# Digital Image Sharpening Using Integral Image Representation and Laplacian Operator

Su-Ling Lee[1] and Chien-Cheng Tseng[2]
[1]Chang Jung Christian University, Tainan, Taiwan
[2]National Kaohsiung University of Science and Technology, Kaohsiung, Taiwan

***Abstract*-In this paper, an image sharpening method using integral image representation and Laplacian operator is presented. First, a parallel algorithm is proposed to compute the integral image of the original image. Then, the integral image is used to compute the Laplacian image by subtracting the center pixel from its surround average in a rectangular window. This method can achieve a constant number of operations per rectangle. Next, the sharpened image is obtained by adding the Laplacian image to the original image. Finally, one numerical example is demonstrated to show the effectiveness of the proposed image sharpening approach.**

## I. INTRODUCTION

In our daily life, it is often to capture a blurring image using camera. In this case, it is desirable to highlight transients in intensity of image by a sharpening method. The applications range from industrial inspection, medical imaging, electronic printing and autonomous guidance. In the literature, several methods have been presented to solve the image sharpening problem. In [1], the sharpening operation is accomplished by using the second order derivatives of image where sharpened image is obtained by adding Laplacian image to the original image. In [2], an unsharp masking method is used to get the sharpened image by subtracting a low-pass filtered version of an image from the original image. In [3], the Grunwald-Letnikov fractional order derivative and Mach band effect are used to develop a digital image sharpening method. The above conventional methods have their unique features, so it is not easy to say which one is the best choice.

On the other hand, integral image has been widely used in the texture mapping, face detection in images, stereo corresponding and real-time adaptive thresholding of image [4][5]. The main feature is that integral image allows us to have very fast feature evaluation, that is, compute feature in constant time. Due to the success of integral image, the purpose of this paper is to use integral image to compute the Laplacian image such that the image sharpening method can be computed in constant time. Fig.1 shows the block diagram of the proposed method. Three steps involved are described below: First, a parallel algorithm is employed to compute the integral image of the original image. Second, the integral image is used to compute the Laplacian image by subtracting the center pixel from its surround average in a rectangular window. Third, the sharpened image is obtained by adding the Laplacian image to the original image. The details of the above three steps will be described in next section. And, image sharpening experiment will be demonstrated to show the effectiveness of the proposed method.

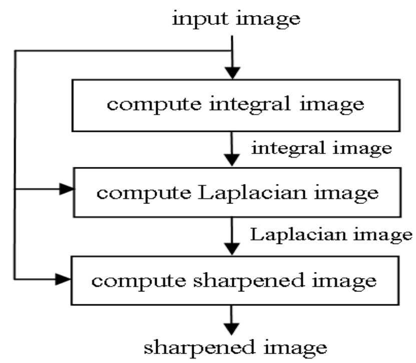


Fig.1 The block diagram of the proposed image sharpening method based on integral image representation.

## II. PROPOSED SHARPENING METHOD

In this section, the details of the proposed image sharpening method will be described. First, the computation of integral image is studied. Given the input image $f(m,n)$, its integral image $i(m,n)$ denotes the sum of all $f(m,n)$ terms to the left and above the pixel $(m,n)$,that is,

$$i(m,n) = \sum_{m' \le m, n' \le n} f(m', n') \tag{1}$$

If the zero initial is set, the integral image $i(m,n)$ can be computed from $f(m,n)$ by the recursive equation [4]:

$$\begin{aligned} i(m,n) = {} & f(m,n) + i(m-1,n) \\ & + i(m,n-1) - i(m-1,n-1) \end{aligned} \tag{2}$$

Taking 2-D z-transform at both sides, the transfer function from $F(z_1,z_2)$ to $I(z_1,z_2)$ is given by

$$\frac{I(z_1,z_2)}{F(z_1,z_2)} = \frac{1}{1-z_1^{-1}-z_2^{-1}+z_1^{-1}z_2^{-1}} = \frac{1}{1-z_1^{-1}}\frac{1}{1-z_2^{-1}} \tag{3}$$

It is clear that the transfer function can be factorized into a cascade connection of two 1-D rectangular integrators. Based on this fact, a two-step parallel algorithm is proposed to compute the integral image below:

Step 1: Perform 1-D integration for each row data of image $f(m,n)$ in parallel. The result is denoted by $t(m,n)$.

Step 2: Perform 1-D integration for each column data of the integral data $t(m,n)$ in parallel. The result is the final integral image $i(m,n)$.

A numerical example of the above computation method of an integral image is shown in Fig.2. Obviously, this result is correct.

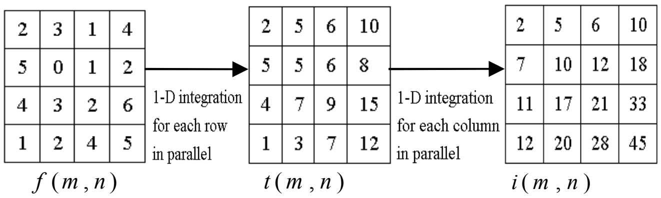


Fig.2 A computation example of integral image from the input image.

Next, let us describe how to compute the Laplacian image from the integral image. From the textbook in [1], the $3\times 3$ mask used to compute Laplacian image is given by

$$\mathbf{H}_1=\frac{1}{8}\begin{bmatrix}-1 & -1 & -1\\ -1 & 8 & -1\\ -1 & -1 & -1\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}-\begin{bmatrix}\frac{1}{8} & \frac{1}{8} & \frac{1}{8}\\ \frac{1}{8} & 0 & \frac{1}{8}\\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8}\end{bmatrix} \tag{4}$$

This equation tells us that the Laplacian is equal to the difference between center value and its surrounding average. The mask in (4) can be rewritten as the form

$$\mathbf{H}_1=\frac{1}{8}\left(\begin{bmatrix}0 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 0\end{bmatrix}-\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}\right) \tag{5}$$

The mask in the right-side of (5) means that the sum of all pixel values in a $3\times 3$ rectangle needs to be computed and it can be solved by using integral image. If we have the integral image $i(m,n)$, the sum of the image $f(m,n)$ for any rectangle with upper left corner $(m_1,n_1)$, and lower right corner $(m_2,n_2)$ can be computed in constant time by [4]:

$$\sum_{m=m_1}^{m_2}\sum_{n=n_1}^{n_2} f(m,n)=i(m_2,n_2)+i(m_2,n_1-1)+i(m_1-1,n_2)-i(m_1-1,n_1-1) \tag{6}$$

Let $(m_1,n_1)=(m-1,n-1)$ and $(m_2,n_2)=(m+1,n+1)$, then the sum of all pixel values in a $3\times 3$ rectangle with center $(m,n)$ can be computed by (6). Based on the above description, the procedure to compute Laplacian image using integral image is listed below:

Step 1: Compute the sum of all pixel values in a $3\times 3$ rectangle with center $(m,n)$ by

$$s_1(m,n)=i(m+1,n+1)+i(m+1,n-2)+i(m-2,n+1)-i(m-2,n-2) \tag{7}$$

Step 2: Compute the Laplacian image using (5) and (7):

$$\nabla^2 f(m,n)=\frac{1}{8}\big(9f(m,n)-s_1(m,n)\big) \tag{8}$$

Similarly, if the following $5\times 5$ mask is used to compute Laplacian image

$$\mathbf{H}_2=\frac{1}{24}\begin{bmatrix}-1 & -1 & -1 & -1 & -1\\ -1 & -1 & -1 & -1 & -1\\ -1 & -1 & 24 & -1 & -1\\ -1 & -1 & -1 & -1 & -1\\ -1 & -1 & -1 & -1 & -1\end{bmatrix} \tag{9}$$

then the procedure to compute Laplacian image is given by

Step 1: Compute the sum of all pixel values in a $5\times 5$ rectangle with center $(m,n)$ by

$$s_2(m,n)=i(m+2,n+2)+i(m+2,n-3)+i(m-3,n+2)-i(m-3,n-3) \tag{10}$$

Step 2: Compute the Laplacian image using (9) and (10):

$$\nabla^2 f(m,n)=\frac{1}{24}\big(25f(m,n)-s_2(m,n)\big) \tag{11}$$

It is clear that the computation loads that use $3\times 3$ and $5\times 5$ masks are the same, so the Laplacian image can be computed in constant time by using integral image.

Finally, let us describe how to compute the sharpened image. The sharpened image can be obtained by adding the Laplacian image to the original image below:

$$o(m,n)=f(m,n)+\alpha\nabla^2 f(m,n) \tag{12}$$

where $\alpha$ is a prescribed constant that uses to control the strength of sharpening.

## III. NUMERICAL EXAMPLE

In this section, a digital image is used to evaluate the performance of the proposed image sharpening method. Fig.3(a) shows the original image. The parameter $\alpha=2$ is chosen and the $3\times 3$ Laplacian mask is adopted. Fig.3(b) depicts the sharpened image. It can be observed that the contrast of image has been enhanced. Finally, it is worth mentioning that the performance of proposed method is the same as that of the conventional Laplacian method. The main contribution is to reduce the computation complexity.

(a) (b)

Fig.3 Experimental results. (a) Original image. (b) Proposed sharpened image.

## IV. CONCLUSIONS

In this paper, an image sharpening method using integral image representation and Laplacian operator have been presented. In the future, the topic that uses integral image to solve other image processing problems may be considered.

## REFERENCES


[1] S. Annadurai and R. Shanmugalakshmi, *Fundamentals of Digital Image Processing*, Pearson Education, 2007.

[2] C.C. Tseng and S.L. Lee, "Color image sharpening using DST-based matrix low-pass Butterworth filter," *IEEE ICCE-TW*, pp.365-366, June, 2017.

[3] C.C. Tseng and S.L. Lee, "Digital image sharpening using fractional derivative and Mach band effect," *IEEE ISCAS*, pp.2765-2768, May, 2012.

[4] D. Bradley and G. Roth, "Adaptive thresholding using the integral image," *Journal of Graphics Tools*, vol.12, pp.13-21, 2007.

[5] B. Bilgic, B.K.P. Horn and I. Masaki, "Efficient integral image computation on the GPU," *IEEE Intelligent Vehicles Symposium*, pp.528-533, 2010.