1 Crayon Combos

Georgette has a box of crayons with six different colors: red, orange, yellow, green, blue, and purple. She is going to color a map and wants to use only three colors.

How many different ways can she pick three colors to use?

Name

2 Fire Up the Grill!

Tammy’s family is having a barbecue for 15 guests. Her mother wants to serve the chicken no later than 6:30 p.m. It takes 20 to 30 minutes for the charcoal to be hot enough to cook the chicken. Then it takes 30 to 40 minutes for the chicken to cook.

At what time will Tammy’s father start the charcoal to be sure the food is ready on time?
1 Crayon Combos

Mathematics Concepts and Skills

Focus: Counting Outcomes
       Combinations
       Sample Space

Related Topic: Probability

Problem-Solving Strategy
✦ Make an Organized List

About the Mathematics

A combination is a selection of items that do not have to be arranged in any particular order. In this problem, students will realize that any arrangement of the same three colors is the same combination; for example, red–yellow–blue is the same as yellow–blue–red or blue–red–yellow, and so on. Students must use an organized approach to listing so that they include each color combination and do not repeat any combinations.

When students begin to study probability, they must take into account all the possible ways that an event might occur. This is usually referred to as the sample space for the event. The listing strategy in this lesson displays all the combinations, so they can be counted and analyzed further if necessary. If students are introduced to this method early in the year, they can then apply it to other situations.

After distributing the problem to the class, have students work in pairs or small groups to find a solution. Once they have had a chance to work, ask students to share their strategies and solutions, or encourage students to find other ways to solve the problem. Use the problem-solving strategies presented in this lesson as a basis for class discussion. Try to relate the Key Questions to students’ methods that may differ from the ones presented here.

Problem-Solving Strategy
✦ Make an Organized List

Each color can be used in combination with two other colors. Start with one color and list all the combinations. Then work with the next color and list all the combinations that do not contain the first color. Continue in this way until all possible combinations are listed.

red–orange–yellow       orange–yellow–green
red–orange–green       orange–yellow–blue
red–orange–blue        orange–yellow–purple
red–orange–purple      orange–green–blue
red–yellow–green       orange–green–purple
red–yellow–blue        orange–blue–purple
red–yellow–purple      yellow–green–blue
red–green–blue         yellow–green–purple
red–green–purple       yellow–blue–purple
red–blue–purple        yellow–blue–purple

There are 20 possible ways for Georgette to choose three colors.

Georgette has a box of crayons with six different colors: red, orange, yellow, green, blue, and purple. She is going to color a map and wants to use only three colors. How many different ways can she pick three colors to use?
Key Questions
1. How many colors will Georgette use to color her map? (Three)
2. How many colors does she have from which to choose? (Six) What are they? (Red, orange, yellow, green, blue, purple)
3. Is red–blue–green a different combination than blue–green–red? (No)
4. If Georgette chooses red for her first color, what colors can she choose for her second color? (Orange, yellow, green, blue, purple)
5. If Georgette chooses red and orange, how many colors does she have left from which to select? (Four)

Teacher Tip
There are six choices for the first color, five choices for the second color, and four choices for the third color, so there are $6 \times 5 \times 4 = 120$ arrangements in all. However, duplications must be eliminated. Since each selection of three different colors can be arranged six ways ($3 \times 2 \times 1 = 6$), the number of different combinations of three colors from six colors is $120 \div 6 = 20$.

Assessing Understanding
Use the following problem to assess students’ understanding of the mathematical concepts and strategies in this lesson.


Key Questions
1. How many choices does Georgette have for the first color? (Six)
2. Once she picks the first color, how many choices does she have for the second color? (Five)
3. Suppose she listed all the different pairs of first color–second color. How many pairs would there be? (30) How do you know? (Each of the six colors could be matched with five other colors, and $5 \times 6 = 30$.)
4. Would all 30 pairs be different? (No)
5. Suppose she listed the pair red–orange. What other pair is the same combination? (Orange–red)
6. How many different combinations of two colors are there? (15) How many duplicates? (15)
Extending the Mathematics

Extending the Mathematics provides opportunities for students to consider the problem with a new condition. This section may also provide opportunities to introduce other mathematical concepts.

[Problem 1] Georgette needed four colors for another map. How many different combinations of four colors can she choose from six crayons? (15)


Key Questions

1. If Georgette picks red, orange, and yellow, how many colors can she choose from for the fourth color? (Three) What are they? (Green, blue, purple)

2. Do you think there will be more or fewer combinations of four colors than there were with three colors? (Fewer) Why? (Possible answer: You must use more colors for each combination.)

Problem-Driven Math

[Problem 2] For the original situation of picking three colors, suppose Georgette picks three crayons without looking.

a. What is the probability that her combination will include a red crayon? ( or )

b. What is the probability that her combination will include a purple crayon? ( or )

c. What is the probability that her combination will be orange–green–blue? ( )

Key Questions

1. How many possible combinations can Georgette pick? (20)

2. Is each combination equally likely? (Yes)

3. How many combinations include red? (10) How many include purple? (10)

Teacher Tip

Point out to students that selecting four colors from six or selecting two colors from six yields the same answer: 15. This is because selecting four colors to use is the same as selecting two colors not to use.
2 Fire Up the Grill!

Mathematics Concepts and Skills

Focus: Elapsed Time
Intervals of Time
Adding and Subtracting with Time

Problem-Solving Strategies
✧ Make a Drawing
✧ Work Backward

About the Mathematics

This problem involves time measurement. Students must be able to use intervals of time and work backward from a final time to determine a starting time. There are several approaches that will lead to a correct answer. Students may use the intervals in steps or add to find the total time required and then subtract that total from the ending time.

Use this lesson to introduce the topics of time measurement and elapsed time. Be sure students know that one hour is equal to 60 minutes and that a.m. hours are from midnight to noon and p.m. hours are from noon to midnight.

After distributing the problem to the class, have students work in pairs or small groups to find a solution. Once they have had a chance to work, ask students to share their strategies and solutions, or encourage students to find other ways to solve the problem. Use the problem-solving strategies presented in this lesson as a basis for class discussion. Try to relate the Key Questions to students’ methods that may differ from the ones presented here.

Problem-Solving Strategies
✧ Make a Drawing

Draw a time line with ten-minute intervals from 5:00 p.m. to 6:30 p.m. (Since each step is less than one hour, the total time is less than two hours.) Put the longer time intervals for each process below the line, and put the shorter time intervals above the line. To be sure the chicken is ready by 6:30 p.m., use the starting times shown for the greatest amounts of cooking times.

Tammy’s dad will start the charcoal at 5:20 p.m.

✧ Work Backward
Method 1

Use the greatest amount of cooking time for each process. Start with the time dinner is to be served and subtract the minutes.

Cooking takes at most 40 minutes.

40 minutes = 30 minutes + 10 minutes
6:30 means 6 hours 30 minutes
       − 30 minutes
       6 hours
Rewrite 6 hours as 5 hours 60 minutes
       − 10 minutes
       5 hours 50 minutes

The heating time is at most 30 minutes.

5 hours 50 minutes
       − 30 minutes
       5 hours 20 minutes

Tammy’s dad will start the charcoal at 5:20 p.m.

**Method 2**
Add the greatest amounts of time for both steps to find the total time.

30 minutes + 40 minutes = 70 minutes = 1 hour 10 minutes

Subtract.

6:30 means  6 hours 30 minutes
       − 1 hour 10 minutes
       5 hours 20 minutes

Tammy’s dad will start the charcoal at 5:20 p.m.

**Assessing Understanding**
Use the following problem to assess students’ understanding of the mathematical concepts and strategies in this lesson.

**[Problem]** It is New Year’s Eve and Marcel’s family is having a late barbecue dinner. It takes 35 to 45 minutes to cook the ribs, and it takes 20 to 30 minutes to heat the charcoal. If the family wants to eat at 12:05 a.m., at what time will Marcel’s brother start the charcoal? (10:50 p.m.)

**Key Questions**
1. What happens to the time at 12:00 midnight? (The hours change from p.m. to a.m.)
2. If Marcel’s brother wants to cook the ribs for the greatest amount of time, when would he put them on the grill? (11:20 p.m.)
3. When could he start the charcoal to allow it the least amount of time to heat? (11:00 p.m.) To allow the greatest amount of time? (10:50 p.m.)

**Key Questions**
1. How many minutes are in one hour? (60)
2. At what time is dinner to be served? (6:30 p.m.)
3. What is the cooking time for the chicken? (30 to 40 minutes)
4. To cook the chicken for the least amount of time, when would Tammy’s mom put it on the grill? (6:00 p.m.)
5. To cook the chicken for the greatest amount of time, when would Tammy’s mom put it on the grill? (5:50 p.m.)
6. What is the greatest amount of time it takes the coals to heat? (30 minutes)
7. Is there any extra information given? (Yes, that 15 people are coming)
Extending the Mathematics

Extending the Mathematics provides opportunities for students to consider the problem with a new condition. This section may also provide opportunities to introduce other mathematical concepts.

[Problem 1] Tyra is making a birthday cake for her sister’s party. It will take her 30 minutes to mix the cake, 50 minutes to bake it, one hour to cool it, and 20 minutes to frost it. If Tyra wants to serve the cake at 4:30 p.m., what is the latest she can start preparing it? (1:50 p.m.)

Key Questions
1. What is the total time in minutes that it takes to make the cake? (160 minutes)
2. How many hours and minutes are there in 160 minutes? (2 hours 40 minutes)
3. If it took exactly three hours to make the cake, when could Tyra start? (1:30 p.m.)
4. How many minutes less than three hours does it actually take to make the cake? (20)
5. What is the latest time Tyra can start to make the cake? (1:50 p.m.)

[Problem 2] Rick is cooking a 5-pound roast. The cookbook said to allow 18 to 22 minutes of cooking time per pound. If Rick puts the roast in the oven at 5:30 p.m., what is the earliest time it will be ready? (7:00 p.m.) The latest time? (7:20 p.m.)

Key Questions
1. What is the least amount of cooking time for the roast? (90 minutes)
2. How many hours and minutes are there in 90 minutes? (1 hour 30 minutes)
3. What is the greatest amount of cooking time for the roast? (110 minutes)
4. How many more minutes is the greatest amount of time than the least amount of time? (20)