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Economic Operation of Distribution Networks with Distributed Generation and Quality of Service

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***Abstract*—Uncertainties in distributed generation with renewable sources, and load fluctuations due to demand response, require the Distribution System Operator to have tools to coordinate supply and demand with an adequate quality of service. A cost optimization method is proposed for this purpose, based on two time-scales: a slow scale were topology changes and transformer taps are varied with their operation costs, and a fast time-scale based on an optimal power flow, in which costs of power supply and quality of service are incorporated. To solve the latter we rely on convex relaxations to the DistFlow model, which allow an efficient solution in radial topologies. We also develop a detailed model in DistFlow of wind and photovoltaic generators operating as subnets of the Distribution network. The method is demonstrated with data from a real network in Uruguay.**

***Index Terms*—Distribution system, Optimal power flow, Distributed generation, Convex optimization.**

I. Introduction

Electric power networks are going through a period of important changes, with the incorporation of new generation technologies, and greater intelligence on the consumer side. The benefit of such changes seems clear if one looks at the system from a wholesale power point of view (at the *Transmission* scale): in particular the desirable change in the energy matrix brought about by renewable sources, or the improved network regulation that load elasticity can provide.

Less is known, however, about the impact of such changes on the *Distribution* network, to which Distributed Generation (DG) sources or Demand Response (DR) mechanisms are often connected. The traditional view of these networks is in terms of relatively predictable quantities of energy flowing "downhill" on a distribution tree. This suggests a relatively rigid network configuration over time, where the occasional changes involve people in the decision loop. This vision will be seriously compromised in networks that are envisioned, with DG injecting highly variable power (wind, solar) at various points in the distribution tree, and more dynamic loads. In this environment, a predominantly passive network will not be capable of meeting quality of service objectives, for instance maintaining voltage within allowed limits. A more active management on the part of a Distribution System Operator (DSO) is called for, taking advantage of the newly available control capabilities: for example, reactive power on inverters connecting DG equipment. The DSO's actions must integrate technical aspects with economic objectives.

The objective of this work is to contribute with the development of mechanisms to allow the DSO to make such decisions with minimal overall cost.

II. Approach and Background

In Figure 1 we present the proposed method to tackle this problem, based on two time-scales:

1) *Fast Scale*: At the finer granularity, indexed by t, we solve an Optimal Power Flow (OPF), given the state of switches, transformers, capacitors, and non-interruptible loads, and including exogenous variables: wholesale prices $\pi(t)$, active/reactive demand $p^c(t)/q^c(t)$, and primary generation $p^g(t)$ with two components, photovoltaic generation $p^{pvg}(t)$ and wind generation $p^{wig}(t)$. This computation returns the optimal cost $J(t)$, the quantities $P^{tras}(t)/Q^{tras}(t)$ purchased in wholesale, the reactive generation $q^g(t)$, and bus voltages $v_i(t)$. The tractability of this optimization problem is based on recently developed convex relaxations, reviewed below.
2) *Slow Scale*: We use the coarser time index T to indicate stages at which we update discrete variables: switches $Sw(T)$ that modify the topology, transformer taps $Tr(T)$, capacitor taps $Cp(T)$, and the start times $start_j$ of non-interruptible loads. This daily planning is based on the OPF cost, fed with time series of prices, demands and primary generation. Since this search is combinatoric it is necessary to restrict it to a moderate number of key switches and transformers.

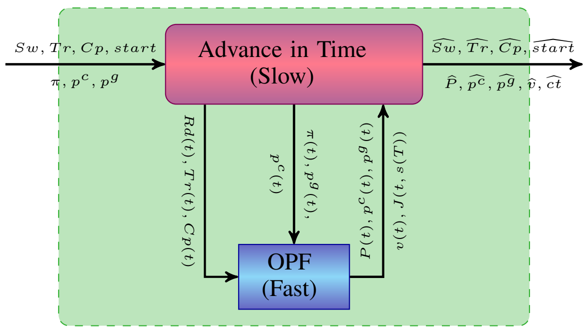

Fig. 1: Diagram of the solution method

A direct precursor to this work is [1], where the object of study is a distribution network with renewable energy, and the above two time-scales are discussed. In [1] the objective function penalizes line losses and energy overconsumption in resistive loads due to delivery voltages; in [2] this is expanded by introducing losses in inverters of the photovoltaic DG. [3] and [4] propose to optimize the network configuration taking as control variables the state of switches. [5] deals with optimizing the location of distributed storage, incorporating in the constraints a linear approximation of the capability curve of the DG, as well as stability limits of the system.

The aforementioned papers are based on the "DistFlow" model for power flow, proposed classically in [6]. The most significant recent advance [1], [7], [8], [9] is, however, the discovery of an adequate relaxation that yields a *convex optimization* problem, amenable to efficient solutions with standard algorithms. In particular for tree networks which are predominant in distribution, the relaxation of DistFlow is often exact, thus yielding a solution to the original problem.

As other related work we mention [10], where a market is developed to pay the DG for the reactive power they provide, through a multi-objective optimization; both synchronous (SG) and doubly fed induction generators (DFIG) for wind energy are considered.

The present paper is based on the proposal of [1], [2], with the following additional contributions. In the first place, the cost function is modified to reflect an economic criterion more relevant to the DSO, including the cost of supply and regulator penalties for voltage violations, as well as other quality of service costs. Secondly, we provide a detailed model of wind generators which, in contrast to [10], are assumed controlled by the DSO. Tests are carried out with data from a real medium voltage network in Uruguay, to which the wind generators were incorporated.

III. System Model

We present here the model that was used, beginning with the fast time-scale optimization, defined by the expressions (1-21). The corresponding explanations are given later in the section.

A. DistFlow and its Convex Relaxation

Distribution networks are typically operated in the form of a tree. For such networks it is convenient to use the DistFlow model of [6] for power flow, given by equations (11), (12), (13), where: v_i with $i \in N$ is the square of the voltage in each bus; p_i^c, q_i^c are the active and reactive powers consumed exogenously and p_i^{wi}, q_i^{wi}, p_i^{pv}, q_i^{pv} the generated ones for respectively wind and solar energy. Cap_i is the unit reactive energy of each shunt capacitor connected at node i, multiplied by an integer variable N_i^{cp} that is adjusted at the slow time-scale. Analogous definitions apply to the integer variables N_i^{Tr} and Tap_i (transformer taps). P_{ij} and Q_{ij} are the active and reactive powers on the line from i to j, while $l_{ij} = \frac{P_{ij}^2+Q_{ij}^2}{v_i}$ is the square of the current. Note however that in (16) the previous equation is replaced by an *inequality*: this relaxation makes the constraint fit into the class of *second order cone programming* (SOCP, see [11]). A relaxed form is also used for constraints (14), (15) on consumed active and reactive powers, to be described below. In the following we will also progressively describe the linear constraints that define the wholesale market, the regulatory penalties, as well as models for wind and solar generators that involve linear and second order cone constraints.

Overall, these relaxations yield a convex optimization problem, solvable with very efficient methods [12]. As long as the solution obtained satisfies the relaxed constraints with equality, we have a solution to the original problem. In our experience and that of others in the literature, for the tree networks under consideration the result is favorable in almost all cases.

$$\min \sum_i J_i\left(t, s\left(T\right)\right) \tag{1}$$

$$J_i\left(t, s\left(T\right)\right) = C_i^{tras}\left(t, s\left(T\right)\right) + C_i^{cs}\left(t, s\left(T\right)\right) \ \forall i \in N \tag{2}$$

$$C_i^{tras} = cp_i^{tras} + cq_i^{tras} \quad \forall i \in N^{tras} \tag{3}$$

$$C_i^{cs} = c_i^{\delta v} + c_i^f \quad \forall i \in N \tag{4}$$

$$cp_i^{tras} = \pi p_i^{tras} P_i^{tras} \quad \forall i \in N^{tras} \tag{5}$$

$$cq_i^{tras} \geq \pi q_i^{+,tras} Q_i^{tras};\ cq_i^{tras} \geq -\pi q_i^{-,tras} Q_i^{tras} \ \forall i \in N^{tras} \tag{6}$$

$$c_i^{\Delta v} \geq k_i^v m^v\left(v_i - \left(1+\delta^v\right)\right) \quad \forall i \in N \tag{7}$$

$$c_i^{\Delta v} \geq k_i^v m^v\left(v_i - \left(1-\delta^v\right)\right); \quad c_i^{\Delta v} \geq 0 \quad \forall i \in N \tag{8}$$

$$c_i^f = k_i^f m^f\left(\delta_i^f - 1\right); \quad c_i^f \geq 0 \quad \forall i \in N \tag{9}$$

$$\delta_i^f\left(s\left(T\right)\right) = \frac{1}{f_0} \sum_{jk \in \Omega_{li}(T)} f_{jk} L_{jk} \quad \forall i \in N \tag{10}$$

$$p_j^c - \left(p_j^{pv} + p_j^{wi}\right) = \sum_{i \in \pi(j)} \left(P_{ij} - r_{ij} l_{ij}\right) - \sum_{k \in \delta(j)} \left(P_{jk}\right) \quad \forall (i,j) \in E \tag{11}$$

$$q_j^c - \left(q_j^{pv} + q_j^{wi}\right) - Cap_j N_j^{cp}\left(T\right) v_j = \sum_{i \in \pi(j)} \left(Q_{ij} - x_{ij} l_{ij}\right) - \sum_{k \in \delta(j)} \left(Q_{jk}\right) \forall (i,j) \in E \tag{12}$$

$$v_j\left(t\right) = \left(1 + Tap_i N_i^{tr}\left(T\right)\right)^2 v_i - 2\left(r_{ij} P_{ij} + x_{ij} Q_{ij}\right) + \left(r_{ij}^2 + x_{ij}^2\right) l_{ij} \quad \forall (i,j) \in E \tag{13}$$

$$p_j^c \geq p_j^{cn}\left(1 - \alpha_j + \alpha_j v_j\right) \quad \forall j \in N \tag{14}$$

$$q_j^c \geq q_j^{cn}\left(1 - \alpha_j + \alpha_j v_j\right) \quad \forall j \in N \tag{15}$$

$$l_{ij} \geq \frac{P_{ij}^2 + Q_{ij}^2}{v_i} \quad \forall (i,j) \in E \tag{16}$$

$$l_{ij} \leq \bar{l}_{ij} \quad \forall (i,j) \in E \tag{17}$$

$$\underline{P}_j^{tras} \leq P_j^{tras} \leq \overline{P}_j^{tras} \quad \forall j \in N^{tras} \tag{18}$$

$$\underline{Q}_j^{tras} \leq Q_j^{tras} \leq \overline{Q}_j^{tras} \quad \forall j \in N^{tras} \tag{19}$$

$$P_j^{tras} = \sum_{(j,k) \in E} P_{jk} \quad \forall j \in N^{tras} \tag{20}$$

$$Q_j^{tras} = \sum_{(j,k) \in E} Q_{jk} \quad \forall j \in N^{tras} \tag{21}$$

B. Supply and Demand

In the interconnection buses N^{tras} to the transmission network, active power P_i^{tras} (5) and reactive (inductive or capacitive) power Q_i^{tras} (6) is exchanged, constrained by the technical limits (18), (19). The wholesale market defines the price πp_i^{tras} for buying/selling active power, and the prices $\pi q_i^{+,tras}$, $\pi q_i^{-,tras}$ for buying capacitive or inductive power, for each of the interconnection buses. The corresponding portion of the cost is specified in (3), (5) and (6).

DG of the different kinds (photovoltaic pv, wind wi) is considered to be under contract of the DSO, who can act directly on their control systems to optimally dispatch reactive power. This requires integrating the components of the generator in the DistFlow as a subnet of the distribution network, all the way to the power sources p_j^{pvg} and p_j^{wig}. The terms p_j^{pv} y p_j^{wi} present in (11) correspond to active power delivered at the connection point, once discounted the internal losses of generators, see Sections III-D , III-E.

To model different kinds of loads we follow [1]: $p_j^c(v_j) = p_j^{cn}(v_j)^{n_j/2}$ and $q_j^c(v_j) = q_j^{cn}(v_j)^{n_j/2}$, where p_j^{cn} and q_j^{cn} are active and reactive powers at nominal voltage, and $0 \leq n_j \leq 2$, for instance $n_j = 0$ for constant power, $n_j = 1$ for constant current, and $n_j = 2$ for constant impedance loads. Through the linear approximation $p_j^{cn}(v_j)^{n_j/2} \approx p_j^{cn}(1 + \frac{n_j}{2}(v_j - 1))$, and defining $\alpha_j := \frac{n_j}{2}$ we arrive at $p_j^c(v_j) \approx p_j^{cn}(1 - \alpha_j + \alpha_j v_j)$. Analogously $q_j^c(v_j) \approx q_j^{cn}(1 - \alpha_j + \alpha_j v_j)$. The relaxed version of these constraints appear in (14), (15).

C. Penalties on the Distribution System Operation

C_i^{cs} in (4) represents for each node, the penalties the DSO receives for deviations from the quality of service conditions. Voltage penalties c_i^v imposed by (7), (8) imply a tolerance δ^v, and have linear growth outside this range, proportional to loads in each node k_i^v. Penalties for *frequency of service interruptions* occur when this quantity exceeds a maximum tolerance f_0, and are also proportional to the consumed energy k_i^f (9). Its expected cost c_i^f is computed assuming a branch failure rate f_{jk} per unit length, where the protections are installed in the feeder heads. Therefore a failure in any branch causes the interruption of the entire tree; the expected number of failures in each node is compared to the allowed limit (10). c_i^f will depend on the state of the switches.

D. Model of Solar Photovoltaic Generators

A PV generator is modeled as a branch in the DistFlow in which we include active power generation p_j^{pv}, and both resistive and semiconductor losses in the inverter (22). The power variables ξ_j^{pv} (24) and s_j^{pv}(23) are relaxed as in [2]:

$$p_j^{pv} = p_j^{pvg} - \left(cv_j^{pv} s_j^{pv} + cr_j^{pv} \xi_j^{pv}\right); \tag{22}$$

$$s_j^{pv} \geq \sqrt{\left(p_j^{Pv}\right)^2 + \left(q_i^{pv}\right)^2};\ s_j^{pv} \leq \overline{s}_j^{pv}; \tag{23}$$

$$\xi_j^{pv} \geq \left(p_j^{pv}\right)^2 + \left(q_i^{pv}\right)^2. \tag{24}$$

E. Doubly Fed Induction Generator (DFIG) for Wind Power

Based on [13] we consider the DFIG as a subnet composed of three branches: stator je, converter jf and rotor or. Two branches depart from the interconnection point j, one towards the stator e and another towards the converter f, both of which exchange active and reactive power with the network (25). In je we model a generator of active power $p_j^{wig}/(1 - n_j)$ and reactive power q_e^{wig} at node e (26), (27). In or we have a "generator" of active power $-n_j p_j^{wig}(1 - n_j)$ and reactive power $n_j q_e^{wig}$ (30), (31), (32). Between e and r we have a "transformer" of ratio $(n_j)(N_{e,r})$ (37). Branch jf is represented by equations (28), (29); power consumed at f comes from the power balance in the converter system and corresponds to the consumption of converters and (36). On the other hand, converters transfer the reactive power Q_{or} required by the rotor and delivers q_f^g at f. Equations (33), (34) and (35) correspond to the conic relaxations of DistFlow. The capability curve will be determined by the component constraints $\overline{l}_{ij}$ and $\overline{s}_{ij}$, respectively windings and inverters (38).

In equation (39) we report an expression based on [16] for the mechanical power p_i^{wig} as a function of wind speed W. This implies a certain control of the angular velocity of the rotor as a function of wind speed; the angular velocity in turn will have impact on slip n_j of the induction machine. Space considerations preclude us from a detailed explanation of this model, we refer to [16] for details.

$$p_j^{wi} = -\left(P_{je} + P_{jf}\right);\quad q_j^{wi} = -\left(Q_{je} + Q_{jf}\right) \tag{25}$$

$$P_{je} = r_{je} l_{je} - \frac{p_j^{wig}}{(1 - n_j)};\quad Q_{je} = x_{je} l_{je} - q_e^{wig} \tag{26}$$

$$v_e = v_j - 2\left(r_{je} P_{je} + x_{je} Q_{je}\right) + \left(r_{je}^2 + x_{je}^2\right) l_{je} \tag{27}$$

$$P_{jf} = r_{jf} l_{jf} + p_f^c;\quad Q_{jf} = x_{jf} l_{jf} - q_f^{wig} \tag{28}$$

$$v_f = v_j - 2(r_{jf} P_{jf} + x_{jf} Q_{jf}) + (r_{jf}^2 + x_{jf}^2) l_{jf} \tag{29}$$

$$P_{or} = r_{or} l_{or} - \left(\frac{n_j}{(1 - n_j)} p_j^{wig}\right) \tag{30}$$

$$Q_{or} = n_j x_{or} l_{or} - \left(n_j q_e^{wig}\right) \tag{31}$$

$$v_r = v_o - 2(r_{or} P_{or} + n_j x_{or} Q_{or}) + (r_{or}^2 + n_j x_{or}^2) l_{or} \tag{32}$$

$$l_{je} \geq \frac{P_{je}^2 + Q_{je}^2}{v_j};\ l_{jf} \geq \frac{P_{jf}^2 + Q_{jf}^2}{v_j};\ l_{or} \geq \frac{P_{or}^2 + Q_{or}^2}{v_o} \tag{33}$$

$$s_f \geq \sqrt{\left(p_f^c\right)^2 + \left(q_f^c\right)^2};\ \xi_f \geq \left(p_f^c\right)^2 + \left(q_f^c\right)^2 \tag{34}$$

$$s_r \geq \sqrt{\left(P_{or}\right)^2 + \left(Q_{or}\right)^2};\ \xi_r \geq \left(P_{or}\right)^2 + \left(Q_{or}\right)^2 \tag{35}$$

$$p_f^c = P_{or} + \left(cv_r s_r + cr_r \xi_r\right) + \left(cv_f s_f + cr_f \xi_f\right) \tag{36}$$

$$v_r = \left(n_j\right)^2 \left(N_{e,r}\right)^2 v_e \tag{37}$$

$$\overline{l}_{je} \geq l_{je};\ \overline{l}_{jf} \geq l_{jf};\ \overline{l}_r \geq l_{or};\ \overline{s}_f \geq s_f;\ \overline{s}_r \geq s_r \tag{38}$$

$$p_j^{wig} = \frac{1}{2} \rho \pi R^2 C_p\left(\lambda \beta\right) W^3. \tag{39}$$

In this paper the DSO has the capability of deciding, in accordance to a global optimization criterion, how much reactive power to request from the stator and the rotor in each of the connected DFIGs.

F. Slow time-scale modeling and optimization

The slow time-scale optimization is formulated as in [1], by means of a temporal variable T which indexes the "state" $s(T)$ of switches, transformers, capacitors, and the dispatch of non-interruptible loads.

1) Switching costs: Our optimization includes penalties for loss of useful life due to frequent operation of the equipment. Let $\Delta N_i^K(T) = \left|N_i^K(s(T)) - N_i^K(s(T-1))\right|$ denote the change in a switch, transformer or capacitor K. If Cm_i^K are the maintenance costs, and Nm_i^K the maximum number of operations [14], the prorated cost between $T-1$ and T is

$$C^S_{T-1,T} = \sum_{K=Sw,Tr,C_p} \sum_i \frac{Cm_i^K}{Nm_i^K} \Delta N_i^K(T). \qquad (40)$$

2) Non-interruptible loads: We consider here loads that can be scheduled by the DSO, but are non-interruptible (for instance a certain industrial process). Scheduling is specified by a binary variable $on_j(T)$ at the slow time scale, that is goes from 0 to 1 at the start time denoted by $start_j$, and back to 0 at $T = start_j + D_j$; here D_j is the process duration.

Within the operating window, three time series are specified at the fast time-scale: (a) nominal active power demanded $\{p_j^{cn}(t)\}$; (b) power factor $\{\Phi_j(t)\}$, and (c) fraction of power at constant impedance $\{\alpha_j(t)\}$. For buses with non-interruptible loads, equation (14) is replaced by $p_j^c \geq (1-\alpha_j+\alpha_j v_j) p_j^{cn} on_j(T)$, and (15) by $q_j^c \geq (1-\alpha_j+\alpha_j v_j) p_j^{cn} \tan(\Phi_j^c) on_j(T)$.

Preference for a certain interval of operation is expressed by a cost $C_j^{nl}(T)$ that takes the value c_j^{nl} at the preferred times $T \in [\underline{H}, \overline{H}]$, increased by a factor k_j^{nl} outside this interval.

3) Slow-scale optimization: The overall optimization objective includes the cost of the fast scale, the switching costs and non-interruptible load penalties across all times and buses:

$$\min \sum_{T=1}^{M} \left\{ \sum_{t \in T} J(t, s(T)) + C^S_{T-1,T} + \sum_j on_j(T) C_j^{nl}(T) \right\}.$$

IV. Implementation

The optimization algorithms for both time-scales were implemented in Matlab. For the fast time-scale the convex optimization package CVX [12] was used. The slow time-scale is solved by dynamic programming, taking as states the integer variables (network switches, transformer and capacitor taps, active non-interruptible loads) and as cost for each configuration the optimal value of the fast time-scale cost at that state. Since the combinations of discrete variables are exponential in number, there are limitations as to how many of these can be optimized over.

For the dynamic programming we must take special care with non-interruptible loads, because once a load j is dispatched the variable $on_j(T)$ must stay on until the duration is completed. While $on_j(T)$ is on, only the remaining discrete variables N_i^{tr}, N_i^{cp} y Sw can change.

To find the minimum cost under these circumstances we proceed as follows. First, for each instant t of the fast time-scale we evaluate the cost of all possible states (N_i^{tr}, N_i^{cp}, Sw and on_j). Then for each choice $(start_1, ..., start_J)$ of the tuple of start times for the J non-interruptible loads, we carry out the optimization over the remaining variables (i.e. in a relevant portion of the state space).

For example in Fig. 2 we have loads C_1, C_2 with $start_1 = T_1$ and $start_2 = T_2$. In the state matrix we identify the subset of allowable states (marked in green) that correspond to the tuple (T_1, T_2) so as to compute the values of $N_i^{tr}(T)$, $N_i^{cp}(T)$ and $Sw(T)$ that minimize the cost for those load start times. This leads to the tuple $(start_1^*, ..., start_J^*)$ that minimizes the total cost.

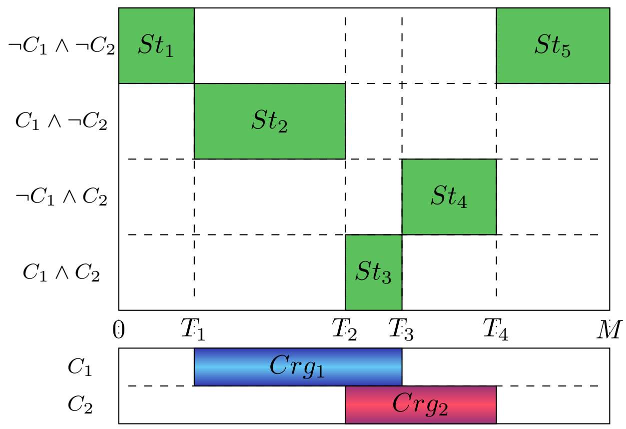

Fig. 2: Non-interruptible loads.
The state-space for dynamic programming (above) is partitioned in accordance to the load dispatch (below). The best trajectory is found in the subset St_i.

V. Case Study

For the analysis we consider a medium-voltage network in the suburban region of Montevideo, its main features depicted in Fig. 3. At the top we have the connection to Transmission, through a 150KV/30KV transformer that allows regulation under load. The sub-transmission portion feeds two 30kV/6kV stations, which are two medium voltage feeders, respectively designed for the rural and urban portions of the network. The rural portion, depicted on the left and including the bottom branch, is a long line supported on wooden posts, feeding an area of small farms, with a high failure rate. The typical configuration is with switch $sw2$ closed, and $sw1$ open, thus separating this rural network from the urban portion, depicted on the right, which corresponds to a subterranean cable, and branches of low failure rate, feeding residential, commercial and industrial loads. The alternate position of the switches is viewed as a backup option for contingencies.

The low voltage portion of the network is represented in our model by incorporating the aggregate loads at the level of the medium voltage buses; the figure indicates the number of substations involved at each point. All in all we have a total of 110 nodes (clients and medium/low voltage substations). We also highlight the nodes which feed two non-interruptible industrial loads of 1.5MVA and 1MVA with respective durations of 6hs and 8hs. The preferred time range of operation is between 7 AM and 10PM, outside this range labor costs triple. Our wind DFIG of 1.5MVA is included close to these loads. The sub-transmission and transmission networks were modeled in the power flow through their Thévenin equivalent.

The wholesale price of active power increases by 30% at 7 PM. Wind speeds and loads are taken from historical records of the peak load day in 2013. For modeling loads we considered two scenarios: one where all loads are of constant power, another where $\alpha = 40\%$ are of constant impedance, the remainder of constant power.

The parameters of the quality of service costs C^{cs} were adjusted to reflect as faithfully as possible the existing rules in the Uruguayan Distribution market [15]. For generator parameters we follow [16], whereas the data for load, network and wind were provided by UTE.

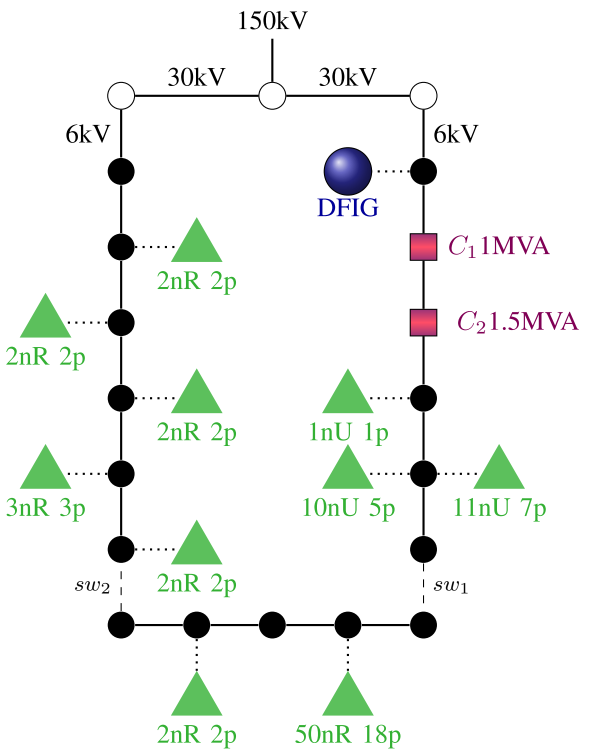

Fig. 3: Network example
Regular buses in black, in red those where non-interruptible loads are connected, in white transformers. Switches are dashed lines. In blue the DFIG wind source. In green the connected subnets, each labeled by XnY Zp where X is number of buses, Y the type (rural or urban) and Z the branch depth.

Results

In both scenarios (Fig. 5 and Fig. 6) the optimizer modifies the network topology from the aforementioned current practice, shifting load from the rural feeder to the urban one; given that the former has a higher failure rate, this implies accepting the risk of larger expected costs for frequency of interruption of service.

In both cases, cost signals of industrial loads overcome the tariff incentives, making them remain active after 7PM when the wholesale price goes up. In particular the 1.5MVA load starts at 7AM, then after the typical mid-day "wind valley" the 1MVA load starts, staying on until after 7PM. Reactive power in wind generators is always delivered by the stator.

For the case where a fraction of loads has constant impedance (Figs. 4 and 5), the transformer tap is kept at minimum setting, voltage is regulated through reactive power. As wind goes down, wholesale purchases increase to cover demand, and reactive generation increases to avoid voltage penalties. After the entrance of the second industrial load, active power generation increases but so does consumption, so power drawn from transmission stays above the 2MVA range, and to avoid under-voltage penalties, so does reactive generation.

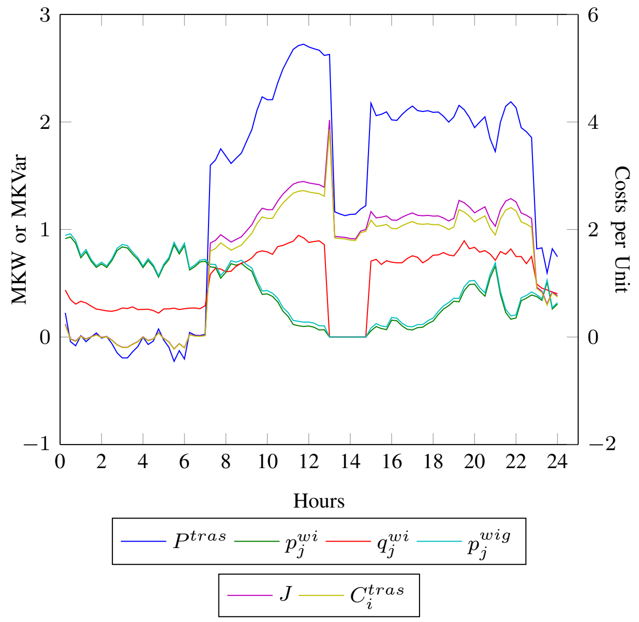

Fig. 4: Fraction of constant impedance loads
Power and costs

Fig. 6 shows the discrete variable choices for the case of constant power loads[1]. Here, as load increases the system chooses to operate the transformer taps. The tap starts at minimum ratio but it goes up jointly with the first industrial load, to avoid penalties for low voltage. Although reactive generation increases, it does not suffice to control voltage, so when active generation gets close to zero the tap is modified again. Once the "valley" of wind generation is past, the tap gradually moves back to the minimum transformation point.

[1]The graphs of power and cost are very similar to the other scenario.

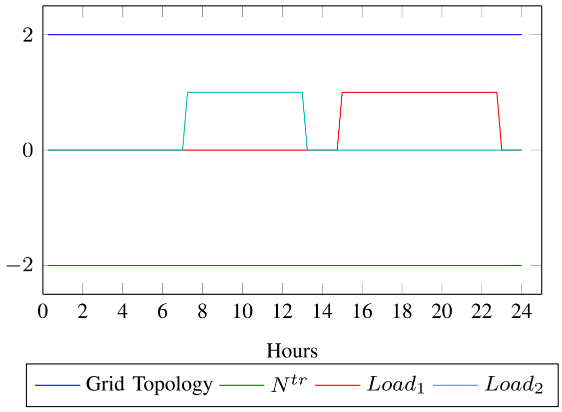

Fig. 5: Fraction of constant impedance loads
Integer variables

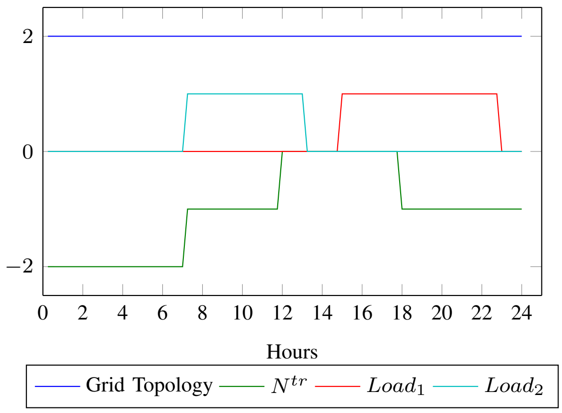

Fig. 6: Constant power loads
Integer variables

VI. Conclusions

We have demonstrated a methodology to find the optimal operation of a distribution network, based on the convex relaxation to OPF in its DistFlow formulation, together with a slow-scale programming of discrete decision variables. Our main contributions to the previous work of [1], [2] are: (i) a cost function relevant to the economics of a distributor, combining wholesale energy costs, regulatory and quality of service penalties, and the cost preferences of certain dispatchable industrial loads; (ii) a model of a DFIG for wind power based on the same optimization framework; (iii) the inclusion of both constant power and impedance loads; and (iv) a test on a real suburban network owned by UTE in Uruguay, with the addition of a wind power source and the industrial loads.

As a general remark on the methodology, we note that in all cases tried the OPF relaxations were verified with equality, including those of the DFIG, lending credence to the potential of this approach to tractably compute complex scenarios.

Regarding the conclusions of the experimental test, we note that results were not always in line with the traditional operation of this network, or with commonly held assumptions on appropriate demand response. In particular, the network topology was altered, and industrial loads did not entirely move to the region of low wholesale prices. Quality of service costs have a decisive impact on the operation strategy, with the DFIG playing a significant role supplying reactive power.

While some of the above observations may be anecdotal to the specific scenario and economic valuations we used, the more general message is that economic operation of "smart" distribution networks is by no means a straightforward task, but there are new optimization tools which offer promise in supporting future DSOs in making rational decisions.

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