Optimal Allocation of Photovoltaic Plants in Electric Power Distribution Networks

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***Abstract*—In this article, the problem of allocation of photovoltaic generators in a distribution network while losses are minimized considering network operating constraints is formulated and solved. Three formulations are considered. In order to take into account adequately the imbalance of distribution networks, a three-phase approach is considered. To solve the three formulations a stochastic algorithm based on an evolution strategy is used showing good results in minimizing technical losses. The results obtained illustrate the proposals and their practical use.**

***Index Terms*—Allocation, distributed, photovoltaic, evolutionary strategy, covariance**

I. INTRODUCTION

Renewable energy sources arise as an important tool to reduce emissions and to diversify the energy matrix as well, decreasing the use of fossil fuels. In the distributed generation they can be installed close to the major centers of consumption, thus helping to minimize the power losses and avoiding investments in electricity transmission.

Most renewable sources are intermittent, compared to conventional energy sources, so the investment becomes strategic and important for the future, in particular to meet global targets of reducing greenhouse gas emissions [1].

Among the various sources of renewable energy, photovoltaic sources (PV) appear with with technological maturity and great potential for economic exploitation. PV arrays are already being successfully used in isolated microgrids and integrated to the grid [2].

However, with the increase of PV power injection directly into the distribution network close to the load, in addition to the reduction in losses, beneficial and harmful effects are expected especially for buses located near the PV generators [3]. Insertion of PV generators in a power grid brings some complications that arise from the intermittent nature of primitive source added the uncertainty of the load [4]. In the case of electricity distribution networks, the imbalance between phases is another complication that must be considered.

Whereas the largest irradiation happens for a few hours and a few days in the year, the dimensioning of photovoltaic generators based on this condition may lead to less generation than expected for the year.

Other studies have proposed reducing losses in the power system by using distributed generation (DG). In [5] a genetic algorithm to scale the DG and minimize losses for single-line systems is used. In [6] a methodology based on sensitivity factor to find optimum active power injection by a generic DG that minimizes losses in the system is proposed. In [4] and [7], is analyzed the benefits of PV penetration on the grid, considering the variation of load and generation over time using the open source software OpenDSS. In [1] and [8] a great design is studied in order to achieve the optimal number of PV generation in an annual planning period.

In this paper, we have formulated and solved the problem of allocation of photovoltaic sources, so that losses are minimized by increasing the PV injection into the network, while network operating constraints are met. Three problem formulations are presented: the first one,named here as canonical and as described above; the second formulation takes into account a maximum value of investment available (in MWp) to be installed; the latter formulation promotes balanced geographical distribution of photovoltaic plants in the grid. Thus, the impact of passing clouds in PV generation is softened.

To consider properly the imbalance of distribution networks, a three-phase approach of network and loads is considered. Three optimization problems are formulated and solved using a stochastic algorithm based on Evolution Strategy with Covariance Matrix adaptation.

The main contributions of this paper are:

- The proposal of three allocation formulations of photovoltaic generators of practical interest for the industry;
- The problem is modeled in a horizon of one year of solar radiation;
- The realistic modeling of the network and the loads, taking into account their asymmetries.
- The optimization problem is solved using metaheuristic based on evolutionary strategies adapted for this type of application.

II. PROBLEM FORMULATION

The goal of this paper is the optimal allocation of photovoltaic power sources in three-phase distribution network

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while technical losses are minimized over a simulation horizon. The problem can be formulated as follows:

$$\min F = \sum_{t} \sum_{NL+NT} \sum_{p} P_{loss,p,t} h \tag{1}$$

Where,
P_{loss} is the active power losses $[kW]$;
NL is the number of transmission lines;
NT is the number of transformers of the distribution;
p is the phase index of the lines and transformers;
t is the time interval index;
h is the time interval $[hour]$, constant and equal to 1 hour.

In this first formulation, referred as canonical, the objective function in Equation 1 represents the sum of losses in all branches of the network at all time stages.

The active power losses in lines and transformers are calculated as follows:

$$P_{loss,p,t} = P_{km,p,t} + P_{mk,p,t} \tag{2}$$

Where,
k and m are nodes connected by phase p;
$P_{km,p,t}$ and $P_{mk,p,t}$ are the active power flow in kW from node k to node m at phase p in the time interval t.

The active power is calculated by power flow:

$$S^*_{km,p,t} = P_{km,p,t} - jQ_{km,p,t} = E^*_{k,p,t}.I_{km,p,t} \tag{3}$$

Where,
$S_{km,p,t}$ is the complex power flow in $[kVA]$;
$Q_{km,p,t}$ is the reactive power flow in $[kVAr]$;
$E_{k,p,t}$ is the complex voltage in node k at phase p;
$I_{km,p,t}$ is the complex current in branch k-m at phase p.

A. Operation constraints

1) Voltage: The bus voltage modules must be within specified limits defined by the following constraint:

$$V_{\min,k} < V_{p,k,t} < V_{\max,k} \tag{4}$$

Where,
$V_{p,k,t}$ is the complex voltage module of bus $[kV]$;
$V_{\min,k}$ and $V_{\max,k}$ are minimum and maximum limits.

2) Power flows: The power flows in a phase p must be within established limits described by the following expression:

$$P_{\min,km,p} < P_{km,p,t} < P_{\max,km,p} \tag{5}$$

3) Load balance: The objective function 1 is subject to the balance of the generation − load modeled through the three-phase load flow equations). In summary, the power balance can be described as follows:

$$P_{sub,t} + P_{PV,t} = P_{L,t} + P_{loss,t} \tag{6}$$

$$Q_{sub,t} = Q_{L,t} + Q_{loss,t} \tag{7}$$

Where,
$P_{sub,t}$ is the active power supplied by substation $[kW]$;
$Q_{sub,t}$ is the reactive power supplied by substation $[kVAr]$.

B. Minimum plant to be installed

Due to economic and operational issues, the installation of very small PV plants is not feasible, due to high maintenance and operating costs compared to it generation capacity.

$$P_{PV,p,k} > P_{\min,PV} \tag{8}$$

C. Load modeling

The load is modeled as constant power considering balanced and unbalanced single-phase and three-phase as well. Therefore for each phase:

$$S_{L,p,t} = V_{L,p,t}.I^*_{L,p,t} \tag{9}$$

Where, L is the index that identifies the load.

The loads are a monthly average consumption based on residential consumer profile [9].

D. Model of photovoltaic generators

The model of the PV generator is composed by the PV panels and the inverter. In this work it is assumed the inverter always works at maximum power point tracking [7][4]. The active power injected by the PV array to the grid is defined as follows:

$$P_{PV,p,k,t} = P_{mpp,p,k}.Ir_{PV,p,k,t}.F_{temp,p,k}.Eff_{p,k} \tag{10}$$

Where,
$P_{mpp,p,k}$ is the rated power of PV panel at the point of maximum power[1] $[kW]$;
$Ir_{PV,p,k,t}$ is the solar irradiation in $[kW/m]$;
$F_{temp,p,k}$ is the temperature factor of PV panel[2];
$Ephf_{p,k}$ is the inverter efficiency.

E. Control variables

The control variables are the photovoltaic to be installed per phase (kW), while losses are minimized. In principle, all buses are candidates, with the exception of the substation bus. This model allows the incremental installation of PV sources in modes single-phase, two-phase and three-phase.

The better performance of PV generation occurs in the smaller time interval h, but the computational burden is quite heavy for simulations in annual horizons analysis. On the other hand, a single average irradiation considering only an average value for the entire month is not sufficient to represent the time variation of the load voltage, which can lead to poor sizing of PV plants.

F. Solar irradiation and temperature

We used the monthly average irradiation to represent the irradiation of a year of planning. Therefore, each day represents one month both for irradiation and temperature. Figure 1 illustrates the monthly average solar radiation and daily temperature in January[3].

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[1] At 25 řC, when subjected to solar irradiation of $1kW/m^2$

[2] It associates the temperature on the panel surface with the power output. Thus, the higher temperature, the lower the power to be injected to the network

[3] Based on solar irradiation of $1000W/m^2$

The simulation of twelve months is grouped into continuous average 12 days, or 288 hours of simulation, involving the use of 12 pairs similar to those found in Figure 1, one for each month of the year. For reasons of space, the eleven remaining pairs were omitted, but are available in [10].

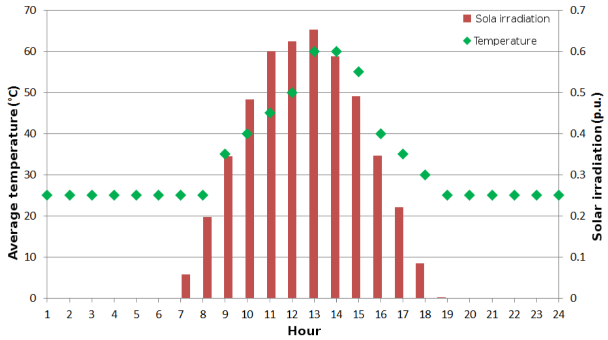

Fig. 1. The monthly average solar radiation and daily temperature in January

1) Formulation 2: In the previous formulation, called canonical, the allocation of photovoltaic injection is unlimited, depending exclusively on the configuration that provides minimal loss. This formulation is useful because it provides the largest technical amount of PV generation that could be installed on a network to reduce losses to a minimum. However, this solution may not be economically interesting due to the capacity factor of photovoltaic plants and idleness of feeder substations in times of high supply of power from the PV systems.

On the other hand, in Formulation 2 the maximum capacity allowed to be installed is a given parameter. Its usefulness is associated with the common practice of companies and regulators in planning incremental investments over the years. The maximum capacity allowed is described in kWp, and added to the canonical formulation as a constraint:

$$\sum_{N} P_{PV,p,k} \leq P_{\max,PV} \tag{11}$$

Where,
$P_{PV,p,k}$ is the rated power of the set of PV panels $[kW]$;
$P_{\max,PV}$ is the total kW allowed to be installed $[kW]$;
N is the number of PV generators in the network.

2) Formulation 3: This formulation promotes balanced allocation of photovoltaic sources on the network. It is expected that the allocated generators are not widely differing capabilities within geographically distributed. This formulation is useful in reducing the impact of shading caused by passing clouds on the photovoltaic arrays and/or in reducing the dependence on local climatological factors.

This action performed by selection process of the evolutionary algorithm. Thus selection process promotes solutions with more homogeneous genes. A measure of homogeneity is given by the following expression:

$$\Delta P_{PV} = \max\left(P_{PV,p,k}\right) - \min\left(P_{PV,p,k}\right) \tag{12}$$

Note that the constraint 8 is considered in all formulations.

III. Resolution technique

In this paper, minimizing losses involves using an evolutionary algorithm based on Evolution Strategy (ES). The ES is a version known by CMA-ES (Covariance Matrix Adaptation - Evolution Strategy), which has the characteristic of each new solution candidate is improved using a covariance matrix based on previous generations.

This version makes use of the weighted intermediate recombination, evolutionary adaptation of global standard deviation and covariance matrix updated with $(rank-one)$ and $(rank-\mu)$ information[11]. The individual of next generation is given by:

$$x_k^{(z+1)} = m^{(z)} + \sigma^{(z)}.\mathcal{N}\left(0, C^{(z)}\right) \tag{13}$$

Where, $x_k^{(z)}$ is the vectorial individual k^{th} and $m^{(z)}$ is the mean individual, both at generation z. The population has λ individuals.

All definitions of values of constants, parameters, size of population of parents and/or offsprings and other features of evolutionary algorithm are as described in [12][11].

A. Fitness function

The form of the fitness function is given as following:

$$\begin{aligned} F_{fit} = {} & \frac{F}{max(F_{fit})} + pen(\Sigma(\Delta V))^2 \\ & + pen(\Sigma(\Delta PV))^2 + pen(\Delta P_{PV})^2 \end{aligned} \tag{14}$$

Where,
F are the losses during the period of study, defined in Equation 1; $pen(\Sigma(\Delta V))^2$ is the penalty for the sum of violations of constraints 4; $pen(\Sigma(\Delta PV))^2$ represents constraint 11 in a penalized form; $pen(\Delta P_{PV})^2$ represents constraint 12 in a penalized form in order to promote homogeneity among sizing of PV generators; $max(F_{fit})$ is the maximum fitness value obtained in a complete run of evolutionary algorithm. This normalization avoid using weights in the other penalty terms.

In Formulation 1, $pen(\Sigma(\Delta PV))^2$ and $pen(\Delta P_{PV})^2$ are disregarded. The term $pen(\Delta P_{PV})^2$ is considered only in Formulation 3.

B. Power flow

Three-phase distribution network model is considered in order to take account of phase imbalances. The three-phase power load flow is solved by the open source software OpenDSS [7]. Once a candidate solution is generated, a power flow calculation is carried out in order to check the load balancing and operational limits.

C. Optimization flowchart

Figure 2 illustrates the optimization algorithm proposed in this paper. The optimization block interacts with the power flow in the validation of candidate solutions. Step called "refining" is responsible for removing solutions that lead to values to be allocated less than the minimum allowed.

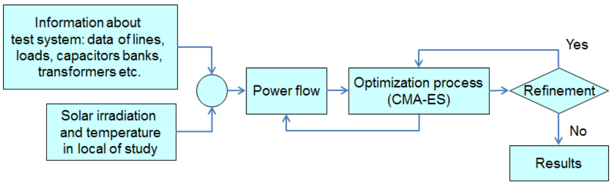

Fig. 2. Optimization process flowchart

IV. Case study

The IEEE 13-bus test system [13] was used. It is an 4.16 kV unbalanced three-phase distribution network with single-phase and three-phase loads. All lines and transformers are represented by models proposed in [13]. The original system data can be found in [14]. In our modeling, we assume all loads as constant active and reactive powers.

In this work, the test system was located in the geographical coordinates of 2° 31’ 48” S, 44° 18’ 10” W [15], in Brazil’s Northeastern region. It was considered a planning horizon of one year, divided into 12 months, with each month has its average solar irradiation in this location, as shown in Figure 3. For the temperature on the panel surface, it was considered a single monthly average throughout the year, as Figure 1.

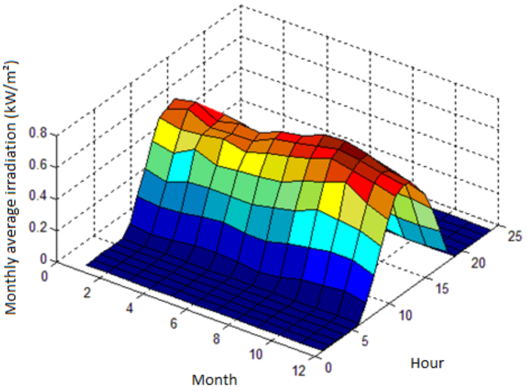

Fig. 3. Average daily solar radiation throughout the year

The hourly consumption profile of loads follows a pattern of residential consumption. Loads are presented by a typical curve of hourly consumption in pu [9], where the peak values are power specified in the IEEE 13-bus file. The curves are derived from a shift of the original consumption curve. For example, Figure 4 illustrates the average daily three phase load profile for a day of January in bus 634. The same curve of average daily consumption for the 12 months of the year was considered.

The nominal amount of PV generators to be distributed and installed on the system can be determined by using one of formulations described in previous sections. Initially, there are several buses candidates for installation of panels. For minimum criteria of economy of scale for installation, operation and maintenance, a minimum kW to be installed in a given bus was established. The bus voltages should remain within their limits [16], being minimum of 0.93 pu and maximum of 1.05 pu.

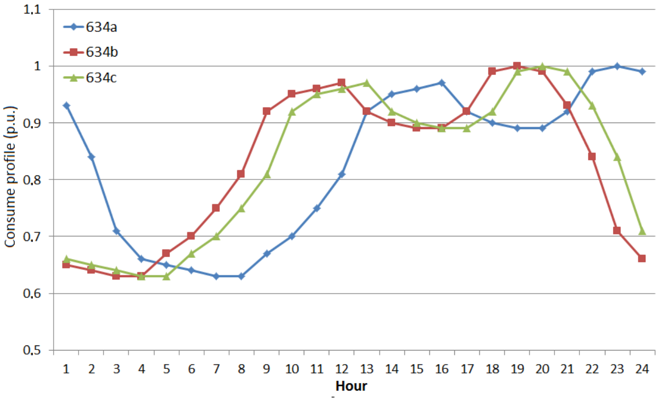

Fig. 4. Average monthly consumption in bus 634

The inverter efficiency curves and the power-temperature curves are the same provided in [7]. The results are shown in amount of energy and organized on a monthly basis The number of days for each month is given below:

TABLE I
Number of days for each month

Jan	Feb	Mar	Apr	Mai	Jun	Jul	Aug	Sep	Oct	Nov	Dec
31	28	31	30	31	30	31	31	30	31	30	31

V. Results

In this section the results achieved by the optimization process are presented. All bus voltages are within their limits established.

A. Base case

In Table II the consumption of the substation and technical losses in the lines and transformers are summarized. Notice that the monthly losses are constant, because the consumption profile is the same for each month. All the energy consumed by loads is supplied by the substation.

TABLE II
Consumption and losses in the base case

Month	Generation SS (MWh)	Losses (MWh) Transf	Losses (MWh) Line	Load (MWh)	Losses (%)
1	2,126.39	3.04	52.18	2,071.16	2.60%
2	1,920.61	2.74	47.13	1,870.73	2.60%
3	2,126.39	3.04	52.18	2,071.16	2.60%
4	2,057.79	2.94	50.5	2,004.35	2.60%
5	2,126.39	3.04	52.18	2,071.16	2.60%
6	2,057.79	2.94	50.5	2,004.35	2.60%
7	2,126.39	3.04	52.18	2,071.16	2.60%
8	2,126.39	3.04	52.18	2,071.16	2.60%
9	2,057.79	2.94	50.5	2,004.35	2.60%
10	2,126.39	3.04	52.18	2,071.16	2.60%
11	2,057.79	2.94	50.5	2,004.35	2.60%
12	2,126.39	304	52.18	2,071.16	2.60%
Year	**25,036.48**	**35.77**	**614.42**	**24,386.26**	**2.60%**

B. Formulation 1

Results with Canonical formulation are presented in Table III, where the energy consumed by the load is complemented by PV generation reducing the de energy provided by the grid. Losses were reduced, however it varied during the year due to the behavior of solar irradiation. Notice that the first few months have more losses than the latter, because for these coordinates the latter months of year have higher daily solar radiation.

TABLE III
Formulation 1: Generation, consumption and losses.

Month	**Generation (MWh)**		**Losses (MWh)**		**Load**	**Losses**
	SS	**PV**	**Transf**	**Line**	**(MWh)**	**(%)**
1	1,421.15	688.35	2.97	35.36	2,071.16	1.82%
2	1,266.22	639.01	2.68	31.82	1,870.73	1.81%
3	1,417.38	692.06	2.97	35.31	2,071.16	1.81%
4	1,343.05	697.56	2.88	33.38	2,004.35	1.78%
5	1,418.99	690.63	2.97	35.49	2,071.16	1.82%
6	1,315.60	725.06	2.87	33.44	2,004.35	1.78%
7	1,312.56	796.02	2.97	34.44	2,071.17	1.77%
8	1,233.35	875.08	2.97	34.29	2,071.16	1.77%
9	1,169.39	871.2	2.88	33.36	2,004.35	1.78%
10	1,231.29	877.33	2.97	34.48	2,071.17	1.78%
11	1,231.47	808.77	2.87	33.01	2,004.35	1.76%
12	1,328.83	779.45	2.97	34.15	2,071.17	1.76%
Year	**15,689.28**	**9,140.51**	**34.97**	**408.52**	**24,386.30**	**1.79%**

The result of the optimization process is presented in Table IV. PV generators with single-phase and three-phase have been installed and the phase is specified in column 2 of the table. The minimum power rating to be installing was 500 kWp.

Losses were reduced compared to the base case and the total kW installed was 16,004.26 kWp.

TABLE IV
Formulation 1, PV power ratings installed

Bus	**Phase**	**Rated power (kWp)**
633	3	730.18
645	1 - b	1,232.39
646	1 - b	2,786.44
692	3	1,511.84
675	3	1,488.72
684	1 - a	3,274.11
684	1 - c	1,349.37
611	1 - c	2,354.41
652	1 - a	1,276.81
Total		**16,004.26**

C. Formulation 2

This formulation is based on the canonical formulation but adding an upper limit of 5,000 kWp for the total installed capacity. That is, the sum of the individual PV units installed can not be greater than this limit.

From results a readjustment of the rate power in relation to Case 1 is observed. Losses were reduced in relation to base case, but in lesser percentage in comparison with case 1. The optimal solution is presented in Table VI, with the majority of the PV generators being three-phase and the major part was installed on the bus 671, which is located in the main feeder.

TABLE V
Formulation 2: Generation, consumption and losses.

Month	**Generation (MWh)**		**Losses (MWh)**		**Load**	**Losses**
	SS	**PV**	**Transf**	**Line**	**(MWh)**	**(%)**
1	1,803.35	313.83	2.99	43.04	2,071.16	2.17%
2	1,621.09	291	2.7	38.67	1,870.73	2.16%
3	1,801.87	315.29	2.99	43.01	2,071.16	2.17%
4	1,730.50	318.03	2.89	41.28	2,004.35	2.16%
5	1,802.68	314.59	2.99	43.12	2,071.16	2.18%
6	1,717.91	330.37	2.89	41.04	2,004.35	2.14%
7	1,753.78	362.33	2.98	41.97	2,071.16	2.12%
8	1,717.08	398.27	2.98	41.21	2,071.16	2.09%
9	1,650.55	396.4	2.88	39.72	2,004.35	2.08%
10	1,716.34	399.2	2.98	41.39	2,071.17	2.10%
11	1,679.25	368.31	2.89	40.32	2,004.35	2.11%
12	1,761.22	355.01	2.98	42.08	2,071.16	2.13%
Year	**20,755.62**	**4,162.63**	**35.12**	**496.85**	**24,386.28**	**2.13%**

TABLE VI
Formulation 2, PV power ratings installed

Bus	**Phase**	**Rated power (kWp)**
671	3	2,977.47
675	3	1,522.51
684	1 - a	500.02
Total		**5,000.00**

D. Formulation 3

This formulation minimizes losses but promoting a homogeneous geographical allocation over network. The results are summarized in Table VII. The minimum power to be installed was set to 300kWp and the total power installed should not exceed 5,000 kWp. The results reflect well the sense of objective of this formulation while all operational constraints are satisfied. In fact, there was a reduction of losses in relation to the base case, and good distribution of installed capacity was obtained, especially in relation to the formulation 2. The PV generators have relatively small difference among them in terms of kWp.

TABLE VII
Formulation 3: Generation, consumption and losses.

Month	**Generation (MWh)**		**Losses (MWh)**		**Load**	**Losses**
	SS	**PV**	**Transf**	**Line**	**(MWh)**	**(%)**
1	1,651.31	463.19	2.98	40.36	2,071.16	2.05%
2	1,480.21	429.47	2.69	36.26	1,870.73	2.04%
3	1,649.15	465.33	2.98	40.33	2,071.16	2.05%
4	1,576.42	469.38	2.88	38.57	2,004,35	2.03%
5	1,650.36	464.31	2.99	40.52	2,071.17	2.06%
6	1,557.97	487.61	2.88	38.35	2,004.35	2.02%
7	1,578.53	534.83	2.98	39.22	2,071.16	2.00%
8	1,524.61	587.98	2.97	38.45	2,071.16	1.96%
9	1,459.05	585.27	2.88	37.09	2,004.35	1.95%
10	1,523.32	589.38	2.97	38.57	2,071,16	1.97%
11	1,501.08	543.7	2.88	37.54	2,004.35	1.98%
12	1,589.45	523.98	2.98	39.29	2,071.16	2.00%
Year	**18,741.48**	**6,144.42**	**35.06**	**464.55**	**24,386.29**	**2.01%**

TABLE VIII
Formulation 3: power ratings installed

Bus	**Phases**	**Rated power (kWp)**
633	3	853.28
632	3	318.77
670	3	524.6
680	3	881.35
645	1 - c	480.89
692	3	872.24
684	1 - a	668.75
611	1 - c	400.11
Total		**5,000.00**

E. Discussion

Table IX summarizes a comparison among the formulations concerning to kWp installed and losses. Note that the best solution in terms of loss reduction is achieved with the Formulation 1, where financial resources to install PV generators is unlimited. However, this approach may not be practical due to the high amount of financial resources needed to deploy the solution. Considering a installed-kW cost of 5.000 USD / kW , the required investment will be approximately 8 million dollars. On the other hand, the formulation 2 considers that investment resources are limited. Considering the 5.000 kW as limit, which corresponds to an investment of 25 million dollars, a significant loss reduction was achieved, but with less investment.

Interesting results were obtained with the Formulation 3. The evolutionary algorithm fractionated and geographically distributed PV units. A minimum limit for installing lesser than the previous formulations was allowed (300 kW) and good loss reduction was achieved, better than with the Formulation 2.

TABLE IX
COMPARATIVE AMONG FORMULATIONS.

Objective	**Rated PV Capacity (kWp)**	**Losses MWh**	**%**	**Reduction % off base case**
Base case	-	650.19	2.60	-
Form. 1	16,004.26	443.49	1.79	31.22%
Form. 2	5,000.00	531.97	2.13	18.08%
Form. 3	5,000.00	499.61	2.01	22.69%

In all formulations the installation of photovoltaic generation led to loss reduction. However, this is not just the only benefit. The installation of load near sources reduces the need for investments in distribution, and when combined with storage devices, contributes to the operation as isolated microgrids (autonomous islands) against contingencies in the grid.

Figure 5 shows the voltage profile obtained for all cases. All voltage are within operational limits. The best voltage profile was obtained with Formulation 1. All formulations provide better voltage profiles than the base case.

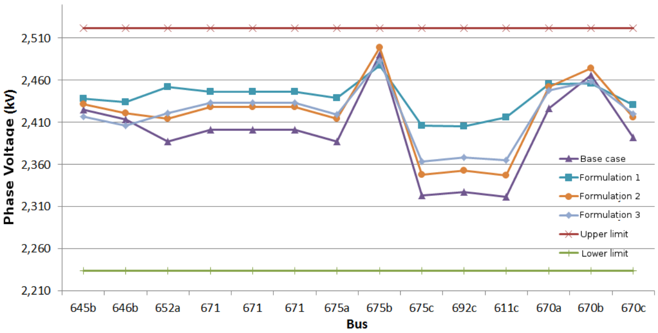

Fig. 5. Voltage profile.

VI. CONCLUSION

In this paper three formulations were proposed for the allocation of photovoltaic generators in a distribution system. Suggested formulations have practice applicability for both planners and investors. Other formulations are also possible depending on the specific scenario to be modeled. Significant results were obtained, illustrating suitably the philosophies of the proposed formulations as well as its practical utility.

The reduction of losses is significant throughout one year and represents gains for the operation and maintenance of the electrical system, and consequently provides financial return to amortize the capital invested.

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