Suspended Load Swing Stabilization

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***Abstract* – This research proposes an interdisciplinary collaboration to improve hoist stabilization for medical evacuation and successful rescues. This paper would include the collaborative efforts from a diverse range of fields to include Systems Engineering, Mechanical Engineering, Physics, and the Special Collections & Archives Division of the United States Military Academy Library. The research objective of this effort is to create an algorithm which could limit the displacement angle of a suspended individual below a helicopter. This would be accomplished by changing the relative length of the cable at different points within the swing of the slung mass. This could all be done while reeling in the hoisted individual to the helicopter by changing the rate at which the hoist is lessening its cable. Elements of the mathematical principles that this research is built on are illustrated through Edgar Allen Poe's application of the pendulum in his short story "The Pit and the Pendulum". Poe was an individual who attended, but did not graduate from, USMA; however, his education at the Military Academy and his subsequent writings are the birthplace of this research endeavor. It is a multi-semester goal, and this paper will present an initial proof of concept.**

Index Terms – MEDEVAC stabilization, second order differential equation of motion, sling load and hoist stabilization, pendulum, Poe, cycloidal pendulum, Atwood's machine.

BACKGROUND

In helicopter rescue situations, minutes, and even seconds matter, especially when dealing with combat casualties that need to be evacuated as soon as possible. The ultimate goal of our research project is to create an algorithm which could limit the displacement angle of a suspended individual below a helicopter. What this research could do is revolutionize the way that the United States Army evacuates their casualties from the battlefield [1-4].

Figure 1: Army MEDEVAC Helicopter

Figure 1 shows the current problem that hoisting casualties into a medical evacuation (also referred to as MEDEVAC) helicopter presents. Currently the only way to limit any swing in the cable is for the crew chief to reach out and take hold of the cable from a seated position within the helicopter. As seen, this human intervention is a highly dangerous posture and could lead to accidents within the aircraft [5]. In this research, it is proposed that as the casualty is hoisted upwards, the rate at which the hoist reels in the casualty could be adjusted at different points of the swing. Thus, this approach could limit the displacement angle and the length of the cable at the same time, thereby saving precious minutes that would otherwise be spent getting shot at by enemy small arms fire or rocket-propelled grenades.

APPROACH

The first part of this paper will explain the motivation of the research through a real-world example. The paper then reviews a model for a simple pendulum with time-varying length. Next, the paper presents simulation results for a specific scenario that reduces the swing angle as a function of time as the cable length is changed. In the second part of the paper, the paper discusses the historical background of the peer-reviewed literature that analyzes a related scenario in Edgar Allen Poe's short story *The Pit and the Pendulum*, where the swing angle increases as a function of time as the cable length is increased. The analysis in the second part of this paper is informed by artifacts in the Special Collections & Archives Division of the United States Military Academy Library, which houses textbooks, a letter, a check, and records of books that were in print at the time that Poe attended the Academy and may have learned about the pendulum.

REAL WORLD EXAMPLE

The loss of life is always disastrous in any scenario, but the loss of a life that could have been avoided is not only unacceptable but overwhelmingly crushing. In the infancy of this research project, current Army pilots were interviewed to discuss ways that this research could affect current Army operations. A previous Army pilot Jacob Capps had the following to say about this MEDEVAC research:

> "The impetus of the problem… was based on real life occurrences. In Afghanistan a night mission to save a Soldier on the ground proved to be fatal as an oscillating rescue hoist [resulted in] the Soldier and rescue specialist losing their lives. This project seeks to reduce the risks inherent in an unstable hover." [6]

This event was caused by a wild swing in displacement angle and could possibly have been avoided. It is the goal of this research to discover and implement this solution as soon as possible to bring home all our Soldiers.

MODEL

In order to solve this complex problem, it must first be simplified down to its simplest level. A two-dimensional planar pendulum with length l and bob with mass m is that simplest level, as shown in Fig. 2 [7]. A few assumptions must be made before moving forward in the analysis. First, we assume that there is no drag caused by air resistance, and second, we assume that the pivot point is frictionless.

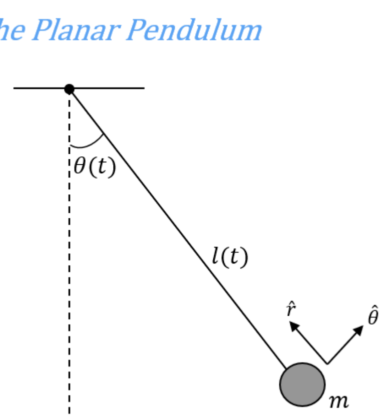


Figure 2: Planar Pendulum. Adapted from [7].

SECOND ORDER DIFFERENTIAL EQUATION OF MOTION

From the planar pendulum shown in Fig. 2, it can be seen that the equation for the speed of the pendulum, in Eqn. (1), where

$$v = l\dot{\theta}, \tag{1}$$

where the terms v, l, and $\dot{\theta}$ represent the speed, cable length, and angular speed, respectively. These are important to note as they are going to be substituted in the equation for kinetic energy, T, as shown in Eqn. (2), such that,

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m(l\dot{\theta})^2. \tag{2}$$

The following step is to write the Lagrangian equation. (Eqn. 4), where we subtract the potential energy (Eqn. 3), of the system, V, from the kinetic energy of the system in order to obtain the Lagrangian (Eqn. 5), according to the expressions,

$$V = -mgl\cos\theta, \tag{3}$$

$$\mathcal{L} = T - V, \tag{4}$$

and

$$\mathcal{L} = \frac{1}{2}m(l\dot{\theta})^2 + mgl\cos\theta. \tag{5}$$

The final step in deriving our final second order equation of motion is substituting our equation for Lagrangian, Eqn. 5, into Lagrange's second order equation of motion (Eqn. 6) given by the expression,

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\theta}} - \frac{\partial\mathcal{L}}{\partial\theta} = 0. \tag{6}$$

By substituting Eqn. 5 into Eqn. 6, we obtain the following expression,

$$\frac{d}{dt}\frac{\partial}{\partial\dot{\theta}}\left(\frac{1}{2}m(l\dot{\theta})^2\right) - \frac{\partial\mathcal{L}}{\partial\theta} = 0. \tag{7}$$

We evaluate Eqn. 7 by taking the partial derivative with respect to the rate of change of the angle with respect to time and obtain Eqn. 8, such that,

$$\frac{d}{dt}\left(\frac{2}{2}m\left(l^2\dot{\theta}\right)\right) + mgl\sin\theta = 0, \tag{8}$$

and

$$\frac{d}{dt}(ml^2\dot{\theta}) + mgl\sin\theta = 0. \tag{9}$$

It is here that it must be remembered that the length of the cable, l, is varying as a function of time as the hoist reels in the casualty. Therefore, we can evaluate the derivative such that,

$$ml^2\ddot{\theta} + m\dot{\theta}2l\dot{l} + mgl\sin\theta = 0, \tag{10}$$

and

$$l\ddot{\theta} + 2\dot{\theta}\dot{l} + g\sin\theta = 0. \tag{11}$$

Last, we divide each term by the length, l, to obtain the second order differential equation of motion shown in Eqn. 12, such that

$$\ddot{\theta} + \frac{2}{l}\dot{\theta}\dot{l} + \frac{g}{l}\sin\theta = 0. \qquad (12)$$

SIGNIFICANCE

Now that there is a clearly-defined equation of motion for the planar pendulum system, it can be dissected to explain a few phenomena which allow intuitive discussion and realization of the minimization of displacement angle while lessening the cable length. Specifically, the middle term, $\frac{2}{l}\dot{\theta}\dot{l}$, is referred to as the dampening term [8]. This term accounts for the rate of change in length and the rate of change in displacement angle therefore allowing the opportunity for both to be minimized. Furthermore, when the derived second order differential equation of motion, (Eqn. 12), is compared to a basic pendulum's equation of motion shown in Eqn. 13, it can be seen that this term is key to solving the problem of limiting displacement angle while lessening cable length, where the basic equation can be written as,

$$\ddot{\theta} + \frac{g}{l}\sin\theta = 0. \qquad (13)$$

PRIOR WORK

Prior research conducted by A. Kavanaugh and T. Moe [9] attempted to model the motion of a scythe that was in the Edgar Allen Poe short story "The Pit and the Pendulum" [10]. In this story, the scythe was swinging more and more wildly as it was being slowly lowered down to its victim. In the case of the MEDEVAC research discussed in this paper, the exact opposite is desired. Rather than the widening of displacement angle, the lessening is desired. Thus initially, attempting the opposite of what A. Kavanaugh and T. Moe [9] modeled was attempted. They postulated that "Pulling up in the middle and letting out on the end" would increase the displacement angle. Thus, in order to lessen the displacement angle, such as shown in Fig. 3, the approach attempted in this research was letting out in the middle and pulling up in the end.

Additional prior work by M. McMillan [11] suggested that two pairs of pulses (changes in the hoists ramp rate) both right before and right after the mass reaches its maximum swing angle would both lessen the displacement angle and the cable length.

In this research, MATLAB code in [9] was modified and expanded on for the situation in which the cable length changes as function of time (shown in Fig. 4), solving Eqn. 12 using ODE45. For the example discussed in this paper, we consider the seconds pendulum, for which the initial length is one meter (for a period of approximately one second), and $g = 9.807\ \frac{\text{m}}{\text{s}^2}$.

Our findings, shown below in Figs. 3 and 4, highlight the relationship that is evident when the change in ramp rate is applied right before and right after the mass reaches its maximum displacement angle.

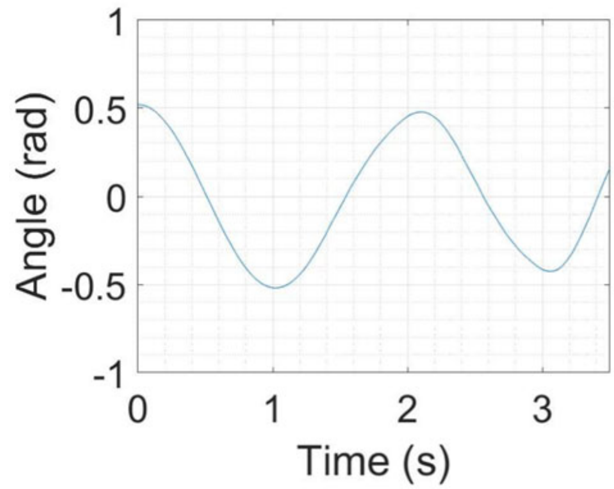


Figure 3: Lessening of displacement angle as the cable length is changed as shown in Fig. 4.

Figure 3 shows how the model has accomplished decreasing the displacement angle. To better orient the reader to Fig. 3, observe how the y-axis shows time and the output displacement angle, found on the x-axis, grows tighter and tighter to the middle value of zero. Therefore, Fig. 3 visually depicts how over time the displacement angle grows closer and closer to zero.

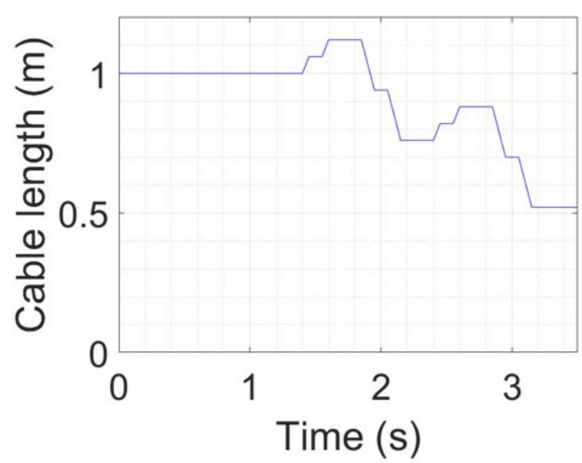


Figure 4: Visualization of the approach to "let out in the middle and pull up at the ends".

Figure 4 presents the proof-of-concept that the overall decreasing cable length as a function of time can produce a decreasing displacement angle, shown in Fig. 3. Keep in mind that the extension of cable length would coincide with the time at which the swing reaches its midpoint, and the shortening of the cable length would occur when the swing reaches a maximum, as the approach suggests.

CONSTRAINTS

We are currently exploring the most efficient placement of these changes of ramp rates and changing the initial conditions to better reflect a real-world scenario. Table I shows the constraints within which the final system must operate [5].

Constraint Type	Constraint	Value
Hoist	Maximum weight	2668.9 N
	Usable cable length	88.4 m
	Cable winding speed (fast, slow)	$(1.778, 0.0508)\,\frac{m}{s}$
Helicopter	Maximum acceleration (forward, rear, side, down, upward)	$(7.3, 9.0, 11.6, 1.7, 3.0)\,\frac{m}{s^2}$
	Acceleration time to maximum velocity (forward, rear, side, down)	(7, 2, 2, 3) s
	Maximum velocity (forward, rear, side, down, upward)	$(51.4, 18.0, 23.2, 5.1, 8.2)\,\frac{m}{s}$
Wind	Wind speed (Disturbance force)	Est. $5.0\,\frac{m}{s}$ (20N)

Table 1: Constraints

THE PENDULUM

The earliest experiments with the pendulum, a mass attached to a string or rod that rotates around a pivot point at one end, are attributed in the early 1600s to Galileo (Italy) who discovered the property of isochronism [12-14]. The pendulum clock, invented by Huygens (Dutch Republic) in 1656, was used for timekeeping through the 1930s.

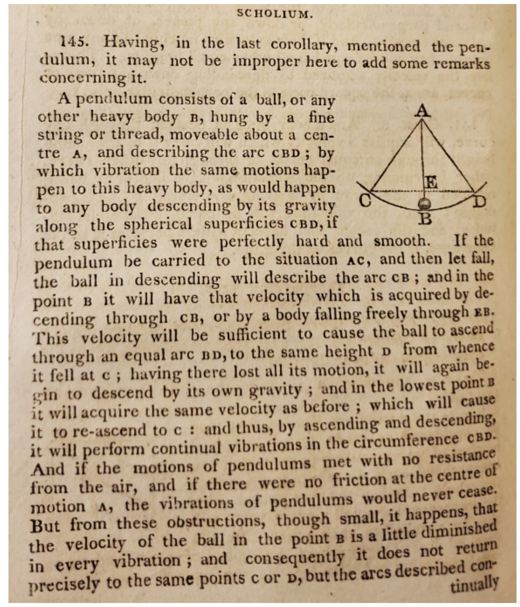
The length of a pendulum with a one-second swing was proposed as the standard unit of length in 1671 (but not retained because the length changes slightly at different locations on Earth). The variation can be measured today through the internet in the "World Pendulum" (Portugal, Germany, and Latvia) Remote Controlled Laboratory hosted by the University of Munich [15].

John Harrison (England) improved the pendulum clock during the 1700s and continued to develop clocks for sailors to use at sea to calculate longitude [16]. This type was made of wood that generated its own oil and minimized friction.

In 1851 Foucault constructed a 67-m-long pendulum in Paris and demonstrated the rotation of Earth [17]. Modern MEDEVAC rescues with hoists are conducted by alpine rescue teams such as Air Zermatt [18] and the United States Coast Guard [19].

PENDULUMS AT UNITED STATES MILITARY ACADEMY ARCHIVES

Figure 5 [20] shows a snapshot of Charles Hutton's 1812 textbook, "A Course of Mathematics", that is located in the USMA Archives today and was used at The United States Military Academy to teach Mathematics. In Fig. 5 [20] the definition of the Pendulum is written at (the beginning of the second paragraph) as a "ball, or any other heavy body, B, hung by a fine string or thread, moveable about a centre A, and describing the arc CBD".


SCHOLIUM.

145. Having, in the last corollary, mentioned the pendulum, it may not be improper here to add some remarks concerning it.

A pendulum consists of a ball, or any other heavy body B, hung by a fine string or thread, moveable about a centre A, and describing the arc CBD; by which vibration the same motions happen to this heavy body, as would happen to any body descending by its gravity along the spherical superficies CBD, if that superficies were perfectly hard and smooth. If the pendulum be carried to the situation AC, and then let fall, the ball in descending will describe the arc CB; and in the point B it will have that velocity which is acquired by decending through CB, or by a body falling freely through EB. This velocity will be sufficient to cause the ball to ascend through an equal arc BD, to the same height D from whence it fell at C; having there lost all its motion, it will again begin to descend by its own gravity; and in the lowest point B it will acquire the same velocity as before; which will cause it to re-ascend to C: and thus, by ascending and descending, it will perform continual vibrations in the circumference CBD. And if the motions of pendulums met with no resistance from the air, and if there were no friction at the centre of motion A, the vibrations of pendulums would never cease. But from these obstructions, though small, it happens, that the velocity of the ball in the point B is a little diminished in every vibration; and consequently it does not return precisely to the same points C or D, but the arcs described continually

Figure 5: An 1812 Academic Definition of a Pendulum [20].

In 1842, Edgar Allen Poe published a short story, *The Pit and the Pendulum* [10, 11]. The textbooks that were used to teach mathematics at the United States Military Academy at the time he was enrolled were Legendre's *Elements of Geometry*, Andesite Press (1825) [21] and Lacroix and Bezout's *Elementary Treatise on Plane and Spherical Trigonometry, and on the Application of Algebra to Geometry; From the Mathematics of Lacroix and Bezout* (1833) [22].

The historical setting for the piece is the Spanish inquisition, during which a bound prisoner is faced with a bladed pendulum as an instrument of impending death. Among many questions raised by the story is where Poe found the inspiration to feature this scientific and technical apparatus in this novel and macabre way. How did the mechanics of the pendulum even enter Poe's consciousness?

One possibility is Poe's brief period of education at the U.S. Military Academy at West Point [23-26]. A decade before Poe's tale of the pendulum was published, he was admitted to the U.S. Military Academy as a cadet. While the idea of Cadet Poe might seem incongruous in the context of the writer Poe's ultimate reputation, it was a sound choice for young Poe who had previously served two years in the Army. Despite his prior military experience, Poe did not fare well at West Point and left pursuant to the sentence of a general court martial without completing the first year of study. Instead Poe went on to publish a book of poems [24].

The Academy in 1830 was a small and Spartan place with only a few buildings, inadequately and perilously heated; there were only 232 in the Corps of Cadets. First year cadets (plebes) studied French and mathematics, which included an element of surveying. Poe spent about eight months immersed in this rigorous course. We know that while he was at West Point the Academy was in possession of an Atwood's machine with a cycloidal pendulum because Albert E. Church, an 1828 graduate who stayed on to teach Mathematics and thus overlapped with Cadet Poe's tenure, wrote in his 1878 memoir of "a tall Atwood's machine standing in one corner of the philosophical room…a cycloidal pendulum with which we used to amuse ourselves when the professor was late coming in…." [27]. It is hard to imagine where else young Poe, who also briefly studied languages at the University of Virginia, would have been exposed to such an apparatus.

WORK ON PENDULUMS AT THE UNITED STATES MILITARY ACADEMY

In 2019, one of the coauthors and colleagues proposed a stabilization system for low-mass sling loads [5]. These researchers proposed a delayed active feedback control method for sling load stabilization [5]. This method called for the lateral movement of the hoist's pivot point along the length of the helicopter. The results of the simulations show that the swing angle can be reduced as the cable length is reduced [Fig. 5, 5].

CONCLUSION

In summary, this research aims primarily to decrease the amount of time that it takes for a combat casualty to be evacuated from a combat zone. By eliminating the need for the crew chief to take hold of the cable, and lean out of the helicopter, and stabilize the swing of a casualty, the automation of this process has the potential to increase the safety of the medical and helicopter personnel who evacuate casualties. This research is vital to the progression towards future fully automated drone helicopters that would be able to conduct medical evacuations in both military and civilian settings on their own.

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