Modal Parameter Estimation using Synchrophasors

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***Abstract*—The development and diversification of the Uruguayan power electric infrastructure, with the significant incorporation of renewable sources is leading to changes in the operation criteria. With all these efforts to make the power system more efficient and smarter, real-time analysis are becoming more required and necessary.**
This paper investigates a method of estimating the electromechanical modes from multiple synchrophasors. The application includes a method with the objective to provide the maximum observability information of the electromechanical modes of interest.
The study specifically analyzes the performance of the method in the 10 machine 39 bus IEEE test system and in the Uruguayan power system.

***Index Terms*—Synchrophasors, Phasor Measurement Unit, Prony Analysis, Autoregressive Model.**

I. INTRODUCTION

A Synchrophasor is a phasor measurement with respect to an absolute time reference. With this measurement it is possible to determine the absolute phase between phase quantities at different locations of the power system.
The introduction of phasor measurement units (PMU) in power system significantly improves the possibilities for monitoring and real-time analysis. Improved monitoring and remedial action capabilities allow network operators to utilize the power system in a more efficient way.
Uruguayan Electric Power System is growing and despite being small it is becoming more complex with the connection of distributed generation such as wind, solar and biomass.
Power system inter-area oscillation is becoming a major concern to the Uruguayan power system utility. It imposes constraints and, therefore, can negatively impact the economical operation of the power system. Without some remedial action, the inter-area oscillation can result in major blackouts.
Because of this, new techniques and innovative technologies for *Online* Stability Analysis are motivated. *Offline* Stability Analysis techniques are no longer enough for the present and future power system.
The analysis is going to be done in IEEE New England System which has 39 busbars and 10 machines. Software tools are PSSE for dynamic simulations and MATLAB for signal processing.

II. PMU PLACEMENT

Modal estimation analysis is focused on generation busbars. The first step in modal estimation is to gather measurement data from substations. These measurements must be sufficient to make the system observable. The aim of the method is to provide the maximum observability information of the modes of interest.

A. Method Description

The method consists on the construction of a Matrix *A* and a binary optimization problem. Matrix *A* contains system nodes interconnection information and it is constructed as follows:

$$a_{ij} = \begin{cases} 1 & \text{if } i = j \\ 1 & \text{if i and j are connected} \\ 0 & \text{other} \end{cases} \tag{1}$$

Binary Optimization Problem statement is:

$$\begin{cases} \min \; f(x) = \sum_{i=1}^{N} x_i \\ Ax \geq b \\ x \in \{0, 1\} \end{cases} \tag{2}$$

Where vector $b = \begin{pmatrix} 1 & \cdots & 1 \end{pmatrix}$ and the function to optimize given by

$$f(x) = \sum_{i=1}^{N} x_i \tag{3}$$

is the total amount of PMUs and its location. Therefore $x_i = 1$ if and only if there is a PMU located at node i.

The solution to this problem is obtained by solving the Binary Optimization Problem by any of the known techniques.

It is important to note that if equality restraints are added the solution will include nodes where PMUs must be located and nodes where PMUs must not be located. So, it is possible to force the solution to locate PMUs in generation busbars and the solution will include this nodes and others in order to get an observable system.

III. OSCILLATION MODES ESTIMATION.

Two signal processing methods are presented for oscillation modes estimation:

- Prony Analysis.
- Autoregressive Model.

A. Prony Analysis

Prony Analysis fits a function $y(n)$ from a set of N samples $y(0) \cdots y(N-1)$ and T_s sample rate by a series of damped complex exponentials:

$$\hat{y}(n) = \sum_{k=1}^{M} a_k e^{\alpha_k (n-1) T_s} \cos(2\pi f_k (n-1) T_s + \theta_k) \tag{4}$$

Where

- $1 \leq n \leq N$.
- M is the model order.
- $\lambda_k = \alpha_k + j2\pi f_k$ is the $k-th$ estimated mode.

Prony equation can be written as a difference equation with the following expression:

$$\hat{y}(n) = \sum_{k=1}^{M} h_k z_k^{n-1} + h_k^* {z^*}_k^{n-1} \tag{5}$$

Where

- $h_k = \frac{A_k}{2} e^{j\theta_k}$
- $e^{(\alpha_k + 2\pi f_k)\delta t}$
- M is model order.
- T_S is sample time.
- A_k, θ_k, α_k, f_k are amplitude, phase, attenuation factor and frequency.

In order to solve the algorithm, it is necessary to uncouple h_k and z_k.

Polynomial with roots z_k and z_k^* is:

$$\prod_{k=1}^{M} (z - z_k)(z - z_k^*) = \sum_{m=0}^{2M} a(m) z^{(2M-m)} = 0 \tag{6}$$

Where $a(0) = 1$ and $a(1) \cdots a(2M)$ are real coefficients.

As

$$y(n) = \sum_{k=1}^{M} h_k z_k^{n-1} + h_k^* {z^*}_k^{n-1} \tag{7}$$

then

$$\sum_{m=0}^{2M} a(m) y(n-m) = \tag{8}$$

$$\sum_{m=0}^{2M} a(m) \sum_{k=1}^{M} h_k z_k^{n-1-m} + h_k^* {z^*}_k^{n-1-m} \tag{9}$$

But $z_k^{n-m-1} = z_k^{n-1-2M} z_k^{2M-m} \Rightarrow$

$$\sum_{m=0}^{2M} a(m) \sum_{k=1}^{M} h_k z_k^{n-1-2M} z_k^{2M-m} + h_k^* {z^*}_k^{n-1-2M} {z^*}_k^{2M-m} = \tag{10}$$

$$= \sum_{m=0}^{2M} a(m) 2\Re \left(\sum_{k=1}^{M} h_k z_k^{n-1-2M} z_k^{2M-m} \right) = \tag{11}$$

$$= 2\Re \left(\sum_{k=1}^{M} h_k z_k^{n-1-2M} \underbrace{\sum_{m=0}^{2M} a(m) z_k^{2M-m}}_{=0} \right) = 0 \tag{12}$$

$$\Rightarrow \sum_{m=0}^{2M} a(m) y(n-m) = 0 \Rightarrow y(n) = \sum_{m=1}^{2M} a(m) y(n-m) \tag{13}$$

Let λ_i be the eigenvalues of matrix Π defined as follows

$$\Pi = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \\ -a(2M) & -a(2M-1) & -a(2M-2) & \cdots & -a(1) \end{pmatrix} \tag{14}$$

Modes $\sigma_k = \alpha_k + j2\pi f_k$ are calculated from λ_k by the following expressions:

$$\alpha_k = \frac{\Re(\log(\lambda_k)}{T_S} \tag{15}$$

$$f_k = \frac{\Im(\log \lambda_k)}{2\pi T_S} \tag{16}$$

B. Autoregressive Model

Autoregressive Model aproximates sample $\hat{y}_k$ from the $k-1$ previous samples:

$$y_k = \omega + \sum_{i=1}^{p} A_i y_{k-i} + \epsilon_k \tag{17}$$

Where $A_1 \cdots A_p$ are $m \times m$ matrix coefficients of the autoregressive model, ω represents mean value and the model error is ϵ_k.

Autoregressive model and Prony analysis are very similar when calculating oscillation modes.

Equations 13 and 17 are equivalent when $2M = p$. It is important to note that equation 13 is not generally a determined linear system and there is always going to be an error when calculating $a(m)$ coefficients. Equation 17 includes the error in the expression. Mean value is not a problem because it may be previously calculated and then a zero mean value signal can be analyzed.

For solving 13 or 17 it is possible to use Least-Square method or similar, like any other over determined linear system.

C. Algorithm Tests

Before using the algorithm for modal analysis in the IEEE New England System, a test is performed. The data used in the comparition is from a known signal and the results for both algorithm are compared. The signal for the test is described in Equation 18.

$$x(t) = e^{0.3t}\cos(2\pi f_1 t) + 2e^{-0.3t}\cos(2\pi f_2 t) + e^{-0.1t}\cos(2\pi f_3 t) + e^{-10t}\cos(2\pi f_4 t) \tag{18}$$

Where $f_1 = 1.33Hz$, $f_2 = 1Hz$, $f_3 = 1.5Hz$ and $f_4 = 0.32Hz$.

The result are presented in a table showing the modes estimation.

Prony Analysis:

$$PRmodes =$$
$$0.3000 - j1.3300 \cdot 2\pi$$
$$0.3000 + j1.3300 \cdot 2\pi$$
$$-0.3000 - j1.0000 \cdot 2\pi$$
$$-0.3000 + j1.0000 \cdot 2\pi$$
$$-0.1000 - j1.5000 \cdot 2\pi$$
$$-0.1000 + j1.5000 \cdot 2\pi$$
$$-10.0000 - j0.3200 \cdot 2\pi$$
$$-10.0000 + j0.3200 \cdot 2\pi$$

Autoregressive Model:

$$ARmodes =$$
$$-0.1000 - j1.5000 \cdot 2\pi$$
$$-0.1000 + j1.5000 \cdot 2\pi$$
$$0.3000 - j1.3300 \cdot 2\pi$$
$$0.3000 + j1.3300 \cdot 2\pi$$
$$-0.3000 - j1.0000 \cdot 2\pi$$
$$-0.3000 + j1.0000 \cdot 2\pi$$
$$-10.0000 - j0.3200 \cdot 2\pi$$
$$-10.0000 + j0.3200 \cdot 2\pi$$

The results obtained in both algorithms indicates that either algorithms can be used to perform modal analysis.

IV. Application.

PMUs can measure electrical magnitudes such as currents, voltages, frequencies and rates of change of frequencies. The signals to perform mode estimation analysis are generators rotor speed or angles. However, these signals are not available from PMU measurements. Due to the impossibility of measure the rotor speed or angle, the study is focused in analyzing one of the synchrophasor measurement: generator-busbar frequency.

Figure 1 is showing bus frequency and rotor speed. The relative error in the modal estimation due to the use of frequencies instead of rotor speeds is calculated later in this document for each specific analysis.

A. Dynamic Simulation

First, dynamic simulation to the New England IEEE system is performed. Channel outputs selection is done according to the result of PMUs placement algorithm. Simulations include both stable and unstable disturbances.

- Unstable Disturbance: generation loss in busbar 39. Figure 2 show frequency and rotor speed for busbar 31 and figure 3 the rotor angle for same busbar. The error of using frequency instead of rotor speed for this signal is 0.7%.

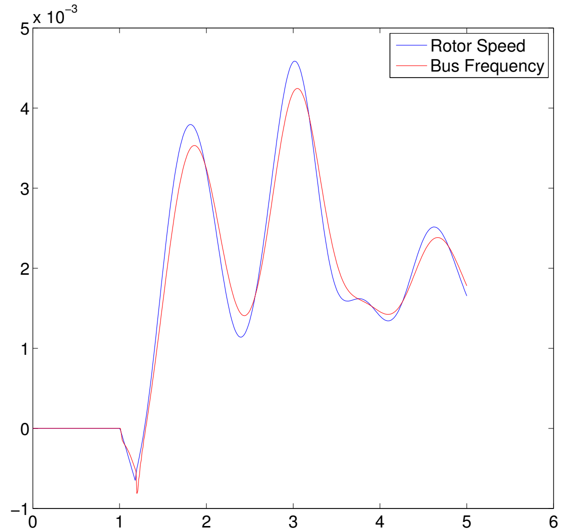

Fig. 1. Rotor speed and Bus Frequency.

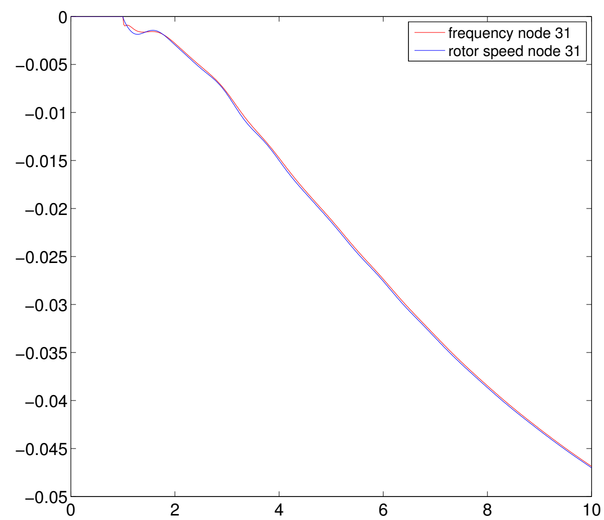

Fig. 2. Rotor speed and Frequency busbar 31 stable disturbance.

- Stable Disturbance: protection tripping in line 1-39. Figure 4 show frequency and rotor speed for busbar 31 and figure 5 the rotor angle for same busbar. The error of using frequency instead of rotor speed for this signal is 11%.

B. Modal Analysis

MATLAB software tool is used for signal processing. Modal estimation is observed and results are compared for both methods.
In order to use same time sample as PMUs, it is defined as $T_s = 20ms$.
To take advantage of the history of the signal, the order of the model is defined as high as possible and equivalent for

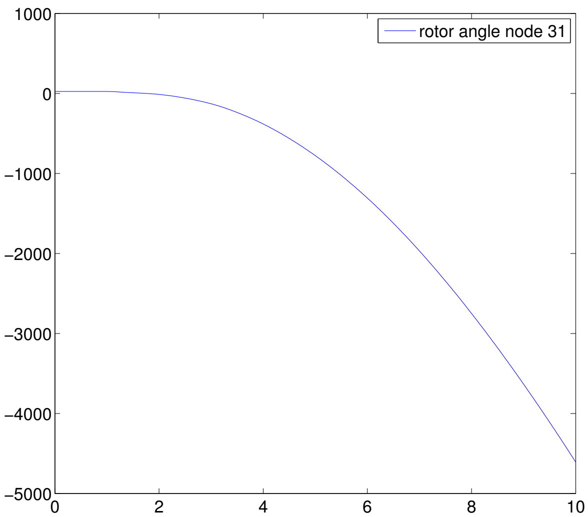

Fig. 3. Rotor angle busbar 31 unstable disturbance.

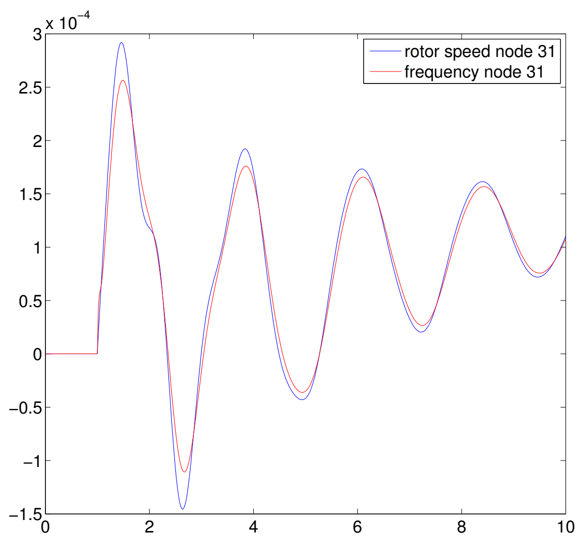

Fig. 4. Rotor speed and Frequency busbar 31 stable disturbance.

both models, making some considerations in Autoregressive Model in order to determine it.
Busbar 31 is taken as an example for the comparison between both methods.
All modal estimation for stables disturbance have shown negative real part and for unstable disturbance the analysis have shown at least one mode with positive real part.

The following results are for the unstable disturbance, because only modes with positive real part are taken intro consideration.

Prony Analysis: figure 6 shows prony signal and the signal obtained by PSS/E simulation for an unstable disturbance.

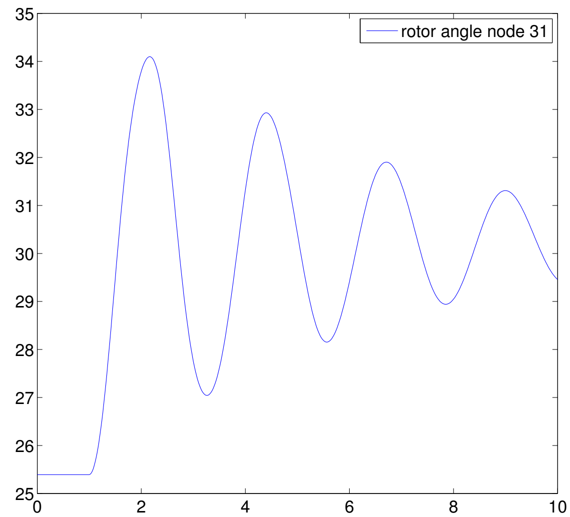

Fig. 5. Rotor angle busbar 31 stable disturbance.

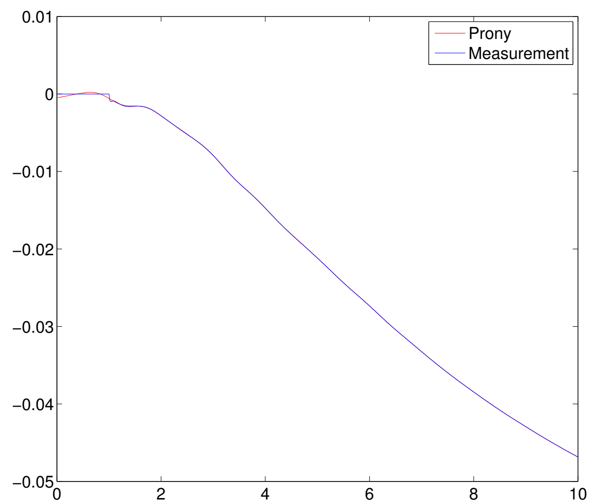

Fig. 6. Loss of Generation at node 39 - Prony Analysis at busbar 31

The relative error between both signals is 0.16%

Only modes with positive real part are presented. The result of the analysis is:

$$\sigma = 0.0092 \pm j0.2325$$

The frequency is

$$f = 0.037Hz$$

and the damping factor is:

$$D = \frac{-0.0092}{\sqrt{0.0092^2 + 0.2325^2}} = -0.0395$$

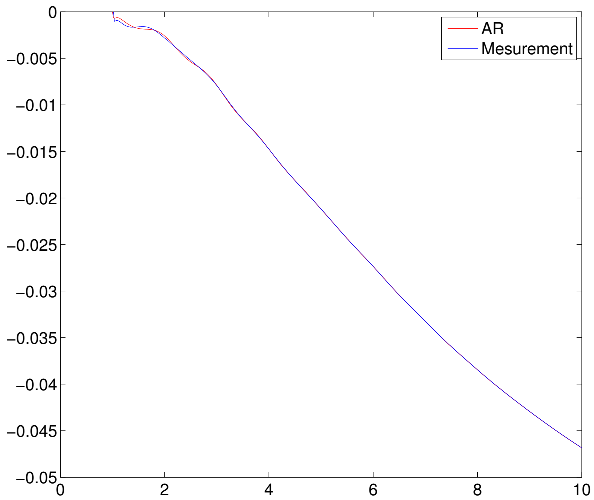

Fig. 7. Loss of Generation at node 39 - Autoregressive Model at busbar 31

Autoregressive Model: figure 7 shows AR signal and the signal obtained by PSS/E simulation for an unstable disturbance. The relative error between both signals is $1.2e^{-13}\% \approx 0\%$

Only modes with positive real part are presented. The result of the analysis is:

$$\sigma = 0.0092 \pm j0.2325$$

The frequency is

$$f = 0.037Hz$$

and the damping factor is:

$$D = \frac{-0.0092}{\sqrt{0.0092^2 + 0.2325^2}} = -0.0395$$

The modal analysis result is the same for both methods, but autoregressive model is more efficient than prony analysis and it possible to determine an appropriate order of the model considering different orders and the results of the modal analysis.

V. Conclusion

This paper investigates the following technical point:

- Complete observability of the power system, including all the generators busbars.
- Real-time modes estimation using synchrophasors frequency measurement.

The investigation is based through transient stability studies covering many possible operating conditions.
Early investigations have shown that with 13 PMUs, the 39-New England system has complete observability and with 16 PMUs the system is observable with one PMU per generator busbar. The AR and Prony method show identical behaviour for small disturbances in the power system, and the AR method performs better than Prony method for large disturbances.

The following step in the investigation is to perform online analysis with the data obtained from PMUs. The principal difference is that for these analysis autoregressive or prony algorithms have to estimate oscillation modes online, recalculating them as soon as a new sample is obtained.

Synchrophasor technology has the potential to improve utility operators ability to conduct real- time grid operations and, detect and respond to potential disturbances. In this paper is shown how phasor systems will help operators to improve dynamic security assessment, detecting earlier the unstable conditions.

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