Vector-sensor array DOA estimation using spatial time-frequency distributions

Hai-Yan Song[1,3,*], Chang-Yi Yang[2], Jin-Ping Qin[3], Ming Diao[1]
1. College of Information And Communication Engineering, Harbin Engineering University, Harbin,China.
2. Department of Computer Science and Information Engineering, National Penghu University of Science and Technology, Magong, Taiwan, R.O.C.
3. School of Electrical and Information Engineering, Heilongjiang Institute of Technology, Harbin, China.
*Corresponding author: songhaiyan0508@hrbeu.edu.cn

***Abstract*—By making use of the extra particle velocity information, an array of vector sensors can achieve better Direction-of-arrival (DOA) estimation performance than a conventional array of pressure sensors. In this paper, we develop a new approach which exploits the inherent time-frequency-space characteristics of the underlying vector-sensor array signal to achieve better DOA estimation performance even in a noisy and coherent environment with few snapshots. Computer simulations with several frequently encountered scenarios, such as a single source and multiple closely spaced coherent sources, indicate the superior DOA estimation resolution of our proposed approach as compared with existing techniques.**

I. INTRODUCTION

Direction-of-arrival (DOA) estimation is an area of active interest with a broad range of applications in sonar, radar, and other fields [1]. By making use of the extra particle velocity information, an array of vector sensors can achieve better performance than a conventional array of pressure sensors [2]. Recently, the interest in DOA estimation with vector-sensor array increased exponentially[3].

However, most of the previous work on DOA estimation with vector-sensor array does not exploit the difference in the time-frequency signatures of the sources. The definition of spatial time-frequency distribution (STFD) was first introduced by Belouchrani and Amin in [4], where the diagonalization of a combined set of STFD matrices was used to solve the problem of blind source separation for non-stationary signals. Different STFD-based methods for DOA estimation have been developed separately in [5,6].

Different from most other DOA estimation methods, we make utilize of multiple time-frequency points, instead of a single one, to reduce the effect of noise and achieve better DOA estimation performance. The simulation results show that our proposed method has superior DOA estimation resolution and higher estimation accuracy than the conventional methods.

II. THE ACOUSTIC VECTOR-SENSOR ARRAY SIGNAL MODEL

Let the uniform linear vector-sensor array in consideration consist of N vector sensors with equal spacing d. Consider K narrowband signals $s_k(t)$, $k=1,\cdots,K$, impinging on the array by azimuth angles θ_k. Then the array signal model can be represented as

$$\mathbf{y}_{pv}(t)=\sum_{k=1}^{K}\begin{bmatrix} e^{-j2\pi f_k\tau_{k1}} \\ e^{-j2\pi f_k\tau_{k2}} \\ \vdots \\ e^{-j2\pi f_k\tau_{kN}} \end{bmatrix}\otimes\begin{bmatrix} 1 \\ \cos\theta_k \\ \sin\theta_k \end{bmatrix}s_k(t)+\mathbf{n}_{pv}(t) \quad (1)$$

where f_k is the frequency of the k-th narrowband signal, $\otimes$ is the Kronecker product, $\tau_{kn}=(n-1)d\cos\theta_k/c$ represents the time delay of the k-th source signal between the n-th sensor and the reference sensor, c is the signal propagation speed, $\mathbf{y}_{pv}(t)$ denotes the $3N\times1$ dimensional vector-sensor array outputs, and $\mathbf{n}_{pv}(t)$ represents the noise field.

Further let $\mathbf{a}(\theta_k)=\begin{bmatrix} e^{-j2\pi f_k\tau_{k1}} \\ e^{-j2\pi f_k\tau_{k2}} \\ \vdots \\ e^{-j2\pi f_k\tau_{kN}} \end{bmatrix}\otimes\begin{bmatrix} 1 \\ \cos\theta_k \\ \sin\theta_k \end{bmatrix}$ denotes the array steering vector corresponding to the k-th source signal, then Eq.(1) can be represented as a compact matrix expression

$$\mathbf{y}_{pv}(t)=\mathbf{X}(t)+\mathbf{n}_{pv}(t)=\mathbf{A}(\boldsymbol{\theta})\mathbf{S}(t)+\mathbf{n}_{pv}(t) \quad (2)$$

where $\mathbf{A}(\boldsymbol{\theta})=[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\cdots,\mathbf{a}(\theta_K)]$ is called the array manifold matrix, $\mathbf{S}(t)=[s_1(t),s_2(t),\cdots,s_K(t)]^{\mathrm{T}}$ represents the original sources, and $\mathbf{X}(t)=\mathbf{A}(\boldsymbol{\theta})\mathbf{S}(t)$ denotes the signal field.

III. SPATIAL TIME-FREQUENCY ANALYSIS FOR DOA ESTIMATION

The concept of STFD has been introduced by [4], and the spatial pseudo Wigner-Ville distribution matrix can be given by

$$\mathbf{PWV}_{\mathbf{yy}}(t,f)=\sum_{\tau=-(L-1)/2}^{(L-1)/2}\mathbf{y}_{pv}(t+\tau)\mathbf{y}_{pv}^{\mathrm{H}}(t-\tau)e^{-j4\pi f\tau} \quad (3)$$

Substitute Eq.(2) into Eq.(3) and assume a noise-free environment, then we obtain

$$\mathbf{PWV}_{\mathbf{yy}}(t,f)=\mathbf{A}(\theta)\cdot\mathbf{PWV}_{\mathbf{SS}}(t,f)\cdot\mathbf{A}^{\mathrm{H}}(\theta) \quad (4)$$

In order to reduce the effect of noise and achieve better DOA estimation performance, we should combine all of the relevant STFD points (t_k,f_k), $k=1,\cdots,K$, to allow more information of the source signal t - f signatures to be included into their respective formulation.

Now consider all of the relevant STFD points (t_k, f_k), corresponding to Eq.(4)

$$\mathbf{PWV}_{\mathrm{yy}}(t_k, f_k) = \mathbf{A}(\theta)\cdot\mathbf{PWV}_{\mathrm{SS}}(t_k, f_k)\cdot\mathbf{A}^{\mathrm{H}}(\theta) \qquad (5)$$

The DOA estimation problem can be converted into the following optimization problem[7]

$$\hat{\mathbf{A}}(\theta) = \arg\min_{\mathbf{A}(\theta)} \sum_{k=1}^{K} \mathrm{off}\left(\mathbf{A}^{\mathrm{H}}(\theta)\cdot\mathbf{PWV}_{\mathrm{yy}}(t_k, f_k)\cdot\mathbf{A}(\theta)\right) \quad (6)$$

Then the joint eigenvalue estimation is

$$\lambda_l = \sum_{k=1}^{K}\left|\mathbf{PWV}_{\mathrm{SS}}(t_k, f_k)^{(l,l)}\right| \Big/ K \quad (l = 1, 2, \cdots, N) \quad (7)$$

So the STFDs-based MVDR spatial spectrum based on joint diagonalization using Jacobi rotations (which is called STFD-JD-MVDR below) can be defined as

$$P_{MVDR}^{TF-JD}(\theta) = \frac{1}{\boldsymbol{a}^{\mathrm{H}}(\theta)\sum_{l=1}^{N}\left(\frac{1}{\lambda_l}\hat{\mathbf{A}}_l(\theta)\hat{\mathbf{A}}_l^{\mathrm{H}}(\theta)\right)\boldsymbol{a}(\theta)} \qquad (8)$$

where $\hat{\mathbf{A}}_l(\theta)$ is the l-th column of $\hat{\mathbf{A}}(\theta)$, which is corresponding to the l-th eigenvalue λ_l.

IV. Numerical Simulations

Consider a uniform linear vector-sensor array with eight sensors. Three narrowband sources impinge on the array from DOA -60°, 0° and 20°. The signal frequency is 20Hz, the total number of snapshots is 1024, and the SNR is 20dB. The numerical results are shown in Fig.1. For convenience, “PSA” represents “Pressure Sensor Array” and “VSA” represents “Vector Sensor Array”.

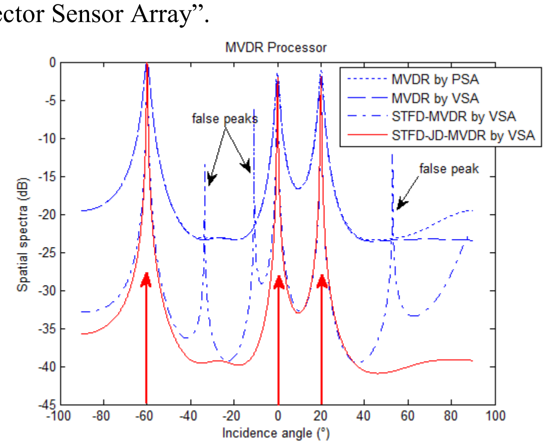

Fig. 1. Spatial spectrum results

It is observed that all the algorithms have three separated spatial spectrum peaks and can resolve the three closely spaced sources. However, it should be noted that both MVDR by PSA and MVDR by VSA have higher sidelobes (about -20dB). In addition, there exist some strong false peaks as indicated by the black arrows in the spatial spectrum curve for STFD-MVDR by VSA, which will submerse the peaks corresponding to the true sources. In comparison, the STFD-JD-MVDR by VSA makes use of not only the extra particle velocity information but also the multiple relevant STFD points to clearly distinguish the three sources as shown by the sharper spectrum peaks. Thus, under the challenging scenario of more closely spaced coherent sources, our proposed algorithm can achieve superior DOA estimation resolution and higher estimation accuracy.

V. Conclusion

In this paper, a vector-sensor array DOA estimation algorithm based on the STFD information has been proposed. Different from most other DOA estimation methods, our proposed algorithm can efficiently combine all of the relevant STFD points by joint approximation diagonalization using the Jacobi rotation technique in order to achieve better DOA estimation performance. The computer simulation results show that our method can achieve better DOA estimation performance, such as the superior resolution and higher accuracy, as compared with existing techniques.

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