

HAME507: Mastering the Time Value of Money

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This course includes

- Eight self-check quizzes
- Three discussions
- Two tools to download and use on the job
- One course project
- One video transcript file

Completing all of the coursework should take about five to seven hours.

What you'll learn

- Explain the importance of the timing of future cash flows
- Use a cash-flow timeline to conceptualize TVM problems
- Use a financial calculator to solve TVM problems, including future and present values of lump-sum payments, perpetuities, and annuities

What you'll need

One of the following:

- Hewlett-Packard 12C
- Texas Instruments BA II Plus
- Texas Instruments BA II Plus app for iPhone and iPad



Course Description

Managing a business means managing its financial resources. While the company controller and accounting professionals

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oversee day-to-day transactions, your ability to make smart decisions about projects relies on your understanding of timelines and cash-flow calculations to track cash flow and payments, the value of securities and investments, and how to determine overall cost-effectiveness.

Using these financial management tools to make informed financial decisions means you need a good working knowledge of a number of financial concepts. You'll also need to know how to compute the values that drive good decision making. This course introduces you to those concepts and shows you the steps for performing important calculations using financial calculators and popular spreadsheet applications. It helps you develop an intuitive understanding of the concepts and formulas and gives you a chance to practice applying the tools. You will come away with a toolbox of approaches for examining key project metrics that you can use to make sure that your company has the best possible chance of project success through managing its financial resources wisely.



Steven Carvell
Professor and Associate Dean for Academic Affairs, School of Hotel Administration, Cornell University

Steven Carvell has taught finance courses at Cornell University since 1986. Professor Carvell is the co-author of *In the Shadows of Wall Street* (Prentice Hall, Inc. Strebel, Paul and Steven Carvell, 1988). Carvell has worked for professional money managers in applied strategy in the equity market and served as a consultant to the Presidential Commission on the 1987 stock market crash. Professor Carvell has conducted numerous specialized Executive Education seminars for some of the largest hotel companies in the world. Carvell holds a Ph.D. from the State University of New York, Binghamton.



Scott Gibson
Zollinger Professor of Finance, Mason School of Business, College of William and Mary

Scott Gibson is the Zollinger Professor of Finance at the College of William and Mary Mason School of Business, and previously held academic appointments at Cornell University and the University of Minnesota. He holds a B.S. and Ph.D.

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in Finance from Boston College. Prior to his academic career, he worked as an analyst with Fidelity Investments and as a credit team leader serving Fortune 500 clientele with HSBC Bank. He's won outstanding teaching awards on numerous occasions, including being named an outstanding faculty member in *BusinessWeek Guide to the Best Business Schools*. His finance research has appeared in top academic journals and has been featured in the financial press, including the *Wall Street Journal*, *Financial Times*, *New York Times*, *Barons*, *BusinessWeek*, and *Bloomberg*.

Start Your Course

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Module Introduction: The Time Value of Money

The time value of money (TVM) is a critical element of financial management within organizations, and the principles being discussed here have relevance for personal financial management, as well. As a non-financial manager within your company, you want to be conversant in the ways that the time value of money affects your company's ability to borrow, invest, and expand in general, as well as to fund your projects. Professors Steve Carvell and Scott Gibson explain.

Note to students:

Your final course project will be to conduct a brief (15 minute) interview with someone either within your organization or beyond who is willing to speak to you about how the content of this course relates to everyday business. To avoid last-minute scheduling problems, you may want to schedule that interview now.

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Tool: Calculators and Spreadsheets

Tools include

- Professor Gibson's Quick guide to using the HP12C
- TA BI II Plus emulator for Apple devices
- HP 12C emulator for Android
- HP 12C emulator online
- Using MS Excel as a financial calculator
- Calculator tutorials

In addition to teaching you the methodology of TVM (time value of money), this course teaches the key strokes for two financial calculators: the HP 12C and the TI BA II Plus. Financial calculators are not the same as scientific calculators or standard calculators. Financial calculators include all of the following five functions:

N: term
PV: Present value
i: interest rate
FV: Future value
Pmt: Payment

Calculators that do not have these five functions will not be able to compute time value of money solutions other than algebraically, which is much harder. Also note that other financial calculators may not use the same keystrokes to perform calculations as the two recommended for this course. If you do not have one of the two recommended calculators, you can download an emulator to your mobile device. You can also perform these calculations in Excel by following the instructions on the link above.

The HP 12C Calculator

The HP 12C is one of only two calculators permitted on Chartered Financial Analyst Program exams (with the TI BA II Plus being the other). You may buy the actual HP 12C calculator, or you can buy an HP 12C iOS or Android app. You'll be able to solve most problems in this course much quicker using the HP 12C or another financial calculator than by using Excel.

The TI BA II Plus Calculator

To find out more about the TI BA II Plus calculator, visit the Texas Instruments website.

Faculty Notes

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The HP 12C is HP's longest- and best-selling product, in continual production since its introduction in 1981. You may buy the actual HP 12C calculator (which sells new for about $60 US or less for a used one), or you can buy an HP 12C app (for less than $10 US-I've used good HP 12C apps costing as little as $0.99 US). Several versions of the HP 12C exist:

- The Classic Gold HP 12C uses only reverse Polish notation* (RPN).
- The Platinum HP 12C allows the user to choose either RPN or algebraic notation (AN) mode. ("Normal" calculators you're familiar with are based on AN. So, the Platinum HP 12C in AN mode might be easiest for you.)
- The Limited Edition 25 th Anniversary HP 12C Platinum edition allows users to choose either RPN or AN mode. HP claims it has a high-quality keyboard similar to the keyboard of the original 1980's Classic Gold HP 12C.
- The Limited Edition 30 th Anniversary HP 12C edition uses only RPN. HP again claims a high-quality keyboard.

*The "Polish" in reverse Polish notation refers to the nationality of logician Jan ukasiewicz, who invented Polish notation in the 1920s. Polish notation is parentheses-free and the inspiration for the idea of the recursive stack, a last-in, first-out computer memory store. Studies show that RPN calculators are superior to AN calculators in terms of speed and accuracy of operation. However, as noted above, you'll likely be able to work faster with a Platinum HP 12C in the familiar AN mode. You are free to use these or any alternative financial calculator if you choose.

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Read: The Price of Money

Key Points

The available interest rate determines how hard the money will work for you.

The value of money changes with respect to time - $1 million dollars received today is worth more to us than the same amount of money received in the future. This might be called the "now factor."

There *are* times when we might choose to have money at a later time, especially if we will be rewarded for doing so. When we put money in a certificate of deposit or a savings account, for instance, we are choosing "later" in exchange for the reward of interest. We save now in exchange for having more money and more to consume later on.

Now let's consider the more typical scenario where the lottery payout is offered in either a smaller lump sum or annuity payments over a fixed period for a larger cash payout.



The available interest rate determines how hard the money will work for you, and that is the missing key piece of information in this scenario. Without it, it's not possible to make this decision.

For instance, if you invested the cash payout of $104.1 million at an interest rate of 10% a year, you would be able to make 25 withdrawals of $10.4 million each. This amount is greater than the 25 payments of $7.8 million that you would get if you took the annuity option, making the cash option the better choice. However, an interest rate of 3% would net you only $5.8 million available for withdrawal in each of the 25 years. In that case, the annuity would be the better option.

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In other words, the choice of which option to take hinges on the interest rate paid by the bank. At an interest rate of 6.18% per year, the values of the two options are the same. If the interest rate is less than 6.18%, the annuity option is better. If the interest rate is greater than 6.18%, the cash option is better.

We will explore how to make these calculations later in the course.

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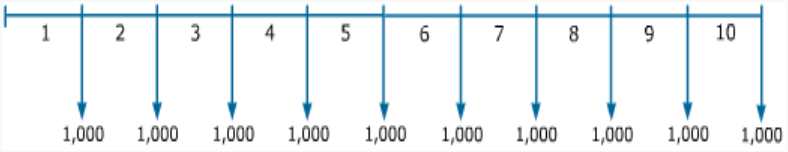
Read: Constructing a Basic Timeline

Key Points

When approaching a time value of money problem, it can be helpful to create a timeline.

Your financial advisor creates a savings plan for you. She says this plan is represented in the timeline she has drawn, which demonstrates when cash flows are expected to occur and how much is expected in each year.

When approaching a time value of money problem, it can be helpful to create a timeline like this:



Constructing a Basic Timeline

First, start with a line like this one:



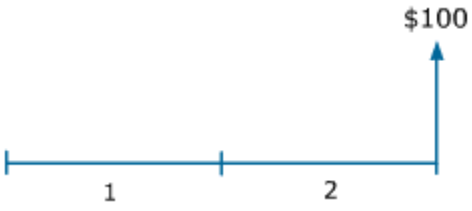
Usually, the timeline begins with today and ends with some time in the future.



By convention, we say that today is time period 0 (zero). The time in the future might be year 2.



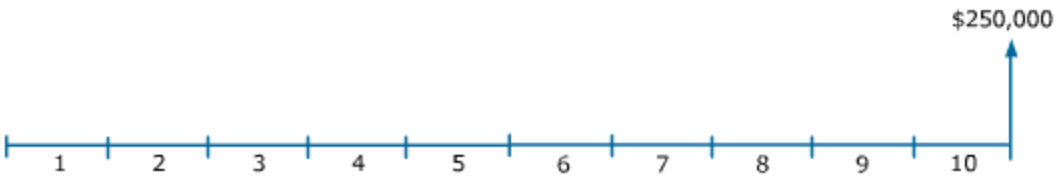
Now try using this timeline to show some transaction. For example, say you expect to receive $100 2 years from now.



Note that the up arrow signifies that money is coming into the account or is being received. That is, the up arrow shows a positive

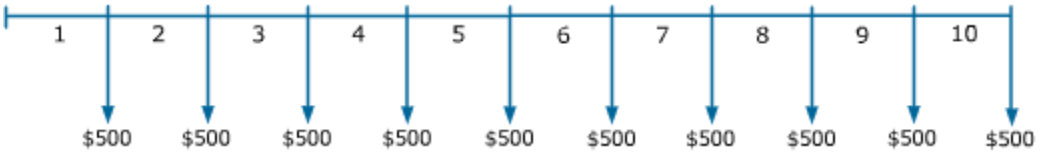
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cash flow. A down arrow signifies that money is going out of the account or is being paid. The down arrow shows a negative cash flow.



For example, what if you expect to receive $250,000 in 10 years? The up arrow shows that you are receiving $250,000 in year 10.

Next, take the example where you are expecting to repay a loan each year for the next 10 years and the loan repayment amount is $500 per year. The down arrows denote that you are paying out $500 per year for 10 years.



Now you are ready to draw your own timelines!

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Listen: Conceptualizing TVM Problems



Steve Carvell
Professor
Cornell University School of Hotel Administration

Do you have any tips for conceptualizing TVM problems?	*Click play to listen.*
How do you keep inflows and outflows straight?	*Click play to listen.*
How do you know if you want a present value or a future value?	*Click play to listen.*

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Watch: The Importance of Timelines

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Activity: Timelines

Download the Tool

TVM Overview Chart

Now that you have learned about the importance of timelines in financial planning, you can complete the Timeline row in your course project document.

To complete this activity:

1. Download the TVM Overview Chart
2. Add the following information to the Timeline row:
    - How to Calculate/Create: Describe the basic steps to create a timeline in your own words (do not just copy/paste from the course)
    - Business use example: In one sentence, describe a scenario where you would use a timeline in your business
    - Personal use example: In one sentence, describe a scenario where you would use a timeline in your personal life

You will complete this chart as you progress through the course materials. By the time you reach the end of the course, you will have developed an overview chart of the TVM tools that you can print for future reference. You will not be required to submit this chart for grading, but can use it for your own professional development.

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Module Wrap-up: The Time Value of Money

In this module, you explored concepts related to the time value of money and the impact that has on an organization's financial health. You examined the importance of the timing of future cash flows, as well how you can use a cash-flow timeline to conceptualize TVM problems.

Note to students:

Your final course project will be to conduct a brief (15 minute) interview with someone either within your organization or beyond who is willing to speak to you about how the content of this course relates to everyday business. To avoid last-minute scheduling problems, you may want to schedule that interview now.

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Module Introduction: Build Your TVM Toolbox

"Building your TVM toolbox" is another way of saying that you will identify critical tools and calculations commonly used in this area of financial management. It also means how you think about the time value of money and what it means to your organization, as Professors Carvell and Gibson explain.

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Read: Find the Future Value of a Lump Sum

Key Points

A lump sum is a set amount of money paid out at one point in time

Find the future value of any lump sum with the formula: $FV=PV(1+i)^n$

Let's say you want to invest a sum of money that will earn interest. It is useful to know what the value of the investment will be at some time in the future, after earning that interest. We call that the future value of a lump sum (FV).

Variables

As with all general equations, the numbers you will need for performing financial analysis calculations are represented by variables. The variables used to calculate FV are:

PV = the amount to be invested
i = interest rate
n = number of time units invested

The unit of time (years, months, days) that determines n must be the same unit reflected in i. Use of a *monthly* interest rate requires that time be expressed in *months* when determining n. Use of a *yearly* interest rate requires that time be expressed in *years* when determining n .

The logic here is straightforward: the value of the balance in the account will increase by 5% each year. So if you multiply the beginning value by 1.05 , you'll get the value at the end of the first year. You can repeat this process to get the value for each subsequent year.

Beginning value:	5,000,000.00
Value after 1 year:	5,000,000 *x* 1.05 = 5,250,000.00
Value after 2 years:	5,000,000 *x* 1.05 *x* 1.05 = 5,512,500.00

Since 1.05 x 1.05 = $(1.05)^2$, notice the pattern that emerges -- the value after n years is found using $(1+i)^n$. Since it

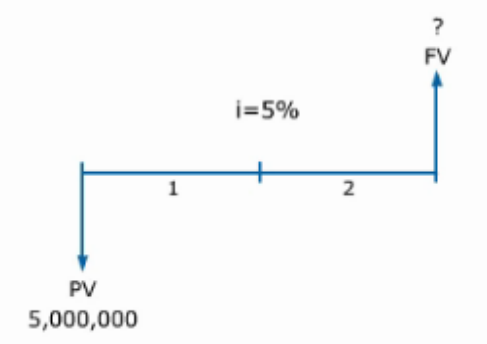
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calculates "compound interest," it is called the "compounding factor." To find the future value of any lump sum, we can then simply use the following general formula:

$$FV=PV(1+i)^{n}$$

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Activity: Calculate the Future Value of a Lump Sum



Let's now see how to perform the calculations for finding the future value of a lump sum using the scenario from the prior page: you have 5 million euros that you wish to invest for two years at a 5% annual interest rate. How much money will you have at the end of the two years?

This demonstration shows you how to use your calculator to find the future value of a lump sum cash flow using the HP 12C.
Download *Printer Friendly Steps for the HP 12C* .

STEPS	KEY SEQUENCE	DISPLAY
CLEAR the financial registers.	[f][REG]	0.00
ENTER the number of periods.	2 [n]	2.00
ENTER the periodic interest rate.	5 [i]	5.00
ENTER the present value.	5,000,000.00 [CHS][PV]	-5,000,000.00
ENTER the payment amount.	0 [PMT]	-0.00
CALCULATE the future value.	[FV]	5,512,500.00

This demonstration shows you how to use your calculator to find the future value of a lump sum cash flow using the TI BA II Plus.
Download *Printer Friendly Steps for the TI BA II Plus* .

Note to Texas Instruments calculator users: If your calculator produces a different answer than the one in this example,

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you may have to change your "payments per year" settings. In all of the problems and examples in this course, payments are made and interest is compounded annually. However, by default, the TI calculator bases its calculations on monthly payments and compounding. You may find these TI Calculator Basics helpful.

STEPS	KEY SEQUENCE	DISPLAY
CLEAR the financial registers.	[2nd][FV]	0.00
ENTER the number of periods.	2 [N]	2.00
ENTER the periodic interest rate.	5 [I/Y]	5.00
ENTER the present value.	5,000,000 [+/-][PV]	-5,000,000.00
ENTER the payment amount.	0 [PMT]	-0.00
CALCULATE the future value.	[CPT][FV]	5,512,500.00

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Read: Find the Present Value of a Lump Sum

Key Points

A lump sum is a set amount of money paid out at one point in time

Find the present value of any lump sum with the formula: $PV=FV/(1+i)^n$

Suppose that you need to plan now to have a sum of money available at some future date. For instance, suppose you'll need to have 100 million euros available in 5 years. If you know that you can earn interest at 5% annually, you can calculate how much money you'll need to set aside now to have 100 million euros at the end of 5 years.

Variables

The variables used to calculate FV are:

FV = the future value of the lump sum
i = interest rate
n = number of time units invested

The unit of time (years, months, days) that determines n must be the same unit reflected in i . Use of a *monthly* interest rate requires that time be expressed in *months* when determining n . Use of a *yearly* interest rate requires that time be expressed in *years* when determining n .

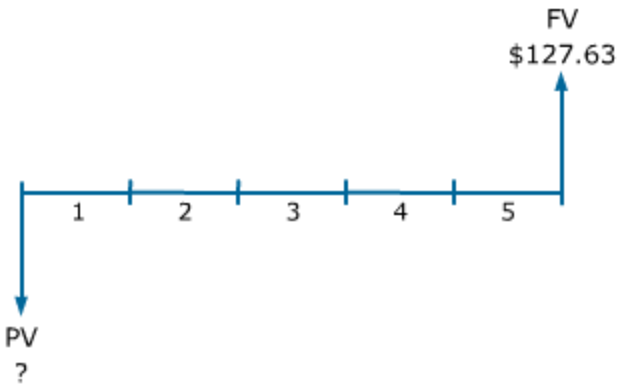
Remember that future value is found with $FV=PV(1+i)^n$.

Therefore, by dividing both sides of the equation by $(1+i)^n$, we see that the present value can be found using the formula: $PV=FV/(1+i)^n$

When we rearranged the future value formula to write the present value formula, the factor relating future value and present value is the discounting factor, which is the reciprocal of the compounding factor: $1/(1+i)n$

Suppose you need $127.63 in 5 years. You know you can earn 5% per year on your money. How much do you have to put up today?

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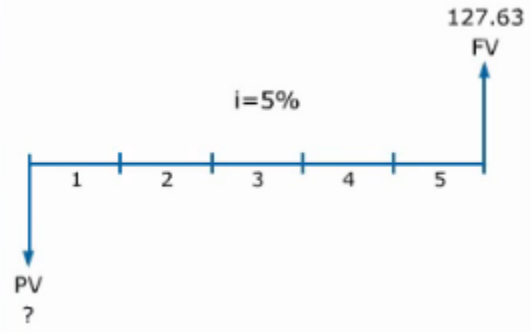
We are given that FV = $127.63, n = 5, i = 5%. So,

PV = $127.63 / (1.05) ^{5} PV = $100

Given the interest rate of 5%, the amount of money you need to set aside today in order to have $127.63 in 5 years is $100.

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Activity: Calculate the Present Value of a Lump Sum



Let's find the present value of $127.63 received in 5 years. Assume a 5% interest rate. (See the timeline to the right.)

This demonstration shows you how to use your calculator to find the present value of a lump sum cash flow using the HP 12C. Download *Printer Friendly Steps for the HP 12C* .

STEPS	KEY SEQUENCE	DISPLAY
CLEAR the financial registers.	[f][REG]	0.00
ENTER the number of periods.	5 [n]	5.00
ENTER the periodic interest rate.	5 [i]	5.00
ENTER the future value.	127.63 [FV]	127.63
ENTER the payment amount.	0 [PMT]	0.00
CALCULATE the present value.	[PV]	-100.00

This demonstration shows you how to use your calculator to find the present value of a lump sum cash flow using the TI BA II Plus. Download *Printer Friendly Steps for the TI BA II Plus* .

Note to Texas Instruments calculator users: If your calculator produces a different answer than the one in this example, you may have to change your "payments per year" settings. In all of the problems and examples in this course, payments are made and interest is compounded annually. However, by default, the TI calculator bases its calculations on monthly payments and compounding. You may find these TI Calculator Basics helpful.

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STEPS	KEY SEQUENCE	DISPLAY
CLEAR the financial registers.	[2nd][FV]	0.00
ENTER the number of periods.	5 [N]	5.00
ENTER the periodic interest rate.	5 [I/Y]	5.00
ENTER the future value.	127.63 [FV]	127.63
ENTER the payment amount.	0 [PMT]	0.00
CALCULATE the present value.	[CPT][PV]	-100.00

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Activity: Present and Future Values of a Lump Sum

You can now complete the Present Value of a Lump Sum and Future Value of a Lump Sum rows in your chart.

To complete this activity:

1. Open the TVM Overview Chart template you downloaded earlier in the course.
2. Add the following information to the Future Value of a Lump Sum and Present Value of a Lump Sum rows:
    - How to Calculate/Create: Briefly describe how to perform the calculation in your own words (do not just copy/paste from the course) and include the formula
    - Business use example: In one sentence, describe a scenario where you would use the calculation in your business
    - Personal use example: In one sentence, describe a scenario where you would use the calculation in your personal life

You will complete this chart as you progress through the course materials. By the time you reach the end of the course, you will have developed an overview chart of the TVM tools that you can print for future reference. You will not be required to submit this chart for grading, but can use it for your own professional development.

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Module Wrap-up: Build Your TVM Toolbox

In this module, you defined the "TVM toolbox." You calculated the future value of a lump-sum payment and the present value of a lump-sum payment, and you identified instances in which these calculations will be helpful.

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Module Introduction: Perpetuities

What are perpetuities, and what is their significance for non-finance people? In this module, you will examine perpetuities, how they function, and why they're important, as Professors Carvell and Gibson explain.

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Read: Find the Present Value of a Perpetuity

Key Points

A perpetuity is an ongoing stream of equal payments that continue forever

Find the present value of a perpetuity with the formula:

$PV_{PERPETUITY} = PMT/i$

A stream of equal payments that lasts forever is a perpetuity. Whereas some regularly scheduled payments seem to go on forever, the perpetuity actually does. Imagine a scenario in which your company wishes to establish a foundation that subsidizes daycare services for community families in need. In this scenario, the foundation intends to provide the equivalent of $100,000 per year in perpetuity for daycare services. The company plans to make a one-time cash contribution now to capitalize the foundation. If invested funds can earn a return of 8% per year, how much money must your company contribute to the foundation today to provide daycare assistance of $100,000 at the end of the first year and every year after?

Variables

The variables used to calculate $PV_{PERPETUITY}$ are:

PV = the amount to be invested
i = interest rate
PMT = annual payment

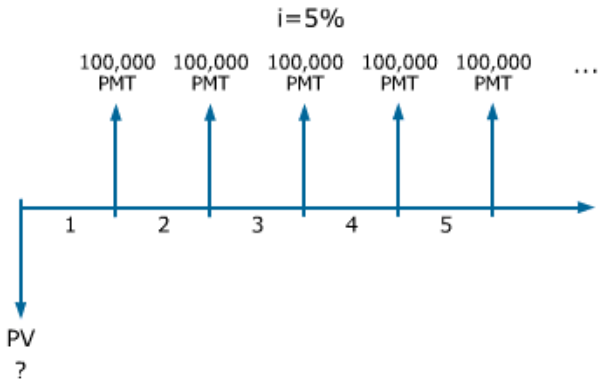
Quantifying the present value of payments received in the future, as in the example above, is a problem commonly encountered in finance. In the case of a perpetuity, the question is, What is the present value of a stream of equal payments that lasts forever? To calculate that value, we begin with an invested amount. Once again, it is important to consider the interest earned in order to understand how this money changes through time. The annual interest paid on the invested amount represents the payment of the perpetuity.

The relationship of the present value of a perpetuity to its annual payment is therefore characterized by the interest rate: $PV_{PERPETUITY} \times i = PMT$.

Rewriting the formula to solve for PV, we derive: $PV_{PERPETUITY} = PMT/i$

Consider the following example: A foundation wishes to pro

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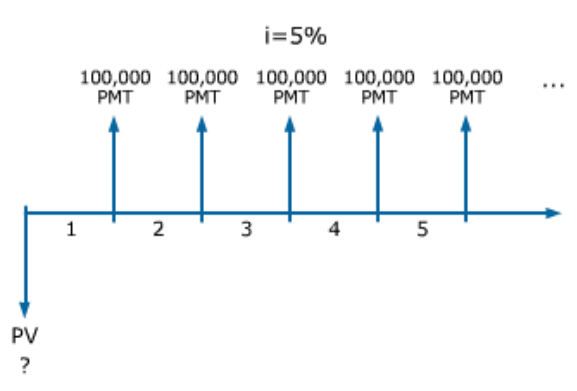
forever, to provide scholarships to finance students. If the i
this donation to Cornell be?

Assume that the first payment will be made one year from t

The donation required is: PV = $100,000 / 0.05 = $2,000,000.

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Activity: Calculate the Present Value of a Perpetuity



You wish to establish a fund that will allow for annual payments of $100,000 forever. The interest rate available is 5%. Calculate how much you must set aside now for this fund so these payments can be made.

This demonstration shows you how to use your calculator to find the present value of a perpetuity using the HP 12C. Download *Printer Friendly Steps for the HP 12C* .

STEPS	KEY SEQUENCE	DISPLAY
ENTER the annual cash flow.	100,000 [ENTER]	100,000
ENTER the interest rate.	0.05	0.05
PERFORM the operation.	[ ]	2,000,000.00

This demonstration shows you how to use your calculator to find the present value of a perpetuity using the TI BA II Plus. Download *Printer Friendly Steps for the TI BA II Plus* .

Note to Texas Instruments calculator users: If your calculator produces a different answer than the one in this example, you may have to change your "payments per year" settings. In all of the problems and examples in this course, payments are made and interest is compounded annually. However, by default, the TI calculator bases its calculations on monthly payments and compounding. You may find these TI Calculator Basics helpful.

STEPS	KEY SEQUENCE	DISPLAY

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ENTER the annual cash flow.	100,000	100,000.00
DIVIDE by the interest rate.	[ ] 0.05	100,000.00
COMPLETE the calculation.	=	2,000,000.00

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Read: Find the Present Value of a Growing Perpetuity

Key Points

Simple perpetuity payments stay the same over time

Growing perpetuity payments increase at a constant rate over time

Find the present value of a growing perpetuity with the formula:

$$PV = PMT / (i-g)$$

Something has been troubling you about your company's plan to subsidize daycare services for community families in need. According to the plan, the company will establish a foundation to provide the equivalent of $100,000 per year in perpetuity. However, in the financial report that you saw, the value of the one-time cash contribution that will capitalize the foundation has been calculated as a simple perpetuity. What about inflation?

You advise the foundation to take into account the projected 3% per year inflation and rework this plan. You are in agreement that invested funds can earn a return of 8% per year. Now you'd like to determine for yourself how much money your company must contribute to the foundation today to provide daycare assistance of $100,000 at the end of the first year and payments that then grow at the inflation rate of 3% per year in perpetuity. Can you do it?

A growing perpetuity is a series of payments that grow at a constant rate over fixed intervals in perpetuity. It differs from a simple perpetuity in that the payments grow at a constant rate, rather than remain the same. Both simple and growing perpetuities continue forever.

Recall that the present value of a perpetuity could be written: $PV = PMT / i$

In the case of a growing perpetuity, you must consider the rate of growth of the payment amounts *in addition to* the rate of growth of the initial capital due to interest earnings. The rate at which the payments grow is g. The interest rate is i .

The formula for the present value of a growing perpetuity is:

$PV = PMT / (i-g)$, where PMT is the equivalent payment amount.

For example, imagine that you wish to endow a chair in finance at your alma mater. The interest rate is 5% per year and your aim is to provide the equivalent of $200,000 per year in perpetuity. If the expected inflation rate is 2.5%, can you determine how much must be set aside today?

According to the formula above, the amount you must put aside today is given by:

$PV = PMT / (i-g) = \$200,000 / (0.05 - 0.025) = \$8,000,000.$

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Activity: Present Value of a Perpetuity and Growing Perpetuity

You can now complete the Present Value of a Perpetuity and Present Value of a Growing Perpetuity rows in your chart.

To complete this activity:

1. Open the TVM Overview Chart template you downloaded
2. Add the following information to the Present Value of a Perpetuity and Present Value of a Growing Perpetuity rows:

- How to Calculate/Create: Briefly describe how to perform the calculation in your own words (do not just copy/paste from the course) and include the formula
- Business use example: In one sentence, describe a scenario where you would use the calculation in your business
- Personal use example: In one sentence, describe a scenario where you would use the calculation in your personal life

You will complete this chart as you progress through the course materials. By the time you reach the end of the course, you will have developed an overview chart of the TVM tools that you can print for future reference. You will not be required to submit this chart for grading, but can use it for your own professional development.

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Watch: How We Use Perpetuities in Companies

In this video, Professors Carvell and Gibson will provide context and meaning to perpetuities as they relate to a measure of a company's value.

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Module Wrap-up: Perpetuities

In this module, you examined perpetuities. You identified strategies for finding the present value of a perpetuity as well as the present value of a growing perpetuity. You also identified why perpetuities are significant to an overall understanding of financial management for non-financial managers.

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Module Introduction: Annuities

Annuities are important to business for a number of reasons, and as a non-finance professional, you should have a working understanding of annuities, including how they function and why they matter. In this module, you will examine annuities and their relevance, as Professors Carvell and Gibson explain.

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Read: Find the Present Value of an Annuity

Key Points

An annuity is a set cash flow paid out in equal amounts over a set period of time

Find the present value of an annuity:

$$PV_{ANNUITY} = PMT\ [1/i - 1/i(1+i)^{n}]$$

Once again, consider the scenario in which a company wishes to establish a foundation that subsidizes daycare services for community families in need. This time, the foundation intends to provide the equivalent of $100,000 per year for 10 years for daycare services. The company plans to make a one-time cash contribution now to capitalize the foundation. If invested funds can earn a return of 8% per year, how much money must the company contribute to the foundation today to provide daycare assistance of $100,000 at the end of the first year and each year after, for 10 years?

Variables

The variables used to calculate $PV_{ANNUITY}$ are:

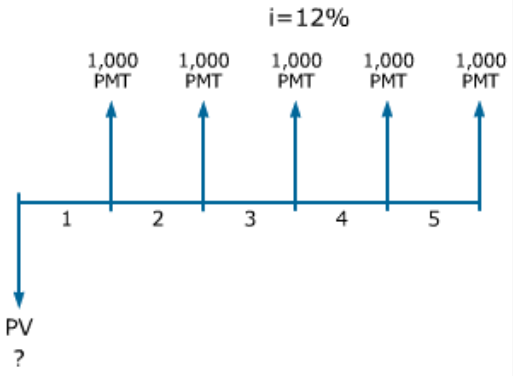
PV = the amount to be invested
i = interest rate
PMT = annual payment

n = number of time units invested

The unit of time (years, months, days) that determines n must be the same unit reflected in i . Use of a *monthly* interest rate requires that time be expressed in *months* when determining n . Use of a *yearly* interest rate requires that time be expressed in *years* when determining n .

This common scenario of a series of equal payments at fixed intervals for a specified number of periods is an annuity. To get a sense of how an annuity works, imagine that you must make a series of $1,000 payments at the end of each of the next 5 years. What is the value today of these payments if the discount rate is 12%?

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The present value of a single payment can be written as: $PV_{ANNUITY} = FV / (1+i)^n$

where FV is the payment amount. Using this relationship, it is possible to find the present value of each of the $1,000 payments (see table).

YEAR	FV/(1+i)n	PV
1	$1,000 / (1.12)1	= $892.86
2	$1,000 / (1.12)2	= $797.19
3	$1,000 / (1.12)3	= $711.78
4	$1,000 / (1.12)4	= $635.52
5	$1,000 / (1.12)5	= $567.43

Since you will have made all five payments, take the sum to get the present value of the entire annuity. The sum is $3,604.78, which is the present value of the payment stream.

Alternatively, this sum can be rewritten to make life a little simpler: $PV_{ANNUITY} = PMT\ [1/i - 1/i(1+i)^n]$

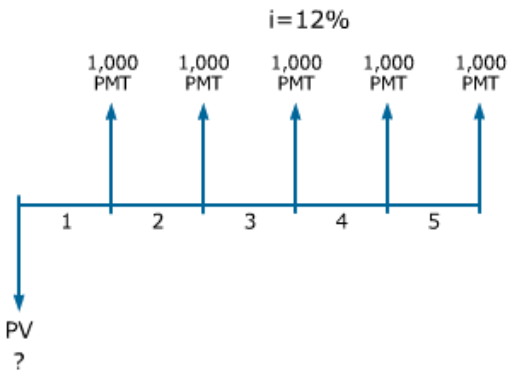
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Watch: Annuity Interpretation

In this video, Professor Scott Gibson discusses some of the intricacies of annuities.

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Activity: Calculate the Present Value of an Annuity



You must make a series of $1,000 payments at the end of each of the next 5 years. What is the value today of these payments if the discount rate is 12%?

This demonstration shows you how to use your calculator to find the present value of an annuity using the HP 12C. Download *Printer Friendly Steps for the HP 12C* .

STEPS	KEY SEQUENCE	DISPLAY
CLEAR the financial registers.	[f][REG]	0.00
ENTER the number of years.	5 [n]	5.00
ENTER the interest rate.	12 [i]	12.00
ENTER the payment amount.	1,000 [PMT]	1,000.00
ENTER the value after 5 years.	0 [FV]	0.00
CALCULATE the present value.	[PV]	-3,604.78

This demonstration shows you how to use your calculator to find the present value of an annuity using the TI BA II Plus. Download *Printer Friendly Steps for the TI BA II Plus* .

Note to Texas Instruments calculator users: If your calculator produces a different answer than the one in this example,

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you may have to change your "payments per year" settings. In all of the problems and examples in this course, payments are made and interest is compounded annually. However, by default, the TI calculator bases its calculations on monthly payments and compounding. You may find these TI Calculator Basics helpful.

STEPS	KEY SEQUENCE	DISPLAY
CLEAR the financial registers.	[2nd][FV]	0.00
ENTER the number of years.	5 [N]	5.00
ENTER the interest rate.	12 [I/Y]	12.00
ENTER the payment amount.	1,000 [PMT]	1,000.00
ENTER the value after 5 years.	0 [FV]	0.00
CALCULATE the present value.	[CPT][PV]	-3,604.78

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Activity: Present Value of an Annuity

You can now complete the Present Value of an Annuity row in your chart.

To complete this activity:

1. Open the TVM Overview Chart template you downloaded
2. Add the following information to the Present Value of an Annuity row:

- How to Calculate/Create: Briefly describe how to perform the calculation in your own words (do not just copy/paste from the course) and include the formula
- Business use example: In one sentence, describe a scenario where you would use the calculation in your business
- Personal use example: In one sentence, describe a scenario where you would use the calculation in your personal life

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Watch: Annuities

Understanding how annuities work will have significant impact on your ability not only to perform better as a non-financial manager within your organization but to take advantage of opportunities to manage your personal finances better, as well. Annuities are significant to many aspects of financial management, as Professors Carvell and Gibson explain.

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Module Wrap-up: Annuities

In this module, you identified strategies for finding the present value of an annuity. You examined the importance of calculating the present value of an annuity, and you saw the relevance of annuities to overall best practices for financial management strategies.

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Future Values

What are future values, how do we calculate them, and why are they relevant? In this video, Professors Carvell and Gibson discuss another tool for the TVM toolbox: future values.

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Read: Find the Future Value of an Annuity

Key Points

An annuity is a set cash flow paid out in equal amounts over a set period of time

Find the future value of an annuity with the formula:

$$FV_{ANNUITY} = PMT\ [\ (1+i)^{n}-1\ /\ i\ ]$$

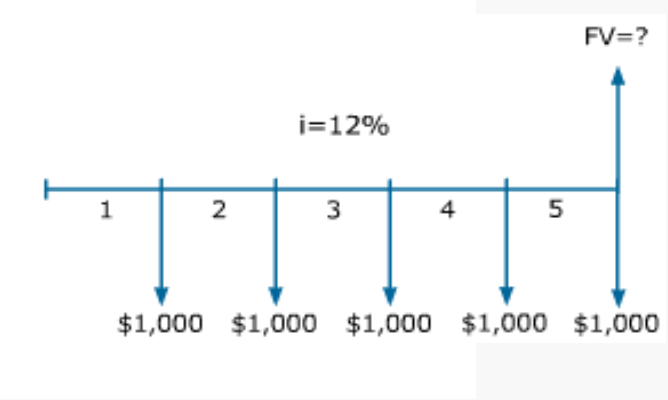
A company has issued $1,000,000 in bonds due in 10 years. It wishes to set up a sinking fund to be used to repay bondholders. The plan is to put aside an amount of money each year so that at the end of 10 years it will have $1,000,000 in the fund. The company wishes to know what the amount is that must be put aside each year.

In order to find this payment amount, the company must know the relationship between the future value of an annuity (in this case, $1,000,000), the number of years, the discount rate, and the payment amount. Assume that the interest rate you can earn on this account is 8%. Can you find the payment amount?

Recall that an annuity is a series of equal payments made at fixed intervals for a specified number of periods. Previously, the present value of an annuity was found starting with a simple mathematical relationship between payment amount, number of years, the future value of a lump sum, and the present value of a lump sum. Now, the future value of a series of equal payments is found by starting with the same relationship.

Consider a simple example in which a series of $1,000 payments will be paid at the end of each of the next 5 years. The appropriate discount rate is 12%. What is the value of the annuity in 5 years?

As in the case of the present value of an annuity, the future values of each of these $1,000 payments are easily found using the lump sum relationship (this time solved for the future value): $FV = PV\ (1+i)^{n}$



In this relationship, as before, the exponent *n* refers to the number of years from the time the payment was made until it is evaluated. Because the Year 1 payment is being evaluated at Year 5, *n* for the Year 1 payment is equal to 4 (Year 5 is 4 years after the Year 1 payment). The Year 1 payment at Year 5 will be worth $1,000 x $(1 + 0.12)^{4}$ = $1,573.52

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PAYMENT YEAR	AMOUNT	VALUE at YEAR 5
1	$1,000	$1,573.52 (4 years later)
2	$1,000	$1,404.93 (3 years later)
3	$1,000	$1,254.40 (2 years later)
4	$1,000	$1,120.00 (1 years later)
5	$1,000	$1,000.00 (0 years later)
	TOTAL:	$6,352.85

The Year 5 values of the remaining annual payments can be found the same way. The future value of this cash flow stream is the sum of these values, $6,352.85.

Alternatively, we can make life a little simpler by using the formula: $FV_{ANNUITY} = PMT\ [\ (1+i)^{n}-1\ /\ i\ ]$

With this relationship between future value, discount rate, payment amount, and number of years, it is easy to solve for any one value if the other three are known.

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Activity: Calculate the Future Value of an Annuity



Consider a simple example in which a series of $1,000 payments will be paid at the end of each of the next 5 years. The appropriate discount rate is 12%. What is the value of the annuity in 5 years?

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STEPS	KEY SEQUENCE	DISPLAY
CLEAR the financial registers.	[f][REG]	0.00
ENTER the number of years.	5 [n]	5.00
ENTER the interest rate.	12 [i]	12.00
ENTER the current value.	0 [PV]	0.00
ENTER the payment amount.	1,000 [CHS][PMT]	-1,000.00
CALCULATE the value in 5 years.	[FV]	6,352.00

This demonstration shows you how to use your calculator to find the future value of an annuity using the TI BA II Plus. Download *Printer Friendly Steps for the TI BA II Plus* .

Note to Texas Instruments calculator users: If your calculator produces a different answer than the one in this example, you may have to change your "payments per year" settings. In all of the problems and examples in this course, payments are made and interest is compounded annually. However, by default, the TI calculator bases its calculations on monthly

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payments and compounding. You may find these TI Calculator Basics helpful.

STEPS	KEY SEQUENCE	DISPLAY
CLEAR the financial registers.	[2nd]][FV]	0.00
ENTER the number of years.	5 [N]	5.00
ENTER the interest rate.	12 [I/Y]	12.00
ENTER the current value.	1,000 [PMT]	1,000.00
ENTER the payment amount.	1,000 [+/-][PMT]	-1,000.00
CALCULATE the value in 5 years.	[CPT][FV]	6,352.00

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Activity: Future Value of an Annuity

You can now complete the Future Value of an Annuity row in your chart.

To complete this activity:

1. Open the TVM Overview Chart template you downloaded
2. Add the following information to the Future Value of an Annuity row:

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Watch: Combining TVM Tools

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Module Wrap-up: Future Values

In this module, you examined future values and calculated the future value of an annuity. You saw the relevance of future values to an overall sound understanding of financial management practices.

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Read: Thank You and Farewell

Congratulations on completing *Mastering the Time Value of Money*. We hope that you now feel completely comfortable with the topics we've covered here. We hope that the material covered has met your expectations and prepared you to better interact with the financial managers in your firm. From all of us at Cornell University and eCornell, thank you for participating in this course.

Sincerely,





Professor Steve Carvell Professor Scott Gibson

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Glossary

annuity

An annuity is a series of equal payments made over a finite number of periods. The equal stream of cash flows can either be paid out, as in the case of a mortgage payment, or received, as in the case of a retirement annuity.

cash flow

The cash amount paid out or received over a period of time.

compounding

Compounding involves moving cash flows from the present into the future, and is the way we show how an initial deposit earns interest on interest over time. For example, if you put $100 in the bank today and earned 10% interest on the funds at the end of the year, you would have earned $10 of interest. If you then decided to leave it in the bank for a second year at 10%, you would earn another $10 on your original deposit and another $1 interest on your interest, for a total interest of $11. The process of earning interest on interest results in the balance growing progressively after each successive year, and continues until you withdraw money from the account.

compounding factor

Defined as
$(1+i)^n$
. The compounding factor is used to make calculations that move cash flows forward in time.

discounting

Discounting involves moving cash flows from the future to the present, and is the way we calculate the value of an amount of money to be received in the future in today dollars. For example, if you were to receive $100 in ten years, how much is that worth to you in today dollars? Also, the further in the future that an amount is to be received, the less it is worth (at any given interest rate) in today dollars.

discounting factor

Defined as 1/
$(1+i)^n$
. The discounting factor is used to make calculations that move cash flows backward in time.

financial calculator

A calculator that has financial functions, such as the time-value-of-money keys: n, i, PV, PMT, and FV.

lump-sum cash flow

A lump-sum cash flow is a one-time cash flow.

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perpetuity

A perpetuity is a series of equal payments made at a fixed interval forever (in perpetuity).

perpetuity, growing

A growing perpetuity is a series of payments that grow at a constant rate over a fixed interval in perpetuity. It differs from a simple perpetuity in that the payments grow at a constant rate, rather than remaining the same.

sinking fund

A sinking fund is a stipulation found in some bond contracts that requires the bond issuer to pay off part of the bond issue at regular intervals before maturity. On some occasions, the bond issuer may be required to set aside money with a trustee, who invests the funds and then uses the accumulated savings to pay off the bonds at maturity.

TVM

Abbreviation for time value of money.

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