A New Low Complexity Mathematical Algorithm for Harmonic Analysis in Industrial Areas Networks of Smart Grids

Washington Fernández R.
Dept. Electric and Electronic Engineer
Universidad del Bio Bío
Concepción-Chile
wfernand@ubiobio.cl

José Mahomar J.
Dept. Electric and Electronic Engineer
Universidad del Bio Bio
Concepción-Chile
jmahomar@ubiobio.cl

Abstract— **In this paper we show a new low complexity mathematical algorithm to analyze electric power systems, which also allows locating optimally active filters in the distribution networks of industrial areas for a smart power grid**.

Index Terms-- ***Filter, active, Harmonic analysis, IAN, smart grid***

I. Introduction

Smart grid is a term used to refer to the next generation of electric networks in which distribution and management of electricity needs two-way communication and computational capabilities for monitoring, efficiency, reliability and safety [1] - [4]. The smart grid is expected to modernize the current grid, which involves monitoring, protection and automatic optimization for the operation of the grid interconnected systems. It must be able to interconnect different systems of electricity generation (renewable, non-renewable) [5]. A smart grid is characterized by a bidirectional connection of electricity and information flows to create an automated, widely distributed transport network [6]. More specifically, a smart grid is regarded as an electrical system that uses information, bidirectional communication security technology and computational intelligence integrated through the generation, transmission, substations, distribution and power consumption, to obtain a "clean," safe, workable, resilient, efficient and sustainable system. This description covers the entire power system from generation to the end point which is the consumption of electricity. From a technical perspective a smart grid must have: *an intelligent infrastructure, management system, protection system.* An intelligent infrastructure requires a bidirectional communication system, which should also provide confidence and security to the data being transmitted. The model given above has three types of consumers: Home Area Networks (HAN), Building Area Networks (BAN) and Industrial Area Networks (IAN).

From the point of view of the smart grid, the industrial area networks comprising companies such as factories, fisheries, metal and oil are highly polluting as they cause power quality issues, such as: current harmonics, a poor power factor, unbalance, voltage sag, swell, etc. That is why this paper shows a new low complexity algorithm for analyzing industrial distribution systems to compensate harmonics present in IANs. Below, we analyze the research conducted in this connection.

Generally, different formulations can be grouped into three methodologies to analyze the electric power systems. The first one is known as harmonic penetration [7], which does not consider the interaction between the linear network and nonlinear elements. Thus, although the formulation is simple, the results are often inaccurate.

The second one is the most recurrent in literature, [8] - [12], and consists of unfolding the problem in a load sharing to fundamental frequency and in an analysis of harmonic interaction so that each is resolved separately and successively until reaching the solution.

The third line of analysis, which is the least referenced, [13], [14] - [15], corresponds to a generalization of the load flow without the presence of harmonics. In it, the corresponding equations of conventional load flow arise by performing a balance of powers in the nodes of the network.

It have to mention that [16] uses a hybrid active filter to eliminate harmonics for non-linear loads with a control scheme. In [17] we describe the bandwidth influence of harmonics in smart grids and give a strategy for their elimination. In [18] we discuss the compensation strategy of shunt active filter and see two situations of compensation strategy. In [19] we research active filters for harmonic elimination and hybrid active filters.

In [20] we propose a hybrid active filter composed of a capacitor and an active filter to eliminate harmonic resonance in an industrial environment.

978-1-4673-6605-2/15/$31.00 ©2015 IEEE

This new algorithm is based on the nodal analysis of the power distribution system and the transfer matrix, which relates the input and output of the distribution system variables.

The input variables are the voltage branches and the current bus, while the output variables are the voltage bus and the current branches.

The low complexity of the algorithm is due to the proper values of the transfer matrix instead of the traditional methods of analysis. The algorithm allows analytically determining the effectiveness of different types of compensation and thus to determine the best points of connection of active filters. It show a new method to graphically calculate the resonance frequencies of the power system. For that purpose we use the concept Hermitian matrix in which its singular values are determined. The maximum value of the latter corresponds to resonance frequencies.

This paper is organized as follows: In section II we show the low complexity algorithm for harmonic analysis. We give the results in graphical form in Section III and finally we conclude the paper in section IV.

II. Low complexity mathematical algorithm

The matrix that relates the input variables to the output variable is called transfer matrix. The output matrix is given by:

$$[\mathbf{Y}] = [\mathbf{H}][\mathbf{U}] \tag{1}$$

Where:

X: Output variable.

H: Transfer matrix.

U: Input variable.

$$Y = \begin{bmatrix} V_N \\ I_b \end{bmatrix} \tag{2}$$

Where:

V_N: Node voltage.

I_b: Branch current.

$$U = \begin{bmatrix} V_b \\ I_N \end{bmatrix} \tag{3}$$

Where:

V_b: Branch voltage.

I_N: Node current.

The transfer matrix is given by:

$$[\mathbf{H}] = \begin{bmatrix} H_{VV} & H_{VI} \\ H_{IV} & H_{II} \end{bmatrix} [\mathbf{A}]^U \tag{4}$$

Where:

H_{VV}: Relates branch voltage (input variable) with node voltage (output variable).

H_{VI}: Relates node current (input variable) with node voltage (output variable).

H_{IV}: Relates branch voltage (input variable) with branch current (output variable).

H_{II}: Relates node current (input variable) with branch current (output variable).

In this case, transfer matrix is composed of four sub-matrices.

Where:

$[\mathbf{A}]^U$: Connection matrix of the input variable.

The connection matrix is defined as:

$$[\mathbf{A}]^U = \begin{bmatrix} A^{UV} & 0 \\ 0 & A^{UI} \end{bmatrix} \tag{5}$$

Where:

A^{UV}, A^{UI}: Voltage sources and connection currents respectively.

The connection matrix A^{UV} has the number of rows equal to the number of branches of the distribution system and the number of columns equals the number of independent sources of voltage. On the other hand, A^{UI} has the same number of rows as the number of nodes and the number of columns equals to the number of independent current sources that are connected to the electrical distribution system.

The dimension of the connection matrix is *n* by *m*, where *n* is the number of nodes plus the branches and *m* is the number of independent voltage sources plus the number of independent sources of current.

In the connection matrix A^{UV}, the element values are: 1, -1 and 0, the element is equal to 1 if the polarity of the voltage has the same direction as the flow of the current branch. The value is -1 if the polarity of the voltage and current flow are not equal and 0 if there are not input and output variables.

The primitive distribution matrix $[Z_{pij}]$ connects all the impedance branches. This is a diagonal matrix of order b by *b*, where *b* is the number of branches of the power system and the elements are defined as:

$$Z_{pij} = \begin{cases} 0 & \forall i \neq j \\ Z_i & \forall i = j \end{cases} \tag{6}$$

The branch connection matrix $[\mathbf{A}]$ of order *b* by *n* provides information about how the branches and nodes are connected. The element is 1 if the branch reaches the node, -1 if the branch comes from the node and 0 if they are not connected.

The matrix H_{VV} has a dimension n by b, H_{VI} has a dimension n by n, H_{IV} has a dimension b by b and H_{II} has a dimension b by n. They are defined as:

$$H_{VV} = -\left[A^T Z_p^{-1} A\right]^{-1} A^T Z_p^{-1} \quad (7)$$

$$H_{VI} = \left[A^T Z_p^{-1} A\right]^{-1} \quad (8)$$

$$H_{IV} = -\left[Z_p^{-1} A\left[A^T Z_p^{-1} A\right] A^T - I\right] Z_p^{-1} \quad (9)$$

$$H_{II} = Z_p^{-1} A\left[A^T Z_p^{-1} A\right]^{-1} \quad (10)$$

Variables of the distribution system are defined for evaluating the system response to different perturbations.

The original disturbance is represented as G^p plus the reference signal which is used to analyze active compensations. We have independent input variables U^R. This representation allows analyzing the distribution system using two transfer matrices for obtaining the system response to each perturbation applied.

$$[\boldsymbol{Y}] = [\boldsymbol{G}][\boldsymbol{U}] \quad (11)$$

Where:

$[\boldsymbol{Y}]$: Output variable vector, without connected active filter, containing bus voltages and branch currents.

$[\boldsymbol{G}]$: Transfer matrix relating the outputs to the inputs without connected active filters. It relates the effect of the disturbances sources on the outputs without the presence of an active filter.

To determine the matrix $[\boldsymbol{G}]$, we have:

$$[\boldsymbol{G}] = [\boldsymbol{H}]\left[\boldsymbol{A}^U\right] \quad (12)$$

Where:

$[H]$; Network structure matrix which considers the connection between the branches, bus and impedance of the system (transformers, power suppliers, generators and nonlinear loads) without connected active filters.

$\left[A^U\right]$: Connection matrix of voltage sources and independent currents without connected active filter.

When active filters are used it is suitable redefine the system for evaluating the response to the original imputs defined as U^p which are considered perturbations, with:

$$\left[U^p\right] = \begin{bmatrix} V_r \\ I_b \end{bmatrix} \quad (13)$$

Where:

V_r: Branch voltage.

I_b: Bus current.

The topology of the grid is modified by replacing the active filters for their equivalent circuits and the reference variables are considered as a new set of input variables U^R formed by taking the references of bus voltages V_b^* and branch currents I_r^*, where:

$$\left[U^R\right] = \begin{bmatrix} V_b^* \\ I_r^* \end{bmatrix} \quad (13)$$

This representation describes the system through two transfer matrices and studies their structure on the influence of disturbances before and after the installation of the active filters assessing whether there is resonance or not, what its magnitude is and how it moves. The output variables are defined as:

$$[Y] = \left[G^p\right]\left[U^p\right] + \left[G^R\right]\left[U^r\right] \quad (14)$$

$$[Y] = \left[H^p\right]\left[A^p\right]\left[U^p\right] + \left[H^p\right]\left[A^R\right]\left[U^R\right] \quad (15)$$

Where:

$\left[H^p\right]$: Network structure matrix with connected active filters considering the effect of disturbances (original imputs). It has the dimension of the number of bus plus the number of branches by the number of bus plus the number of branches.

$\left[A^p\right]$: Connection matrix of independent voltage and currents sources with connected active filters. It considers the effect of disturbances, dimensions of the number of bus by the number of branches by the number of branches plus the number of independent voltage sources plus sources of nonlinear currents.

The development of a fast and easy way is necessary to find the possible changes that are introduced in the resonant frequency defined as $\mathbf{G}^p$ due to nonlinear loads. In this paper we evaluate the magnitude of $\mathbf{G}^p$ which corresponds to the largest singular value. The singular values of the matrix $\mathbf{H}$ are equal to the square root of the first k eigenvalues of the Hermitian:

$$\rho_i = \sqrt{\lambda_i\left(H(s)^T \; H(s)\right)} \quad (16)$$

Where:

λ_i: Singular Eigenvalue.

$H(s)^T$: Transposed matrix.

The gain for an input variable u is defined as the ratio between the magnitude of H in the direction and magnitude of u.

$$\min(\rho(H)) < \frac{\|Hu\|_2}{\|u\|_2} < \max(\rho(H)) \quad u \neq 0 \quad (17)$$

If the gain of the system is increased for a particular frequency or the resonant frequency is associated with a large change in gain, both cases may be the source of a major operational problem in the distribution system and it is related with the maximum singular value.

Below, we give the algorithms for the analysis of low complexity electrical power systems developed in this paper.

Algorithm for calculating current, voltage and Fourier spectrum (determine THD)

Step 1

Define matrices A, A^p, Af, Af^p, $A2$, AR, *vector F and write* $k=1$

Step 2

Calculate $Z_k, H_k, P_k, Zf_k, Hf_k, Pf_k$

$Y_k = H_k P_k$

Step 3

If $k=1$, *calculate* $Z2$, $H2$, U_k

$$Yf_k = Hf_k P_k + H2_k U_k$$

If not

$Yf_k = H_k P_k$

Step 4

Write $k = k+1$, *Si* $k = n$ *stop*

If not go to step 2

Where:

A, A^p, Af, Af^p, $A2$, AR: *Are incidence matrices, connection matrix of disturbance , incidence matrix with filter, connection matrix of disturbances with filter, incidence matrix by reference effect, connection matrix of disturbances per reference effect respectively .*

We obtain Y_k *and* Yf_k *which represent the voltage and current outputs in the different bus and branches for the given frequencies either filtered or unfiltered, respectively, where the waveform is obtained and expressed in the unit of time:*

$$S_i = |Y_k|\, sen(2\pi f_k t_i + \theta_k)$$

Where:

t: Time in seconds.

F_k*: Frequency in Hertz.*

θ_k : Y_k *angle.*

The algorithm for calculating the frequencies of resonance is as follows:

Step 1

Define matrices A, A^p, Af, Af^p, $k=1$, *vector F and write* $k=1$

Step 2

Calculate Z_k, H_k, Zf_k, Hf_k

$$VS_k = \sqrt{\rho\left(H_k^T H_k\right)}$$

$$VSf_k = \sqrt{\rho\left(Hf_k^T\, Hf_k\right)}$$

Step 3

Write $k = k+1$, *Si* $k = n$ *stop, otherwise go to step 2.*

Where:

A, A^p, Af, Af^p *are the connection matrix of disturbances, incidence matrix with filter, connection matrix of disturbances with filter respectively.*

F: Frequency vector of dimension n.

Z_k, H_k, Zf_k, Hf_k : Matrices previously described.

VS_k : Resonance frequency for the system without a filter.

VSf_k : Resonance frequency for the system with filter

III. RESULTS AND DISCUSSION

A. Results

The program is implemented in MATLAB, which contains different types of windows. In windows 1 and 2 the matrix is associated to the system topology. Window 3 allows sign in values of current sources, voltage percentage on each bus and angle, values of resistors and reactances, both inductive and capacitive. Window 4 allows entering information related to the type of filter used. Table I shows the types of filters and their options.

TABLE I Types of filters and their location.

Type of filter	Location
Shunt	Bus where a **bf** shunt filter is installed.
Series	Bus where a **Rf** series filter is installed.
Hybrid shunt-passive	Bus where a **bf** shunt filter is installed. Branch where a passive filter and its value are installed.
Hybrid series-passive	Branch where a **Rf** series filter is installed. Branch where a passive filter and its value are installed.
Shunt-series	Branch where a **bf** shunt filter is installed. Branch to apply **Rf series filter**.

Finally, window 5 describes the different graphics that can be deployed. Figs. 1 to 2 show the MATLAB software windows.

Figure 1 Window 2, allows entering each bus and branch coming in out (construction of incidence matrix).

Figure 2 Window that gives the options of scheduled graphics.

Figs. 3, 4 and 5 give the results given by the software.

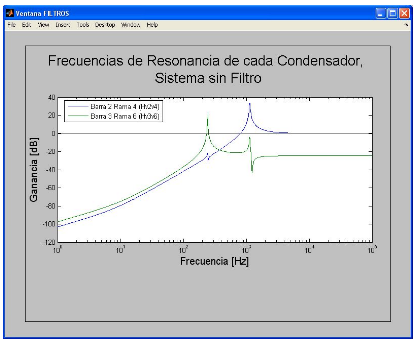

Figure 3 Resonance frequency of each capacitor, system without filter.

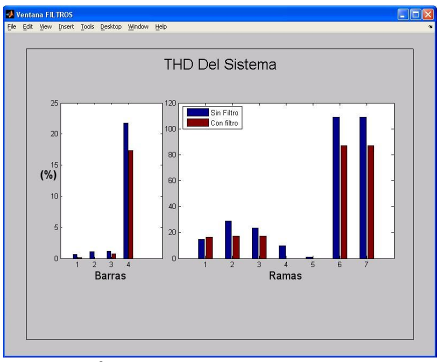

Figure 4 THD current of power system.

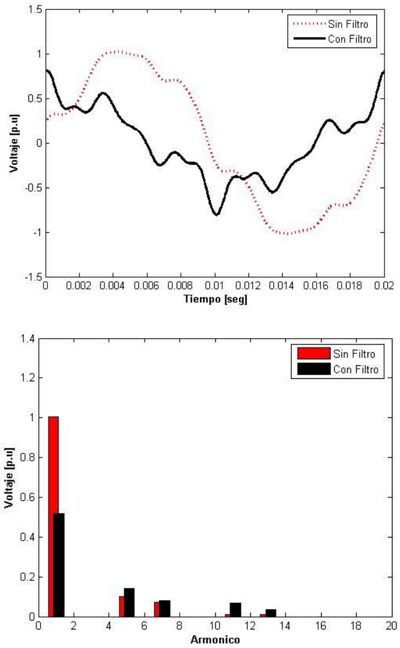

Figure 5 Voltage signals and frequency spectrum in bus 6 (a) Voltage in bus 6. (b) Fourier spectrum in bus 6.

With the software it can achieve a simulation with networks of up to 300 bus, 600 branches, 150 pollutant sources and 50 characteristic harmonic orders and/or inter-harmonic in a time window of no more than 30 seconds in processing time (CPU) in a computer with the following characteristics: Pentium IV, 2.66 GHz and 512 MB RAM.

Table II gives the number and execution time required.

TABLE II Number of bus and runtime.

Number of bus	C.P.U time (seg)
10	0,031
30	0,062
80	0,593
100	0,890
130	2,406
150	3,515
200	7,812
250	16,093
280	22,203
290	22,844
300	27,234

B. Discussion

It show a new low complexity algorithm for analyzing electric power systems with linear and non-linear loads, capacitors and passive filters. For this we use a transfer matrix. The method uses the concept of singular value of these matrix arrangements, which allow relating output variables with system disturbances.

The algorithm is developed to analyze the performance of compensation with respect to a reactive power and eliminate current and voltage harmonics of shunt and series power filters. We show that the arrangement of the transfer functions together with the data of contaminated bus provides enough information to find the best place for compensation of the active power filter. Through a complete analysis of the power system, we can evaluate the effectiveness of compensation without reducing the grid to a simplified or equivalent model, as it has been done in many papers, where some important features are missing, as the possible resonances in different bus. In addition, the proposed method allows visualizing the currents distribution in branches with and without connected active filters, so that current injected by the nonlinear load is not injected into the system.

This systematic approach reduces the possibility of degrading the performance of the system as a whole when filters are added, facilitating the process of selecting the most appropriate bus for their connection. By analyzing the results, we can say that the proposed method allows evaluating in a simple and efficient way an electrical network to find resonance frequencies, bus more contaminated and get the response of the system when connecting any modeled load as a current source.

IV. Conclusions

It developed an efficient analytical mathematical tool, which does not require iterative numerical methods for solutions, thereby avoiding convergence problems. The programming language used is MATLAB whose application in matrix management arrangements, is perfect for large power systems, and where required programming and processing time is small. For example, for a system of 300 bus, the time of computational processing is less than one minute (Pentium IV processor of 2.66 GHz 512 MB RAM), therefore, the low complexity algorithm to analyze power systems can be used in networks of industrial areas to meet the regulations of the smart grid.

Future works: The following topics are suggested for analyzing active filters in power systems:

- Analysis of the compensation effectiveness considering the shunt active filter as a current source, and the series active filter as a voltage source.
- Development of the proposed method for voltage and current unbalanced power systems.

Implementation of solution algorithms for systems greater than 300 bus to work with matrices with few nonzero imputs (dispersive matrices) significantly reducing their processing times and thus the total runtime.

Acknowledgment

The authors would like to thanks CONYCIT (the Chilean Research Council) for the financial support given through project FONDEF IDeA CA13I103318.

References

[1] Departament of energy, “The Smart Grid and Introduction”, disponible en: www.energy.gov/SmartGridIntroduction.html

[2] Y. Kim, H. Thottan and W. Lee, “A Secure Descentralized Data Centric Information Infrastructure for Smart Grid”, IEEE Communications Magazine, pp. 58-65, June 2010.

[3] H. Gharavi, R. Ghafurian, “Smart Grid the Electric System of Future”, Proceeding of IEEE, pp. 917-921, August 2011.

[4] K. Chen, P. Yeh, H. Hsi and S. Chang, “Communication Infrastructure of Smart Grid”, 4th International Symposium Control and Signal Processing (ISCCP) 2010, pp. 1-5.

[5] D. Wollman, G. Fizpatrick and P. Boynton, “NIST Coordination of Smart Grid Interoperabbility Standards”, Conference on Precision Electromagnetic Measurements, Daejson, Korea 2010, pp. 531-532.

[6] Z. Fan, P. et. al., “Smart Grid Communication: Overview of Research Challenge Solutions, and Standarization Activites”, IEEE Communications Survey & Tutorial, Vol. 25, N° 1, pp. 21-33, First Quarter 2012.

[7] J. Pileggi, N. H. Chandra, A. E. Emanue1, "Prediction of harmonic vo1tages in distribution systems", IEEE PAS-100, No. 3, , pp. 1307-1315, March 1981.

[8] J. Arrillaga, C. P. Arnold, "Computer analysis of power systems", John Wiley and Sons, 1990.

[9] L. Arrillaga, C. D. Callaghan, "Three phase AC-DC 1000 and harmonic load flows", IEEE Transactions on Power Delivery, Vol. 6, N° 1, pp. 238-244, January 1991.

[10] C. D. Callaghan, J. Arrillaga, "Convergence criteria for iterative harmonic analysis and its application to static convertors", IV-ICHPS, Budapest, 4-6 October 1990.

[11] R. Carbone, M. Fantauzzi, F. Gagliardi, A. Testa, "Some consideration on the Iterative Harmonic Analysis Convergence", IEEE Transactions on Power Delivery, Vol. 8, N°. 2, April 1993.

[12] R. Carbone, F. Gagliardi, A. Testa, "A parallel compensation technique to improve the Iterative Analysis Convergence", IEEE VI-ICHPS, Bologna, pp. 332-340, 21-23 Sept. 1994.

[13] J. Pedra, "El problema del flujo de cargas en redes con armónicos", Tesis Doctoral, ETSIB, Universidad Politécnica de Catalunya, 1986.

[14] J. Usaola, "Régimen permanente de sistemas eléctricos de potencia con elementos no lineales mediante un procedimiento híbrido de análisis en los dominios del tiempo y de la frecuencia", Tesis Doctoral, ETSII de Madrid, 1990.

[15] D. Xia, G. T. Heydt, "Harmonic Power Flow Studies. Part I. Formulation and solution", IEEE Transactions PAS-l Vol, N°. 6, pp. 1257-1265, June 1982.

[16] J. Pedra, M. Salichs, "Harmonic power flow studies. Part l. Formulation and solution", International Symposium on "Electrical energy conversion in power systems", W-H.l, Capri, May 1989.

[17] N. Prasad, P. Gupta, D. Masand, “Performance evaluation of hybrid active power filter”, International Conference on Communication System and Network Technologies, pp. 573-576, 2012.

[18] Z. Shuai, D. Liu, J. Shen, Ch. Tu, Y. Cheng and A. Luo, “Series and parallel resonance problem of wideband frequency harmonic and its elimination strategy”, IEEE Transactions on Power Electronics, Vol. 29, Nº 2, pp. 1941-1952, April 2014.

[19] L. Shuming, Z. Yongqiang, S. Ji, X. Yonghai, X. Xiangning, “The research of shunt active filter compensation strategy”, International Conference on Energy and Environment Technology, pp. 361-364, 2009.

[20] H. Akagi, “Active harmonic filters”, Proceedings of the IEEE, Vol. 93, Nº 12, pp. 2128-2141, December 2005.

[21] T. Lin, Y. Ching, J. Cheng, “Design of a hybrid active filter for harmonics suppression in industrial facilities”, Interntional Conference on Power Electronic and Drive System, PEDS 2009, pp. 121-126, 2009.