Optimal Demand Response based on time-correlated utility in forward power markets

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***Abstract*—This paper is concerned with integration of demand response from small loads such as residential small appliance into forward electricity market. We use a simple model and formulate the problem as a dynamic program that maximizes social welfare. We focus particularly in those electric appliances with intertemporal consumption constraint. The satisfaction of end-users from operating them are captured through a utility function, which is also time-correlated. We pursued Lagrange decomposition in order to solve the high complexity problem optimization of the complete system. The main purpose is to optimally schedule a day in advance Local Service Entity's (LSE) procurement and final-users consumption in the forward electricity market.**

Index Terms- Demand Response, Smart Grid, decomposition algorithms, LSE.

I. INTRODUCTION

Demand Response (DR) is an important, manageable task, promising to enable interaction of end-users and the utility company or LSE, maximizing capital profit an efficiency of the power grid [1]. On the supply side, the LSE participates in several power markets, particularly purchasing electricity on the wholesale electricity market and reselling it on the retail market to end-users. On the demand side, DR shapes the amount of end-user's power consumption in response to time-varying energy prices imposed by the LSE, reducing their demand when the market is short of energy or shifting the demand to another time when supply is abundant, improving the efficiency of consumption. The main purpose of the LSE is to provide the aggregate loads in the wholesale electricity market so it can operate efficiently, while, on the other hand, hide the complexity and absorb the risk impossed on the electricity market for their clients. This volatility present in the electricity market is a consequence of the uncertainty in not dispatchable renewable sources that can fluctuate rapidly and by large amounts.

The context of this paper includes DR from residential end-users with small amounts of loads. Especially we model DR program for electric appliance whose particularity are the intertemporal consumption constraint and a time-correlated utility function to model end-user satisfaction. Examples of this residential appliance are air-conditioning units or heaters and any kind of energy storage. We model the electricity market as a forward capacity market. The LSE makes his bids for power consumption for the following day accordingly with their customer's demand predictions. The LSE also participates in several wholesale markets (such as real-time balancing, ancillary service, etc.) to purchase electricity from generators and accomplish the real-time demand of their customers, selling it in the retail market. We consider end-users as price-takers in the retail market, offering bidding strategies to maximize their profits, achieving at the same time system-wide benefits as social welfare maximization and reduction of system marginal prices. We assume that the LSE is regulated such that his objective is not to maximize his capital profit by selling electricity, but cooperate with the end-users to maximize the social welfare of the complete system. In this article, we complement the works exposed in [2] and [3]. The model proposed in [2] maximizes utility profit or minimizes deviation from user desired consumption sometimes integrated with unit commitment and economic dispatch. The end-users react and adapt their forecast of power consumption following a price signal sent by the utility company. This behavior is achieved by coordination of different appliance indirectly by the real-time prices, so as to flatten the total demand at different time as much as possible. In [3] two-period procurements are considered, day-ahead and real-time. An amount of power generation is scheduled by the LSE in the wholesale market for the next

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operating day accordingly his customer's power consumption forecast. At the dispatchable time, the LSE participates in a real-time market to balance the actual power demands of their customers, the undispatchable renewable generation and the power scheduled during the day-ahead market. A distributed algorithm is proposed, with the LSE and end-users exchanging information and maximizing social welfare. However, they used a simple utility function that measures deviation from an estimated power consumption curve. In addition, we integrate storage capacity on the demand side, which helps to reap more benefits from DR, reducing the peak load and flattening the entire profile and the variation in demand. We include decomposition methodologies of manageable complexity for DR coordination across the grid in the retail market, preserving end-users preferences, constraints, and privacy. We assume a decentralized communication between the participants of the optimization problem, exchanging adequate signal prices.

II. System Model

We use a discrete-time model with a finite horizon that models a day. Each day is divided in T periods of equal duration, indexed by $t \in \mathscr{T} = \{1,..,T\}$. The time duration of each period depends on the market, corresponding with the time resolution the energy can be dispatched or the DR decision can be made. We consider the LSE only participating in day-ahead market; it does not cover real-time balancing market or auxiliary service such as regulation or reserves.

A. LSE Model

The LSE has to decide how much power procure for each consumption period t for the next day to supply his aggregated demand. Lets $P_d := (P_d(t), t \in \mathscr{T} = \{1,..,T\})$ be a non negative vector variable representing day-ahead dispatchable power scheduled or reserved for the next day by the LSE in the wholesale market. Schedule $P_d(t)$ incurs a cost to the LSE of $C_d(P_d(t);t)$. The design of the retail prices imposed for LSE to end-users are a consequence of $C_d(\cdot,\cdot)$, which summarizes the cost to at least recover the running costs of supplying aggregates demand, including the payment of the wholesale market. The modeling of this cost function is an active research issue which is not treated here. We just assume that $C_d(\cdot,\cdot)$ is a convex increasing function for each t over P_d. The LSE must derive the optimal decision for power reserved on the day-ahead market and set the prices in the retail market $w := (w(t), t \in \mathscr{T} = \{1,..,T\})$. This problem can be mathematically expressed as follows.

$$\underset{P_d}{Max} \sum_t [w(t) \cdot P_d(t) - C_d(P_d(t);t)] \quad (1)$$

$$\text{s.t} \quad P_d(t) \geq 0, \quad \forall t \in \mathscr{T} = \{1,..,T\}$$

B. End-User Model

We consider N users that are served by a unique LSE. Without loss of generality, we consider each user has only one appliance whose general characteristics are the intertemporal consumption restriction and a time-correlated utility function that measures the client's satisfaction. During the day, user i uses an amount of power $y_i := (y_i(t), t \in \mathscr{T} = \{1,..,T\})$ in this appliance, obtaining for each period t of the day a level of satisfaction summarized by the utility function $u_i(y_i;t)$.

Appliance Model: We model thermal appliances such as heating, ventilation, air conditioning, etc., which control the temperature inside a room or environment. Let's $T_i^{in}(t)$ and $T^{out}(t)$ be the inside and outside temperatures of the place where appliance's user i works at time t, and let's define $\mathscr{A}_{it}$ the set of time user's i is at home. The temperature inside the room evolves accordingly the following equation:

$$T_i^{in}(t) = T_i^{in}(t-1) + \alpha(T_{out}(t) - T_i^{in}(t-1)) + \beta y_i(t)$$

where α and β represent thermal characteristics of the appliance and the environment. The third term of the dynamic transfer equation represents the thermal efficiency of the system: for $\beta > 0$ the appliance is a heater, while for $\beta < 0$ is a cooler. We also define $T_i^{in}(0)$ as the temperature at the end of the previous day. Rewriting the expression in a matricial way we obtain:

$$T_i^{in} = T_i^{in}(0) \cdot (1-\alpha)^t + A \cdot T_{out}^T + B \cdot y_i^T$$

where A and B are constant matrices and T_i^{in} and T_{out} are the inside and outside temperatures of each gap of time $t \in \mathscr{T} = \{1,..,T\}$.

The satisfaction of user i can be captured as the sum of deviations from a comfort or desired temperature in the environment for each period of time the user is at home. We define the utility function $u_i(y_i(t);t) = -b_i\|T_i^{comf}(t) - T_i^{in}(y_i;t)\|^2$ $\forall t \in \mathscr{A}_{it}$, being $\mathscr{A}_{it}$ the set of times the user is at home.

Battery model: In addition to thermal appliance, customer i also has a battery which provides further flexibility in consumption across time. The presence of storage capacity in demand side optimizes the use of energy, according to signal prices of electricity given by the LSE in the retail market. We denote as $b_i(t)$ the energy capacity of the battery at time t, and by $r_i(t)$ the power charged/discharged from the battery at time t. We model the dynamics of the battery energy level as:

$$b_i(t) = \sum_{\tau=0}^{t} r_i(\tau) + b_i(0)$$
$$\text{s.t} \quad r_i \in \mathscr{R}_i \tag{2}$$

where $\mathscr{R}_i$ is the feasible set of $r_i := (r_i(t), t \in \mathscr{T} = \{1,..,T\})$ such as $\forall\ t$, :

$$\begin{aligned} &0 \leq b_i(t) \leq B_i \\ &r_i^{min} \leq r_i(t) \leq r_i^{max} \\ &b_i(T) \geq \gamma \cdot B_i, \gamma \in (0,1] \end{aligned}$$

where B_i is the capacity of user's i battery, r_i^{max} and r_i^{min} are upper and lower limits of charged/discharged power ramp. In order to make sure that there is a certain amount of energy at the beginning of the next day, we impose a minimum charge for time period T of the previous day, which means $b_i(T) \geq \gamma \cdot B_i$. The cost of operating the battery is modeled by the function $D_i(r_i)$ that depends on the vector of charged/discharged of power r_i along the following day. This cost may correspond to the amortized purchase and maintenance cost of the battery lifetime, which depends on how fast/much/often/deep its is charged and discharged. We assume that the cost function $D_i(r_i)$ is a convex function over the vector r_i. The next function attains its maximum at the given desired operating point, which yields maximum satisfaction to the end-user. Assuming the signal prices $w(t)$ imposed by the LSE, each user chooses its own power demand and charging schedule to maximize its own net benefit.

$$\underset{r_i, y_i}{Max} \sum_t \left[u_i(T_i^{in}(y_i, t); t) - w(t)x_i(t) \right] - D_i(r_i) \tag{3}$$

$$\begin{aligned} \text{s.t} \quad &0 \geq x_i(t) \geq Q_i^{max} \\ &T_i^{comf}(t) - T_i^{in}(y_i, t) \leq \epsilon\ \forall t \in \mathscr{A}_{it} \\ &r_i \in \mathscr{R}_i \\ &y_i(t) \geq 0 \end{aligned}$$

being ϵ the maximum tolerance of deviation from the comfort temperature in the environment and $x_i(t) = y_i(t) + r_i(t)$ the amount of power scheduled in the retail market by the user, $x_i := (x_i(t), t \in \mathscr{T} = \{1,..,T\})$. It is reasonable to assume that user's i batteries cannot discharge more power than the amount needed by the appliances.

C. System model

We would like to design an energy procurement and a demand response scheme to find y, r and P_d that maximize the social welfare of the complete system.

$$\begin{aligned} &W_{total}(y, r, P_d) := \\ &\underset{y,r,P_d}{Max} \textstyle\sum_t \left[\sum_i u_i(T_i^{in}(y_i, t); t)) \right] - \sum_i D_i(r_i) \\ &\qquad\qquad - \textstyle\sum_t C_d(P_d(t); t)) \end{aligned} \tag{4}$$

$$\begin{aligned} \text{s.t} \quad &0 \geq x_i(t) \geq Q_i^{max} \\ &T_i^{comf}(t) - T_i^{in}(y_i, t) \leq \epsilon\ \forall t \in \mathscr{A}_t \\ &r_i \in \mathscr{R}_i \\ &P_d(t), y_i(t) \geq 0 \\ &\textstyle\sum_i x_i(t) = P_d(t) \end{aligned}$$

The last equality constraint stands for the power balance of the system.

III. DISTRIBUITED ALGORITHM

To maximize the social welfare, we must find the optimal energy procurement and DR scheme of the LSE and end-users, but there are certain constrains on the information they can exchange. For example, is not desirable for the LSE to make public its cost structure. Furthermore, each end-user has preferences abut its controllable operation and it would not be manageable for the LSE to solve an optimization problem directly coordinating a large number of end-users. The approach here uses a dual decomposition of the optimization problem after introducing appropriate multipliers to decouple constrains [4]. Lagrange relaxation in coupling constraint will decompose the principal problem into several manageable problems with the appropriate structure to be solved by each of the participants. The LSE solves its problem in coordination with the forecast power consumption predicted by each end-user. This is accomplished with the appropriate information exchange, which do not need to reveal end-users preferences and LSE's cost function. The only coupling constrains consider will be the power balance one, while the remaining constraints will kept implicitly. Let $w^t = (w(t), t \in \mathscr{T} = \{1,..,T\})$ be the Lagrange multipliers associated to the power balance constraint. The Lagrangian function is:

$$\begin{aligned} &\mathcal{L}_{total}(y, r, P_d, w^t) := \\ &\underset{(w^t)}{Min}\ \underset{(y,r,P_d)}{Max}\, W_{total}(y, r, P_d) + w^t \cdot (P_d - \textstyle\sum_i (y_i + r_i)) \end{aligned} \tag{5}$$

$$\begin{aligned} \text{s.t} \quad &w^t \cdot (P_d - \textstyle\sum_i (y_i + r_i)) = 0 \\ &0 \geq x_i(t) \geq Q_i^{max}; \forall t \in (1,..,T) \\ &T_i^{comf}(t) - T_i^{in}(y_i, t) \leq \epsilon; \forall t \in \mathscr{A}_{it} \\ &r_i \in \mathscr{R}_i \\ &P_d(t), y_i(t) \geq 0; \end{aligned}$$

After a straightforward re-arrangement, the Lagrangian funtion can be written as:

$$\mathcal{L}_{total}(y,r,P_d,w^t) = \underset{(w^t)}{Min}\ \underset{(y,r,P_d)}{Max} \sum_i L_{user_i}(y_i,r_i,w^t) + L_{LSE}(P_d,w^t) \quad (6)$$

where

$$\mathcal{L}_{LSE}(P_d,w^t) = \sum_t \left[w^t(t)P_d(t) - C_d(P_d(t);t)\right] \quad (7)$$

$$\mathcal{L}_{user_i}(y_i,r_i,w^t) = \sum_t \left[u_i(T_i^{in}(y_i,t);t)\right] - D_i(r_i) - \sum_t \left[w^t(t)(y_i(t)+r_i(t))\right] \quad (8)$$

It is clear from (6) that optimization problem (5) can be decoupled into $1+N$ min-max problems, one performed by the LSE (7) and the remaining (8) solved by each of the N end-users. Furthermore, we can see that (7) and (8) are the same problems as (1) and (3) respectively. Utilizing a gradient descent algorithm, the LSE solves (7) with the consumption information sent by end-users, updating the power to purchase P_d in the wholesale market. The LSE must solve not only the optimal solution, but also the Lagrange multipliers or signal prices w^t of (7) and report it to end-users. Each end-user receives the signal prices from the LSE and solves its own maximimization problem (8). After several iterations, the system converges to the optimal decision for the LSE and end-users, P_d^*, y_i^* and r_i^* respectively.

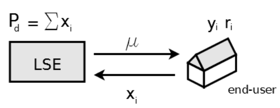

Figure 1. Information exchanged by LSE and end-users.

The information exchange is described in Fig 1, where the proposed decomposition and solution method respects end-users privacy and the cost function structure of the LSE. The model starts at the begining of each day; the LSE and end-users iteratively exchange information through a bidirectional communication. Each of them compute the signal prices w^t, optimizing the consumption and charging profile y_i and r_i for each period t of the next day. Here is the algorithm:

Initially user i chooses $y_i^0 \in [y_i^{min}, y_i^{max}]$ and $r_i^0 \in \mathscr{R}_i$ and the LSE chooses $P_d \in [0, P_d^{max}]$.
In iteration $k+1 = 1,2,...$ do the following:

1) Each end-user updates their forecast consumption x_i and report it to the LSE. The LSE computes the aggregate demand and actualize his reserve in the whole sale market and also the signal prices solving.
   - $P_d^{k+1} = P_d^k + \alpha^m(-\frac{\partial C_d}{\partial P_d}(P_d^k,t) + w^k)$
   - $w^{k+1} = w^k - \alpha_2^m(P_d^k - \sum_i(y_i^k + r_i^k))$

   Then, LSE broadcasts the signal price w^{k+1} to end-users.
2) Each end-user i solves its optimization problem (7) with the updated signal prices and actualize its forecast consumption.
   - $y_i^{k+1} = y_i^k + \alpha^m(\frac{\partial U_i}{\partial Tin_i} \cdot \frac{\partial Tin_i}{\partial y_i} - w^k)$
   - $r_i^{k+1} = r_i^k + \alpha^m(-\frac{\partial D_i}{\partial r_i} - w^k)$

   where α^m is the step size.

IV. NUMERICAL AND ANALYSIS RESULTS

We consider a scheduling horizon of twenty-four periods of one-hour, starting at 1 A.M. until 12 P.M.. We consider four households attended by the LSE. Users are not at home during office hours (from 8:00 A.M to 18:00 A.M). Each of the households is assumed to have one heater and an electric battery. The basic parameters used for the simulation are shown as follows.

1) *Heater*: The outside temperature of end-user environments is shown in Fig.2, which capture a typical day. Also, the period of time the residents are out of home is what we called office hours. We assume the comfortable temperature in the room is 22°C and a comfortable deviation $\epsilon = \pm$ 1°C is tolerated by the DR scheme.
2) *Battery*: The storage capacity is chosen randomly from $[8000, 10000]W/h$ and the maximum charging/discharging rate is $2000W/h$ and set $\sigma_i = 0.5$. We model the cost function of the battery in the following form [2]:

   $$D_i(r_i) = \eta_1 \sum_{t=1}^{T}(r_i(t))^2 - \eta_2 \sum_{t=1}^{T-1}(r_i(t)r_i(t-1))$$

   Where η_1 and η_2 are positive constants. The first term model the damaging effect of fast charge/discharge. The second term penalizes charging/discharging cycles assuming that if $r_i(t)$ and $r_i(t+1)$ have different signs there will be a cost. If $\eta_1 > \eta_2$ the cost function D_i is a positive convex function over the vector r_i .
3) *User's Utility Function*: For each household we assume $Q^{max} = 30kW/h$, and the utility function $u_i(y_i(t);t) = -b_i\|T_i^{comf}(t) - T_i^{in}(y_i;t)\|^2$ where b_i is a positive constant. We assume three cases for the appliance.

- There is always a resident at home with the heater working all time, mantaining the temperature of the environments at 22°C.
- There is a resident at home $\forall t \in \mathcal{A}_{i,t}$ with no DR scheme, we modeled the heater as on/off appliance scheduled by the user.
- There is a resident at home $\forall t \in \mathcal{A}_{i,t}$ with DR scheme controlling the thermal appliance and the battery. The DR scheme keeps the temperature inside user's environment closed to the comfort temperature. Furthermore, the DR will absorb energy from the grid to charge/discharge the battery in a convenient way.

4) *LSE's Cost Function* $C_d(P_d; t)$: We assume $C_d(\cdot; t)$ is an increasing and convex function that satisfy $C_d(0; t) = 0$. In this way, we consider for each time period a cost function $C_d(P_d(t)) = [a_1 P_d(t)^2 + a_2 P_d(t)]/2$ [3]. We also assume that there exists $P_{max}(t) \geq \sum_i Q_i^{max}, \quad \forall t \in \mathscr{A}_{it}$. This implies that the LSE can support the maximal possible demand $\sum_i Q_i^{max}$ using only day-ahead energy, with finite marginal cost.

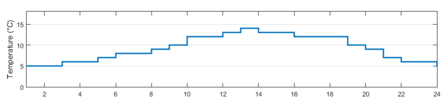

Figure 2. Outside Temperature along the day.

A. Temperature inside user's enviroment.

We show here a comparison between different operation schemes of the thermal appliance in order to evaluate the performance of the proposed DR strategy.

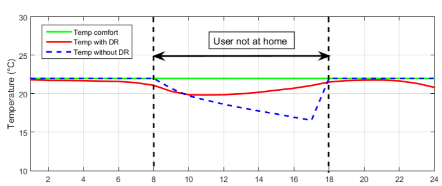

Figure 3. Temperature inside a user's enviroment with and without DR strategy.

In Fig 3 and Fig 4 we can see the performance of the DR scheme at an end-user environment, the evolution of $T_{in}(t)$ in the three cases and the power consumed. In the first scheme, light line in Fig 4, the end-user just allocates energy consumption to keep the environment's temperature as close

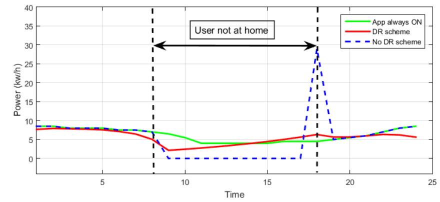

Figure 4. Power consumed from the grid by an end-user in the different schemes.

as possible to a comfortable temperature. The end-user does not pay attention to the signal prices and he just optimizes his utility function. In the second scheme, dark dotted line in Fig 4, the end-user controls the thermal appliance as ON/OFF, and just allocates energy consumption to keep the temperature closed to the comfort temperature when he is at home. We can see a pronounced peak when the therml appliance is turned on close to the arrival time. The electricity demand over a day under the DR scheme is shown in dark solid line in Fig 4. We can observe an elastic behavior of the controlled appliance, absorbing energy even when the resident is not at home. The temperature in the environment is free to move for $\forall t \notin \mathcal{A}_{it}$ but the DR keeps it into certain limits and does not leave it to deviate far from the comfort temperature. In addition, the energy consumed by the end-user in those periods of time is not only used in the thermal appliance, but also for charging the battery.

The DR scheme helps to reduce the overall electricity prices of the system. The end-user with DR profile not only consumes in an intelligent way, shaping the demand along the day and flattening the consumption curve, but also maintains the comfort in the environment.

B. Presence of storage capacity.

The battery can storage energy in periods of low demand, and release the energy at periods of high demand, reducing the peak load. Also, the presence of batteries helps the LSE to shift load from peak to base load periods in order to reduce the peak of power in the end-users grid. Fig 5 shows the energy capacity of an end-user's battery along the day and also the charging/discharging rates for each period t. It is clear that the DR scheme uses part of the capacity storage in the battery to keep the temperature closed to the comfortable temperature in the room when there is someone at home, and charge it when the environment's temperature is free to move. Fig 6 shows that the battery in the end-user's DR scheme also has the capacity to smooth the end-user's consumption curve, by absorbing or delivering power to limit fluctuations due to the thermal appliance. It is clear that DR scheme with battery increases load factor in demand side without affecting the end-user's utility.

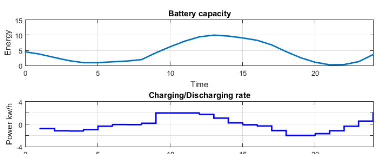

Figure 5. Battery storage capacity and charging/discharging rate along the day.

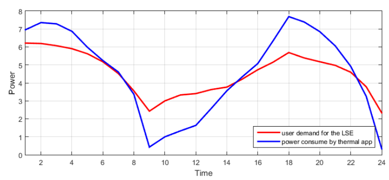

Figure 6. Power consumed by end-user's DR scheme with and without battery.

C. DR scheme in the supply side.

From the LSE or supply side point of view, it is interesting to observe in Fig 7 the important reduction of the power peaks and the smoothed total power load curve (not only smaller peaks, but also valleys). The load factor with DR is 0.7304 and 0.6789 with and without battery respectively, and drops even more to 0.2395 with no DR scheme in the end-users profile.

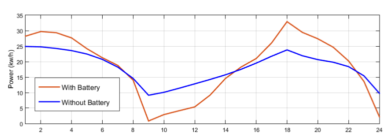

Figure 7. Total LSE supply to accomplish demand with and without battery.

Not only the DR scheme with battery can increase the load factor greately, but also reduces the peak and total cost of the complete energy system. This reduction in the power reserved by the LSE in the wholesale market could also be reflected in a reduction of energy prices for end-users.

V. Conclusion and Future Work

This work was motivated by the promising benefits of the incorporation of DR scheme in thermal appliances, both in

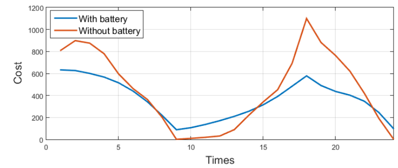

Figure 8. Total costs incur by LSE in the wholesale market to accomplish demand side.

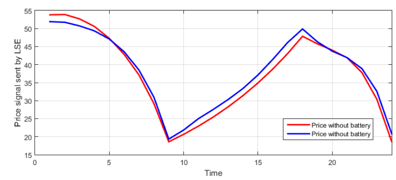

Figure 9. Marginal signal prices imposed by the LSE to end-users.

supply side and demand sides. We use a thermal appliance with time correlated constrains and also time correlated utility function. End-users interact with LSE who participates in a wholesale market to procure the aggregated demand. The DR scheme schedules end-user power consumption accounting their preferences and satisfaction, maximizing not only their own profits but also the social welfare. The primal-dual method used to decouple some constraints and optimize the system, preserves the integrity of the private information of the LSE and also captures end-user's preferences and restrictions. It is interesting to pursue extensions in other markets such as real-time and ancilliary services, and to improve the model for the wholesale market clearing formulation for the LSE. The inclusion of uncertainty both in supply and demand side and the inclusion of stochastic constraints would be also future lines of research.

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