsytem *might* in reality implement when instructed to implement a target setpoint. It accounts for the uncertainty of the subsystem operation. For subsystems such as PV/wind farms, or loads, the belief function returns larger sets, to account for their volatility.

It is important to observe that these three elements (i) are the only information needed for real-time control in the

Commelec framework and (ii) abstract the specific internal state of the resource.

B. Operation of a Resource Agent

For any resource agent in the Commelec framework, the sending/receiving message cycle is repeated continuously and endlessly in normal operation. In this sense, a RA computes an advertisement just after receiving a request. The whole cycle of a RA is described next (see Fig. 2).

1) Reception of a new setpoint request $(P, Q)_{t_0 \to t_1}$ at t_0. The request is to maintain this power fixed from t_0 to $t_1 = t_0 + \Delta t$. Δt is defined by the GA.
2) Check the feasibility of the requested setpoint given the actual instantaneous measured/estimated state in the resource (not available to the GA). Note that for resources with high uncertainty, the implemented power may significantly differ from the requested setpoint, while it will be exactly the same (ideally) for fully controllable resources. This uncertainty has been advertised in the previous belief function. This action also serves to maintain the resource safe in case the GA requests an invalid power setpoint.
3) Instruct the resource to implement the verified setpoint $(\tilde{P}, \tilde{Q})_{t_0 \to t_1}$.
4) Compute and send the advertisement $(\mathcal{A}, BF, C)_{t_0}$ based on local measurements. This advertisement should ensure that $(P, Q)_{t_1 \to t_2}$ maintains the resource safe until t_2.
5) Wait for a new request from GA.
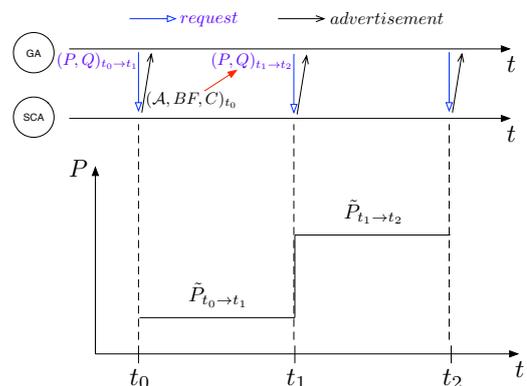

Fig. 2.   Representation of the messages between GA and RA in time. The red arrow indicates that $(P, Q)_{t_1 \to t_2}$ is strongly dependant on $(\mathcal{A}, BF, C)_{t_0}$. The lower diagram shows the implementation of the requested setpoint in time. For simplicity only active power is shown.

III. DESIGN OF SUPERCAPACITOR AGENT (SCA)

In this section we present the design of a SuperCapacitor Agent (SCA), which is a resource agent associated with a supercapacitors array.

A. Specificity of the SCA

For storage systems characterized by relatively high-energy densities (e.g., batteries), the update rate of the resource setpoint imposed by the Commelec framework (around 100ms)

is small enough so that the state of charge can normally be considered constant between two consecutive setpoints. In contrast, for a supercapacitor array, this assumption is no longer true. Unlike battery energy storage systems, supercapacitors are characterized by a higher power density and a smaller energy density [12]. Therefore, operation capabilities of a supercapacitors array for the next time step will depend on the last implemented power setpoints.

As a result, the SCA must have an accurate model in order to assess the electrical state of its resource. For this purpose, we rely on the supercapacitor cell model presented in [13], where the redistribution of residual charge phenomena is taken into account. A brief description of the model is given in the next section in order to provide the elements needed for the SCA definition.

As the Commelec framework controls AC power flows, the SCA needs to consider the presence of a power converter. A generic model of a power converter is presented in [14]. It is composed by an algebraic transfer function that represents the losses taking place in the device. In view of this assumption, a setpoint (P, Q) on the AC side of the converter is transformed into a setpoint p on the DC side of the converter and, therefore, on the supercapacitors array. The mapping between a (P, Q)-setpoint and a p-setpoint is presented in [14] for resources with AC/DC converters and it is no further discussed here.

We focus now in the computation of the limitations of the storage device on the DC side assuming a constant power request between two subsequent deployed setpoints (see Sections III-C and III-D). More precisely, the SCA needs to compute the set of DC power setpoints that can be safely implemented.

B. Supercapacitor cell model

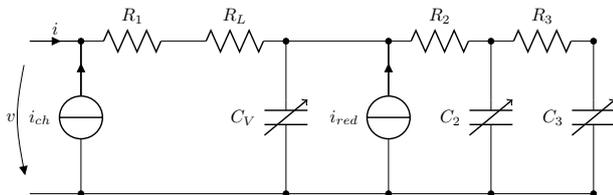Fig. 3 shows the model of a supercapacitor cell [13].

Fig. 3.   Adopted supercapacitor model.

The peculiarity of this model is that it is capable to represent the so-called redistribution of the residual charge taking place in each cell of the array. The residual charge (Q_R) is the amount of charge that remains stored into the SC after a discharge that resulted into a SC terminals voltage (v) achieved at the end of the discharge [13]. The modeling of Q_R redistribution is accounted by two current sources i_{ch} and i_{red} in the supercapacitor equivalent circuit (Fig. 3). Note that i_{ch} is only active when the SC is in operation, namely $i > 0$ (charging) or $i < 0$ (discharging), while i_{red} is inactive. On the contrary, i_{red} is active when $i = 0$ and i_{ch} is inactive. The supercapacitor cell can then be represented as a system

of 6 state variables $\mathbf{x} = (v_{C_V}, v_{C_2}, v_{C_3}, i_{ch}, i_{red}, \dot{i}_{red})$, where v_{C_V}, v_{C_2} and v_{C_3} are the internal capacitors voltages and, i_{ch} and i_{red} the current magnitude of the new current sources.

The system can be defined as a linear control ODE problem where $\mathbf{x}$ is the state, i the input and v the output[1].

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}i \tag{1}$$

$$v = \mathbf{C}\mathbf{x} + Di \tag{2}$$

$$\mathbf{x}(t_1) = \mathbf{x}_1 \tag{3}$$

For the implementation of the SCA, we consider the following assumptions:

- There is access to the instantaneous measurement of v and i. With this measurement available, and given initial conditions, we can compute the state of the model at any time step.
- The parameters of the model are known by means of the process described in [13].
- The SC array is made of identical cells connected in series so that we can directly aggregate them. Therefore, the status of a cell inherently identifies the one of the entire array[2].

C. The SCA Problem

The SC array has operational limits in terms of voltage and current magnitudes: $v_{max} > 0, v_{min} > 0, i_{max} > 0$ and $i_{min} < 0$. In practice the current limits will be mostly defined by the dc-dc stage of the power converter interface.

Observation III.1. For the cell model presented in [13], both the voltage and the current are monotonic with respect to the power. In Fig. 4, we present a numerical validation of this property applied to the cited model. Each curve represents the evolution of v and i with respect to p for different initial conditions. The power-range of each curve is defined by the operational limits of the cell.
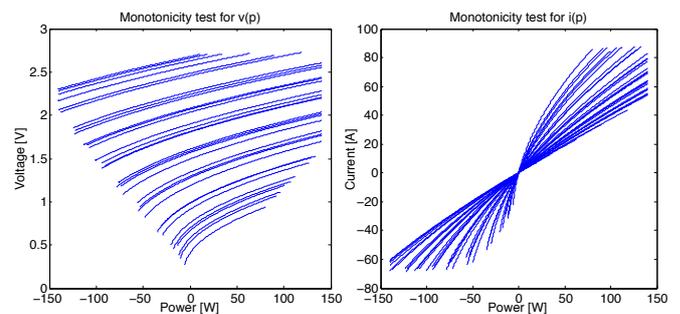

Fig. 4.   Results of the computation of $v(p)$ and $i(p)$ for different initial conditions. The values are computed for a single cell of [13].

The SCA problem is to compute, at time t_0, the set of all DC-side power setpoints $p = p_{t_1 \to t_2}$ that are **feasible**, i.e. for

---

[1]The details on the equivalence of (1) and (2) with the supercapacitor model are discussed in the Appendix.
[2]Note that the distribution of charge among cells is out of the scope of the used model.

which v and i are inside their operational constraints during the whole interval $[t_1, t_2]$. It follows that, given Obs. III.1, the set of feasible DC-power setpoints is an interval, and it is sufficient to compute the maximum and minimum DC-powers p_{min} and p_{max}. The SCA problem at t_0 is therefore:

$$
\begin{aligned}
&\text{minimize (resp. maximize)} && p_{t_1 \to t_2} \\
&\text{subject to} && p(t) = v(t)i(t) \\
& && v_{min} \le v(t) \le v_{max} \\
& && i_{min} \le i(t) \le i_{max} \\
& && \text{Eqs. (1), (2) and (3)} \\
& && \text{for } t \in [t_1, t_2]
\end{aligned}
$$

D. Solution of the SCA Problem

First recall that the power p is kept fixed for the considered time-window. We can develop Eq. (2) to compute v as a function of p, when $i \neq 0$:

$$v = \mathbf{Cx} + D\frac{p}{v}$$
$$v^2 - \mathbf{Cx}v - Dp = 0$$
$$v = \frac{\mathbf{Cx} \pm \sqrt{(\mathbf{Cx})^2 + 4Dp}}{2}. \tag{4}$$

Assuming that v is always between the bounds, it is always positive. Then, the only valid solution of Eq. (4) when $p > 0$ is the one linked to the positive sign, since $D > 0$ (see the Appendix). Note that the argument of the square root is negative for $p < p_c = -\frac{(\mathbf{Cx}(t_2))^2}{4D}$. Therefore, the correct operation of the supercapacitors array requires $p \ge p_c + \delta$, where δ is a safety margin suitably defined. The physical meaning of this power is the following: for values lower than p_c, the electrical state of the array is so deteriorated that no actual power can be provided by the device. As a consequence the solution with the negative sign in Eq. (4) can be never reached by continuity argument, i.e., we always use the positive sign also when $p < 0$. Applying this to Eq. (1) the non-linear ODE problem to be solved is:

$$
\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{B}i \\
&= \mathbf{Ax} + \frac{\mathbf{B}}{D}(v - \mathbf{Cx}) \\
&= \left(\mathbf{A} - \frac{\mathbf{BC}}{D}\right)\mathbf{x} + \frac{\mathbf{B}}{D}v \\
&= \left(\mathbf{A} - \frac{\mathbf{BC}}{D}\right)\mathbf{x} + \frac{\mathbf{B}}{D}\left(\frac{\mathbf{Cx} + \sqrt{(\mathbf{Cx})^2 + 4Dp}}{2}\right) \\
\dot{\mathbf{x}} &= \left(\mathbf{A} - \frac{\mathbf{BC}}{2D}\right)\mathbf{x} + \frac{\mathbf{B}}{2D}\sqrt{(\mathbf{Cx})^2 + 4Dp},
\end{aligned}
\tag{5}
$$

which solution is only obtained through numerical methods assuming a given value on the input p.

When $p = 0$, then $i = 0$ and we can directly solve Eq. (1):

$$
\begin{aligned}
\mathbf{x} &= e^{\mathbf{A}t}\mathbf{x_0} \\
v &= \mathbf{C}e^{\mathbf{A}t}\mathbf{x_0}
\end{aligned}
\tag{6}
$$

The computation of the initial conditions $\mathbf{x_0}$ for both cases ($i = 0$ and $i \neq 0$) is presented in the Appendix.

We now present how to compute the DC power bounds for the period $t_1 \to t_2$ at time t_0. Recall that from the viewpoint of the resource, $p > 0$ represents charging and $p < 0$ discharging.

1) At t_0 the implemented *constant* power from $t_0 \to t_1$ is known. With this, compute the state $\mathbf{x}(t_1)$ by solving Eq. (5) with $p = p_{t_0 \to t_1}$.
2) Use (6) in the window $[t_1, t_2]$, with the purpose of verifying that the redistribution of charges does not violate the voltage limits. Store the value $v_{test} = v(t_2)|_{p=0}$[3].
3) **Computation of p_{max}:** first, note that in a charging phase when $p > 0$, v always increases (see Obs. III.1). As p is constant during this phase, the maximum magnitude of i is reached exactly at the beginning of the new operation window. Hence, the corresponding power that will attain this condition is given by $p_{max}^i = \mathbf{Cx}(t_1)i_{max} + Di_{max}^2$.
   Then, by performing a binary search, we look for the value of p that gives $v(t_2) = v_{max}$. This power is named p_{max}^v. If $v_{test} > v_{max}$, we perform the search in the range $[v_{max}i_{min}, 0]$, otherwise in $[0, v_{max}i_{max}]$. We finally compute $p_{max} = \min(p_{max}^i, p_{max}^v)$.
4) **Computation of p_{min}:** as for p_{max}, we perform a binary search for the value of p such that it reaches $i = \max\left(i_{min}, \frac{v_{min} - \mathbf{Cx}(t_2)}{D}\right)$. If $v_{test} < v_{min}$ the search is done in the range $[0, v_{max}i_{max}]$, otherwise in $[\max(p_c, v_{max}i_{min}), 0]$.

IV. SCA ADVERTISEMENT

As mentioned in section II the advertisement sent by the RA is composed by three elements: PQ profile $\mathcal{A}$, belief function BF and virtual cost C.

A. PQ profile

The PQ power capabilities of the supercapacitors array are defined by the previously computed limits, suitably transferred to the AC terminal through the power converter. The power limits on the AC side are called P_{min} and P_{max}[4]. As in [14] we model the converter as a static element with fixed efficiency. The intersection between these limits and the power converter capabilities -which is considered as the disk in the PQ plane with center in the origin and radius its rated power- will define the PQ profile (for simplicity only the P-coordinate is presented): $\mathcal{A}(P) = [P_{min}, P_{max}] \cap [P_{min}^C, P_{max}^C]$.

B. Belief function

Considering that the supercapacitors array is connected through a power converter to the grid, we assume that it is *fully controllable*, namely, it can follow exactly the requested

---

[3]For example, if the redistribution phase causes the voltage to increase and violate v_{max}, p_{max} will be negative (i.e., the array should be discharged).
[4]Note that in the AC side, $P > 0$ is for discharging.

power. This means that we can define $BF(P,Q) = \{(P,Q)\}$, where the singleton $\{(P,Q)\}$ is the power request.

C. Virtual cost

We define C, as a function of the current state-of-charge (SoC) and therefore only depending on the active power P. We assume that there is a desirable or target value (SoC_T) defined externally (e.g. system-wise with a long-term optimizer). Therefore, we set a higher cost for powers that make $\Delta SoC = SoC - SoC_T$ larger. A suitable quadratic function is desirable for providing convexity.

We consider that the SoC is computed as the ratio between the charge stored in all the capacitors of the model (C_V, C_2 and C_3) and the nominal capacity Q_n[5] $= (C_V + C_2 + C_3)(v_{max} - v_{min})$:

$$SoC = \frac{1}{Q_n}\left(C_V v_{C_V} + C_2 v_{C_2} + C_3 v_{C_3} + Q_R\right)$$

$$Q_R = \begin{cases} \Delta Q_1 & \text{if } i \neq 0 \\ \Delta Q_3 & \text{if } i = 0 \end{cases}\text{[6]}$$

Hence, the cost function that we have adopted is:

$$C(P,Q) = \frac{\Delta SoC}{3}\left(\text{sgn}(\Delta SoC)P^2 - 2P\right).$$

V. SIMULATION RESULTS

In this section we show the application of our method to a supercapacitors-based energy storage device of 120kVA and 300Wh (equivalent to 853 cells of [13] in series).

In Fig. 5 we present the results of a 50s simulation where different power setpoints ("Request") are provided by an external controller (the GA in the Commelec framework). The DC current limit of the converter is set to 80A. We show the evolution of v, i, the AC power, the power limits and the state-of-charge.

The system starts in a resting phase for 1s ($p = 0$) with a $SoC = 0.5pu$. At 2s the GA requests to charge at full power (120kVA) but soon (around 7s) the SCA advertises to the GA a power limitation, thus avoiding the violation of v_{max}, the charge continues reducing the current magnitude while maintaining $v = v_{max}$. This can be also seen in the SoC plot, as the supercapacitors array gets almost full in this phase. At 12s the request is null, and the array goes into a redistribution phase. The voltage decreases slowly due to the redistribution of charges in the cells. At 22s a discharging phase with constant power (inside the limits) causes the voltage to decrease and then the current to increase. The current limitation is soon reached (around 25s) and the power magnitude decreases accordingly. At around 28s the voltage magnitude is so deteriorated that the p_c limit is reached and the current magnitude is reduced. At 32s a new charge starts but as the voltage magnitude is so low, the i_{max} limit is rapidly activated. Since the voltage starts increasing, both

---

[5]Q_n represents the effective charge that the supercapacitors array can store since it cannot be operated until $v = 0$ as the DC voltage is bounded by the power electronics interface.
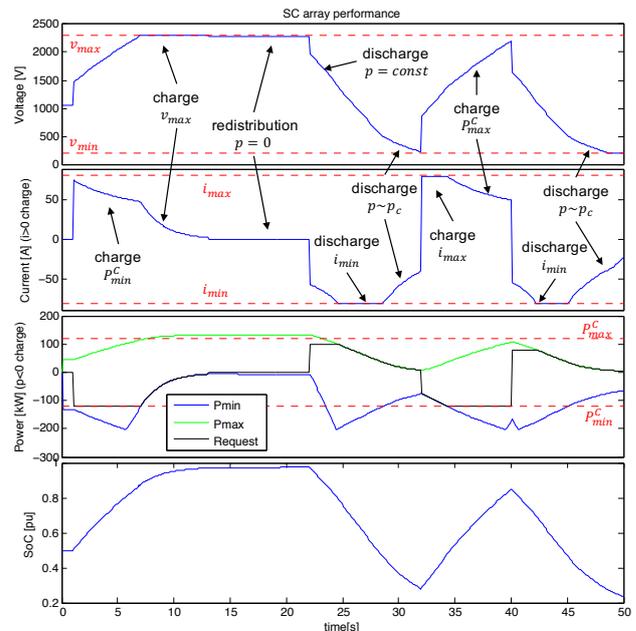

Fig. 5. Simulation results. The red dashed lines represent the operation limits for each variable. In the case of the power plot, it represents the power converter capabilities (120kVA). The actual PQ profile (in the P-coordinate) is $\mathcal{A}(P) = [P_{min}, P_{max}] \cap [P^C_{min}, P^C_{max}]$.

P_{min} and P_{max} increase in magnitude and the system is charged at constant power from second 34. At 40s the GA requests again the injection of power into the grid and the voltage starts decreasing while the power is limited first by i_{min} and then by p_c.

It is interesting to see the rapid evolution of the SoC during short-time windows. As a function of this rapid SoC dynamic, the proposed process is rapidly capable to compute and advertise the internal state of the supercapacitors array through the Commelec framework.

VI. CONCLUSION

We have proposed a method to expose the power operation limits of a supercapacitors-based energy storage system by maintaining both voltage and current magnitudes always inside feasible values. The method also allows to follow explicit power setpoints sent from external controllers in order to provide power services to the grid. We have shown that given the rapid dynamic of this kind of storage system, the implementation of such a method is necessary for allowing real-time control. More specifically, the proposed method, allows for the definition of a supercapacitors array agent within the Commelec control framework.

APPENDIX

Model equivalence

We present in this section how to derive equations (1) and (2) according to the SC model [13]. As previously mentioned, the current sources in the model of Fig. 3 are active or inactive depending upon the current magnitude i. Indeed,

when $i = 0$ only i_{red} is active and when $i \neq 0$ only i_{ch} is. We differentiate this two cases as *redistribution mode* and *charging/discharging mode* respectively. In general **A** can be expressed as:

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \\ \mathbf{0}_{3x3} & A_{ch} & \mathbf{0}_{1x2} \\ & \mathbf{0}_{2x1} & \mathbf{A}_{red} \end{pmatrix}$$

For both cases $\mathbf{A}_{11}$ can be defined as:

$$\mathbf{A}_{11} = \begin{pmatrix} \frac{-1}{C_V R_2} & \frac{1}{C_V R_2} & 0 \\ \frac{1}{C_2 R_2} & -\left(\frac{1}{C_2 R_2} + \frac{1}{C_2 R_3}\right) & \frac{1}{C_2 R_3} \\ 0 & \frac{1}{C_3 R_3} & -\frac{1}{C_3 R_3} \end{pmatrix}$$

For the redistribution mode ($i = 0$):

$$\mathbf{A}_{12} = \begin{pmatrix} 0 & \frac{1}{C_V} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \mathbf{A}_{red} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{\tau_2 \tau_3} & -\left(\frac{1}{\tau_2} + \frac{1}{\tau_3}\right) \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

A_{ch}, **B** and D can be defined as zero.
For the charging/discharging mode ($i \neq 0$):

$$\mathbf{A}_{12} = \begin{pmatrix} \frac{1}{C_V} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \mathbf{B}^T = \begin{pmatrix} \frac{1}{C_V} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_{ch} = -\frac{1}{\tau}, \quad \mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & R_1 + R_L & 0 & 0 \end{pmatrix}$$

$$D = R_1 + R_L$$

$\mathbf{A}_{red}$ can be defined as zero. The parameters τ, τ_1 and τ_2 are the time constants of the two current sources.

Initial conditions

Given the duality of the problem, new initial conditions for the state variables i_{ch}, i_{red} and $\dot{i}_{red}$ have to be computed on each mode transition. In [13] the details for computing these values are discussed. Here we present the results useful for our formulation.

For the transition to the redistribution mode:

$$i_{red}(0) = \left(\frac{\eta_2}{\tau_2} + \frac{\eta_3}{\tau_3}\right)\Delta Q_4, \dot{i}_{red}(0) = -\left(\frac{\eta_2}{\tau_2^2} + \frac{\eta_3}{\tau_3^2}\right)\Delta Q_4,$$

where η_2 and η_3 are parameters of the model and ΔQ_4 is the quota of charge contributing to the redistribution phase due to the diffusion phenomena. This is the effective charge that will be redistributed in the cell during this phase. It is important to note that this charge is the result of the difference between the available charge for redistribution at the beginning of the last charging/discharging phase (ΔQ) and the charge that was indeed redistributed (ΔQ_1) and the amount of residual charge that the cell would have in case it stays in open-circuit conditions (ΔQ_2), i.e., $\Delta Q_4 = \Delta Q - \Delta Q_1 - \Delta Q_2$. We define as ΔQ_3 the charge is effectively redistributed during this phase, i.e., the integral of i_{red}:

$$\Delta Q_3 = \eta_2 \Delta Q_4 \left(1 - e^{-\frac{t}{\tau_2}}\right) + \eta_3 \Delta Q_4 \left(1 - e^{-\frac{t}{\tau_3}}\right).$$

For the transition to the charging/discharging mode:

$$i_{ch}(0) = \frac{\Delta Q}{\tau},$$

where $\Delta Q = \Delta Q_4 - \Delta Q_3 + \Delta Q_2$. As mentioned before, ΔQ_1 is the charge that was redistributed during the charging/discharging mode, i.e., the integral of i_{ch}:

$$\Delta Q_1 = \Delta Q \left(1 - e^{-\frac{t}{\tau}}\right).$$

The initial conditions for all other state variables are simply taken as the state in the previous step.

REFERENCES

[1] K.C. Divya and J. Ostergaard. Battery energy storage technology for power systems - an overview. *Electric Power Systems Research*, 79:511–520, 2009.
[2] J. A. Peças Lopes, C. L. Moreira, and A. G. Madureira. Defining control strategies for microgrids islanded operation. *IEEE Transactions on Power Systems*, 21(2):916–924, 2006.
[3] F. Katiraei and M. R. Iravani. Power management strategies for a microgrid with multiple distributed generation units. *IEEE Transactions on Power Systems*, 21(4):1821–1831, Nov. 2006.
[4] José Matas, Miguel Castilla, Luis García de Vicuña, Jaume Miret, and Juan Carlos Vásquez. Virtual impedance loop for droop-controlled single-phase parallel inverters using a second-order general-integrator scheme. *IEEE Transactions on Power Electronics*, 25(12):2993 – 3002, December 2010.
[5] D.E. Olivares, A. Mehrizi-Sani, A.H. Etemadi, C.A. Cañizares, R. Iravani, M. Kazerani, A. H. Hajimiragha, O. Gomis-Bellmunt, M. Saeedifard, R. Palma-Behnke, G.A. Jimenez-Estevez, and N.D. Hatziargyriou. Trends in microgrid control. *IEEE Transactions on Smart Grid*, 5(4):1905–1919, 2014.
[6] A. Bernstein, L. Reyes-Chamorro, J.-Y. Le Boudec, and M. Paolone. A composable method for real-time control of active distribution networks with explicit power setpoints. Part i: Framework. *Electric Power Systems Research*, 125:254–264, 2015.
[7] A. Rufer and P. Barrade. A supercapacitor-based energy-storage system for elevators with soft commutated interface. *IEEE Transactions on Industry Applications*, 38(5):1151–1159, 2002.
[8] N. Mendis, K. M. Muttaqi, and S. Perera. Management of battery-supercapacitor hybrid energy storage and synchronous condenser for isolated operation of pmsg based variable-speed wind turbine generating systems. *IEEE Transactions on Smart Grid*, 5(2):944–953, 2014.
[9] R. E. Araújo, R. de Castro, C. Pinto, P.Melo, and D. Freitas. Combined sizing and energy management in evs with batteries and supercapacitors. *IEEE Transactions on Vehicular Technology*, 63(7):3062–3076, 2014.
[10] A. Kuperman, M. Mellincovsky, C. Lerman, I. Aharon, N. Reichbach, G. Geula, and R. Nakash. Supercapacitor sizing based on desired power and energy performance. *IEEE Transactions on Power Electronics*, 29(10):5399–5405, 2014.
[11] M. Kaus, J. Kowal, and D. U. Sauer. Modelling the effects of charge redistribution during self-discharge of supercapacitors. *Electrochimica Acta*, 55(25):7516–7523, 2010.
[12] T.A. Smith, J.P. Mars, and G.A. Turner. Using supercapacitors to improve battery performance. In *Proceedings of the 2002 IEEE 33rd Annual Power Electronics Specialists Conference*, volume 1, pages 124–128, 2002.
[13] D. Torregrossa, M. Bahramipanah, E. Namor, R. Cherkaoui, and M. Paolone. Improvement of dynamic modeling of supercapacitor by residual charge effect estimation. *IEEE Transactions on Industrial Electronics*, 61(3):1345 – 1354, April 2013.
[14] L. Reyes-Chamorro, A. Bernstein, J.-Y. Le Boudec, and M. Paolone. A composable method for real-time control of active distribution networks with explicit power setpoints. Part ii: Implementation and validation. *Electric Power Systems Research*, 125:265–280, 2015.