# Phase Difference Algorithm and Its FFT Implementation for High-accuracy Power System Frequency Monitoring

Xiaolong Ma[1,2], Cheng-Hung Lin[2], *Member, IEEE*, and Tao Jin[1], *Member, IEEE*

[1] College of Electrical Engineering and Automation, Fuzhou University, Fuzhou 350116, China

[2] Department of Electrical Engineering, Yuan Ze University, Jungli, Taiwan 32003

***Abstract*--To track the frequency of power system signals under noisy and harmonic circumstances quickly and precisely, a reliable phase difference algorithm based on discrete Fourier transform and its key module implementation are introduced in this paper. By utilizing main spectral line phase difference of two adjacent data windows, the frequency can estimate easily. The introduced algorithm is suitable for FPGA implementation, and the simulation result shows that it has high-accuracy.**

## I. INTRODUCTION

The frequency estimation of power system signals as prerequisite and foundation to calculate other parameters like the voltage amplitude and phase has attracted widely attention. In the literature, frequency-tracking methods can be divided into two categories: hardware phase-locked loop and software tracking method [1]. The software tracking approaches are favored for their flexibility and low cost. Because of the digital implementation convenience and good inhibitory effect of harmonic, discrete Fourier transform (DFT) is suitable for tracking frequency. However, when frequency is fluctuated around the normal value, non-integer sample points of per cycle leads to spectrum leakage problems [2] [3]. Thus, two different measures are adopted to solve these. One is from the magnitude spectrum point of view, like window functions and spectrum line interpolation [4]. These methods obtain good improvement effect at significant frequency fluctuate situations, but the implementation complexity is also increased. The other improvement is from the phase spectrum point of view. Actually, the offset of frequency also sensitive to the phase change. Therefore, this paper analysis the phase error of DFT in detail and establish the phase expression of main spectral line under spectrum leakage condition. Then by observing the phase relationship between two adjacent data windows, a phase-difference algorithm that based on DFT, called pd-DFT, is introduced to monitor the frequency.

## II. PHASE DIFFERENCE ALGORITHM

Initially, we consider a cosine signal of discrete series $x(n)$ that the frequency is fluctuated around the normal value. Its DFT $\dot{X}$ is also described as follows:

$$x(n) = \sqrt{2}X_m\cos[2\pi(1+\Delta\lambda)n/N+\varphi]\ , \tag{1}$$

$$\dot{X} = X_P e^{j\varphi} e^{j\pi\Delta\lambda\frac{N-1}{N}} + X_N e^{-j\varphi} e^{-j\pi(2+\Delta\lambda)\frac{N-1}{N}}, \tag{2}$$

$$X_P = \frac{\sqrt{2}}{N}X_m\sin(\pi\Delta\lambda)/\sin(\pi\lambda/N)\ , \tag{3}$$

$$X_N = \frac{\sqrt{2}}{N}X_m\sin(\pi\Delta\lambda)/\sin(\frac{2\pi}{N}+\frac{\pi\Delta\lambda}{N})\ , \tag{4}$$

where $X_m$, $\varphi$, $N$ and $\Delta\lambda$ are the amplitude, phase, sampling number, and frequency fluctuation ratio respectively. $X_P$ and $X_N$ are the magnitude spectrum values of positive and negative frequency. Focused on the relationship between the phase of DFT $\varphi'$ and the real phase $\varphi$ of signal as follows:

$$\varphi' \approx \varphi + \Delta\lambda\pi\frac{N-1}{N} + \frac{\Delta\lambda\pi}{N\sin(2\pi/N)}\sin(\frac{2\pi}{N}-2\varphi')\ . \tag{5}$$

Equation (5) reveals the relation between the frequency fluctuation ratio and phase difference. In addition, according to the phase shift characteristics of DFT, two adjacent data windows have phase difference of $2\pi$. Thus, the real phase of signal must satisfy the following relation:

$$\varphi_2 - \varphi_1 = 2\pi(1+\Delta\lambda)\ , \tag{6}$$

where $\varphi_2$ and $\varphi_1$ are the real phases of adjacent data windows. Combined with the (5) and (6), the frequency fluctuation ratio $\Delta\lambda$ can be deduced as follows:

$$\Delta\lambda \approx \frac{\varphi_2' - \varphi_1' - 2\pi}{2\pi + \frac{\pi[\sin(\frac{2\pi}{N}-2\varphi_2') - \sin(\frac{2\pi}{N}-2\varphi_1')]}{N\sin(2\pi/N)}}\ . \tag{7}$$

Thus, according to $\Delta\lambda$ in (7) and the sampling frequency $f_s$, the real frequency can be obtained as follows:

$$f = (1+\Delta\lambda)\, f_s/N. \tag{8}$$

## III. SIMULATION RESULTS

The simulation of tracking the frequency of power system signal was conducted using MATLAB to verify the algorithm. The nominal frequency and the sampling frequency are set to 50Hz and 1600Hz, respectively. In order to be closer to real circumstance, Gaussian white noise and harmonic also considered. Then, the following part will give the simulation results about three aspects of pd-DFT performances.

### *A. Tracking Accuracy*

The single frequency signal that from 49.5 Hz to 50.5 Hz mixed with 50 dB signal-to-noise ratio was considered to test and verify the tracking accuracy. Fig. 1 shows the trend of pd-DFT error. In Fig. 1, we find the tracking error of pd-DFT is in order of $10^{-5}$Hz, which means it has high-accuracy. Simultaneously, we can also observe that as the frequency fluctuation increases, the tracking accuracy also gradually decreases which caused by approximate processing of the algorithm phase calculation. Therefore, if the sampling points are enough, the approximate processing is reasonable and the decrease trend of tracking accuracy will be relieved. Thus, the accuracy of pd-DFT can be improved by properly increasing the sampling frequency.

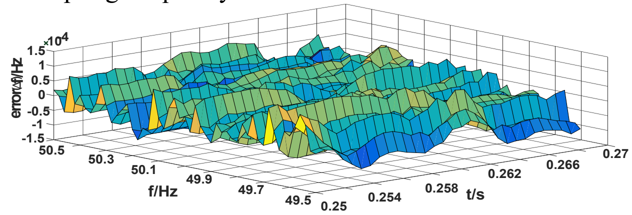



Fig. 1. Frequency tracking errors of pd-DFT.

### B. Reliability and Simplicity

To discuss reliability and simplicity of pd-DFT, the interpolated DFT (Ip-DFT) [5] and the all-phase DFT (ap-DFT) [6] are considered. Then, consider two conditions: small frequency fluctuation of 0.1Hz and large frequency fluctuations of 3.5Hz to conduct comparative analysis. Fig.2 shows the simulation results among different algorithms. As shown in Fig.2, simulation results reveal two main effects. Firstly, pd-DFT compared with ap-DFT has higher accuracy in the small fluctuation condition and the tracking error of Ip-DFT is not converged. This is because Ip-DFT uses the ratio between highest and second highest magnitudes of the DFT spectrum to estimate the deviation of normal frequency. However, the existence of noise distorts the spectrum relation between side-lobe and main-lobe, so the opposite interpolation direction may occur. Thus, the reliability of pd-DFT is higher than Ip-DFT and this is very important for a practical application. Secondly, the tracking error of ap-DFT is the minimum in the large fluctuation condition. However, its accuracy gain is obtained at the expense of increasing the processing data window length. Besides, the ap-DFT increases preprocessing process and its implementation need multiple multiplications and additions. These make the execution of pd-DFT simpler than ap-DFT. Thus, compared with the ap-DFT algorithm, pd-DFT algorithm has the advantage of simple implementation and real-time response.

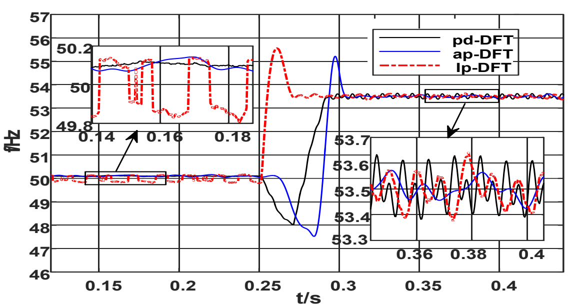



Fig. 2. Frequency tracking comparison of different algorithms.

### C. Inhibitory Effect on the Harmonics

As mentioned above, the signal in the actual power system generally is mixed with noise and harmonics, so the inhibitory effect on the harmonics and noise is a key metrics of measurement algorithm. In Table I, a series of tests about harmonics and noise have done to verify the pd-DFT's performance. In addition, the results in Table I are the average of 100 tests, so the accidental factors can exclude. In Table I, the presence of noise and harmonics has slight influence on the frequency tracking accuracy. Moreover, pd-DFT achieves the accuracy less than $3\times10^{-3}$Hz despite the existence of harmonics. Thus, the results show that pd-DFT meets the requirement of IEEE std.C37.118-2011 (< $5\times10^{-3}$ Hz).

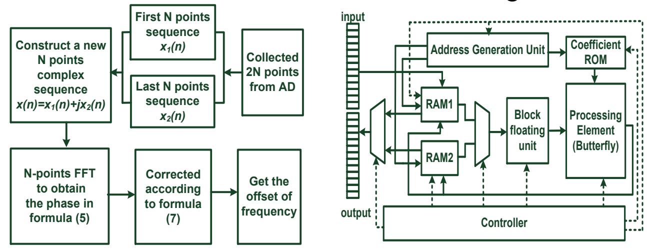
Table I. TRACKING ERROR UNDER HARMONIC AND NOISE EXISTENCE.

| SNR(dB) / THD (%) | 60 | 50 | 40 | 30 |
|---|---|---|---|---|
| 0 (pure) | $7.47\times10^{-5}$ | $2.37\times10^{-4}$ | $7.51\times10^{-4}$ | $2.37\times10^{-3}$ |
| 5 ($3^{th}$) | $1.17\times10^{-4}$ | $2.61\times10^{-4}$ | $7.59\times10^{-4}$ | $2.34\times10^{-3}$ |
| 9.43 ($3^{th}$,$5^{th}$) | $2.37\times10^{-4}$ | $3.72\times10^{-4}$ | $8.31\times10^{-4}$ | $2.33\times10^{-3}$ |
| 13.7 ($3^{th}$,$5^{th}$,$7^{th}$) | $4.43\times10^{-4}$ | $5.39\times10^{-4}$ | $9.74\times10^{-4}$ | $2.46\times10^{-3}$ |

* THD: total harmonic distortion. * SNR: signal to noise ratio.

## IV. IMPLEMENTATION

For implementing the pd-DFT, the key modules are the DFT of two adjacent data windows. However, the DFT can be replaced by the fast Fourier transform (FFT) in hardware implementation to obtain better real-time performance and low complexity. As shown in Fig. 3(a), we could construct an N-point complex sequence to achieve the FFT of two real sequences using the symmetry of FFT. Then, half of time can be saved to correct frequency. A block diagram of radix-2 FFT FPGA implementation is also shown in Fig. 3(b). It uses two dual-ports RAM to support the processing element pipelining operation and employs a block-floating unit to solve overflow problems. Experiment shows the processing speed can reach 5.3 µs, and maximum phase error is only 0.25%. In Table II, the synthesis result shows that this design takes up about 2% hardware resources and it has low hardware usage.




(a) (b)

Fig. 3. (a) Flow chart of pd-DFT and (b) block diagram of radix-2 FFT.

Table II. Hardware sources consumption.

| Device | Cyclone II EP2C35F672C6 |
|---|---|
| Total logic elements | 575 / 33,216 ( 2 % ) |
| Total combinational functions | 522 / 33,216 ( 2 % ) |
| Dedicated logic registers | 259 / 33,216 ( < 1 % ) |
| Total memory bits | 5,668 / 483,840 ( 1 % ) |
| Embedded Multiplier elements | 8 / 70 ( 11 % ) |

## V. CONCLUSION

In this paper, a phase difference algorithm based on DFT introduced to track the frequency of power system. The introduced pd-DFT algorithm is simple, reliable and has good inhibitory effect on the harmonics. The simulation results show it has high-accuracy. Simultaneously, FFT as the key module was designed based on FPGA. The implementation results show it has low hardware usage.

## REFERENCES


[1] W. Wang, Z. W. Zhang, *et al.*, "Tracking frequency based on all digital phase-locked loop in power system," *J. Power Electronics*, vol.44, pp. 89-91, Feb. 2010.

[2] D. Agrez, "Spectrum analysis of waveform digitizers by IDFT and leakage minimization," in *Proc. Instrum. Meas. Technol. Conf. (IMTC)*, vol.3, pp. 1717-1722, 2005.

[3] T. Radil and P. M. Ramos, "New spectrum leakage correction algorithm for frequency estimation of power system signals," *IEEE Trans. Instrum. Meas.*, vol. 58, no. 5, pp. 1670-1679, May 2009.

[4] B. Y. Qing *et al.*, "Approach for electrical harmonic analysis based on Nuttall window double-spectrum-line interpolation FFT," *Proceedings of the CSEE*, vol. 28, no. 25, pp.153-158, 2008.

[5] D. Belega and D. Petri, "Frequency estimation by two or three-point interpolated Fourier algorithms based on cosine windows," *J. Signal Processing*, vol.117, pp.115-125, 2015.

[6] J. Z. Liu, Z. X. Hou, and C. Y. Wang, "Windowed all-phase DFT modulated in time domain and its application in spectrum analysis," *Networking and Mobile Computing IEEE Press,* pp.1865-1868,2009.