Abstract—In an orthogonal frequency division multiplexing (OFDM), the pilot signal is utilized for the channel estimation (CE). The scattered pilot (SP) reduces the number of pilot signals, but the interpolation is required in the frequency band without the pilot signal. The linear interpolation (LI) is achieved simply. However, many errors are occurred in a multipath fading. The combined complex time frequency interferometry (C-CTFI) is achieved by combining the real and imaginary pilot signals, and the accuracy of the CE is improved by averaging the channel impulse response (CIR) of the different time window. However, a large number of pilot signals is required. To solve these problems, in this paper, we propose the CE based on the LI-CTFI for a SP-OFDM.

Index Terms—OFDM, SP, LI-CTFI, CE

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a kind of digital modulation techniques, and is adopted in many techniques such as long term evolution advanced (LTE-A) and wireless local area network (WLAN) [1]. Moreover, OFDM is also expected in 5G [2]. In wireless communications, since the channel variance is occurred due to a multipath fading, the channel estimation (CE) and the equalization are necessary. For these operations, the pilot signal is utilized in an OFDM [3]. The orthogonal pilot (OP) assigns the pilot subcarrier to achieve the CE in all utilized frequency band. However, since the pilot signal is a redundancy, the transmission rate is deteriorated. Since the scattered pilot (SP) assigns the pilot subcarrier sparsely in the utilized frequency band, the number of pilot signals is reduced, and the transmission rate is improved [4]. On the other hand, the SP requires the interpolation in the frequency band without the pilot signal. The linear interpolation (LI) is interpolated between the adjacent pilot subcarriers linearly, but many errors are occurred in a multipath fading [5]. As the other method, the time frequency interferometry (TFI) and the complex TFI (CTFI) have been proposed. The TFI assigns the pilot subcarrier such as the SP, and the accuracy of the CE is improved by averaging the channel impulse response (CIR) of the different time window [6]. In the CTFI, we have proposed the distributed and combined CTFIs. The distributed CTFI (D-CTFI) separates the pilot signal for the real and imaginary signals to treat the multi propagation system such as a multiple-input multiple-output (MIMO) [7]. In this paper, since we consider a single-input single-output (SISO), the TFI and the D-CTFI are same. The combined CTFI (C-CTFI) combines the pilot signal for the real and imaginary signals, and the accuracy of the CE is more improved by increasing the number of time windows [8]. However, since the number of pilot signals is increased, the transmission rate is deteriorated as same the OP. To solve these problems, in this paper, we propose the CE based on the LI-CTFI for a SP-OFDM.

II. SYSTEM MODEL

A. Channel Model

Firstly, the CIR between the transmitter and the receiver is given by

\[ h(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l), \]  

where \( h_l \) and \( \tau_l \) are the complex channel gain and the delay time of the \( l \)th propagation path with \( \sum_{l=0}^{L-1} E[|h_l|^2] = 1 \), respectively, \( L \) is the number of paths, and \( E[\cdot] \) is the ensemble average operation. And then, the channel response (CR) is obtained by the fast Fourier transform (FFT) operation as

\[ H(k) = \int_{0}^{\infty} h(\tau) \exp(-j2\pi k\tau) d\tau \]

\[ = \sum_{l=0}^{L-1} h_l \exp(-j2\pi k\tau_l), \]

where \( j = \sqrt{-1} \).

B. System Model

Figure 1(a) shows the block diagram of the proposed system at the transmitter. Firstly, the binary data signal is generated, and is coded by the convolutional code with the interleaving. Next, after the serial-to-parallel (S/P) conversion, the coded signal is modulated as \( x(k,i) \) for the \( k \)th subcarrier and the
ith symbol, and the pilot signal is inserted. In this paper, we define the pilot signal as
\[
x(k, i) = \begin{cases} 
1 & \text{for } \mod(k, \alpha) = 0 \\
0 & \text{otherwise}, 
\end{cases}
\]  
(3)

where \( \alpha \) is the interval between the adjacent pilot subcarriers for \( 0 < \alpha \leq N - 1 \), and \( N \) is the number of subcarriers. And then, after the inverse FFT (IFFT) operation, the guard interval (GI) is inserted to prevent the inter-symbol interference (ISI). Finally, after the parallel-to-serial (P/S) conversion, the transmitted signal is sent to the receiver.

Figure 1(b) shows the block diagram of the proposed system at the receiver. Firstly, after the S/P conversion and the GI elimination, the received signal of the time domain \( y(t) \) is converted to the frequency domain signal by the FFT operation as
\[
y(k, i) = \frac{2S}{N} \sum_{t=1}^{(i+1)N-1} y(t) \exp \left( -j2\pi \frac{kt}{N} \right)
= \frac{2S}{N} H(k, i) x(k, i) + z(k, i),
\]  
(4)

where \( S \) is the average transmission power, \( z(k, i) \) is additive white Gaussian noise (AWGN) with zero-mean and variance \( 2N_0/N \), and \( N_0 \) is single side power spectral density. And then, to eliminate the channel \( H(k, i) \) from (4), the received signal \( y(k, i) \) is detected by the zero-forcing (ZF) equalization as
\[
\tilde{x}(k, i) = \sum_{m=0}^{P-1} \frac{y(k, i)}{H(k, m)/P},
\]  
(5)

where \( P \) is the number of pilot symbols, and \( \tilde{H}(k, i) \) is the estimated channel. \( \tilde{H}(k, i) \) will be shown in Section III. Finally, after the demodulation, the P/S conversion, and the decoding with the deinterleaving, the bit signal is output.

III. PROPOSED LI-CTFI

A. LI

From (3), the OP and the SP assign the pilot subcarrier in \( \alpha = 1 \) and \( \alpha \geq 2 \), respectively. Therefore, the SP for \( \alpha \geq 2 \) reduces the number of pilot signals. However, the SP requires the interpolation to achieve the CE in the frequency band without the pilot signal. The LI [5] has been proposed as the simple interpolation as
\[
\tilde{H}(k, i) = \begin{cases} 
y(k, i) & \text{for } \mod(k, \alpha) = 0 \\
H(k, i) & \text{otherwise}, 
\end{cases}
\]  
(6)

where \( H(k, i) \) is the operation for the LI as
\[
H(k, \alpha + \beta, i) = \frac{(\alpha - \beta)g(\alpha k, i) + \beta g(\alpha + 1, i)}{\omega \alpha}
\]  
for \( 1 \leq \beta < \alpha \),
\]  
(7)

and \( \omega \) is the weight for the LI.

B. TFI and C-CTFI

Firstly, in the TFI [6], the pilot signal is given by (3) for \( \alpha = 2 \). And then, after the IFFT operation, the received pilot signal of the time domain is obtained from (4) between \( 0 \leq i \leq P - 1 \) as
\[
y(t) = \sqrt{\frac{2S}{N}} \sum_{k=0}^{N-1} y(k, i) \exp \left( \frac{j2\pi kt}{N} \right)
= \sqrt{\frac{2S}{N}} \sum_{l=0}^{L-1} h_l \zeta(\tau) + z(t),
\]  
(8)

where \( S_p \) is the average transmission power of the pilot signal, and \( z(t) \) is AWGN. In this case, \( \zeta(\tau) \) for the TFI is given by
\[
\zeta(\tau) = \delta(\tau - \tau_i) + \delta(\tau - \tau_i - 2G),
\]  
(9)

where \( G \) is the GI length as \( N/4 \). Finally, (8) is converted to the frequency domain signal by the FFT operation as
\[
\tilde{H}(k, i) = \sqrt{\frac{Np}{2Sp}} \frac{(i+1)N-1}{\sum_{i=1}^{N}} y(t) \exp \left( j2\pi \frac{kt}{N} \right)
= \sqrt{\frac{Np}{2Sp}} \sum_{i=0}^{L-1} \int_{-\infty}^{\infty} h_l \zeta(\tau) \exp \left( \frac{j2\pi kt}{N} \right) d\tau + \eta(k, i),
\]  
(10)

where \( \eta(k, i) \) is AWGN with \( E[|\eta(k, i)|^2] = E[|z(k, i)/\rho|^2] = \sigma_n^2/\rho, \sigma_n^2 \) is the noise variance, and \( \rho \) is the number of time windows. Observing (9) and (10), the noise variance is to be half by the averaging for \( \rho = 2 \). Next, in the C-CTFI [8], the pilot signal is given by
\[
x_cctfi(k, i) = \begin{cases} 
1 & \text{for } \mod(k, 2) = 0 \\
j & \text{for } \mod(k, 4) = 1 \\
-j & \text{for } \mod(k, 4) = 3.
\end{cases}
\]  
(11)

And then, at the receiver, the time domain signal is obtained as shown in (8), and \( \zeta(\tau) \) for the C-CTFI is given by
\[
\zeta(\tau) = \{ \delta(\tau - \tau_i) - \delta(\tau - \tau_i - G) \\
+ \delta(\tau - \tau_i - 2G) + \delta(\tau - \tau_i - 3G) \}/2.
\]  
(12)

Observing (10) and (12), the noise variance is to be quarter by the averaging for \( \rho = 4 \). Therefore, the noise variance for the C-CTFI is reduced compared with the TFI.

C. Proposed LI-CTFI

By using the C-CTFI, the good CE is achieved, but the transmission rate is deteriorated since the pilot subcarrier is assigned in all utilized frequency band as same the OP. To solve this problem, in this paper, we propose the CE for the LI-CTFI based on the SP with \( \alpha \geq 2 \).

Figure 2 shows the example of the proposed LI-CTFI for \( \alpha = 2 \) in a multipath fading. For the proposed LI-CTFI, we consider the combination for the LI and the C-CTFI. In this case, (8) is rewritten as
\[
y_{pro}(t) = \sqrt{\frac{2S}{N}} \sum_{k=0}^{N-1} x_cctfi(k, i) \tilde{H}(k, i) \exp \left( \frac{j2\pi kt}{N} \right)
= \sqrt{\frac{2S}{N}} \sum_{l=0}^{L-1} h_l \zeta(\tau) + z(t) + \tilde{z}(t),
\]  
(13)
where \( \tilde{z}(t) \) is the error component due to the LI. Since the proposed method utilizes the C-CTFI, \( \zeta(\tau) \) equals (12) in (13). And then, (13) is converted to the frequency domain signal by the FFT operation as

\[
\hat{H}_{pro}(k, i) = \sqrt{\frac{N\rho}{2S_p}} \sum_{l=1}^{N-1} y_{pro}(t) \exp \left( -j2\pi \frac{kt}{N} \right) \\
= \sqrt{\frac{N\rho}{2S_p}} \sum_{l=0}^{L-1} \int_{-\infty}^{\infty} h_{\zeta}(\tau) \exp(-j2\pi k\tau) d\tau + \eta(k, i) + \Delta(k, i), \tag{14}
\]

where \( \Delta(k, i) \) is the error component due to the LI. Observing (13) and (14), the proposed method achieves the CE by using the LI and the C-CTFI, but the error component \( \Delta(k, i) \) is included. Therefore, the proposed method reduces \( \Delta(k, i) \) by using the repeat process. Firstly, the proposed method estimates \( \Delta(k, i) \) for the \( n \)th repeat process as

\[
\hat{\Delta}_n(k) = \frac{1}{D} \sum_{m=0}^{P-1} \sum_{i=0}^{D-1} \{ \hat{H}_{pro,n-1}(k, m)/P \} \hat{x}_{n-1}(k, i),
\]

where \( \hat{H}_{pro,n}(k, i) \) and \( \hat{x}_{n-1}(k, i) \) are the estimated CR and the detected signal in the \( n \)th repeat process, and \( D \) is the number of data symbols. And then, (6) is rewritten by using (15) as

\[
\hat{H}_{lin,n}(k, i) = \begin{cases} y(k, i) & \text{for } \mod(k, \alpha) = 0 \\ \hat{\Delta}_n(k) H_{lin}(k, i) & \text{otherwise.} \end{cases}
\]

Observing (16), the proposed method adds the influence of the estimated error component \( \hat{\Delta}_n(k) \) in the conventional operation for the LI. By applying (16) in (13) (i.e., \( \hat{H}_{lin,n}(k) = \hat{H}_{lin}(k) \)), the proposed method achieves the CE with the repeat process. This process is repeated until \( n = \theta \).

IV. COMPUTER SIMULATION RESULTS

In this section, we evaluate the system performance of the proposed method by using the computer simulation. Table I shows the computer simulation parameters. As shown in Figure 1, at the transmitter, the original data signal is coded by the convolutional code for the rate \( R = 1/2 \) and the constraint length \( K = 7 \) with the interleaving. After the S/P conversion, the parallel signal is modulated by a quadrature phase shift keying (QPSK), and the SP is inserted as shown in Figure 3. The modulated and pilot signals are converted to the time domain signal by the IFFT operation. And then, after the GI insertion and the P/S conversion, the time domain signal is sent to the receiver via the propagation channel. At the receiver, after the S/P conversion and the GI elimination, the time domain signal is converted to the frequency domain signal by the FFT operation. And then, after the pilot signal separation, the channel state is estimated by the proposed LI-CTFI with the repeat process, and the received signal is detected by the ZF equalization. Finally, after the QPSK demodulation and the P/S conversion, the demodulated signal is decoded by the Viterbi soft decoding algorithm with the deinterleaving.

Figure 4 shows the bit error rate (BER) performance for the conventional and proposed methods. Here, \( \text{Pro}(\alpha, P, \omega) = (4, 1, 2) \) means the proposed LI-CTFI, and \( \text{Pro}(\alpha, P, \omega; \theta) = (4, 1, 2, 8) \) means the proposed LI-CTFI with the repeat process. The conventional method of the OP for \( P = 2 \) shows about 1.5 dB gain compared with the OP for \( P = 1 \) at

<table>
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<th>COMPUTER SIMULATION PARAMETERS.</th>
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**BER** = 1 × 10^{-5} by averaging the two pilot symbols. The conventional method for the TFI shows about 1.5 dB gain compared with the OP for P = 2 by reducing the average power of the pilot signal. Moreover, the conventional method for the C-CTFI shows about 0.5 dB gain compared with the TFI by averaging the four time windows. On the other hand, the conventional method for the LI shows the floor at BER = 4 × 10^{-2} due to many estimated errors. The proposed method for the LI-CTFI shows about 16 dB gain compared with the LI. This is because the error due to the LI is reduced by using the averaging for the C-CTFI. Moreover, the proposed method for the LI-CTFI with the repeat process shows about 9 dB gain compared with the proposed method without the repeat process at BER = 2 × 10^{-4}, and shows the about same performance as the OP for P = 2 at BER = 1 × 10^{-5}. This is because the error due to the LI is more reduced by using the repeat process.

Figure 5 shows the throughput performance for the conventional and proposed methods. The conventional method for the LI shows the worst performance in all Eb/N0 due to many errors. The conventional method of the OP for P = 1 and 2 has the tradeoff between low and high Eb/N0 due to the number of pilot symbols and the accuracy of the CE. The conventional method for the TFI and the C-CTFI shows the good performance compared with the OP by improving the accuracy of the CE. On the other hand, the conventional method for the C-CTFI is deteriorated for the maximum throughput compared with the TFI due to a large number of pilot signals. The proposed method for the LI-CTFI shows the about same performance as the OP for P = 2 in low Eb/N0. And then, in high Eb/N0, it shows the about same performance as the C-CTFI. Moreover, the proposed method for the LI-CTFI with the repeat process shows the best performance by reducing the number of pilot signals and the error due to the LI in all Eb/N0.

**V. CONCLUSION**

In this paper, we have proposed the CE based on the LI-CTFI for a SP-OFDM. The SP reduces the number of pilot signals. And then, the LI achieves the interpolation between the adjacent pilot subcarriers linearly, but many errors are occurred. The C-CTFI is improved for the accuracy of the CE by averaging the four time windows, but the pilot signal is assigned in all utilized frequency band. The proposed method utilizes the SP, and combines the LI and the C-CTFI. Moreover, the proposed method applies to the repeat process. From the computer simulation results, the proposed method has shown the good BER and throughput performances. In the future plan, we will consider the reduction of the computational complexity.

**REFERENCES**