Math & Crafts, Educational Activities: Ancient Math Methods, Future Directions.

Ernesto Vega Janica
IEEE Standards Association, e.vegajanica@ieee.org

***Abstract* - This paper summarizes multiple numerical systems, math concepts and historical references, and deeply focus on a hands-on exercise where children, parents and educators participate. The main goal of the hands-on experience is to engage our communities around Math, Science and Culture. Kids and adults can have a direct experience with tools, numbers, cultural historical references, and more. This combination of theory and practice should help our kids appreciate technical concepts by their own means and methods, as well as providing a wide-range of learning possibilities for the audience. The intent is to implement some of these "Math & Crafts" activities in local libraries and elementary schools.**

Index Terms – abacus, ancestral numerical systems, Arhuaco, ASCII, Aztecs, base 2, base 10, base 20, base 60, binary code, decimal system, Incas, Kiphu/Quipu, mathematics, Maya, Nepohualtzintzin, numerical systems, pre-Hispanic, STEM, Soroban, Suan Pan, vigesimal system, Yupana.

Introduction to Most Commonly Used Ancient Math Methods

The research and analysis for this paper is limited to a number of numerical and abacus systems that could offer practical educational models, as well as, hands-on activities for children, educators and parents. The intent is to craft a few practical presentations to promote the use of mathematical analysis, as well as cognitive processes, while teaching basic mathematical concepts to kids. The research also plans to launch these initiatives at the local libraries and expose some of these concepts to a broader audience.

This article is designed to present basic background information related to some relevant mathematical concepts and possible uses. These concepts include ancient methods used in China, Japan, Mesopotamia, Europe and the Americas. Then, the article will provide our first "Math & Crafts" activity, using a not-so-conventional numerical system, with the sole intent to provide a new prospective, a different starting point to be considered. Similar concepts and mathematical models can then be used with the help of the audience, parents and educators.

Abacus and How Relevant are they Still Today.

For many, abacuses are considered the precursors of the modern digital calculator and computers. Abacuses are very simple instruments used to perform mathematical calculations such as: addition, subtraction, multiplication and division, as well as extraction of the square root and cubic root. Very simple instruments, yet very useful and able to perform complex calculations, even when using infinitesimal or astronomical magnitudes.

Many ancient cultures around the world developed various types of abacuses, and probably many more have been lost through time. From China and Japan, to India, Mesopotamia and the new world with the Mayan, Aztecs and Incas, many civilizations developed multiples versions of these instruments. But, most importantly, is how we can use these mechanisms to teach math today. There are several types of abacus, and we will analyze some of them in the following sections.

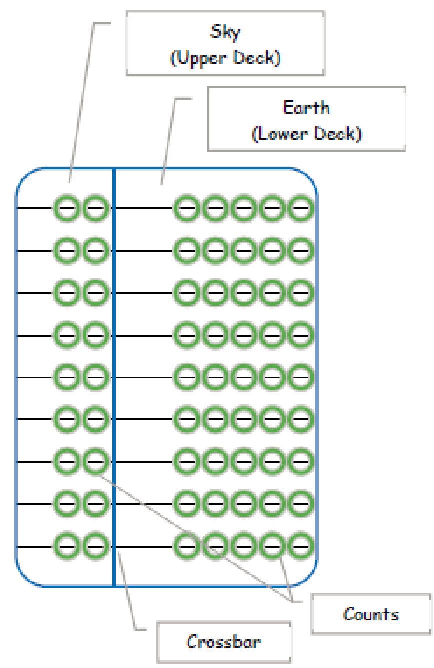

FIGURE I [1]
ABACUS

An abacus usually consists of 9 rows of beads strung together in a frame of parallel wires. The counts are separated by a crossbar, forming 2 sections called "sky" and "earth", also known as upper deck and lower deck. There are two counts on each column of the sky, where each count has a value of 5 units. Likewise, in each ground column, there are 5 counts. Here each count has a value of 1 unit. Each column of counts has a value that increases with a power of 10, just like in our decimal system.

How To Use The Abacus

1. Let's use the European abacus as an example. The European abacus, unlike the oriental versions, organizes the beads or counts horizontally, and they are usually composed of 10 rows of 10 counts each, for a total of 100 beads. These beads are usually of different colors so that people who are learning mathematics can understand more clearly and visually recognize the functioning of the abacus.

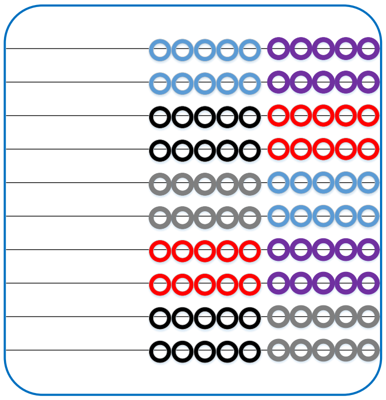

FIGURE II [1]
EUROPEAN ABACUS

2. The simplest abacus consists of a value of 1 for each bead, therefore, the only thing we will do when moving the beads will be to add or subtract 1 count.

3. However, for more complex calculations, other values are usually given to the counts, thus increasing the capacity of our calculations. The numbers that can be reached in the abacus in its most advanced form is up to ten billion, if we distribute the values like this:
   - Bottom row: the counts have a value of 1
   - Second row from below: the counts are worth 10 each
   - Third row from below: each count is worth 100
   - Fourth row from below: the counts have a value of 1000
   - And so on, to the tenth row, to a value of ten billion.

4. To add with the abacus we will only have to place all the counts to the left and, when we want to add, we will have to move the counts to the right, in this way, it will be very easy to calculate the total.

5. Remember that each level has a different value, therefore if we have a number that is 250, we will have to move two counts of level 3 and then five counts of level 2, in this way we will have 200 + 50 = 250

6. If we want to add 1,000 to the number 250, we will only have to move one count of the fourth row and, thus, the counts that accumulate to the right will add up the total of the amounts that we are adding.

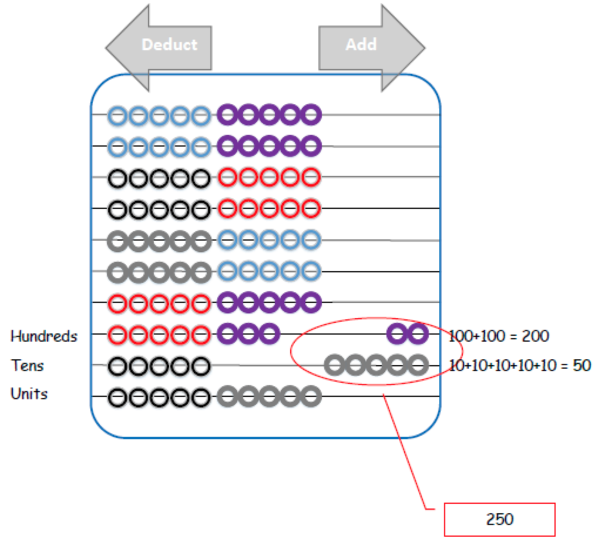

FIGURE III [1]
EUROPEAN ABACUS, ADD AND DEDUCT EXAMPLE

7. To be able to subtract with the abacus, the procedure will be the same but the movement of the counts will be from right to left. In this way, the chips that remain on the right, will be the total after subtracting the amounts to be deducted.

8. The results will always have to be read from top to bottom.

Could we use the abacus to multiply and divide?
Yes, of course, let's see an example of a multiplication:

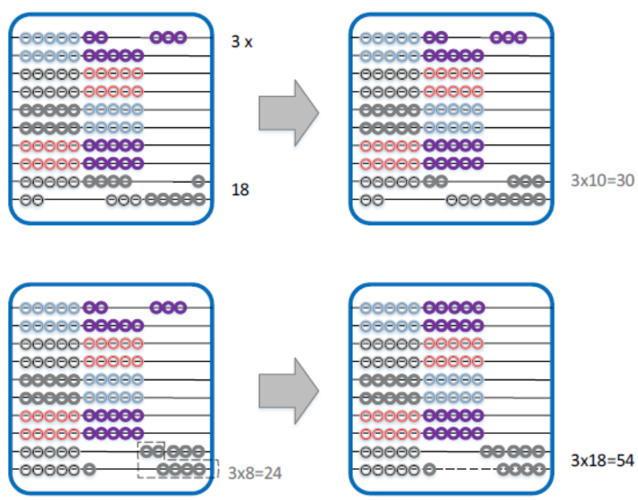

FIGURE IV [1]
EUROPEAN ABACUS, MULTIPLICATION EXAMPLE

Similarly occurs with division cases, and when using other types of abacus.

Types of Abacus

Chinese Abacus or Suan-Pan
It generally uses rounded counts; it has more than 7 columns: 2 counts in each upper deck and 5 in the lower deck. Most of them are usually made of a hardwood or bamboo. Counts are counted by moving them up or down the columns. If you move them close to the crossbar (horizontal bar in the center, or sometimes off-center), its value is counted or added. If you move such count away from the crossbar, they are deducted or with no value. The Suan-Pan can be returned to the starting position instantly by a rapid movement on the horizontal axis to rotate all the beads off the crossbar.

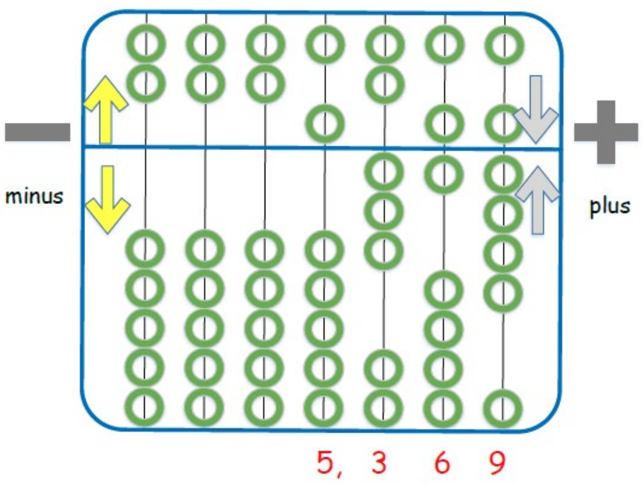

FIGURE V [1]
CHINESE ABACUS (SUAN-PAN)

Japanese Abacus, Soroban or Schoty
Similar to the Chinese abacus in terms of the orientation of the counts, it is a decimal instrument that has 9 counts per horizontal rod and is able to work with units, tens, etc. The counts tend to be of different colors to differentiate the quantities and thus facilitate their use.

Mesopotamian abacus
Perhaps one of the first abacus in history of humankind. It dates from the period of 2700-2300 A.C. approximately. They consisted of a table of successive columns that delimited the successive orders of magnitude of their numerical system, which was a sexagesimal type (base 60).

The Mesopotamian Abacus was built in a flat stone and a kind of sand mixture. In which letters were drawn to identify the magnitude of the counts (i.e. hundred, tens, etc.), and placed stones on the sand that represented the value, quantity or numbers.

Other ancient abacuses include Egyptian, Persian or Iranian, Greek, Roman, Abacus of India, Russian, among others.

Mayan Abacus or Nepohualtzintzin
Ne-Pohuali-Tzintzin means, "counting with small similar items by a person"
This calculation tool is based on the vigesimal system (base 20). A grid made with rods and seeds represented the numbers. There are four counts on the lower section, which in the first row have unit values (or 1, 2, 3 and 4), and in there are three accounts on the upper section, with values of five units (or 5, 10 and 15), respectively. The counts in the additional columns have assigned values depending on the numerical system in base 20, like shown in Figure VI.

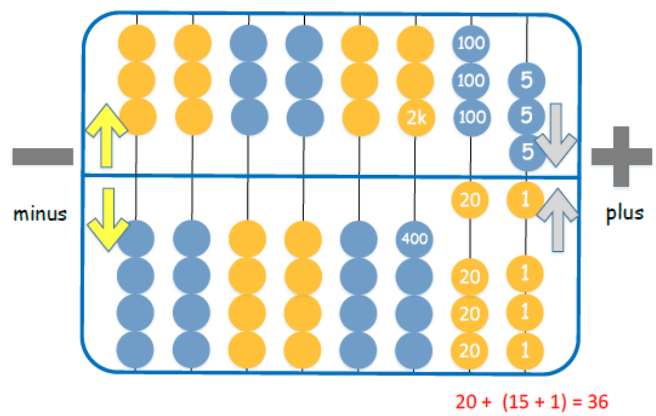

FIGURE VI [1] [2]
MAYAN ABACUS (NEPOHUALTZINTZIN)

For the Maya and Aztecs, counting in base 20 was completely natural, since the use of native sandals allowed them to also use their toes to perform calculations.

A complete Maya abacus would have 13 rows with 7 accounts in each row, representing 91 accounts in each Nepohualtzintzin. This number, 91, is a very important number to be able to understand the close relationship between Maya counts and certain natural phenomena they observed and recorded. For example [1] [2]:

1 Nepohualtzintzin (91) = Number of days in each season of the year

2 Nepohualtzintzin (182) = Number of days of the corn cycle

3 Nepohualtzintzin (273) = Number of days of a women pregnancy

4 Nepohualtzintzin (364) = Completes a cycle, and approximately 1 year

It is worth noting that in the Maya Abacus, both spatial (big) quantities and infinitesimal (small) quantities can be calculated with absolute precision.

Other very ancient abacus attributed to the Olmec culture have been found, and even some bracelets of Maya origin, as well as a variety of shapes and materials in other cultures.

MATH & CRAFTS ACTIVITY 1: BUILDING A BASE 20 CALCULATOR

Do you want to build your own "Mayan Calculator"? portable, without batteries and very economical? Well, you can use seeds of different types or colored beads if you wish. Organize the beads (or counts) in the following pattern and you would have built your Mayan calculator.

FIGURE VII [1]
MAYAN CALCULATOR (ABACUS ON BASE 20)

This activity is based on the following concept, see Figure VIII.

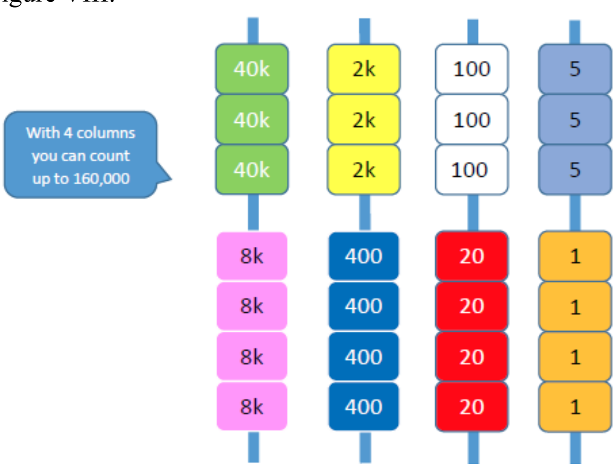

FIGURE VIII [1]
BASE 20 COUNTS-ABACUS THEORY

Inca Abacus: The Yupana and the Quipu (or Khipu)
The **Quipu** of the Incas was a system of knotted ropes used to record numerical data. The quipus were like an advanced accounting book of events, so to speak. Some think that the type of wool or thread, its color and even the type of knot had a special meaning for each of the Andean ethnic groups that used the quipus.

However, the quipus were most likely not used to perform the mathematical calculations. The calculations were most probably made using a Yupana (Quechua for "counting tool").

The set of systems that includes the quipu and the yupana altogether are considered as the Incas Abacus. This abacus is based on the decimal system as we know it today. Although, some historians and researchers still analyzing their possible uses and connections with the system in base 40, a possible relation with the Fibonacci sequence (1, 1, 2, 3, 5 ...), and powers of 10, 20 and 40 as possible values for the different fields in the instruments found.

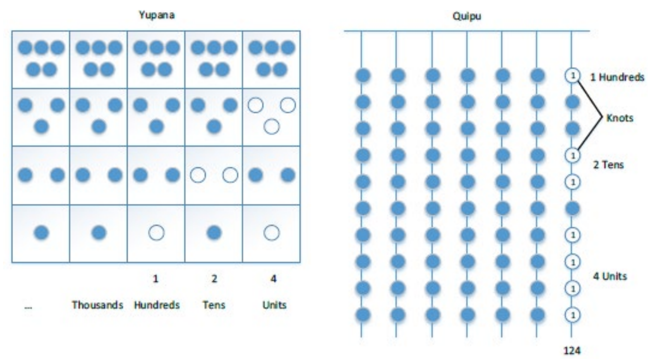

FIGURE IX [1]
INCA ABACUS

It is important to highlight that **the Maya, Aztecs and Incas had knowledge of Zero (0)**; which represent a major milestone in evolution, and indicates that they arithmetical knowledge was substantial in the history of humankind.

Binary Abacus
The abacus in Base 2 or Binary Abacus uses a series of parallel wire beads arranged in three different rows. The counts represent a change of action, either in the form of One (1) or "on-ignition"; or in the form of zero (0) or "off-off".

This type of abacus is the basis for the technology used in computers today. This abacus can represent any numbers, letters and signs that then can be stored in a computer; these operations are performed using a binary software known as the ASCII programming language.

IMPACT ON CURRENT NUMERICAL SYSTEMS AND APPLIED SCIENCE

It is important to highlight that the current math teaching methodologies, and even our current Gregorian calendar and uses of current binary code can use some improvements. For example, Pope Gregory VIII only corrected the current globally used calendar in 1582 and it was only adopted in the new world in 1752. The Pope basically corrected the Julian calendar by adding leap years every four years except in those years when a new century begins (1900, 2000, etc...) unless 400 cannot evenly divide the new century's number. This is more of a mathematical adjustment, but it works very well. For example, the year 1900 was not a leap year (1900/400 = 4.75), but the year 2000 was a leap year (2000/400 = 5). This adjustment by Pope Gregory VIII has an accuracy of approximately sixteen seconds per year. This is why we still use it today.

In binary code, the numerical system in base 2 or binary abacus uses a series of parallel wire beads arranged in three different rows. The accounts represent a change of action in the sequence, either in the form of One (1) or "on-ignition;" or in the form of zero (0) or "off-off."

Amazingly, this type of abacus is the basis for the technology used in computers today. This numerical system represents any number, letters, words, signs, and commands as a combination of ones and zeros that can be stored and used on computer languages or through the ASCII programming language (see FIGURE X).

Computers can only understand numbers, so ASCII is the numerical representation of any character such as 'a' or '@' or 'numbers' or any action or command of any kind.

Imagine what could happen if we use the system in base 20? Or the Mayan numerical system where several more options could be leveraged to represent the inputs or logic behind the scenes?

For starters, the information that currently takes tens of digits could be represented with far fewer characters, thereby reducing both the amount of memory needed and processing time. The applications would be extremely useful. That is why the Mayan numerical system, or base 20 system, could be a key element for the development of new technologies, managing massive amounts of data, and programing new super computers.

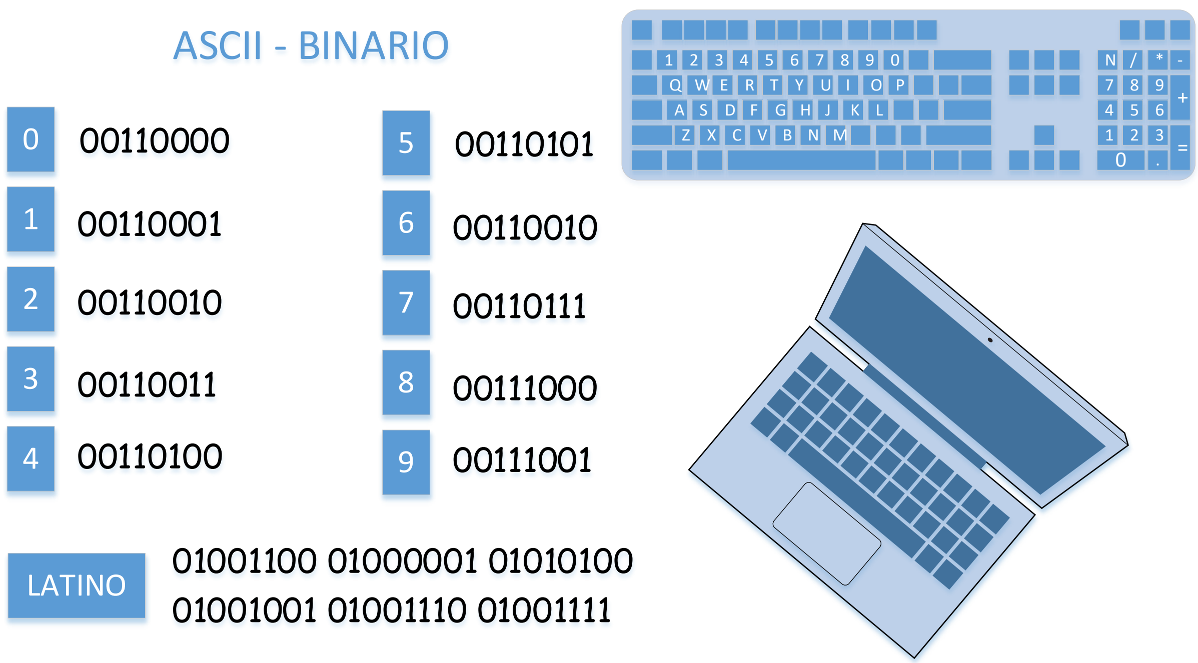

FIGURE X [1]
BINARY CODE AND ASCII CODE USED IN MODERN COMPUTERS. DRAWN BY THE AUTHOR.

IMPLEMENTATION IN GLOBALIZED CURRICULA AND APPLIED SCIENCE

If there is a place for new ways of teaching mathematical systems in current educational programs, all these options, and maybe even other ones, should be included. Not only would this prevent a perceived bias and be against scientific observation rules, but any exclusions might be damaging to education itself.

Therefore, a conscious effort to include these cultures and their principles to current educational programs should be made. For example, analyzing various educational curriculums from Europe, North and South America, it is possible to have certain guidelines for future implementations, whenever possible.

In some cases, curriculums have five levels or grades in elementary schools, and others have six grades, therefore, it is reasonable to include these numerical systems and science skills that best fit each program or curriculum. For the most part, fifth or sixth grade seems like the ideal time; the math and measuring systems needed usually accompany other STEM activities and skills, thus reinforcing young kid's base knowledge and creativity. It also opens the door for new opportunities to be pursued by local boards of education or individual/regional native/indigenous schools.

Other uses may include tactile and play lessons in counting and computing (i.e., build and practice with a Maya calculator as seen in FIGURE XI)

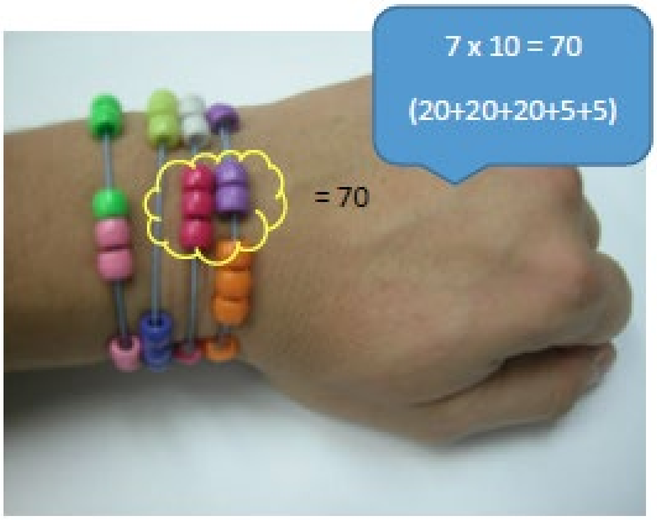

FIGURE XI [1]
MAYA CALCULATOR BY THE AUTHOR.

HOW CAN WE HELP EXISTING INDIGENOUS COMMUNITIES?

Teaching or even interacting with native communities in existence today may require a different approach from those used in urbane societies. Not all native tribes are the same--their languages, geographic locations, and willingness to accept outsiders will vary from tribe to tribe. Therefore, adapting to each community will be essential for a successful implementation of these numerical systems into their culture and daily life. These numerical systems may give those existing tribes a chance to improve their native math skills and overall awareness of other civilizations. Having a documented set of instructions and collected history facts about each approached native community will be a huge add value of further research.

CONCLUSIONS

This paper is a simple attempt to raise awareness of multiple numerical systems and how a few set of instructions and activities could help us promote math among children, parents and educators.

Some of the recommendation to include these numerical systems and their applications in current activities in public libraries, and in educational curriculums is simply an attempt to provide a general guidance, and not to be considered as any type of interventionism to the local educational programs, existing indigenous communities and/or their cultures.

Finally, regardless of curriculum acceptance or not, an effort should be made to preserve and promulgate these advanced math skills and applied science; and to explore how integrating them into our current numerical systems can be beneficial to solve technology challenges facing our current and future societies.

ACKNOWLEDGMENT

I am grateful to all of those with whom I have had the pleasure to work during this and other related projects. I would especially like to thank my parents, whose love and guidance are with me in whatever I pursue. Most importantly, I wish to thank my loving and supportive wife, Adriana, and my wonderful children Sofia and Alejandro, whom provide unending inspiration.

I would like to thank the peer reviewers for their assistance and comments that greatly improved the manuscript, although any errors are my own and should not tarnish the reputations of these esteemed persons.

I am also immensely grateful to the Arhuaco tribe, natives of the Sierra Nevada de Santa Marta, Colombia, for their wisdom and, as they continue to serve as living reminders of native ancestors.

REFERENCES

[1] Vega Janica, E. (2018), "Nativo Matemáticas 5", math textbook for kids, Spanish ed. [ISBN: 978-0-9997757-0-7]. "Native Mathematics 5-6", English ed. [ISBN: 978-0-9997757-1-4].

[2] Vega Janica, E., Vega Murgas, H., Esmeral Ariza, S. (2019), "Pueblo Iku: Ciencia, Naturaleza y Arte del Pueblo Arhuaco", STEM and Historical indigenous textbook for kids, Bilingual Spanish/Iku ed. [ISBN: 978-0-9997757-2-1].

AUTHOR INFORMATION

Ernesto Vega Janica, Senior Manager Opportunities Development, Business Development & Alliance Management, IEEE Standards Association.
https://www.linkedin.com/in/ernesto-vega-janica-50231319/

https://www.amazon.com/Ernesto-Vega-Janica/e/B07VFPG6XY?ref_=dbs_p_pbk_r00_abau_000000