A Robust Control Chart for Monitoring Reliability Systems

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Abstract—**Control charts are one of the significant statistical control tools widely used to monitor processes. The system's reliability is affected by failure processes during its lifetime. Monitoring the failure processes is a crucial issue in complex and repairable systems. To design accurate control charts for monitoring a reliability system, it is necessary to estimate the system parameters properly. Since the presence of outliers in sample data can highly affect the parameter estimation using a classic estimator, a robust estimator can estimate the parameters efficiently and close to the actual value in the presence of contamination. In this research, robust monitoring for reliability systems is designed. To estimate the distribution parameters, the robust method based on the M-estimator is used to estimate the time between failure distribution parameters from the Weibull distribution. Using simulation studies, the efficiency of the classic and robust estimators is compared based on the norm and MSE under different contamination scenarios. Then, the robust lower control limit is designed based on the simulated control limit and mathematical formulation methods. The results show that the robust control limits are close to the actual values in the absence or presence of outliers. However, the classic control limits are highly affected by the contamination and their values are far from the actual when there are shifts in the parameter.**

I. INTRODUCTION

Control charts are useful statistical tools for monitoring processes. They play a significant role in improving the quality of processes and products. Competition in the world market provides consumers the chance to choose a product with higher quality and reliability in its useful life. Thus, the product must be produced so that to attract customers and fulfill their satisfaction. According to [1], reliability is one of the essential qualitative properties of a system. Reliability is defined as the probability that a system performs its intended function in a certain time interval and under certain conditions. A reliable system has less chance of failure. The lack of a quality control program in any production process and system reliability evaluation increases the failure probability in the product or system. This may cause a system failure and disrupt individual activities. In some systems, the damages caused by system or product failure may not be recovered. So, it is essential to estimate the reliability of a system correctly and control it in its lifetime. The failure probability density function ($f(t)$) and failure rate function ($\lambda(t)$) are the two important functions in the reliability analysis. The failure rate curve which is also called the bathtub curve is shown in Figure 1 for three types of systems. The curve marked (a) represents the computer programs, curve (b) represents the mechanical systems, and curve (c) shows the electrical and electronic systems. Also, the bathtub curve is divided into three failure periods including Early Failure, Intrinsic Failure, and Wear-out Failure.

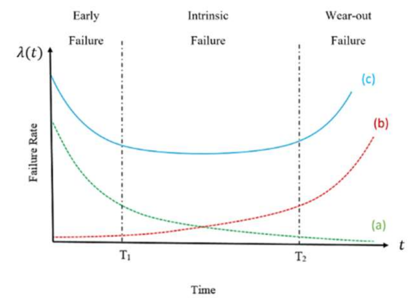

Figure 1. Bathtub curve

According to [2], when the failure probability density function follows an Exponential distribution with parameter θ, the best reliability estimate is the maximum likelihood estimation method (MLE). The MLE method is used to estimate the reliability of a system at a particular time.

During data collection, the presence of outliers, that contaminate the data, is inevitable. So, in the presence of the outliers, the sample mean as a classic estimator of the parameter θ cannot estimate it accurately. To overcome this problem, [3] estimated the parameter θ by using a robust estimator. They suggested using the median of the data as a robust estimator instead of the sample mean when outliers are present in the data. In their research, an M-estimator as a robust estimation method is used, and their suggested standardized median is compared to the classic estimators RCS and RCQ. Despite less asymptotic effeciency of a standardized median estimator than the two other estimators, they recommended the standardized median because of a simpler computation and less net error sensitivity. [4] proposed probability estimation methods for estimating reliability. They used the method of moments estimation to estimate the parameters of the failure density function. [5] found the most optimum factor of the system in the experiment using the analysis of the variance of the system parameters. [6], [7], and [8] introduced minimization schemes that can be used in minimizing the loss function and estimating the system parameters efficiently.

[9] used the time between failures for constructing a control chart instead of utilizing the number of failures or

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failure times during a certain period such as a week or month. They also showed the performance of their suggested control chart through average run length (ARL). [10] constructed a control chart for monitoring a profile while the time between failure follows three-parameter Weibull distribution. To forecast and improve system reliability. [11] studied monitoring the Weibull-distributed time between events. [12] compared some Weibull-distributed time between-events control charts to a CUSUM chart for monitoring the mean shifts in the processes. [13] used a Shewhart control chart for monitoring the Weibull mean using the Gamma distribution.

Researchers in [14] and [15] used advanced methods such as deep learning to estimate the system parameters accurately. [16], and [17] proposed robust control charts for monitoring simple and multiple linear profiles in phase I and phase II, respectively.

In this research, the Weibull distribution function is considered to estimate the reliability systems. The parameters of the Weibull distribution are estimated using the M-estimator and the MLE based on the iterative reweighting algorithm in the absence and presence of outliers, and the efficiency of the classic and robust estimators is evaluated and compared. For the first time, a robust control chart is constructed using a robust estimation method which can be utilized in monitoring reliability systems in the presence of contamination in data. The rest of the paper is organized as follows: In Section 2, the robust estimation method used to estimate the distribution parameters is studied. In Section 3, using simulation studies, the efficiency of the classic and robust methods are compared and a robust control chart is constructed for reliability systems. Conclusions and future research opportunities are discussed in Section 4.

II. M-ESTIMATOR

M-estimators are the particular kinds of robust estimators, which are generalized estimators of MLE. According to [18], they have less sensitivity and more effectiveness against outliers.

A. M-estimator For Regression

According to [18], For a simple linear regression model, $y=\beta_0+\beta_1 x+u;\ u\sim NID(0,\sigma^2)$, where y is a response variable, x is an independent variable, β_0 and β_1 are model parameters, and u is an error term if the model parameters are estimated based on the samples of size n as (x_i,y_i), the estimators of β_0 and β_1 are defined to satisfy (1).

$$\text{Min} \sum_{i=1}^{n} \rho\left(\frac{r_i(\widehat{\underline{A}})}{\hat{\sigma}}\right) \tag{1}$$

where $\widehat{\underline{A}}=(\widehat{B_0},\widehat{B_1})$ is a model estimates vector, $\hat{\sigma}$ is an error term standard deviation estimate, and $r_i=(y_i-\hat{y}_i)$; i=1,2,…,n, the ith sampling residual, where y_i is an observed value for the corresponding x_i, and $\hat{y}_i=\widehat{B_0}+\widehat{B_1}x_i;\ _{i=1,2,\ldots,n}$ is a prediction value. The ρ function is defined as:

$$\rho'(\bullet) = \psi(\bullet) \tag{2}$$

where ρ and ψ functions are functions of the Bisquare family ([18]).

B. Estimating Reliability System Parameters Using M-estimator

For estimating the reliability system parameters, the following process should be used if its failure function follows Weibull distribution with parameters α and β. Failure function at time t is defined as:

$$f(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right],\alpha , \beta>0 \tag{3}$$

So, the reliability function at time t is:

$$R(t)= \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right]. \tag{4}$$

If the natural logarithm is taken twice from (4), the results are as follows:

$$Ln(R(t)) = -\left(\left(\frac{t}{\alpha}\right)^{\beta}\right). \tag{5}$$

$$-Ln(R(t)) = Ln(R(t)^{-1}) = \left(\frac{t}{\alpha}\right)^{\beta}. \tag{6}$$

$$Ln[Ln(R(t)^{-1})]= \beta(Lnt-Ln\alpha)= \beta Ln(t)-\beta Ln(\alpha). \tag{7}$$

In (7), t is the failure time, and R(t) is the system reliability at this time, which is calculated by the nonparametric method.

For facilitating the calculations, it is assumed that $y=Ln[Ln(R(t))^{-1})]$ and x=Ln(t). So,

$$y = \beta x-\beta Ln\alpha. \tag{8}$$

If $b = -\beta Ln(\alpha)$ and a=β are introduced, the following linear relationship is obtained.

$$y = ax+b. \tag{9}$$

According to [19], the estimator of the parameter function is equal to the estimator function of that parameter, which means:

$$\widehat{g(\theta)}= g(\hat{\theta}). \tag{10}$$

Thus, by robustly estimating a and b using the regression M-estimator, the robust estimates of parameters α and β can be calculated. The computational algorithm introduced by [18], is modified for estimating the parameters a and b of (9) and is explained as follows:

1- Convert the Weibull reliability function to the regression line, according to (4) and (9), and introduce vectors $\underline{A}=\binom{a}{b}$ and $X=\binom{1}{x}$.

2- Assume $j = 0$, the initial value for $\widehat{A_0} = \binom{\hat{a}}{\hat{b}}$ is determined by a classic method such as ordinary least squares (OLS), and the variance estimate of the dependent variable is obtained using (11).

$$\hat{\sigma} = \frac{1}{0.675} \mathrm{Med}_i \left(|r_i| \mid r_i \neq 0 \right) \quad (11)$$

3- For $j = j+1$, considering $\widehat{A_j}$:

a) The residuals for $i = 1, 2, \ldots, n$, are calculated as:

$$r_{i,j} = y_i - x_i' \widehat{A_j}. \quad (12)$$

b) Assign weights based on the introduced weight functions, Huber or Bisquare in (13).

$$w_{i,j} = W\left(\frac{r_{i,j}}{\hat{\sigma}}\right) \quad i=1,2,\ldots,n \quad (13)$$

c) Obtain $\widehat{A}_{j+1}$ for $(j+1)^{th}$ level as:

$$\sum_{i=1}^{n} w_i x_i \left(y_i - x_i' \widehat{A}\right) = 0. \quad (14)$$

4- If $\mathrm{Max} \frac{\left(|r_{i,j} - r_{i,j+1}|\right)}{\hat{\sigma}} < \varepsilon$, then the algorithm should be stopped, and the robust estimator of the Weibull distribution of α and β are obtained using (15) and (16). Otherwise, the algorithm should be continued from step 3.

$$\hat{\beta} = \hat{a}. \quad (15)$$

$$\hat{\alpha} = e^{-\frac{\hat{b}}{\hat{\beta}}}. \quad (16)$$

In the current research, the Bisquare function is used as a weight function. ρ, ψ, and W functions of the Bisquare family are presented in Table 1.

TABLE I. FAMILY OF BISQUARE AND HUBER FUNCTIONS ([18])

Function	Huber	Bisquare
$\rho_k(x)$	$\begin{cases} x^2 & \text{if } \lvert x\rvert \leq k \\ 2k\lvert x\rvert - k^2 & \text{if } \lvert x\rvert > k \end{cases}$	$\begin{cases} 1-\left(1-\left(\frac{x}{k}\right)^2\right)^3 & \text{if } \lvert x\rvert \leq k \\ 1 & \text{if } \lvert x\rvert > k \end{cases}$
$\rho'_k(x) = \psi_k(x)$	$\begin{cases} x & \text{if } \lvert x\rvert \leq k \\ \mathrm{sgn}(x)k & \text{if } \lvert x\rvert > k \end{cases}$	$\begin{cases} x\left(1-\left(\frac{x}{k}\right)^2\right)^2 & \text{if } \lvert x\rvert \leq k \\ 0 & \text{if } \lvert x\rvert > k \end{cases}$
$W_k(x)$	$\min\{1, \frac{k}{\lvert x\rvert}\}$	$\begin{cases} \left(1-\left(\frac{x}{k}\right)^2\right)^2 & \text{if } \lvert x\rvert \leq k \\ 0 & \text{if } \lvert x\rvert > k \end{cases}$

In the functions presented in Table 1, k is constant. The analysis showed that the maximum asymptotic relative efficiency for the Weibull distribution is obtained when $k=1.5$, which is considered for validating the proposed method in Section 3.

III. SIMULATION STUDIES

A. Efficiency of the Robust Parameter Estimator in the Reliability Systems

In the first step of monitoring a reliability system, the parameters of the Weibull distribution are estimated using the classic and robust estimators in different conditions. In this simulation study, $n = 30$ samples are generated from the Weibull distribution with parameters $\alpha=4$, $\beta=2$ in each run, and the simulation run is repeated 1000 times using MATLAB software. To estimate and compare the classic and robust estimators in the presence of outliers, the following data contamination scenarios are studied.

1. A shift in α, from $\alpha=4$ to $\alpha=3$.
2. A shift in β, from $\beta=2$ to $\beta=1$.
3. A shift in both α and β simultaneously, from $\alpha=4$ and $\beta=2$ to $\alpha=3$ and $\beta=1$.

In this research, P100%=10, 20, 30, 40, and 50 of the generated data are contaminated according to each mentioned scenario. Thus, to compare the classic and robust estimates, P100% of the generated data includes outliers.

The population parameters are estimated using the proposed robust method and the maximum likelihood estimation (MLE), as a classic method based on generated contaminated samples in each scenario.

To compare the accuracy of the classic and robust methods in the presence of outliers, the mean square error (MSE) and norm of the estimators of α and β are used in the current research. The joint estimators' norm of vector (α,β) is obtained as follows: if the estimators of the parameters vector are shown as $(\hat{\alpha},\hat{\beta})$ and the $\underline{b}$ is defined as:

$$\underline{b} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} \hat{\alpha}-\alpha \\ \hat{\beta}-\beta \end{bmatrix}. \quad (17)$$

then the norm of the estimators is obtained as:

$$N = \underline{b}' \underline{\underline{\Sigma}}^{-1} \underline{b}. \quad (18)$$

where $\underline{\underline{\Sigma}}$ is the variance-covariance matrix of $(\hat{\alpha},\hat{\beta})$ which is defined as:

$$\underline{\underline{\Sigma}} = \begin{bmatrix} \mathrm{var}(e_1) & \mathrm{cov}(e_1,e_2) \\ \mathrm{cov}(e_2,e_1) & \mathrm{var}(e_2) \end{bmatrix}. \quad (19)$$

The estimated norm value is obtained as follows:

$$\hat{N} = \underline{b}' \widehat{\underline{\underline{\Sigma}}^{-1}} \underline{b}. \quad (20)$$

In general, if the parameter θ is estimated by $\hat{\theta}$, the MSE of the estimator is defined using (21).

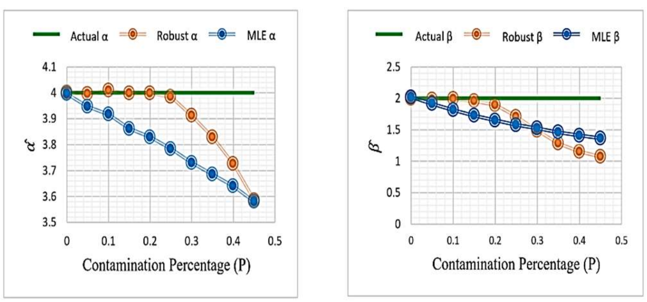
$$\mathrm{MSE}(\hat{\theta}) = E\left[\left(\hat{\theta}-\theta\right)^2\right] = \mathrm{Var}(\hat{\theta}) + \left[E(\hat{\theta})-\theta\right]^2. \quad (21)$$

Since knowledge about the statistical distribution of the estimator is required for obtaining the expectation value, and the distribution of estimators usually used to be unknown, so the MSE is obtained based on the simulation method according to (22).

$$MSE[\widehat{\theta}] = \frac{\sum_{i=1}^{k} (\hat{\theta}_i-\theta)^2}{k}, \; i=1,2,\ldots,k \tag{22}$$

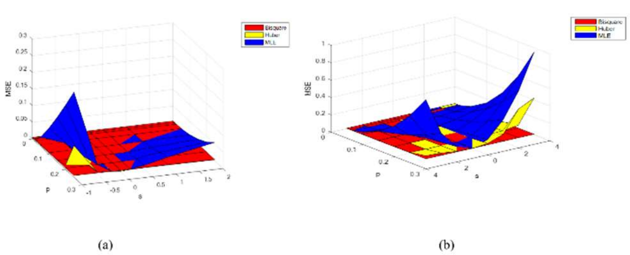
where θ is the parameter and $\hat{\theta}_i$ is its estimate in the i^{th} level of simulation in k simulation runs. Figure 2 compares the actual parameter values to the estimated values using classic and robust estimators in the different contamination percentages when α and β are shifted separately. In Figure 2-a, the robust estimator outperforms the MLE when α is shifted in all contamination percentages. Also, the robust method can estimate the parameter close to its actual value even in the presence of 20% contamination. In Figure 2-b, the robust estimator shows better performance and accuracy than MLE up to 25% of contamination when β is shifted. The MSE of $\hat{\alpha}$ and $\hat{\beta}$ based on the different shift values (s) and contamination percentages (P) presented in Figure 3.

According to the results, different robust methods have a smaller MSE than the MLE method, and the MSE of the classic method gets increased dramatically in large shifts and high contamination. The norm value of $\hat{\alpha}$ and $\hat{\beta}$ based on the shift values (s) and contamination percentages (P), while only one of the parameters is shifted, as shown in Figure 4. Also, Figure 5 presents the norm joint estimator of parameters α and β based on the different shifts and contamination percentages. In both Figures 4 and 5, the norm of the estimated parameters is not affected by the contamination; however, in the presence of outliers, the norm of the MLE method is highly altered.



(a) α is shifted (b) β is shifted

Figure 2. Comparison of the classic and robust estimates in different contamination percentages.

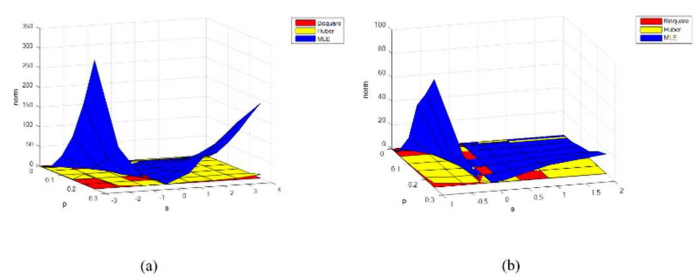


(a) α is shifted (b) β is shifted

Figure 3. MSE of $\hat{\alpha}$ and $\hat{\beta}$ in the different contamination percentages (P).

B. *Designing a Robust Control Chart for the Reliability Systems*

In this research, for monitoring a reliability system, two methods are used to calculate the control limits. In both methods, first, the parameters of the distribution are estimated using the M-estimator as a robust method and the MLE as a classic method. Then, the control charts are constructed based on the estimated parameters. Since in monitoring a reliability system, the failure time is estimated and controlled, the longer failure time is preferred. So, in constructing a control chart for the reliability systems, the upper control limit can be ignored, and the lower control limits which are calculated using the classic and robust method will be compared. The first method is to construct a control chart based on simulated control limits. In this method, m=30 random samples are generated from the Weibull distribution with parameters α=4, β=2. Then, the generated data are sorted in ascending order. If γ is denoted as a type-I error probability, $\left(\frac{\gamma}{2}\right)100\%$ and $\left(1-\frac{\gamma}{2}\right)100\%$ are defined as lower and upper limits of time between failures chart, respectively. For γ=5%, the actual lower and upper limits of the control chart for times between failures in the absence of outliers are 0.6393 and 7.6052, respectively. By shifting the distribution parameter (α) in different contamination percentages, the generated samples are going to be contaminated. Next, the distribution parameters are estimated by the classic and proposed robust methods. Using MATLAB software and parameter estimates, 10000 samples are generated from the Weibull distribution, and the lower control limits are constructed according to the procedure mentioned for the actual control limits. The lower control limit values using the classic and robust estimation methods are calculated for shift values (s) in α and different contamination percentages (P) based on the first method are presented in Tables 2 and 3, respectively.



(a) α is shifted (b) β is shifted

Figure 4. Norm of $\hat{\alpha}$ and $\hat{\beta}$ in the different shift values (s) and contamination percentages (P).

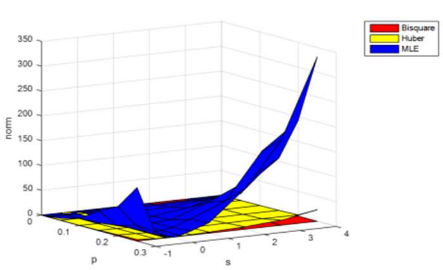


Figure 5. Norm of joint $\hat{\alpha}$ and $\hat{\beta}$ in the different shift values (s) and contamination percentages (P).

TABLE II. Classic lower control limit using MLE for different shifts (s) in α and contamination percentages (P) based on the simulated control limit method

	P=0	P=0.05	P=0.1	P=0.15	P=0.2	P=0.25
s=-1	0.6162	0.5248	0.4788	0.4378	0.3895	0.3814
s=-0.5	0.6231	0.6122	0.5693	0.5926	0.5268	0.5145
s=-0.2	0.5951	0.6153	0.6525	0.6114	0.6016	0.5985
s=0	0.6146	0.6253	0.6314	0.6136	0.6293	0.6355
s=0.2	0.6246	0.6263	0.6789	0.6325	0.6708	0.6561
s=0.5	0.6231	0.6269	0.6334	0.6613	0.7051	0.7209
s=1	0.6446	0.6376	0.6794	0.7133	0.745	0.7706
s=2	0.6179	0.6543	0.7259	0.7351	0.7433	0.8246
s=3	0.6726	0.6591	0.6852	0.7367	0.7845	0.8245
s=4	0.6031	0.6375	0.6637	0.681	0.7291	0.7495

TABLE III. Robust lower control limit using M-estimator for different shifts (s) in α and contamination percentages (P) based on the simulated control limit method

	P=0	P=0.05	P=0.1	P=0.15	P=0.2	P=0.25
s=-1	0.6297	0.6263	0.6166	0.6258	0.6277	0.641
s=-0.5	0.6369	0.6574	0.627	0.6366	0.6347	0.6143
s=-0.2	0.6301	0.628	0.6401	0.6294	0.676	0.6326
s=0	0.6413	0.6695	0.6503	0.6199	0.6505	0.6361
s=0.2	0.6152	0.6408	0.6455	0.613	0.633	0.6303
s=0.5	0.6369	0.6287	0.6303	0.6315	0.6143	0.6373
s=1	0.6254	0.6216	0.6517	0.6383	0.6363	0.6782
s=2	0.6321	0.6128	0.6324	0.6206	0.6209	0.6115
s=3	0.6212	0.6644	0.6238	0.6331	0.603	0.676
s=4	0.6343	0.6613	0.6491	0.6439	0.6379	0.6799

Also, the results for the actual, classic and robust lower control limits based on the largest shift (s=+4) in different contamination percentages are compared in Figure 6.

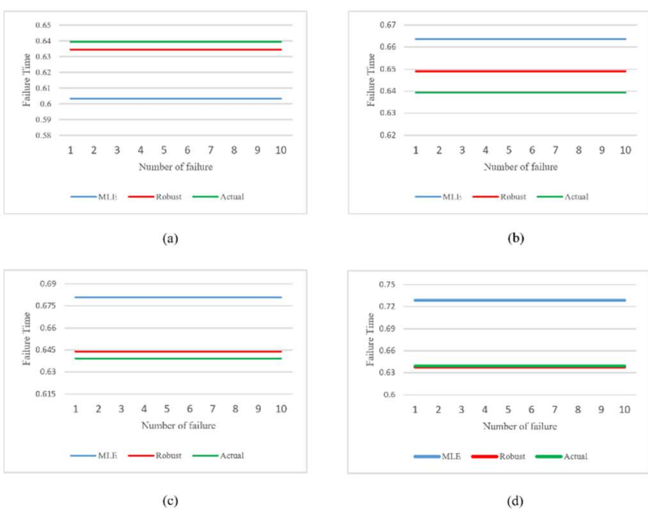


Figure 6. Constructed control charts using the simulated control limit method based on the largest shift in α (s=+4) in Different Contamination percentages (P): (a) P=0%, (b) P=10%, (c) P=15%, (d) P=20%

Considering the results, by increasing the contamination in different parameter shifts, the constructed control limits based on the MLE method are highly affected. However, the presence of contamination does not change the constructed control limits based on the M-estimate method, and their values are close to the actual limits. The second method is to construct the control chart using a mathematical formulation based on [8]. In this method, m=30 random samples are generated from the Weibull distribution with parameters α=4, β=2. Then, the generated data are sorted in ascending order. If γ is denoted as a Type-I error probability, according to [8], the control limits are calculated using (23) and (24).

$$UCL=\alpha\left(Ln\frac{1}{\left(\frac{\gamma}{2}\right)}\right)^{\frac{1}{\beta}}. \quad (23)$$

$$LCL=\alpha\left(Ln\frac{1}{\left(1-\frac{\gamma}{2}\right)}\right)^{\frac{1}{\beta}}. \quad (24)$$

If γ=5%, the actual lower and upper limits of the control chart for times between failures in the absence of outliers are 0.6365 and 7.6826, respectively. Considering the certain shifts for the distribution parameter (α) in different contamination percentages, the contaminated data are generated using MATLAB software, and the parameters are estimated through MLE and M-estimate methods. Then, based on the mentioned procedure for the actual limits, the classic and robust lower control limits are calculated using Equation (24). The lower control limit values using the classic and robust methods for different shifts (s) in α and contamination percentages (P) based on the second method are presented in Tables 4 and 5, respectively.

TABLE IV. Classic lower control limit using MLE for different shifts (s) in α and contamination percentages (P) based on the mathematical formulation method

	P=0	P=0.05	P=0.1	P=0.15	P=0.2	P=0.25
s=-1	0.6373	0.5645	0.5016	0.4468	0.4006	0.3583
s=-0.5	0.6375	0.6144	0.589	0.5669	0.5456	0.5265
s=-0.2	0.6373	0.6299	0.6206	0.6142	0.605	0.5977
s=0	0.6376	0.6365	0.6388	0.6386	0.6394	0.6371
s=0.2	0.638	0.6354	0.6412	0.6468	0.6497	0.6713
s=0.5	0.6395	0.6538	0.669	0.6831	0.7005	0.7169
s=1	0.6361	0.6633	0.6879	0.7147	0.7436	0.7761
s=2	0.6373	0.666	0.7011	0.7377	0.7792	0.8274
s=3	0.6371	0.6543	0.6826	0.7178	0.7592	0.8105
s=4	0.6379	0.6311	0.6446	0.6698	0.7066	0.7569

TABLE V. Robust lower control limit using M-estimator for different shifts (s) in α and contamination percentages (P) based on the mathematical formulation method

	P=0	P=0.05	P=0.1	P=0.15	P=0.2	P=0.25
s=-1	0.6351	0.6333	0.6324	0.6326	0.6362	0.6367
s=-0.5	0.6348	0.6344	0.6304	0.6322	0.6306	0.6354
s=-0.2	0.6334	0.6352	0.6333	0.6351	0.6343	0.6334
s=0	0.6348	0.634	0.6352	0.6361	0.6367	0.6355
s=0.2	0.6357	0.6402	0.6439	0.6489	0.6529	0.6342
s=0.5	0.6371	0.6387	0.6372	0.6372	0.6368	0.6361
s=1	0.6339	0.6368	0.6356	0.6367	0.6371	0.6364
s=2	0.6342	0.6347	0.6362	0.6334	0.6364	0.6373
s=3	0.634	0.6322	0.6343	0.6365	0.6329	0.6344
s=4	0.6354	0.6345	0.6353	0.6369	0.633	0.6361

Also, the actual, classic, and robust lower limits for the largest shift in α, (s=+4), in different contamination percentages are compared and shown in Figure 7.

According to the results of the second method, by increasing the contamination percentage and applying different shifts, the limits of the control chart designed by the proposed robust method are closer to the actual control limits, and they have less sensitivity in the presence of outliers in comparison to the classic method.

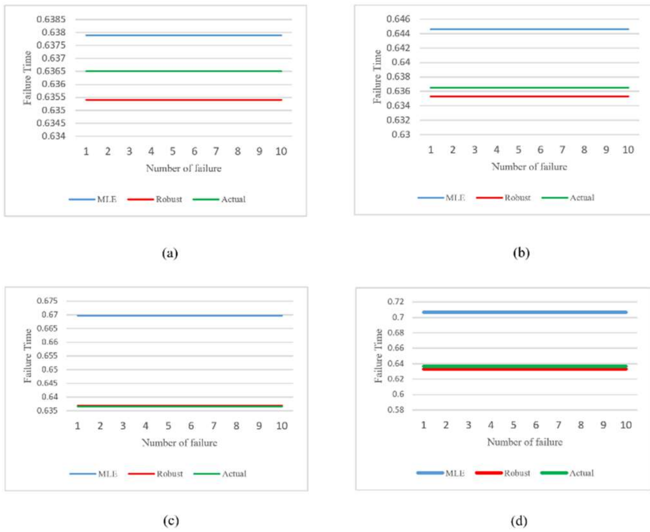


Figure 7. Constructed control charts using the mathematical formulation method based on the largest shift in α (s=+4) in Different Contamination percentages (P): (a) P=0%, (b) P=10%, (c) P=15%, (d) P=20%

IV. CONCLUSIONS

In this research, a robust control chart for monitoring reliability systems was introduced. To estimate the distribution parameters, the robust method based on the M-estimator was used to estimate the time between failure distribution parameters from the Weibull distribution. The efficiency of the robust estimator was investigated based on the norm and MSE criteria, and the results were compared with the MLE as a classic method. The results showed that the robust estimator performed as well as the classic estimator in the absence of outliers. However, in the presence of contamination, the performance of the robust estimator was much better than the classic one and close to the actual values. To construct the control charts two methods were proposed. In the first method, the control limits were found based on the simulated control limits. In the second method, a mathematical formulation was used to calculate the control limits. In both methods, only the lower control limits were considered as a determinant factor because the upper limit for monitoring a failure time in the reliability system can be ignored. The actual, robust, and classic lower control limits in different shifts in parameter α for a range of contamination percentages were compared. The results showed that in the both first and second methods, by increasing the contamination percentages, the classic control limits based on the MLE method were highly affected by the outliers. However, the robust control limits based on the M-estimates method were close to the actual limits in the absence or presence of outliers. So, using robust control charts is highly recommended in the monitoring of reliability systems. Investigating the correlation between the estimators of the Weibull parameters and its effect on the reliability estimation, and considering other types of robust estimators to construct the control charts and comparing their performances could be interesting areas for further research.

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