# Anti-Jamming CRN Transmission Stackelberg Game under Cumulative Interference Constraint

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***Abstract*—The power allocation of a secondary user (SU) in a cognitive radio network (CRN) contending with a smart jammer (JA) is formulated as a zero-sum game, in which the SU as the leader first chooses the transmit power while JA as the follower selects jamming power to interrupt the transmission of the SU. The impact of the cumulative interference constraint for ensuring PU's quality of service (QoS) is investigated. A Stackelberg equilibrium of the anti-jamming CRN game is derived and the conditions assuring its existence are provided.**

## I. Introduction

Jamming defence is a challenging problem in cognitive radio networks (CRNs), wherein smart jammers (JAs) can learn users' transmission strategies and pose a major threat to the whole communications. Game-based methods have been widely used to defend against the jamming [1]–[4]. Among them, authors in [3] evaluated the impact of the observation error of a JA on the performance of a Stackelberg anti-jamming game.

However, to the best of our knowledge, no previous work has considered the cumulative interference (CI) effect caused by SU and JA. Follow this line, we formulate an anti-jamming CRN transmission game and derive the Stackelberg equilibrium (SE) of the game.

## II. System Model

We consider a CRN which consists of a secondary transmiter (ST), a secondary user (SU), a primary user (PU), and a smart *jammer* (JA). The instantaneous channel power gains for the secondary link, the interference link from JA to SU and the interference link from ST to PU, and from JA to PU are denoted as $h_s, h_j, g_s$, and $g_j$, respectively. Suppose $N_0$ be the noise power, $J$ be the transmission power of smart jammer, $P$ be transmission power of ST. Then, the utility of SU is $\varphi_s(P,J) = \frac{h_s P}{N_0+h_j J} - C_s P$, where $C_s$ represents SU's transmission cost per unit power. The jammer's utility is $\varphi_j(P,J) = -\frac{h_s P}{N_0+h_j J} - C_j J$, where $C_j$ denotes jamming cost of per unit power of the jammer. Both of SU and jammer satisfy the CI constraint (CIC):

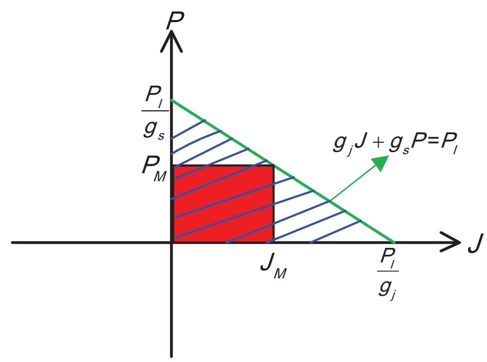


Fig. 1: The cumulative interference constraint vs the separate interference constraints.

$$g_j J + g_s P \le P_I, \tag{1}$$

The CIC (1) is more flexible or '*loose*' than the separate interference constraints (SIC) considered in [3]. In other words, as shown in Fig. 1, the red rectangle is the feasible region confined by the SIC in [3] while the blue triangle is the feasible region constrained by CIC (1). It is evident that the latter is more loose than the former.

## III. Stackelberg Equilibrium in the Game

### A. Follower Sub-game

Considering the observed SU's transmission power $\hat{P}$, the smart jammer's power strategy can be obtained:

$$J^{SE} = \arg\max_{J>0} \varphi_j(\hat{P}, J) \tag{2}$$

$$s.t. \quad g_j J + g_s \hat{P} \le P_I. \tag{3}$$

**Lemma 1.** *The best response of the JA is*

$$J^{SE}=\begin{cases}0 & \hat{P}<\frac{N_0^2C_j}{h_sh_j},\\ \frac{1}{h_j}\left(\sqrt{\frac{h_sh_j\hat{P}}{C_j}}-N_0\right) & \frac{N_0^2C_j}{h_sh_j}\le\hat{P}\le P^{\diamond},\\ \frac{P_I-g_s\hat{P}}{g_j} & \hat{P}>P^{\diamond},\end{cases} \tag{4}$$

*where* $P^{\diamond}=\frac{\left[-g_j\sqrt{h_sh_j/C_j}+\sqrt{g_j^2h_sh_j/C_j+4g_sh_j(N_0g_j+h_jP_I)}\right]^2}{4g_s^2h_j^2}$.

*Proof.* We have $\frac{\partial\varphi_j}{\partial J}=\frac{h_sh_j\hat{P}}{(Jh_j+N_0)^2}-C_j$, and $\frac{\partial^2\varphi_j}{\partial J^2}=\frac{-2h_sh_j^2\hat{P}}{(Jh_j+N_0)^3}\le 0$. Thus $\varphi_j(\hat{P},J)$ is concave with respect to $J$, which maximizes $\varphi_j$ when $\tilde{J}=\frac{1}{h_j}\left(\sqrt{\frac{h_sh_j\hat{P}}{C_j}}-N_0\right)$ if $0\le\tilde{J}\le(P_I-g_s\hat{P})/g_j$. Thus $J^{SE}=\tilde{J}$ when $\frac{1}{h_j}\left(\sqrt{\frac{h_sh_j}{C_j}\hat{P}}-N_0\right)\le(P_I-g_s\hat{P})/g_j$ and $\frac{1}{h_j}\left(\sqrt{\frac{h_sh_j\hat{P}}{C_j}}-N_0\right)\ge 0$, i.e., $\frac{N_0^2C_j}{h_sh_j}\le\hat{P}\le P^{\diamond}$. If $\tilde{J}\le 0$, i.e., $\hat{P}\le\frac{N_0^2C_j}{h_sh_j}$, $\varphi_j$ decreases with $J$ for $0\le J\le(P_I-\hat{P}g_s)/g_j$, obtaining $J^{SE}=0$. If $\tilde{J}>(P_I-\hat{P}g_s)/g_j$, i.e., $\tilde{P}>P^{\diamond}$, $\varphi_j$ increases with $J$, yielding $J^{SE}=(P_I-\hat{P}g_s)/g_j$. □

### *B. Leader Sub-game*

Given the estimated transmit power of smart jammer $\hat{J}$, the user's optimal strategy is obtained by solving:

$$P^{SE}=\arg\max_{P>0}\varphi_j(P,\hat{J}) \tag{5}$$

$$s.t.\quad g_s\hat{J}+g_sP\le P_I. \tag{6}$$

**Lemma 2.** *Denote* $\tilde{P}=\frac{h_sc_j}{4h_jc_s^2}$, $P^o=\frac{P_I+N_0g_j-\sqrt{(h_jh_sg_jP_I+h_sg_j^2N_0)/C_s}}{g_sh_j}$, *and define the functions* $l(x)=\sqrt{h_sxC_j/h_j}-C_sx$ *and* $h(y)=\frac{g_jh_sy}{g_jN_0+h_jP_I-g_sh_jy}-C_sy$*. Then, the best response of the SU is*

$$P^{SE}=\begin{cases}0, & \Pi_0,\\ N_0^2C_j/(h_sh_j), & \Pi_1,\\ \tilde{P}, & \Pi_2,\\ P^{\diamond}, & \Pi_3,\\ P_I/g_s, & \Pi_4,\end{cases} \tag{7}$$

*where*

- *Condition* $\Pi_0$*:* $C_s>h_s/N_0, P^{\diamond}\ge P_I/g_s$ *or* $C_s>h_s/N_0, P^{\diamond}<P_I/g_s, P^o\ge P_I/g_s$ *or* $C_s>h_s/N_0, P^{\diamond}<P_I/g_s, P^o<P_I/g_s, h(P_I/g_s)\le 0$ *;*
- *Condition* $\Pi_1$*:* $P^{\diamond}\ge P_I/g_s, h_s/(N_0)\ge C_s\ge h_s/(2N_0), C_j<h_jh_sP_I/(N_0^2g_s)$ *or* $h_s/(N_0)\ge C_s\ge h_s/(2N_0), P^{\diamond}<P_I/g_s, P^o\ge P_I/g_s$ *or* $h_s/(N_0)\ge C_s\ge h_s/(2N_0), P^{\diamond}<P_I/g_s, P^o<P_I/g_s, l(\frac{N_0^2C_j}{h_sh_j})\ge h(P_I/g_s)$ *;*
- *Condition* $\Pi_2$*:* $P^{\diamond}\ge P_I/g_s, C_s<h_s/(2N_0), C_j<4h_jC_s^2P_I/(h_sg_s)$ *or* $C_s<h_s/(2N_0), P^{\diamond}<P_I/g_s, P^o\ge P_I/g_s, \tilde{P}<P^{\diamond}$ *or* $C_s<h_s/(2N_0), P^{\diamond}<P_I/g_s, P^o<P_I/g_s, \tilde{P}<P^{\diamond}, l(\tilde{P})>h(P_I/g_s)$*;*

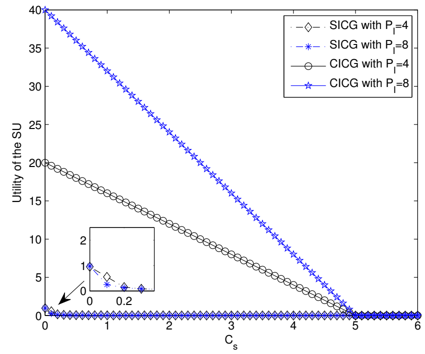


Fig. 2: The influence of the transmission cost of SU.

- *Condition* $\Pi_3$*:* $C_s<h_s/(2N_0), P^{\diamond}<P_I/g_s, \tilde{P}>P^{\diamond}, P^o\ge P_I/g_s$ *or* $C_s<h_s/(2N_0), P^{\diamond}<P_I/g_s, \tilde{P}>P^{\diamond}, P^o<P_I/g_s, l(P^{\diamond})>h(P_I/g_s)$*;*
- *Condition* $\Pi_4$ *corresponds to the other case.*

## IV. SIMULATION RESULTS

In this section, simulation results are presented to evaluate the proposed Stackelberg game. We set $h_s=0.5$, $h_j=0.5$, $g_s=1$, $g_j=1$, $C_j=0.2$, $N_0=0.1$. In addition, we compare the proposed CIC based game (CICG) with SIC based game (SICG) [3]. (Reproducible MATLAB code: https://github.com/gaobingaobingaobin/anti-jamming)

In Fig. 2, both CICG and SICG decrease with the increasing value of $C_s$, and CICG has higher utility of SU than that of SICG. This is as expected because the SU as the leader in the hierarchical game of CICG can achieve higher gain by utilizing the CIC to control and decrease the JA's jamming power.

## V. CONCLUSIONS

In this work, we proposed a Stackelberg game in CRNs by considering the CIC on PU. In the Stackelberg game, the SE was derived and the conditions assuring its existence were provided. The performance of CIC-based power control scheme was slightly improved, comparing with the SIC-based scheme.

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