Compressive Beamforming for Underwater Acoustic Source Direction-of-Arrival Estimation

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***Abstract*-- The Direction-of-Arrival (DOA) estimation under noisy conditions, coherent sources and few snapshots is a challenging problem especially in forward-looking sonar. Compressive beamforming can achieve spatial sparsity, thus improve spatial resolution, by imposing penalties based on the l_1-norm, and has recently become an exciting field that has attracted considerable attention in signal processing. In this work, we investigate the principle of compressive beamforming in detail and further extend this technique to the underwater acoustic DOA estimation case, exactly to say, apply compressive beamforming for effective DOA estimation in Forward-Looking Sonar scenario. Experimental data from the Forward-Looking Sonar Experiment (FLS-Ex) demonstrate the high-resolution capabilities of compressive beamforming.**

I. INTRODUCTION

In the array signal processing domain, the topic of Direction-of-Arrival (DOA) estimation has become an area of active interest with applications in radar, sonar and other areas [1]. Compressive sensing (for short, CS) is a novel sampling paradigm and has already inspired some notable investigation in the context of DOA estimation [2].

By turning to the compressive sensing framework, we are able to exploit the inherent sparsity of the underlying signal in space domains to achieve super-resolution even in a noisy and coherent environment with few snapshots [3]. Gorodnitsky et al. view DOA estimation as an underdetermined problem and use a recursive weighted minimum-norm algorithm FOCUSS to find its sparse solutions [4]. In ocean acoustic signal processing, Gerstoft et al. analyze in detail the performance of CS in DOA estimation in terms of the discretization of the angular space, the coherence of the sensing matrix and the SNR [5].

The motivation of this paper is to investigate the principle and the performance of compressive beamforming in detail, and apply compressive beamforming in practical scenario: Forward-Looking Sonar. To the best of our knowledge, this is the first research that compressive beamforming is applied for effective DOA estimation in Forward-Looking Sonar scenario.

II. COMPRESSIVE BEAMFORMING FOR DOA ESTIMATION

The DOA problem can be formulated as a sparse representation problem. Let $\left\{\tilde{\theta}_1,\tilde{\theta}_2,\cdots,\tilde{\theta}_{\tilde{n}},\cdots,\tilde{\theta}_{\tilde{N}}\right\}$ ($\tilde{\theta}_{\tilde{n}}\in\left[-90°,90°\right]$) be a sampling grid of all directions of interest, and we construct the sensing matrix $\tilde{\mathbf{A}}$ formed by steering vectors corresponding to each potential source directions as its columns

$$\tilde{\mathbf{A}}=\left[\boldsymbol{a}(\tilde{\theta}_1),\boldsymbol{a}(\tilde{\theta}_2),\cdots,\boldsymbol{a}(\tilde{\theta}_{\tilde{N}})\right] \quad (1)$$

where $\boldsymbol{a}(\tilde{\theta}_k)=\left[1,e^{j2\pi f_k d\sin\tilde{\theta}_k/c},\cdots,e^{j2\pi f_k (N-1)d\sin\tilde{\theta}_k/c}\right]^{\mathrm{T}}$ is the steering vectors and $\tilde{\mathbf{A}}$ is an overcomplete representation in terms of all possible source directions. Furthermore, we reformulate the signal field $\mathbf{S}(t)$ by a new $\tilde{N}\times 1$ vector $\tilde{\mathbf{S}}(t)$, where the n-th element $\tilde{s}_n(t)$ is nonzero and equal to $s_k(t)$ if source k comes from $\tilde{\theta}_n$ for some k and zero otherwise. Then, the DOA problem is recast as a sparse representation problem as shown [6]

$$\mathbf{X}(t)=\tilde{\mathbf{A}}\tilde{\mathbf{S}}(t)+\mathbf{N}(t) \quad (2)$$

Generally speaking, the actual number of sources is small compared with all possible source directions of interest, so the underlying spatial spectrum is sparse, and we can solve Eq.(2) by l_1-norm methodology. In the presence of the noise field $\mathbf{N}(t)$, Eq.(2) can be solved as

$$\min\left\|\tilde{\mathbf{S}}(t)\right\|_1 \quad s.t. \quad \left\|\tilde{\mathbf{A}}\tilde{\mathbf{S}}(t)-\mathbf{X}(t)\right\|_2\le\varepsilon \quad (3)$$

where ε is the upper bound for the noise energy (l_2-norm). For convenience, we call Eq.(3) conventional compressive sensing beamforming (for short: CS).

For some particular scenarios, DOA estimation with multiple snapshots is of greater practical importance and has the following form

$$\mathbf{X}(t)=\tilde{\mathbf{A}}\tilde{\mathbf{S}}(t)+\mathbf{N}(t)\,,\quad t\in\left\{t_1,\cdots,t_T\right\} \quad (4)$$

Let $\mathbf{X}=\left[\mathbf{X}(t_1),\mathbf{X}(t_2),\cdots,\mathbf{X}(t_T)\right]$, $\tilde{\mathbf{S}}=\left[\tilde{\mathbf{S}}(t_1),\tilde{\mathbf{S}}(t_2),\cdots,\tilde{\mathbf{S}}(t_T)\right]$, and $\mathbf{N}=\left[\mathbf{N}(t_1),\mathbf{N}(t_2),\cdots,\mathbf{N}(t_T)\right]$. Then Eq.(4) can be further reformulated in a compact form

$$\mathbf{X}=\tilde{\mathbf{A}}\tilde{\mathbf{S}}+\mathbf{N} \quad (5)$$

However, because the signal is generally sparse in space, not in time, the numerical solution to Eq.(5) is a little complex. To accommodate this issue, we should first compute the l_2-norm of all time-samples of a particular spatial index of $\tilde{\mathbf{S}}$,

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such as $\tilde{\mathbf{S}}_i^{l_2} = \left\| \left[\tilde{\mathbf{S}}_i(t_1), \tilde{\mathbf{S}}_i(t_2), \cdots, \tilde{\mathbf{S}}_i(t_T) \right] \right\|_2$, and penalize the l_1-norm of $\tilde{\mathbf{S}}^{l_2} = \left[\tilde{\mathbf{S}}_1^{l_2}, \tilde{\mathbf{S}}_2^{l_2}, \cdots, \tilde{\mathbf{S}}_{\tilde{N}}^{l_2} \right]$ (see Ref.[3] for details). Then Eq.(5) can be solved as

$$\min \left\| \tilde{\mathbf{S}}^{l_2} \right\|_1 \quad s.t. \quad \left\| \tilde{\mathbf{A}} \tilde{\mathbf{S}} - \mathbf{X} \right\|_2 \le \sigma \qquad (6)$$

where σ is the upper bound for the noise energy (l_2-norm).

In effect, both Eq.(3) and Eq.(6) are convex optimization problems and can be readily handled by the CVX toolbox[7].

III. EXPERIMENTAL RESULTS

The experimental data were collected during the Forward-Looking Sonar Experiment (FLS-Ex), which was conducted by Institute of Acoustics, Chinese Academy of Sciences . In FLS-Ex, the transmitting array transmitted a continuous wave (CW) comb signal whose center frequency was $f_c = 200$ KHz, and the data were collected by a vertical uniform array. The spatial spectra results are shown in Fig.1.

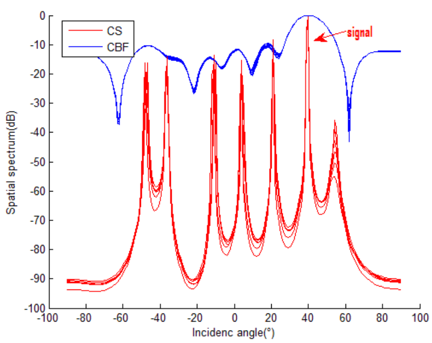

Fig. 1. Spatial spectrum for experimental results

It is noted that the CBF plots suffer from low resolution and artifacts due to sidelobes and noise, while the largest peak illustrates the capability of CS to perform well compared with CBF. The experimental results clearly demonstrate that CS algorithms have an excellent performance compared with CBF in practical scenario.

IV. CONCLUSION

DOA estimation with sensor arrays can be formulated in the CS framework and solved efficiently by the well-established interior point method, such as CVX toolbox. In order to achieve higher resolution than conventional beamforming, we have investigated the principle and the performance of compressive beamforming respectively using single snapshot and multiple snapshots in detail, and then extended the algorithms to the Forward-Looking Sonar scenario. The conclusion that can be drawn out of the experimental results is that the compressive beamforming has superior high-resolution capabilities than CBF and can be applied in practical scenarios effectively.

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EXAMPLES OF REFERENCE STYLES

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