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*Full documents may be downloaded at
Introduction and Purpose

“Computational fluency — having and using efficient and accurate methods for computing — is essential. Students should be able to perform computations in different ways, including mental calculations, estimation, and paper–and–pencil calculations using mathematically sound algorithms… Computational fluency should develop in tandem with understanding.”

Principles and Standards for School Mathematics
NCTM 2000–2004

There has been agreement among educators over countless years that many K–12 students lack accuracy and automaticity in mathematical computation. This is not a new challenge for upper elementary, middle years, and high school teachers, and college professors. Parents also express concern that they cannot help students with computation because many of today’s teachers expect students to demonstrate understanding when using procedures. There has been controversy over how computational fluency is developed, and which is more important: procedural automaticity or conceptual understanding.

An analysis of number sense items on the Washington Assessment of Student Learning (WASL) shows that students meeting the standard tend to respond correctly to number sense items. Lower performing students are also likely to give correct responses. Overall, Washington students do well on number sense. When students are required to use computation in complex problem situations, they do not perform as well as on number sense items alone.

The Computational Fluency Supplemental Program Review is part of a larger computational fluency project that is addressing the gap in student performance related to automaticity and complex problem solving. The goal of the project is to increase K–12 students’ computational fluency with understanding and automaticity as measured by an increasing in the number of students meeting or exceeding standard as defined by Washington mathematics standards.

The objectives of the larger project are to:
- Develop a common, research-based understanding of computational fluency;
- Determine how students develop proficiency that includes both understanding and automaticity;
- Identify instructional models and resources; and
- Develop teacher instructional expertise to help advance student proficiency.

Purpose

The purpose of the program review was two-fold: 1) to report on evidence of alignment to Washington mathematics standards and 2) identify the teacher usability of published computational fluency supplemental programs (resources) submitted for review.

With this purpose in mind, the Office of Superintendent of Public Instruction (OSPI) convened a panel of educators to review commercially available mathematics computational fluency supplemental programs. Members of the OSPI Mathematics Initiative Team coordinated the review. The review panel of forty Washington educators represented classroom teachers, administrators, mathematics coaches, and teachers on special assignment from districts across the state.
This report provides alignment information about the computational fluency supplemental programs submitted for review. The intent of this report is not to endorse specific commercially published programs or materials.

Districts should further investigate programs and take into consideration the needs of their students, resources available, and professional development required to implement a program with fidelity. An additional resource available on the OSPI Web site is *Using Response to Intervention (RTI) for Washington’s Students*. Periodically updated reviews of current research as to the efficacy of mathematics programs can be located at *What Works Clearinghouse*, U.S. Department of Education; Metiri Group, *Technology Solutions That Work*; and *The Best Evidence Encyclopedia*, Johns Hopkins University. (See Appendix A)
Mathematics Teaching and Learning

OSPI subscribes to a philosophy of teaching and learning mathematics in which:

- Mathematics is a language and science of patterns.

- Mathematical content (Essential Academic Learning Requirements - EALR 1: NS, ME, GS, PS, AS) must be embedded in the mathematical processes (EALRs 2–5: SP, RL, CU, MC).

- For all students to learn significant mathematics, content must be taught and assessed in meaningful situations.

Connecting Content and Process in a Meaningful Context

“The relationship between content and process in mathematics is critical in developing mathematically proficient students. The combination of content and process gives students mathematical power. Teachers need to use instructional practices that provide opportunities for students to practice and demonstrate proficiency in both content and process on a regular basis. All process requires mathematical content. Lessons designed by the teacher must have content at or below grade level. A student cannot reason without something to reason about.”

Washington Mathematics Grade Level Expectations (GLEs), 2006
**Scope of the Review**

A Publisher Notice was posted on the OSPI Web site inviting publishers to submit K–12 computational fluency supplemental programs for review during the week of March 26–30, 2007. Publishers submitted Intent to Submit forms that indicated which programs would be submitted for the review. Upon confirmation of inclusion in the review, publishers completed the Program Information Document and Computational Fluency Matrix for each program submitted. Both text-based and/or computer-based programs were included in the review. *(See Appendix B)*

Washington educators were invited to apply to participate in the review. The goal was to have enough reviewers to complete four “reads” for each grade level for each program.

The review week schedule was intensive, with many reviewers staying later or arriving earlier than the scheduled times.

<table>
<thead>
<tr>
<th>Day</th>
<th>Time</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>8 a.m.–5 p.m.</td>
<td>Training/Review</td>
</tr>
<tr>
<td>Tuesday</td>
<td>8 a.m.–5 p.m.</td>
<td>Review</td>
</tr>
<tr>
<td>Wednesday</td>
<td>8 a.m.–5 p.m.</td>
<td>Review</td>
</tr>
<tr>
<td>Thursday</td>
<td>8 a.m.–5 p.m.</td>
<td>Review</td>
</tr>
<tr>
<td>Friday</td>
<td>8 a.m.–3:30 p.m.</td>
<td>Review</td>
</tr>
</tbody>
</table>

**Computer-based Programs (▲)**

The design of many computer-based programs (▲) presented a considerable challenge in the review process. While publishers design computer programs to facilitate student access, they were sometimes not accessible for review purposes. Reviewers needed direct access to assessments, instruction, and practice without having to work through multiple prompts. Unfortunately, the review did not have the additional resources (time, computers, and reviewers) to navigate these challenges efficiently. Some computer programs received three “reads” rather than four “reads” per grade level because of a high total number of grade levels that had to be reviewed by reviewers on one computer.

In some cases a portion of a program received a mark of 0 (No evidence that meets criteria) and a note on the review document of “CNA” indicating, “Could Not Access”, meaning the program was not accessible to reviewers for that data point. “CNA” notations were not included in the report of review data. OSPI strongly recommends that schools and districts consider not only the review results, but also the actual computer-based programs in order to ensure appropriate selection based on the needs of their students.

<table>
<thead>
<tr>
<th>Program</th>
<th>CNAs</th>
<th>Total # Data Points</th>
<th>% CNA</th>
</tr>
</thead>
<tbody>
<tr>
<td>MathFacts in a Flash ▲</td>
<td>73</td>
<td>640</td>
<td>11%</td>
</tr>
<tr>
<td>Larson Learning Math Courseware ▲</td>
<td>89</td>
<td>957</td>
<td>9%</td>
</tr>
<tr>
<td>ALEKS Online Mathematics ▲</td>
<td>103</td>
<td>1187</td>
<td>9%</td>
</tr>
<tr>
<td>Destination Math ▲</td>
<td>42</td>
<td>1036</td>
<td>4%</td>
</tr>
<tr>
<td>Passkey: A Prescriptive Learning System ▲</td>
<td>36</td>
<td>1147</td>
<td>3%</td>
</tr>
<tr>
<td>Math Expeditions ▲</td>
<td>31</td>
<td>1093</td>
<td>3%</td>
</tr>
<tr>
<td>Foundational Mathematics ▲</td>
<td>6</td>
<td>852</td>
<td>1%</td>
</tr>
<tr>
<td>FASTT Math ▲</td>
<td>8</td>
<td>1331</td>
<td>1%</td>
</tr>
<tr>
<td>MathXL ▲</td>
<td>4</td>
<td>687</td>
<td>1%</td>
</tr>
<tr>
<td>Cognitive Tutor Bridge to Algebra ▲</td>
<td>1</td>
<td>592</td>
<td>0%</td>
</tr>
<tr>
<td>Algebra 1, Parts 1 and 2 ▲</td>
<td>0</td>
<td>595</td>
<td>0%</td>
</tr>
<tr>
<td>Number Worlds Text and/or ▲</td>
<td>0</td>
<td>876</td>
<td>0%</td>
</tr>
<tr>
<td><strong>Grand Totals</strong></td>
<td><strong>393</strong></td>
<td><strong>10993</strong></td>
<td><strong>4%</strong></td>
</tr>
</tbody>
</table>
**Grade Level Review**

During the course of the review, it became apparent that some programs submitted might address GLEs at grades other than those indicated by the publisher. While students struggling with computational fluency likely need to begin below grade level, instruction still needs to accelerate students to grade level standards. **It was not in the scope of this review to determine to which grade level the program best aligned. Reviewers reviewed programs at the grade level(s) indicated by publishers.**

**Instruction and Practice or Practice-Only Programs**

While publishers did not indicate whether the computational fluency programs submitted were for the purpose of instruction and practice or solely for additional practice of particular concepts or procedures, the K–12 Mathematics Computational Fluency Supplemental Program Review Panel reviewed both types of programs. OSPI strongly recommends that schools and districts consider not only the review results, but also the actual programs in order to ensure appropriate selection based on the needs of their students that may not be sufficiently addressed by their core/comprehensive program and therefore needs supplementation (e.g., assessment, instruction, and/or practice).

**Professional Development and Implementation Support**

Programs vary in terms of the amount and intensity of the professional development needed to use the program with fidelity. Some programs require little or no training to implement while others require more training with ongoing support. **It was not in the scope of this review to determine the professional development and support needed to implement and use programs with fidelity.** Districts should investigate programs to determine the professional development required to implement a program efficaciously.
# Programs Reviewed

Reviewers reviewed 40 computational fluency programs from 23 publishers. Publishers who followed the Intent to Submit process delineated in the Publishers Notice on the OSPI Web site voluntarily submitted the programs reviewed. *(See Appendix B)* This list does not represent all of the computational fluency supplemental programs that are commercially available.

For more information about these and other programs, districts should periodically visit updates at these Web sites: *Metiri Group: Technology Solutions That Work, the What Works Clearinghouse* from the Department of Education, and *The Best Evidence Encyclopedia* from Johns Hopkins University. *(See Appendix A)*

<table>
<thead>
<tr>
<th>Program (▲ Computer-based program)</th>
<th>Publisher</th>
<th>Copyright Date(s)</th>
<th>Grades Reviewed</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGS Algebra</td>
<td>Pearson AGS Globe</td>
<td>2004</td>
<td>7–12</td>
</tr>
<tr>
<td>AGS Basic Math Skills</td>
<td>Pearson AGS Globe</td>
<td>2003</td>
<td>7–12</td>
</tr>
<tr>
<td>AGS Pre-Algebra</td>
<td>Pearson AGS Globe</td>
<td>2004</td>
<td>7–12</td>
</tr>
<tr>
<td>ALEKS Online Mathematics ▲</td>
<td>ALEKS Corporation</td>
<td>2006</td>
<td>3–12</td>
</tr>
<tr>
<td>Algebra 1, Parts 1–2 ▲</td>
<td>PLATO Learning</td>
<td>2005</td>
<td>7–12</td>
</tr>
<tr>
<td>Boosting Your Math Skills</td>
<td>Wright Group/McGraw-Hill</td>
<td>2007</td>
<td>1–6</td>
</tr>
<tr>
<td>Building Computational Fluency</td>
<td>The Math Learning Center</td>
<td>2007</td>
<td>1–6</td>
</tr>
<tr>
<td>Cognitive Tutor Bridge to Algebra ▲</td>
<td>Carnegie Learning, Inc.</td>
<td>2006</td>
<td>6–8</td>
</tr>
<tr>
<td>Context for Learning Mathematics Series</td>
<td>Heinemann</td>
<td>2007</td>
<td>K–6</td>
</tr>
<tr>
<td>Destination Math ▲</td>
<td>Riverdeep Learning Management System</td>
<td>2002</td>
<td>K–9</td>
</tr>
<tr>
<td>Every Day Counts</td>
<td>Great Source Education Group</td>
<td>2005</td>
<td>K–6</td>
</tr>
<tr>
<td>FASTT Math ▲</td>
<td>Tom Snyder Productions</td>
<td>2006</td>
<td>2–12</td>
</tr>
<tr>
<td>Figure It!</td>
<td>ORIGO Education, Inc.</td>
<td>2006</td>
<td>1–6</td>
</tr>
<tr>
<td>Foundational Mathematics ▲</td>
<td>PLATO Learning</td>
<td>2002</td>
<td>6–12</td>
</tr>
<tr>
<td>Fundamentals</td>
<td>ORIGO Education, Inc.</td>
<td>2003</td>
<td>K–6</td>
</tr>
<tr>
<td>Mastering Math</td>
<td>Harcourt Achieve</td>
<td>2004</td>
<td>1–6</td>
</tr>
<tr>
<td>Program</td>
<td>Publisher</td>
<td>Copyright Date(s)</td>
<td>Grades Reviewed</td>
</tr>
<tr>
<td>-------------------------</td>
<td>---------------------------------------</td>
<td>-------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>Math Around the Clock</td>
<td>Pearson Scott-Foresman</td>
<td>2003</td>
<td>1–6</td>
</tr>
<tr>
<td>Math Expeditions ▲</td>
<td>PLATO Learning</td>
<td>2000</td>
<td>K–8</td>
</tr>
<tr>
<td>Math Skill Builder</td>
<td>Pearson Prentice Hall</td>
<td>2007</td>
<td>6–8</td>
</tr>
<tr>
<td>Mathematics Navigator</td>
<td>America’s Choice</td>
<td>2006</td>
<td>2–10</td>
</tr>
<tr>
<td>Mathmentals</td>
<td>ORIGO Education, Inc.</td>
<td>2003</td>
<td>1–6</td>
</tr>
<tr>
<td>MathFacts in a Flash ▲</td>
<td>Renaissance Learning, Inc.</td>
<td>2006</td>
<td>1–5</td>
</tr>
<tr>
<td>MathXL ▲</td>
<td>Pearson Prentice Hall</td>
<td>2004</td>
<td>6–12</td>
</tr>
<tr>
<td>Nimble with Numbers</td>
<td>Modern Curriculum Press</td>
<td>2002</td>
<td>1–6</td>
</tr>
<tr>
<td>Number Corner</td>
<td>The Math Learning Center</td>
<td>2007</td>
<td>K–5</td>
</tr>
<tr>
<td>Number Worlds Text and/or ▲</td>
<td>SRA/McGraw-Hill</td>
<td>2007</td>
<td>K–6</td>
</tr>
<tr>
<td>ORIGOmath: A Step by Step Approach to Computation</td>
<td>ORIGO Education, Inc.</td>
<td>2007</td>
<td>1</td>
</tr>
<tr>
<td>Skills Intervention Kit</td>
<td>Pearson Prentice Hall</td>
<td>2005</td>
<td>6–9</td>
</tr>
<tr>
<td>The Book of Facts</td>
<td>ORIGO Education, Inc.</td>
<td>2007</td>
<td>1–5</td>
</tr>
<tr>
<td>The Box of Facts</td>
<td>ORIGO Education, Inc.</td>
<td>2007</td>
<td>1–6</td>
</tr>
<tr>
<td>The Think Tank</td>
<td>ORIGO Education, Inc.</td>
<td>2005</td>
<td>2–6</td>
</tr>
<tr>
<td>Top Line Math</td>
<td>Harcourt Achieve</td>
<td>2006</td>
<td>8–12</td>
</tr>
<tr>
<td>Working with Numbers</td>
<td>Harcourt Achieve</td>
<td>2004</td>
<td>1–6</td>
</tr>
</tbody>
</table>
Overview of Review Process

Prior to the review, all submitted programs were pre-screened based on the Program Information Document to confirm they were supplemental computational fluency, and not core/comprehensive mathematics, programs. No programs submitted were excluded before the review.

During the K–12 Mathematics Computational Fluency Supplemental Program Review, the reviewers followed a two-part program review process. Reviewers participated in training, completed their reviews independently, and refrained from sharing information, ratings, and comments with other reviewers. A minimum of four reviewers conducted the two parts of the review for each program and grade level. (See Appendix C)

Part 1: Grade Level Review
For Grade Level Expectations (GLEs) 1.1.3, 1.1.4, 1.1.5, and 1.1.6 (columns in the review document), reviewers located and recorded evidence of assessments, instruction, and practice (rows in the review document) in each program. GLE 1.1.7 was included in the instruction and practice rows (Instruction #5 and Practice #1). (See Appendix D)

The "Total Number of Criteria" refers to the number of specific criteria identified in each of GLEs 1.1.3–1.1.6 at each grade level.

<table>
<thead>
<tr>
<th>Grade Level Review Rubric</th>
<th>Total Number of Criteria (x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td>Indicator</td>
</tr>
<tr>
<td>4</td>
<td>Meets or exceeds all criteria</td>
</tr>
<tr>
<td>3</td>
<td>Sufficiently meets criteria</td>
</tr>
<tr>
<td>2</td>
<td>Minimally meets criteria</td>
</tr>
<tr>
<td>1</td>
<td>Meets some, but not all, criteria</td>
</tr>
<tr>
<td>0</td>
<td>Meets no criteria</td>
</tr>
</tbody>
</table>

Part 2: Program Feature Review
Reviewers used the familiarity with the program materials developed during Part 1 of the review to record evidence for each of the Program Feature Review criteria located in each program and grade level. If not included in the manual, reviewers also looked at student materials. (See Appendix D)

<table>
<thead>
<tr>
<th>Program Feature Review Rubric</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td>Indicator</td>
</tr>
<tr>
<td>0</td>
<td>No</td>
</tr>
<tr>
<td>1</td>
<td>Yes</td>
</tr>
</tbody>
</table>

A full description of each of the two parts of the review process follows.

1 A minimum of three reviewers completed the two parts of the review for each grade level for some computer programs, particularly if there were a high number of grade levels in the program.
Part 1: Grade Level Review

Alignment for Student Achievement

“To ensure student achievement in mathematics, it is critical that the curriculum, instruction, and assessment align. The Essential Academic Learning Requirements, including the Grade Level Expectations, provide the foundation for the definition of the curriculum by the school district. Curriculum documents may take a variety of forms: a district curriculum guide or scope and sequence, a course syllabus, or a unit or lesson plan. Instruction refers to the implementation of the defined curriculum, which includes pedagogy and the use of aligned instructional resources. Assessment should take many forms, including diagnostic, classroom-based formative and summative assessments that measure student proficiency of the defined curriculum. Alignment exists when students are assessed on what they have been taught and when what they have been taught aligns with the state standards.”

Washington Mathematics Grade Level Expectations (GLEs), 2006

The Grade Level Review focused on reviewers locating and citing evidence of computational fluency Assessment, Instruction, and Practice from each of the programs as to its alignment with Grade Level Expectations 1.1.3 (properties), 1.1.4 (ratio and proportion), 1.1.5 (operations), and 1.1.6 (procedures). Grade Level Expectation 1.1.7 (strategies) was included in Instruction data point #5 and Practice data point #1 in the review document. A description of each Grade Level Review elements and the rubric for Grade Level Review follows. (See Appendix D for Grade Level Review documents.)
Assessment

Effective mathematics instruction utilizes different types of assessment for different purposes. Reviewers cited evidence of pre-assessment, formative assessment, and summative/post-assessment for each GLE specified at the grade level reviewed.

- **Pre-assessment** is a process (test, task, or interview) used to establish a starting point prior to instruction by determining what the student already knows and is able to do. Pre-assessments used with post-assessments measure student learning for a specific period of instruction or time span.

  - **Screening assessments** identify both at-risk and at-grade-level students. Teachers may use student work from the core/comprehensive mathematics program for this purpose.
  
  - **Diagnostic Assessment** identifies what a student knows, identifies the nature of a student’s misconceptions, and indicates the intervention/instruction needed to move the student to standard.

- **Formative Assessment** is a process (test, task, or interview) used frequently to identify conceptual and computational errors, to guide learner and teacher to corrective instruction/practice, and to adjust learning and instruction.

  - **Progress Monitoring Assessments** determine a student’s rate of progress toward achieving proficiency on the EALRS/GLES, identify students who are not demonstrating adequate progress in daily lessons, and influence instructional decisions.

- **Summative Assessment, or post-assessment**, is a process (test, task, or interview) used at the end of a period of intervention/instruction, which includes all concepts taught over a given time, unit, and/or strand. They may be used to identify the cumulative understandings for each learner, to indicate strengths and weaknesses of program and instruction provided, and to provide a map of conceptual development and progress. Along with pre-assessments, summative assessments can be used to measure student learning resulting from of a period of intervention/instruction.

  - **Outcome Assessments** can serve as a screening at the beginning of the school year as well as a measure of growth at the end of the year. Data may be used to identify students in terms of those who achieved grade level expectations, those who made progress, and those who did not.

  - **Statewide achievement tests**, such as the Washington Assessment of Student Learning (WASL), measure the level of proficiency that students have achieved based on the state standards (EALRs and the GLEs).

Reviewers sought and cited a minimum number of pieces of evidence for each GLE criterion, as applicable to each grade:

1. **Pre/Diagnostic Assessment** – at least 1 piece of evidence for each GLE criterion

2. **Formative Assessment/Progress Monitoring** – at least 2 pieces of evidence for each GLE criterion

3. **Summative Assessment** – at least 1 piece of evidence for each GLE criterion
_instruction

“Learning mathematics involves learning ways of thinking. It involves learning powerful mathematical ideas…it also entails learning how to generate those ideas, how to express them using words and symbols, and how to justify to oneself and to others that those ideas are true.”

Thomas Carpenter, 2003

Focused instruction helps students whose mathematics achievement is significantly below current grade level standards and accelerates their learning to grade level standard.

Reviewers followed each mathematics concept or procedure specified from the Grade Level Expectations through the program and cited a minimum number of pieces of evidence within the program for each GLE criterion as applicable to each grade for instruction that:

1. **Develops conceptual understanding using multiple representations**
   (e.g., symbolically, with a model, or a real-world situation) – at least 2 pieces of evidence for each GLE criterion

2. **Makes connections explicit among representations** – at least 2 pieces of evidence for each GLE criterion

3. **Uses a variety of problem types** (e.g. start, change, and result unknown; compare; grouping, rate, price, multiplicative comparison; partitive and measurement division) *(See Problem Types and Examples in Appendix B)* – at least 2 pieces of evidence for each GLE criterion

4. **Uses computation in a variety of problem situations** (contexts) – at least 2 pieces of evidence for each GLE criterion

5. **Develops efficient, accurate strategies** *(See Children’s Strategies in Appendix B)* – at least 2 pieces of evidence for each GLE criterion

6. **Develops efficient, accurate procedures and algorithms** – at least 2 pieces of evidence for each GLE criterion
**Practice**

“…evidence clearly suggests that for most students, mastery and understanding come through, not after, meaningful interaction with ideas.”

Carol Ann Tomlinson and Jay McTighe

Integrating Differentiated Instruction and Understanding by Design, 2006

Focused practice opportunities accelerate student learning and facilitate their acquisition of and proficiency with a new concept or procedure. Practice should provide multiple, varied opportunities scheduled over time to develop and strengthen proficiency. Practice should also integrate new learning with previous learning.

Reviewers followed each mathematics concept or procedure specified from the Grade Level Expectations through the program and cited a minimum number of pieces of evidence within the program for each GLE criterion as applicable to each grade:

1. **Use efficient, accurate strategies** – at least 2 pieces of evidence for each GLE criterion

2. **Use efficient, accurate procedures and algorithms** – at least 2 pieces of evidence for each GLE criterion

3. **Solve a variety of problem types** (e.g. start, change, and result unknown; compare; partitive and measurement division) – at least 2 pieces of evidence for each GLE criterion

4. **Solve problems in a variety of situations** (contexts) – at least 2 pieces of evidence for each GLE criterion
Grade Level Expectations

Grade Level Expectations (GLEs) 1.1.3, 1.1.4, 1.1.5, 1.1.6, and 1.1.7 are all related to computational fluency and provided the grade level content foundation for the Grade Level Review.

- **Grade Level Expectations**

  **EALR 1:** The student understands and applies the concepts and procedures of mathematics.
  - GLE 1.1.3 (properties of operations)
  - GLE 1.1.4 (ratio and proportion)
  - GLE 1.1.5 (understanding of operations)
  - GLE 1.1.6 (computational procedures with specific operations and number sets)
  - GLE 1.1.7 (computational strategies)*

  *Included in Instruction and Practice (rows) in the review document (See Appendix C for GLEs 1.1.3–1.1.7 for each grade level.)

- **Organization of the GLEs**

  The organization of the GLEs for the review is shown using GLE 1.1.3 from Grade 4. Reviewers used a document with only GLEs 1.1.3–1.1.6 for grades K–11/12 for reference purposes during the review. The content criteria were underlined and numbered (e.g., properties of addition\(^{(1–3)}\)). In each GLE to further assist reviewers during the review. (See Appendix B – for reporting purposes GLE 1.1.7 is included for each of the grade levels.)

**K–12 EALR Statement:** EALR 1: The student understands and applies the concepts and procedures of mathematics.

**K–12 Component:** Component 1.1: Understand and apply concepts and procedures from number sense.

**K–12 Subcomponent:** Number and Numeration

**Grade Level Expectations (GLEs):**

<table>
<thead>
<tr>
<th>Evidence of Learning:</th>
<th>Explain or illustrate integer values using words, pictures, models, and symbols.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EX Evidence of Learning:</strong></td>
<td>Explain the meaning of integers and gives examples.</td>
</tr>
<tr>
<td><strong>EX Evidence of Learning:</strong></td>
<td>Locate the additive inverse of a given integer on a number line.</td>
</tr>
</tbody>
</table>

**EX Evidences of Learning...**

- Are examples of student demonstrations that provide educators with common contexts of the learning stated in the GLE.
- Are NOT an exhaustive list, checklist, or grade level expectations.
- DO give guidance for the number set or other related information specific to the GLE. (e.g., two-digit times three-digit numbers)
**Part 1: Grade Level Review Results**

**General Findings**

In comparing the data for all programs by grade level (Figure 1 and Figure 2), it is worth noting that, on average, programs for grades K–3 rated significantly higher than any other grade range. Another trend of increased ratings occurred in programs designed for grades 7 and 8. Grades 9–12 had an average rating of less than 1 (Met some, but not all, criteria), as did grades 4–6.

This distribution of ratings may be explained by the relative focus on computational fluency early in the elementary grades and a focus on computational remediation in the pre-algebra grades 7 and 8, along with emphasis on other mathematical concepts in grades 4–6 and 9–12.

**All Programs**

Figure 4 represents an ordered average rating for all programs that cover at least two grades within K–5. It is important to note that small differences between similar programs are not statistically significant, but that a difference of 0.1 or more is statistically significant. Thus, it would not be warranted to say that Harcourt Mathletics has a significantly higher average rating than Number Worlds. However, one could draw the conclusion that Destination Math has a significantly higher overall average rating than FASTT Math, for example.

Note that the overall ratings measure specific criteria for Grade Level Expectations in Assessment, Instruction, and Practice. Many factors influence the relative value of a specific program. Overall ratings are one such factor. It is not necessarily appropriate to claim that one program is better or worse than another, based on this analysis. Rather the claim is that a program may have a better match to the standards-based criteria used in this review than another program.

**Elementary Programs**

Figure 5 and Figure 6 show the average ratings by program for grades K–5, broken down by Grade Level Expectation. Note that the data was split into two graphs of fourteen programs each (28 total), to improve readability. The programs are ordered by their overall average rating across all Grade Level Expectations (GLEs). Overall, in grades 3–5, GLE 1.1.3 (Properties), fared relatively poorly, compared to the other GLEs, in terms of an average rating. GLEs 1.1.5–1.1.7 (Operations, Procedures, and Strategy) were generally stronger and internally consistent than GLE 1.1.3 (Properties). That is, programs that tended to have a higher average rating for GLE 1.1.5 (Operations) also tended to have similar ratings for GLE 1.1.6 (Procedures) and GLE 1.1.7 (Strategy) as well. Slightly more than one-third of the programs had an average rating of 1.5 or greater.

**Middle School Programs**

Figure 7 and Figure 8 show average ratings for middle school grades 6–8. Of the 18 programs reviewed, 5 had an average rating of 1.5 or greater. Statistically significant differences of 0.1 or greater are seen across the broad range of average ratings (0 to 2.1), but many individual comparisons between similarly rated programs do not show a statistically significant difference, like Mathematics Navigator and AGS Algebra.

In grades 6–8, like the elementary grades, GLE 1.1.3 (Properties) had a lower average rating than did the other GLEs 1.1.4–1.1.7 (Ratio & Proportion, Operations, Procedures, and Strategy). For most programs, Ratio & Proportion (GLE 1.1.4) and Procedures (GLE 1.1.6) were the highest rated GLEs, indicating that middle school supplemental computational fluency curricula conforms more closely to Washington State standards in those two GLEs than curricula at grade spans K–5 and 9–12.
High School Programs

Figure 10 and Figure 11 show average ratings for grades 9-12. High school programs had the lowest overall average ratings, compared to elementary and middle school programs. A statistically significant difference of 0.15 is seen between the broad range of programs, but again, similarly rated programs like AGS Mathematics: Concepts and AGS Pre-Algebra do not show a statistically significant difference between their average ratings.

In general, for high school grades, GLE 1.1.6 (Procedures) had the highest average rating, compared to Ratio & Proportion, Operations, and Strategy (GLEs 1.1.4, 1.1.5, and 1.1.7, respectively). It is important to note, however, that compared to elementary and middle school grades, programs at the high school level received a significantly lower average rating across all GLEs. It is difficult to draw conclusions about the reasons for the difference between elementary, middle and high school, however.
Results for All Programs by Grades

Arithmetic mean was used as the statistical average for the grade level review data. (See Appendix E for Data Analysis Approach) Average ratings for all programs are reported two ways:

1. Grades and Program Elements of Assessment, Instruction, and Practice
2. Grades and GLEs 1.1.3–1.1.7

![Average Rating for All Programs by Grade and Program Element](image)

Figure 1. This graph shows the average rating for all programs by grade, broken down by Program Element – Assessment, Instruction, and Practice. Overall, early elementary grades have higher average ratings than other grade levels. There is a smaller peak of higher average ratings during middle school, where presumably, more emphasis on remediation is occurring. (See Appendix D for Grade Level Rubric)
Average Rating for All Programs by Grade and GLE

Figure 2. This graph shows the average rating for all programs by grade, broken down by Grade Level Expectation (GLE). Note that not all GLEs correspond with all grade levels, as shown in the chart. (See Appendix D for Grade Level Rubric)

<table>
<thead>
<tr>
<th>Grade Level Expectation</th>
<th>Grade</th>
<th>K</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9/10</th>
<th>11/12</th>
</tr>
</thead>
<tbody>
<tr>
<td>GLE 1.1.3 – Properties</td>
<td></td>
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<tr>
<td>GLE 1.1.4 – Ratio &amp; Proportion</td>
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<td>MS</td>
</tr>
<tr>
<td>GLE 1.1.5 – Operations</td>
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<td></td>
<td>MS</td>
</tr>
<tr>
<td>GLE 1.1.6 – Procedures</td>
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<td>MS</td>
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<tr>
<td>GLE 1.1.7 – Strategy</td>
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<td></td>
<td>MS</td>
</tr>
</tbody>
</table>

Figure 3. This chart shows which Grade Level Expectations correspond to specific grades. A shaded cell indicates that the GLE is associated with the specific grade level. A blank cell indicates no GLE is associated with the specific grade level. MS = Maintain Skills.
Results by Grade Spans and Programs

Review results are arranged by program from strongest to weakest alignment using the Grade Level Review Criteria. Results are reported by grade span in two ways:

1. Program Elements of Assessment, Instruction, and Practice
2. Grade Level Expectations 1.1.3–1.1.7

Grades K-5 Average Rating for Computational Programs
(▲ - Computer-based)

Figure 4. This graph shows the overall average rating for all Grades K–5 programs reviewed. The program “ORIGOmath: A Step by Step Approach to Computation” is not shown in this graph because it covers only Grade 1. All other programs cover at least two grades within K–5. (See Appendix D for Grade Level Rubric)
Figure 5. Part 1 of 2 graphs showing average ratings by program for Grades K–5, broken down by Grade Level Expectation. Note that the program “ORIGOmath: A Step by Step Approach to Computation” is not shown in this graph series because it covers only Grade 1. All other programs cover at least two grades within K–5. The graphs were split to improve readability. (See Appendix D for Grade Level Rubric)
Figure 6. Part 2 of 2 graphs showing average ratings by program for Grades K–5, broken down by Grade Level Expectation. The graphs were split to improve readability. (See Appendix D for Grade Level Rubric)
Grades 6-8 Average Rating for Computational Programs
(▲ - Computer-based)

Figure 7. This graph shows the overall average rating for all Grades 6-8 programs reviewed. Programs that cover at least two grade levels in this range are included.

Note 1: Larson Learning Courseware Grade 6 was reviewed in the intermediate portion (grades 3–6) of the program only, not in the pre-algebra portion (grades 6–8).
Figure 8. This graph shows the average ratings by program for Grades 6–8, broken down by Grade Level Expectation. Programs that cover at least two grade levels in this range are included. (See Appendix D for Grade Level Rubric)

<table>
<thead>
<tr>
<th>Grade Level Expectation</th>
<th>Grade</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<th>9/10</th>
<th>11/12</th>
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<tbody>
<tr>
<td>GLE 1.1.3 – Properties</td>
<td>K</td>
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<td>MS</td>
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<td>GLE 1.1.4 – Ratio &amp; Proportion</td>
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<td>GLE 1.1.5 – Operations</td>
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<td>GLE 1.1.6 – Procedures</td>
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<td>GLE 1.1.7 – Strategy</td>
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<td></td>
<td></td>
<td>MS</td>
<td>MS</td>
</tr>
</tbody>
</table>

Figure 9. This chart shows which Grade Level Expectations correspond to specific grades. A shaded cell indicates that the GLE is associated with the specific grade level. A blank cell indicates no GLE is associated with the specific grade level. **MS = Maintain Skills.**

Note 2: Larson Learning Courseware Grade 6 was reviewed in the intermediate portion (grades 3–6) of the program only, not in the pre-algebra portion (grades 6–8).
Figure 10. This graph shows the overall average rating for all Grades 9–12 programs reviewed. Programs that cover at least two grade levels in this range are included. (See Appendix D for Grade Level Rubric)
Figure 11. This graph shows the average ratings by program for Grades 9–12, broken down by Grade Level Expectation. Programs that cover at least two grade levels in this range are included. (See Appendix D for Grade Level Rubric)

Figure 12. This chart shows which Grade Level Expectations correspond to specific grades. A shaded cell indicates that the GLE is associated with the specific grade level. A blank cell indicates no GLE is associated with the specific grade level. MS = Maintain Skills.
Results by Programs and Grades

Review Results are reported in alphabetical order by program name and grade span in two ways:

1. Program Elements of Assessment, Instruction, and Practice
2. Grade Level Expectations 1.1.3–1.1.7

This chart shows which Grade Level Expectations correspond to specific grades. A shaded cell indicates that the GLE is associated with the specific grade level. A blank cell indicates no GLE is associated with the specific grade level. **MS = Maintain Skills.**

<table>
<thead>
<tr>
<th>Grade Level Expectation</th>
<th>Grade</th>
<th>K</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<th>6</th>
<th>7</th>
<th>8</th>
<th>9/10</th>
<th>11/12</th>
</tr>
</thead>
<tbody>
<tr>
<td>GLE 1.1.3 – Properties</td>
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<td>MS</td>
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<td>GLE 1.1.4 – Ratio &amp; Proportion</td>
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<td>MS</td>
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<td>GLE 1.1.5 – Operations</td>
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<td>MS</td>
<td>MS</td>
</tr>
<tr>
<td>GLE 1.1.6 – Procedures</td>
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<td>MS</td>
<td></td>
</tr>
<tr>
<td>GLE 1.1.7 – Strategy</td>
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<td></td>
<td>MS</td>
<td>MS</td>
</tr>
</tbody>
</table>
AGS Algebra
Average Rating by Grade and Program Element

AGS Algebra
Average Rating by Grade and GLE
AGS Basic Math Skills
Average Rating by Grade and Program Element

AGS Basic Math Skills
Average Rating by Grade and GLE
AGS Mathematics: Concepts
Average Rating by Grade and Program Element

AGS Mathematics: Concepts
Average Rating by Grade and GLE
AGS Pre-Algebra
Average Rating by Grade and Program Element

AGS Pre-Algebra
Average Rating by Grade and GLE
ALEKS Online Mathematics ▲
Average Rating by Grade and Program Element
▲ - Computer-based

Grade
Average Rating
0.00 0.50 1.00 1.50 2.00 2.50 3.00 3.50 4.00
3 4 5 6 7 8 9/10 11/12

Average Rating by Grade and GLE
▲ - Computer-based

Grade
Average Rating
GLE 1.1.3 - Properties
GLE 1.1.4 - Ratio & Proportion
GLE 1.1.5 - Operations
GLE 1.1.6 - Procedures
GLE 1.1.7 - Strategy
Boosting Your Math Skills
Average Rating by Grade and Program Element

Average Rating by Grade and GLE
Building Computational Fluency
Average Rating by Grade and Program Element

Grade
0.00 0.50 1.00 1.50 2.00 2.50 3.00 3.50 4.00
Average Rating
Assessment
Instruction
Practice

Building Computational Fluency
Average Rating by Grade and GLE

Grade
0.00 0.50 1.00 1.50 2.00 2.50 3.00 3.50 4.00
Average Rating
GLE 1.1.3 - Properties
GLE 1.1.4 - Ratio & Proportion
GLE 1.1.5 - Operations
GLE 1.1.6 - Procedures
GLE 1.1.7 - Strategy
Everyday Counts
Average Rating by Grade and Program Element

Everyday Counts
Average Rating by Grade and GLE
FASTT Math ▲
Average Rating by Grade and Program Element
▲ - Computer-based

Grade vs. Average Rating

- Grade: K-12
- Average Rating: 0.00 to 4.00
- Program Elements: Assessment, Instruction, Practice

FASTT Math ▲
Average Rating by Grade and GLE
▲ - Computer-based

Grade vs. Average Rating

- Grade: K-12
- Average Rating: 0.00 to 4.00
- GLEs: 1.1.3 - Properties, 1.1.4 - Ratio & Proportion, 1.1.5 - Operations, 1.1.6 - Procedures, 1.1.7 - Strategy
Figure it!
Average Rating by Grade and Program Element

Figure it!
Average Rating by Grade and GLE
Foundational Mathematics ▲
Average Rating by Grade and Program Element
▲ - Computer-based

- Average Rating by Grade and Program Element
- Grade: 11/12, 9/10, 8, 7, 6
- Program Elements: Assessment, Instruction, Practice

Foundational Mathematics ▲
Average Rating by Grade and GLE
▲ - Computer-based

- Average Rating by Grade and GLE
- Grade: 11/12, 9/10, 8, 7, 6
- GLEs: 1.1.3 - Properties, 1.1.4 - Ratio & Proportion, 1.1.5 - Operations, 1.1.6 - Procedures, 1.1.7 - Strategy
Harcourt Mathletics
Average Rating by Grade and Program Element

Harcourt Mathletics
Average Rating by Grade and GLE
Note 3: The intermediate grades (3–6) were reviewed in a new online version of the program. Reviewers marked 60% of the data points for those grades “0” with a note of “CNA” – for “could not access.

Note 4: Grade 6 was reviewed in the intermediate portion (grades 3–6) of the program only, not in the pre-algebra portion (grades 6–8).
Mastering Math
Average Rating by Grade and Program Element

Mastering Math
Average Rating by Grade and GLE
Math Expeditions ▲
Average Rating by Grade and Program Element
▲ - Computer-based

Math Expeditions ▲
Average Rating by Grade and GLE
▲ - Computer-based

Assessment ▲
Instruction ▲
Practice ▲
Math Skill Builder
Average Rating by Grade and Program Element

Math Skill Builder
Average Rating by Grade and GLE
MathFacts in a Flash
Average Rating by Grade and Program Element
(▲ - Computer-based)

MathFacts in a Flash
Average Rating by Grade and GLE
(▲ - Computer-based)
MathSteps
Average Rating by Grade and Program Element

MathSteps
Average Rating by Grade and GLE
Nimble with Numbers
Average Rating by Grade and Program Element

Grade | Assessment | Instruction | Practice
--- | --- | --- | ---
1 | | | |
2 | | | |
3 | | | |
4 | | | |
5 | | | |
6 | | | |

Nimble with Numbers
Average Rating by Grade and GLE

Grade | GLE 1.1.3 - Properties | GLE 1.1.4 - Ratio & Proportion | GLE 1.1.5 - Operations | GLE 1.1.6 - Procedures | GLE 1.1.7 - Strategy
--- | --- | --- | --- | --- | ---
1 | | | | | |
2 | | | | | |
3 | | | | | |
4 | | | | | |
5 | | | | | |
6 | | | | | |
Number Worlds Text and/or ▲
Average Rating by Grade and Program Element
▲ - Computer-based

Average Rating

0.00 0.50 1.00 1.50 2.00 2.50 3.00 3.50 4.00

Grade

Assessment
Instruction
Practice

Number Worlds Text and/or ▲
Average Rating by Grade and GLE
▲ - Computer-based

Average Rating

0.00 0.50 1.00 1.50 2.00 2.50 3.00 3.50 4.00

Grade

GLE 1.1.3 - Properties
GLE 1.1.4 - Ratio & Proportion
GLE 1.1.5 - Operations
GLE 1.1.6 - Procedures
GLE 1.1.7 - Strategy
ORIGOmath: A Step by Step Approach to Computation
Average Rating by Grade and Program Element

ORIGOmath: A Step by Step Approach to Computation
Average Rating by Grade and GLE
Average Rating by Grade and Program Element

Grade | Average Rating
-----|-----------------
11/12 | 1.00
9/10  | 0.75
8     | 1.50
7     | 2.00
6     | 2.50
5     | 3.00
4     | 2.50
3     | 2.00
2     | 1.50

Average Rating by Grade and GLE

Grade | Average Rating
-----|-----------------
11/12 | GLE 1.1.3 - Properties
9/10  | GLE 1.1.4 - Ratio & Proportion
8     | GLE 1.1.5 - Operations
7     | GLE 1.1.6 - Procedures
6     | GLE 1.1.7 - Strategy
Skills Intervention Kit
Average Rating by Grade and Program Element

<table>
<thead>
<tr>
<th>Grade</th>
<th>Assessment</th>
<th>Instruction</th>
<th>Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>9/10</td>
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Skills Intervention Kit
Average Rating by Grade and GLE

<table>
<thead>
<tr>
<th>Grade</th>
<th>GLE 1.1.3 - Properties</th>
<th>GLE 1.1.4 - Ratio &amp; Proportion</th>
<th>GLE 1.1.5 - Operations</th>
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<tbody>
<tr>
<td>9/10</td>
<td>0.25</td>
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<td>6</td>
<td>0.5</td>
<td>1.0</td>
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</tr>
</tbody>
</table>
Working with Numbers
Average Rating by Grade and Program Element

Working With Numbers
Average Rating by Grade and GLE
Part 2: Program Feature Review

In addition to mathematical content, assessment, instruction, and practice, an effective mathematics computational fluency supplemental program includes components that make it accessible for teachers and to students. (See Appendix C.)

1. **Assessment**
   The program provides a variety of assessments that measure the understanding and application of concepts, strategies, procedures, and algorithms taught within the program at strategic points in time (pre-assessment, formative, and summative assessment). It provides guidelines to teachers for using assessments to differentiate instruction and report progress.

2. **Explicit Instructional Guidance**
   Explicit instructional guidance includes various aspects of explicit and clear support for the teacher and for the student. Reviewers reviewed programs for ease and clarity of use, guidance for pacing, coherence within its component parts, and instructional options based on frequent monitoring and feedback of student performance.

3. **Program Design**
   The design facilitates easily using all program components in an efficacious manner that is mathematically accurate.

4. **Universal Access/Cultural Relevance**
   The program offers a variety of options for moving students to the standards. Such options include scaffolding techniques, English Language Learner (ELL) strategies, and using age appropriate illustrations, examples, and contexts.
**Part 2: Program Feature Review Results by Category**

Program Feature Review Results are reported by feature category and programs. Results are presented from greatest to least percent of program feature criteria met for each category: Assessment, Explicit Instructional Guidance, Program Design, and Universal Access/Cultural Relevance.

### Assessment Features by Program

▲ - Indicates Computer-based Program

#### Number Worlds Text and/or ▲

#### Building Computational Fluency

#### ALEKS Online Mathematics ▲

#### Cognitive Tutor Bridge to Algebra ▲

#### Mathematics Navigator

#### Harcourt Mathletics

#### PassKey: A Prescriptive Learning System ▲

#### Top Line Math

#### Math Skill Builder

#### MathFacts in a Flash ▲

#### Skills Intervention Kit

#### MathXL ▲

#### Algebra 1, Parts 1 and 2 ▲

#### Boosting Your Math Skills

#### MathSteps

#### Number Corner

#### Everyday Counts

#### FASTT Math ▲

#### ORIGOmath: A Step by Step Approach to Computation

#### Math Around the Clock

#### First Steps in Mathematics: Number

#### Foundational Mathematics ▲

#### AGS Mathematics: Concepts

#### Mastering Math

#### Working with Numbers

#### AGS Pre-Algebra

#### Math Expeditions ▲

#### AGS Basic Math Skills

#### The Book of Facts

#### Context for Learning Mathematics Series

#### AGS Algebra

#### Larson Learning Math Courseware

#### Nimble with Numbers

#### Mathementals

#### Facts That Last; PWR; PYF

#### Destination Math ▲

#### Fundamentals

#### The Think Tank

#### The Box of Facts

#### Figure it!

---

**Figure 13.** This graph shows the percent of Assessment Feature Criteria for each program, across all grades.
Explicit Instructional Guidance
Features by Program

(▲ - Indicates Computer-based Program)

Figure 14. This graph shows the percent of Explicit Instructional Guidance Feature Criteria for each program, across all grades.
Program Design Features

Figure 15. This graph shows the percent of Program Design Feature Criteria for each program, across all grades.
### Universal Access/Cultural Relevance

#### Features by Program

- 0% 20% 40% 60% 80% 100%

The Box of Facts
- Figure it!
- MathFacts in a Flash
- Working with Numbers
- Nimble with Numbers
- The Think Tank
- Mastering Math
- PassKey: A Prescriptive Learning System
- Cognitive Tutor Bridge to Algebra
- AGS Algebra
- Harcourt Mathletics
- AGS Mathematics: Concepts
- Math Expeditions
- Destination Math
- AGS Pre-Algebra
- Foundational Mathematics
- Math Around the Clock
- AGS Basic Math Skills
- Building Computational Fluency
- The Book of Facts
- Skills Intervention Kit
- Facts That Last; PWR; PYF
- FASTT Math
- Boosting Your Math Skills
- Fundamentals
- Mathematics Navigator
- MathSteps
- Context for Learning Mathematics Series
- MathXL
- Number Corner
- MathFacts in a Flash
- Figure it!
- The Box of Facts
- ALEKS Online Mathematics
- ORIGOmath: A Step by Step Approach to
- PassKey: A Prescriptive Learning System
- Top Line Math

#### Figure 16.

This graph shows the percent of Universal Access/Cultural Relevance Feature Criteria for each program, across all grades.
Appendix A: Resources


**Response to Intervention and the Three-Tiered Model**

The Three-Tiered Model is grounded in assessment data that is collected and immediately analyzed and interpreted for strategically planning targeted instruction. Key instructional decisions are based on data from assessments. The Three-Tiered Model includes pre-assessments (screening and diagnostic), formative assessment, progress monitoring, and summative assessments (outcome or performance). In the Three-Tiered Model, assessment is done more frequently with students who struggle (Tier II and III).

**Tier I**
All students receive core classroom instruction and prevention.

**Tier II**
Struggling students receive **strategic instruction approximately three times a week in addition to** core classroom instruction.

**Tier III**
Students who continue to struggle in spite of Tier II services receive **intensive intervention approximately one to two hours a day in addition to** core classroom instruction.

Adapted from the Tri-Level Reading Model, UT Center for Reading and Language Art

The Response to Intervention (RTI) manual, *Using Response to Intervention for Washington’s Students*, explains and provides guidelines for the use of a multi-tiered problem-solving approach (including the Three-Tiered Model). It is designed to bring high quality instruction to all students. “RTI is the practice of (1) providing high-quality instruction/intervention matched to student needs and (2) using learning rate over time and level of performance to (3) make important educational decisions to guide instruction.” (National Association of State Directors of Special Education 2005)
Appendix A: Resources


Reviews of current research as to the efficacy of mathematics programs are periodically updated at these Web sites:

Metiri Group, *Technology Solutions That Work*, [http://www.k12.wa.us/EdTech/whatworks.aspx](http://www.k12.wa.us/EdTech/whatworks.aspx). This database provides a comprehensive analysis of what research says works and does not work with software intended to increase academic achievement in mathematics, especially for at-risk students. It also gives information about the targeted audience for which the programs were designed to impact. This resource is free to educators throughout the state of Washington. Availability of this database does not imply endorsement by OSPI of any software programs listed, but rather is intended to be a resource to assist districts and schools in their consideration of software purchases. In order to access the information contained on this site, it is necessary to obtain a ‘token’ from your district technology or mathematics specialist. If your district does not have a token, please contact your regional Educational Technology Support Center (ETSC) director at [http://www.edtech.wednet.edu/etsccorner/directory.shtml](http://www.edtech.wednet.edu/etsccorner/directory.shtml) or the Mathematics or Educational Technology office at OSPI. For more information on technology and learning, visit the OSPI Education Technology Web site link to [http://www.k12.wa.us/EdTech/whatworks.aspx](http://www.k12.wa.us/EdTech/whatworks.aspx). For additional "how-to" directions on getting started with the database, go to [http://www.edtech.wednet.edu/resources/metiri/](http://www.edtech.wednet.edu/resources/metiri/).

Slavin, Robert E., and Lake, Cynthia, “Effective Programs in Elementary Mathematics: A Best Evidence Synthesis”, *The Best Evidence Encyclopedia*, Johns Hopkins University, [http://www.bestevidence.org/math/math_summary.htm](http://www.bestevidence.org/math/math_summary.htm). What mathematics programs have been proven to help elementary students to succeed? To find out, this review summarizes evidence on three types of programs designed to improve the mathematics achievement of students in grades K–6:

- Mathematics Curricula (MC), such as Everyday Mathematics, Saxon Math, and other standard and alternative textbooks.
- Instructional process programs (IP), such as cooperative learning, classroom management programs, and other approaches primarily intended to change teachers’ instructional strategies rather than curriculum or textbooks.

U.S. Department of Education, *What Works Clearinghouse*, [http://www.whatworksclearinghouse.org/](http://www.whatworksclearinghouse.org/). The What Works Clearinghouse (WWC) review of this topic focuses on math curricula designed for use in elementary schools with attention to student outcomes related to math achievement. Elementary school is defined in this review as a school with any of the six grades, kindergarten through fifth grade. Elementary school math curricula are math programs that specify clear learning goals for students; extend over the course of a semester or more; are central to students’ regular instruction; and are based on text materials, manipulatives, computer software, videotapes, other materials, or any combination thereof. Closely related programs such as supplemental math programs, and instructional practices such as computer-assisted instruction, may be addressed in future WWC reviews, but are not included in the current review.
Publisher Notice: February 9, 2007

In March 2007, the Office of Superintendent of Public Instruction (OSPI) will review supplemental programs that are designed to increase computational fluency for students in kindergarten through high school. As defined in the Principles and Standards for School Mathematics (NCTM 2000–2004), “Computational fluency – having and using efficient and accurate methods for computing – is essential. Students should be able to perform computations in different ways, including mental calculations, estimation, and paper-and-pencil calculations using mathematically sound algorithms... Computational fluency should develop in tandem with understanding.”

Publishers are invited to indicate an Intent to Submit any supplemental program that addresses foundational skills and concepts that develop fluency with operations with at least one of the following: whole numbers, fractions, mixed numbers, decimals, and percentages.

Location: TBA

Submission of Instructional Materials for Evaluation
The program review will include only mathematics computational fluency supplemental instruction materials. The programs should meet this definition: they are tools that teachers use in tandem with, or in addition to, the core/comprehensive mathematics program to ensure that students meet or exceed grade level standards in computation. Core/comprehensive mathematics programs will not be considered in this review.

Master Price Agreement
Publishers must be willing to participate in a Master Price Agreement with OSPI if, as a result of the review process, the programs meet or exceed standards determined in the review process.

Standards for the Review Process
Standards for the review process are being developed from a review of research on the development of computational fluency. Those standards will include:
Appendix B: Publisher Notice

Characteristics of Effective Computational Fluency Programs

A computational fluency program needs to facilitate:
- Development of meaning for numbers and operations.
- Recognizing the relationships among numbers and operations.
- Understanding computational strategies and using them appropriately and efficiently.
- Making sense of numerical and quantitative situations.

A computational fluency program needs to provide:
- Opportunity for students to develop strategies for learning math facts AND explicit strategy instruction.
- Practice of known facts to develop automaticity.
- Practice using known computational procedures on the whole range of problems on which the procedures may be used.
- Knowledge of progress over practice sessions.

The Program Information Document, Program Computational Fluency Matrix and the program materials will serve as the foundation for the review process for each program. (NOT a Publisher Self-Study Document as used previously in some OSPI reviews) accompanies the Publisher Notice – see also the SAMPLE Program Computational Fluency Matrix – both located at http://www.k12.wa.us/CurriculumInstruct/publishernotices.aspx).

The results of the program review are the sole judgment of the OSPI review panel members. Scores assigned by the reviewers shall be final. There is not an appeals process.

Publisher representatives will not present materials to, or have conversations with, any reviewers. Publishers will not be present during the week when panel members are evaluating materials. We ask that publisher representatives identify how the Review Manager, Karrin Lewis, may reach them during the review week to answer any questions that may arise.

Preliminary results of the review will be shared at a Publisher Representative Meeting on May 21, 2007, location and time to be announced.
Appendix B: Publisher Notice

Step One: Publisher Notice of Intent to Submit Materials and Program Information Document

Due: March 5, 2007, 4 p.m.

We respectfully require that publishers complete the following information by email:

1. Notify OSPI via email at leeann.mills@k12.wa.us to confirm your Intent to Submit programs for review. Please send a separate Intent to Submit for each program to be reviewed.
2. Include in your email notice a completed Program Information Document.

NOTES:

- If it is determined that the Program Information Document has not been completed, program materials will NOT be accepted into the review.

- OSPI staff will NOT meet with individual publisher representatives regarding this process. Please do not request to do so. All representatives are respectfully asked to contact the OSPI Mathematics Initiative Office for clarifications regarding the review process. Responses to inquiries will be posted as Frequently Asked Questions (FAQs) on the OSPI Web site: http://www.k12.wa.us/CurriculumInstruct/publishernotices.aspx.

OSPI Contact:
Lee Ann Mills
Mathematics Initiative Office
Office of Superintendent of Public Instruction
600 Washington Street SE
PO Box 47200
Olympia, WA 98504-7200
leeann.mills@k12.wa.us
Phone: 360-725-6445 Fax: 360-725-6023
Appendix B: Publisher Notice

Step Two: Materials Submission and Set-up

Date: March 25, 2007, 2 p.m.–4 p.m.
Location: TBA

Program Submission – Program Review Materials Submission Checklist:

Prior to set-up, publisher representatives will bring all program materials for the review including the completed Program Review Materials Submission Checklist (see page 6) to check-in with the Review Manager. OSPI Mathematics Initiative staff will verify with each publisher representative that each step on the checklist has been completed.

Materials Set-up:
1. Set-up will occur following sign-off by OSPI personnel on the Program Review Materials Submission Checklist.
2. Each publisher sets up his/her own program materials. If the program is computer-based, one computer per program and all accessories necessary to use the program must be provided. The publisher will pay all costs, if any, associated with internet access necessary to use the program.
3. Mark materials clearly, so reviewers understand all the components of the program.
4. Each program will be provided ½ a table (size 3' x 6') only. Please do not bring table skirts, stand up displays or visual advertisement posters.
5. Materials will NOT be accepted after 4 p.m.

Materials Submitted for Review:

- Four copies of the Program Information Document, each in a separate labeled letter-sized file folder. (These four copies are in addition to the one copy submitted with the Publisher's Intent to Submit.)

- Four copies of the Program Computational Fluency Matrix, each in a separate labeled letter-sized file folder (accompanies the Publisher Notice – see also the SAMPLE Program Computational Fluency Matrix – both located at http://www.k12.wa.us/CurriculumInstruct/publishernotices.aspx).

- Four (4) labeled sets of the computational fluency supplemental program that include:
  - All teacher materials for specified grade level(s), K–12
  - All student materials for specified grade level(s), K–12 – needed only if not replicated inside the teacher materials

NOTE: Do not ship any materials to OSPI. OSPI will not be responsible for transporting materials to or from the review site.
Appendix B: Publisher Notice

Step Three: Materials Pick-Up

Dates: Friday, March 30, 2007, 3 p.m.–4:00 p.m.
Location: TBA

- Pick-up ALL program materials between 3 p.m. and 4 p.m. only.
- If materials remain after 4:00 p.m., we will assume that the publisher does not need them back and they will be recycled via the hotel process.

Thank you for your understanding and cooperation in this effort.

Timeline Summary

<table>
<thead>
<tr>
<th>Activity</th>
<th>Date(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Publisher Notice and Program Information Document Posted</td>
<td>February 9, 2007</td>
</tr>
<tr>
<td>Intent to Submit and Program Information Document Due</td>
<td>March 5, 2007</td>
</tr>
<tr>
<td>Letter of Acceptance or Non-Acceptance into the Review emailed</td>
<td>March 12, 2007</td>
</tr>
<tr>
<td>to Originator of Intent to Submit</td>
<td></td>
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<tr>
<td>Publisher Materials Set-up</td>
<td>March 25, 2007</td>
</tr>
<tr>
<td>Publisher Materials Pick-up</td>
<td>March 30, 2007</td>
</tr>
<tr>
<td>Preliminary Report Publisher Meeting</td>
<td>May 8, 2007</td>
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<tr>
<td>Report at OSPI Summer Institutes</td>
<td>Summer 2007</td>
</tr>
</tbody>
</table>
Appendix C: Reviewer Training Resources

Kindergarten GLEs 1.1.3–1.1.7

1.1.3 (NONE at this Grade Level)

1.1.4 (NONE at this Grade Level)

1.1.5 Understand the meaning of addition\(^{(1)}\). (1 criterion)

For example:

✓ Express stories involving addition with words, pictures, and numbers.
✓ Show how addition can be used with objects or situations in the classroom.
✓ …

1.1.6 (NONE at this Grade Level)

1.1.7 (NONE at this Grade Level)
Appendix C: Reviewer Training Resources

Grade 1 GLEs 1.1.3–1.1.7

1.1.3 (NONE at this Grade Level)

1.1.4 (NONE at this Grade Level)

1.1.5 Understand the meaning of subtraction\(^{(i)}\). (1 criterion)

For example:
✓ Express stories involving subtraction with words, pictures, and numbers.
✓ Show relationships between addition and subtraction using physical models, diagrams, and acting out situations.
✓ ...

1.1.6 Use computational procedures for addition\(^{(i)}\) of whole numbers to 12\(^{(a)}\). (1 criterion)

For example:
✓ Apply strategies, including counting on, counting back, and doubling, for addition facts to at least 12.
✓ Use addition procedures in a given situation.
✓ Explain strategies or procedures used to determine sums.
✓ ...

1.1.7 Apply appropriate strategies and use tools for adding whole numbers.

For example:
✓ Use appropriate tools including mental math, paper and pencil, manipulatives, or calculator to add or subtract in a given situation.
✓ ...

Appendix C: Reviewer Training Resources

Grade 2 GLEs 1.1.3–1.1.7

1.1.3 (NONE at this Grade Level)

1.1.4 (NONE at this Grade Level)

1.1.5 Understand the meaning of addition\(^{(1)}\) and subtraction\(^{(2)}\) and how they relate to one another\(^{(3)}\). (3 criteria)

For example:
- ✓ Show relationships between addition and subtraction using words, pictures, and/or numbers.
- ✓ Use joining, separating, part-part-whole, and comparison situations to add and subtract.
- ✓ Describe real-life situations involving addition using physical models and diagrams from various cultures and acting out situations.
- ✓ Illustrate addition and subtraction using words, pictures, and/or numbers.
- ✓ ...

1.1.6 Use computational procedures for addition\(^{(1)}\) and subtraction\(^{(2)}\) of whole numbers to 18\(^{(a)}\). (2 criteria)

For example:
- ✓ Apply strategies, including counting on, counting back, doubling, and halving, for addition and subtraction facts.
- ✓ Use addition and/or subtraction procedures in a given situation.
- ✓ Explain strategies or procedures used to determine sums and differences.
- ✓ Add and subtract with 2- or 3-digit numbers using a calculator and explain the procedures used.
- ✓ ...

1.1.7 Apply appropriate strategies and use tools for adding and subtracting whole numbers.

For example:
- ✓ Apply mental math strategies, including composing and decomposing numbers or finding combinations that are easy to add or subtract, to compute with numbers through 100.
- ✓ Use calculator, manipulatives, or paper and pencil to add or subtract in a given situation.
- ✓ Explain methods to mentally group numbers efficiently.
- ✓ ...

Appendix C: Reviewer Training Resources

Grade 3 GLEs 1.1.3–1.1.7

1.1.3 Understand and use the commutative property\(^{(1)}\) and identity property\(^{(2)}\) of addition on whole numbers\(^{(a)}\). (2 criteria)

For example:
✓ Explain or show how the commutative property works for addition and not subtraction using words, pictures, physical models, or numbers.
✓ Identify equivalent expressions using the commutative property.
✓ Explain the identity property of addition and give examples.
✓ Show whether addition equations are true or false and provide an explanation, based on the commutative or identity property of addition.
✓ …

1.1.4 (NONE at this Grade Level)

1.1.5 Understand the meaning of multiplication\(^{(1)}\) and division\(^{(2)}\) of whole numbers\(^{(a)}\). (2 criteria)

For example:
✓ Illustrate multiplication and division using words, pictures, models, and/or numbers.
✓ Illustrate and explain the inverse relationship between multiplication and division using words, pictures, models, and/or numbers.
✓ Show and explain the relationship between multiplication and repeated addition.
✓ Show and explain the relationship between division and repeated subtraction.
✓ Explain the use of division to find the number of equal shares or the quantity in each equal share.
✓ Identify and explain fact families for multiplication and division.
✓ Explain the meaning of a remainder in a division problem.
✓ Select and/or use an appropriate operation to show understanding of addition, subtraction, multiplication, or division of whole numbers.
✓ …

1.1.6 Use computational procedures for addition\(^{(1)}\) and subtraction\(^{(2)}\) of whole numbers\(^{(a)}\). (2 criteria)

For example:
✓ Apply a variety of strategies to add and subtract 3-digit numbers in a given situation.
✓ Describe and show relationships between strategies and procedures for multiplying and dividing.
✓ Complete multi-step computations that involve addition and subtraction and explain strategies.
✓ Explain and apply strategies or use procedures to add three 2-digit or two 3-digit numbers, and/or subtract numbers with 1, 2, or 3 digits.
✓ Explain and apply strategies or use procedures to add up to 5 one-digit whole numbers.
✓ …
Appendix C: Reviewer Training Resources

1.1.7 Apply strategies and use tools appropriate to tasks involving addition and subtraction of whole numbers.

For example:
✓ Select and use appropriate tools from among mental computation, estimation, calculators, manipulatives, and paper and pencil to compute in a given situation.
✓ Explain why a selected tool is most efficient for a given situation.
✓ …
Appendix C: Reviewer Training Resources

Grade 4 GLEs 1.1.3–1.1.7

1.1.3 Understand and use the associative property of addition\(^{(1)}\) and multiplication\(^{(2)}\), and the commutative\(^{(3–4)}\), identity\(^{(5–6)}\), and zero properties\(^{(7)}\) of multiplication on whole numbers\(^{(8)}\). (7 criteria)

For example:

✓ Explain the commutative property of multiplication and give examples.
✓ Explain the identity property of addition and multiplication and give examples.
✓ Explain why equations are true or false based on one of the properties of addition or multiplication.
✓ Illustrate and demonstrate the use of the commutative, associative, or identity property of addition or multiplication using words, pictures, numbers, or objects.
✓ Use addition or multiplication properties to replicate a computational strategy when given an example.
✓ Illustrate and demonstrate the use of the zero property of multiplication on whole numbers.
✓ …

1.1.4 (NONE at this Grade Level)

1.1.5 Understand the meaning of addition\(^{(1)}\) and subtraction\(^{(2)}\) of like-denominator fractions\(^{(8)}\). (2 criteria)

For example:

✓ Represent addition and subtraction of fractions with like-denominators using numbers, pictures, and models including everyday objects, fraction circles, number lines, and geoboards.
✓ Use joining, separating, part-part-whole, and comparison situations to add and subtract like-denominator fractions.
✓ Translate a given picture or illustration into an equivalent symbolic representation of addition and subtraction of like denominator fractions.
✓ Select and/or use an appropriate operation to show understanding of addition and subtraction of like-denominator fractions.
✓ …

1.1.6 Use computational procedures for multiplication\(^{(1)}\) and division\(^{(2)}\) of whole numbers\(^{(8)}\). (2 criteria)

For example:

✓ Select and develop strategies that help with recall of multiplication and division through 12s.
✓ Select and test algorithms used in computational situations that involve multiplication and division of whole numbers and explain strategies.
✓ Compute with whole numbers using a combination of any two operations in a given situation.
✓ Explain and apply strategies or use procedures to multiply 2-digit numbers by 3-digit numbers and/or divide 3- or 4-digit numbers by 2-digit numbers without remainders.
✓ Appropriately apply and explain the concept of remainder in a given context.
✓ …
Appendix C: Reviewer Training Resources

1.1.7 Apply strategies and use tools appropriate to tasks involving multiplication and division of whole numbers.

For example:
- Select and use appropriate tools from among mental computation, estimation, calculators, manipulatives, and paper and pencil to compute in a given situation.
- Explain why a selected tool is most efficient for a situation.
- ...
Appendix C: Reviewer Training Resources

Grade 5 GLEs 1.1.3–1.1.7

1.1.3 Understand and apply the concept of divisibility including primes\(^{(1)}\), composites\(^{(2)}\), factors\(^{(3)}\), and multiples\(^{(4)}\). (4 criteria)

For example:
✓ Use the concepts of odd and even numbers to check for divisibility.
✓ Illustrate prime or composite numbers by creating a physical model.
✓ Identify prime or composite numbers between 1 and 100 and explain why a whole number is prime or composite.
✓ Explain how to find the least common multiple (LCM) and greatest common factor (GCF) of two numbers.
✓ Use factors, multiples, and prime and composite numbers in a variety of situations.
✓ Factor a number into its prime factorization or into factor pairs.
✓ Explain or show whether one number is a factor of another number.
✓ Explain or demonstrate why a number is prime or composite
✓ …

1.1.4 (NONE at this Grade Level)

1.1.5 Understand the meaning of addition\(^{(1)}\) and subtraction\(^{(2)}\) of non-negative decimals\(^{(a)}\) and fractions\(^{(b)}\). (4 criteria)

For example:
✓ Represent addition and subtraction of fractions with denominators of 2, 4, 8 or 2, 3, 6, 12 or 2, 5, 10.
✓ Represent or explain addition and subtraction of non-negative decimals through thousandths using words, pictures, models, or numbers.
✓ Explain a strategy for adding and subtracting fractions.
✓ Select and/or use an appropriate operation(s) to show understanding of addition and subtraction of non-negative decimals and/or fractions.
✓ Explain the relationship between addition and subtraction of non-negative decimals and fractions.
✓ Translate a picture or illustration into an equivalent symbolic representation of addition and subtraction of non-negative fractions and decimals.
✓ …

1.1.6 Apply strategies or use computational procedures to add\(^{(1)}\) and subtract\(^{(2)}\) non-negative decimals\(^{(a)}\) and like-denominator fractions\(^{(b)}\). (4 criteria)

For example:
✓ Add and subtract non-negative decimals and like-denominator fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, and/or 15.
✓ Find sums or differences of decimals or like-denominator fractions in given situations.
✓ Calculate sums of two numbers with decimals to the thousandths or three numbers with decimals to hundredths.
✓ Calculate difference of numbers with decimals to thousandths.
✓ …
Appendix C: Reviewer Training Resources

1.1.7 Apply strategies and uses tools appropriate to tasks involving addition and subtraction of non-negative decimals or like-denominator fractions.

For example:
✓ Select and use appropriate tools from among mental computation, estimation, calculators, manipulatives, and paper and pencil to compute in a given situation.
✓ Explain why a selected strategy or tool is more efficient or more appropriate than another strategy or tool for a situation.
✓ Describe strategies for mentally adding or subtracting non-negative decimals and/or like-denominator fractions.
✓ …
Appendix C: Reviewer Training Resources

Grade 6 GLEs 1.1.3–1.1.7

1.1.3 Understand and use properties of addition\textsuperscript{(1–3)} and multiplication\textsuperscript{(4–7)} on non-negative decimals and fractions\textsuperscript{(a)}. (7 criteria)

For example:
✓ Illustrate and explain the commutative, associative, and identity properties of addition and multiplication and the zero property of multiplication on non-negative decimals and fractions.
✓ Use addition and multiplication properties to assist in computations.
✓ Determine whether a computation is reasonable based on application of the commutative, associative, and identity properties of addition and/or multiplication.
✓ …

1.1.4 Understand the concepts of ratio\textsuperscript{(1)} and percent\textsuperscript{(2)}. (2 criteria)

For example:
✓ Write or show and explain ratios in part/part and part/whole relationships using words, objects, pictures, models, and/or symbols.
✓ Represent equivalent ratios using objects, pictures, or symbols.
✓ Represent equivalent percentages using objects, pictures, and symbols.
✓ Express or represent percent as a ratio based on 100 equal size parts of a set.
✓ Explain ratio and percents and give examples of each.
✓ Create a ratio equivalent to a given ratio to determine an unknown value for a dimension or a number of events or objects.
✓ …

1.1.5 Understand the meaning of multiplication\textsuperscript{(1)} and division\textsuperscript{(2)} of non-negative decimals\textsuperscript{(a)} and fractions\textsuperscript{(b)}. (4 criteria)

For example:
✓ Explain or show the meaning of multiplying and dividing non-negative fractions and decimals using words, pictures, or models.
✓ Explain the effect of multiplying a whole number by a decimal number.
✓ Explain why multiplication of fractions involves multiplying denominators.
✓ Demonstrate how multiplication and division with decimals affects place value.
✓ Explain remainders of a division problem in a given situation.
✓ Translate a picture or illustration into an equivalent symbolic representation of multiplication and division of non-negative fractions and decimals.
✓ Select and/or use an appropriate operation to show understanding of addition, subtraction, multiplication, or division of non-negative rational numbers.
✓ …
Appendix C: Reviewer Training Resources

1.1.6 Apply strategies or uses computational procedures to add and subtract non-negative decimals and fractions. (4 criteria)

For example:
✓ Find the sums or differences of non-negative fractions or decimals.
✓ Find sums or differences of decimals or fractions in real-world situations.
✓ Use the least common multiple and the greatest common factor of whole numbers to simplify or compute with fractions.
✓ Calculate sums of two numbers with decimals to the thousandths or three numbers with decimals to hundredths.
✓ Calculate difference between numbers with decimals to thousandths.
✓ Complete multiple-step computations requiring addition and/or subtraction.
✓ …

1.1.7 Apply strategies and uses tools appropriate to tasks involving addition and subtraction of non-negative decimals and fractions.

For example:
✓ Select and use appropriate strategies and tools from among mental computation, estimation, calculators, manipulatives, and paper and pencil to compute in a given situation.
✓ Explain why a selected strategy or tool is more efficient or more appropriate than another strategy or tool for a situation.
✓ Describe strategies for mentally adding and/or subtracting non-negative decimals and fractions.
✓ …
Appendix C: Reviewer Training Resources

Grade 7 GLEs 1.1.3–1.1.7

1.1.3 Understand and use the inverse property of addition\(^{(1)}\) on integers\(^{(1a)}\) (W) and the inverse property of multiplication\(^{(2)}\) on non-negative decimals\(^{(2a)}\) or fractions\(^{(2b)}\). (3 criteria)

For example:
✓ Use the inverse relationship between multiplication and division to simplify computations.
✓ Use the inverse properties of addition and multiplication to simplify computations and explain why they work with integers, fractions, and decimals.
✓ Use, represent, or evaluate an application of the commutative, associative, and/or identity properties of addition on non-negative decimals or fractions.
✓ Use, represent, or evaluate an application of the commutative associative, identity, and/or zero properties of multiplication on non-negative decimals or fractions.
✓ …

1.1.4 Understand the concept of direct proportion\(^{(1)}\). (1 criterion)

For example:
✓ Explain or illustrate the meaning of a ratio, percent or proportion.
✓ Express proportional relationships using objects, pictures, and symbols.
✓ Complete or write a proportion for a given situation.
✓ Predict a future situation using direct proportion
✓ Represent equivalent ratios and/or percents using pictures, diagrams, or symbols.
✓ Determine or use a ratio, percent, or proportion in a given situation.
✓ …

1.1.5 Understand the meaning of addition\(^{(1)}\) and subtraction\(^{(2)}\) of integers\(^{(a)}\). (2 criteria)

For example:
✓ Explain or show the meaning of addition and subtraction of integers using words, pictures, or real-world models.
✓ Translate a symbolic addition or subtraction of integers into a real-life situation.
✓ Show addition and subtraction of integers using technology.
✓ Translate a given picture or illustration representing addition or subtraction of integers into an equivalent symbolic representation.
✓ Explain why multiplication of fractions involves multiplying denominators while addition of fractions requires finding common denominators.
✓ Select and/or use an appropriate operation to show understanding of addition and subtraction of integers.
✓ …
Appendix C: Reviewer Training Resources

1.1.6 Apply strategies or uses computational procedures using order of operations\(^{(1)}\) to add\(^{(2)}\), subtract\(^{(3)}\), multiply\(^{(4)}\), and divide\(^{(5)}\) non-negative decimals and fractions\(^{(a)}\). (5 criteria)

For example:
✓ Find the product or quotient using non-negative decimals and fractions.
✓ Use multiplication and division in real world situations involving non-negative rational numbers.
✓ Multiply non-negative decimals and fractions.
✓ Divide non-negative decimal numbers by non-negative decimal numbers to the hundredths place.
✓ Compute with non-negative rational numbers using order of operations.
✓ Interpret and apply the concept of remainder in a given situation.
✓ Complete multi-step calculations requiring two or more operations with non-negative decimals and fractions.
✓ ...

1.1.7 Apply strategies and uses tools to complete tasks involving addition and subtraction of integers and the four basic operations on non-negative decimals and fractions.

For example:
✓ Select and use appropriate strategies and tools from among mental computation, estimation, calculators, manipulatives, and paper and pencil to compute in a given situation.
✓ Explain why a selected strategy or tool is more efficient or more appropriate than another strategy or tool for a situation.
✓ Describe strategies for mentally adding and/or subtracting integers and multiplying and/or dividing non-negative decimals and fractions.
✓ ...

Appendix C: Reviewer Training Resources

Grade 8 GLEs 1.1.3–1.1.7

1.1.3 Understand and use the distributive property\(^{(1)}\) and the properties of addition\(^{(2)}\) and multiplication\(^{(3)}\) on rational numbers\(^{(a)}\). (3 criteria)

For example:
✓ Demonstrate the distributive property of multiplication over addition using an area model or picture.
✓ Use the distributive property to simplify expressions that include integers.
✓ Use the distributive property to factor expressions.
✓ Represent or evaluate the application of the addition and multiplication properties on rational numbers including integers.
✓ Use the addition and multiplication properties, including the distributive property, to assist with computations.
✓ …

1.1.4 Apply the concepts of ratio\(^{(1)}\), percent\(^{(2)}\), and direct proportion\(^{(3)}\). (3 criteria)

For example:
✓ Determine an unknown value for a dimension or a number of events or objects using ratio or proportion.
✓ Determine an unknown value for a dimension or a number of events or objects using percents.
✓ Select and use the most advantageous representation of ratios or percents in a given situation.
✓ Determine a ratio or percent in a given situation.
✓ …

1.1.5 Understand the meaning of addition\(^{(1)}\), subtraction\(^{(2)}\), multiplication\(^{(3)}\), division\(^{(4)}\), powers\(^{(5)}\), and square roots\(^{(6)}\) on rational numbers\(^{(a)}\). (6 criteria)

For example:
✓ Explain the meaning of multiplication and division of integers including remainders using words, pictures, or models.
✓ Explain the meaning of taking whole number powers of integers or square roots of whole numbers using words, pictures, or models.
✓ Represent a situation involving multiplication or division of integers, whole number powers of integers, or square roots of whole numbers.
✓ Explain how the result of dividing a rational number by a fraction between 0 and 1 is different from the result of dividing the same number by a fraction greater than 1.
✓ Translate a given situation, picture, or illustration into a numeric expression or equation involving decimals, fractions, integers, whole number powers, and square roots of whole numbers.
✓ Select and/or use an appropriate operation to show understanding of whole number powers and square roots.
✓ Convert between equivalent forms of rational numbers including whole number powers and square roots of perfect squares.
✓ …
Appendix C: Reviewer Training Resources

1.1.6 Apply strategies or uses computational procedures using order of operations\(^{(1)}\) and addition\(^{(2)}\), subtraction\(^{(3)}\), multiplication\(^{(4)}\), division\(^{(5)}\), powers\(^{(6)}\), and square roots\(^{(7)}\) on rational numbers\(^{(8)}\). (7 criteria)

For example:
✓ Compute with rational numbers using order of operations.
✓ Compute using whole number powers and/or square roots of perfect squares.
✓ Interpret and apply the concept of remainder in a given situation.
✓ Complete multi-step computations using two or more different operations with rational numbers.
✓ …

1.1.7 Apply strategies and uses tools to complete tasks involving computation of rational numbers.

For example:
✓ Select and justify appropriate strategies and tools from among mental computation, estimation, calculators, manipulatives, and paper and pencil to compute in a given situation.
✓ Explain why a selected strategy or tool is more efficient or more appropriate than another strategy or tool for a situation.
✓ Describe strategies for mental computation with integers using powers and square roots.
✓ …
Appendix C: Reviewer Training Resources

Grades 9/10 GLEs 1.1.3–1.1.7

1.1.3 {Maintain skills from previous grade level(s)}

1.1.4 Understand the concept of inverse proportion\(^{(1)}\) and apply direct\(^{(2)}\) and inverse proportion\(^{(3)}\). (3 criteria)

For example:
✓ Explain, illustrate, or describe examples of inverse proportion.
✓ Determine whether a real-world problem involves direct or inverse proportion.
✓ Use direct or inverse proportion to determine an unknown number of objects or an unknown value in a given situation.
✓ …

1.1.5 Compute using scientific notation\(^{(1)}\). (1 criterion)

For example:
✓ Compute using scientific notation.
✓ Use scientific notation to simplify a calculation.
✓ …

1.1.6 Complete multi-step computations with combinations of rational numbers\(^{(a)}\) using order of operations\(^{(1)}\) and addition\(^{(2)}\), subtraction\(^{(3)}\), multiplication\(^{(4)}\), division\(^{(5)}\), powers\(^{(6)}\), and square roots\(^{(7)}\). (7 criterion)

For example:
✓ Calculate using order of operations on rational numbers.
✓ Use properties to reorder and rearrange expressions to compute more efficiently.
✓ Apply strategies to complete multi-step computations fluently.
✓ …

1.1.7 {Maintain skills from previous grade level(s)}
Appendix C: Reviewer Training Resources

Grades 11/12 GLEs 1.1.3–1.1.7

1.1.3  {Maintain skills from previous grade level(s)}

1.1.4  {Maintain skills from previous grade level(s)}

1.1.5  Understand the concept\(^{(1)}\) and symbolic representation\(^{(2)}\) of rational numbers\(^{(a)}\)
including absolute values. (2 criteria)

For example:
✓ Find \(\left(\frac{a}{b}\right)_{\text{integer}}\), where \(a, b \in \text{integers}, b \neq 0\)
✓ Find \(\left(\frac{a}{b}\right)_{\text{simple fraction}}\), where \(a, b \in \text{integers}, b \neq 0\)
✓ Find the absolute value of any real numbers
✓ Represent/interpret expressions involving absolute value
✓ Compare expressions involving absolute value—on or off a number line
✓ Perform arithmetic operations with expressions involving absolute value
✓ …

1.1.6  Complete multi-step computations of real numbers in all forms\(^{(a)}\), including rational exponents\(^{(1)}\) and scientific notation\(^{(2)}\), using order of operations\(^{(3)}\) and properties of operations\(^{(4)}\). (4 criteria)

For example:
✓ Compute using rational numbers
✓ Compute using scientific notation
✓ Solve problems using basic properties of exponents and logarithms
✓ Compute problems with multi-steps using order of operations, properties
  (associative, commutative, and distributive)
✓ …

1.1.7  Applies strategies and uses tools to complete tasks involving computation of real numbers. (aligns with CRS 4.2)

For example:
✓ Select and justify appropriate strategies and tools from among mental computation, estimation, calculators, manipulatives, and paper and pencil to compute in a given situation.
✓ Describe strategies for mentally solving problems using involving real numbers.
✓ …
### Appendix C: Reviewer Training Resources

#### Some Mathematics Properties

<table>
<thead>
<tr>
<th>Operations</th>
<th>Commutative</th>
<th>Associative</th>
<th>Distributive</th>
<th>Identity</th>
<th>Inverse</th>
<th>Zero</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>$a + b = b + a$</td>
<td>$a + (b + c) = (a + b) + c$</td>
<td></td>
<td>$a + 0 = a$</td>
<td>$-a$</td>
<td></td>
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<tr>
<td></td>
<td>Id. element = 0</td>
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</tr>
<tr>
<td>Multiplication</td>
<td>$a \cdot b = b \cdot a$</td>
<td>$a \cdot (b \cdot c) = (a \cdot b) \cdot c$</td>
<td></td>
<td>$a \cdot 1 = a$</td>
<td>$\frac{1}{a}, a \neq 0$</td>
<td>$a \cdot 0 = 0$</td>
</tr>
<tr>
<td></td>
<td>or $ab = ba$</td>
<td></td>
<td></td>
<td>Id. element = 1</td>
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</tbody>
</table>
### Appendix C: Reviewer Training Resources

#### Problem Types and Examples

<table>
<thead>
<tr>
<th>Problem Type</th>
<th>Multiplication</th>
<th>Measurement Division</th>
<th>Partitive Division</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Join</strong></td>
<td>(Result Unknown) Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether?</td>
<td>(Change Unknown) Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether?</td>
<td>(Start Unknown) Connie had some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?</td>
</tr>
<tr>
<td><strong>Separate</strong></td>
<td>(Result Unknown) Connie had 13 marbles. She gave 5 to Juan. How many marbles does Connie have left?</td>
<td>(Change Unknown) Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give to Juan?</td>
<td>(Start Unknown) Connie had some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with?</td>
</tr>
<tr>
<td><strong>Part-Part-Whole</strong></td>
<td>(Whole Unknown) Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?</td>
<td>(Part Unknown) Connie has 13 marbles. 5 are red and the rest are blue. How many blue marbles does Connie have?</td>
<td></td>
</tr>
<tr>
<td><strong>Compare</strong></td>
<td>(Difference Unknown) Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan?</td>
<td>(Compare Quantity Unknown) Juan has 5 marbles. Connie has 8 more than Juan. How many marbles does Connie have?</td>
<td>(Referent Unknown) Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have?</td>
</tr>
</tbody>
</table>

### Appendix C: Reviewer Training Resources

#### CHILDREN’S STRATEGIES FOR SOLVING BASIC FACTS

<table>
<thead>
<tr>
<th>Problem</th>
<th>Direct Modeling</th>
<th>Counting</th>
<th>Derived Facts</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5 + 7 = ?</strong>&lt;br&gt;Join Result Unknown</td>
<td>Makes a set of 5 counters and a set of 7 counters. Pushes the two sets together and counts all the counters.</td>
<td>Counts “5 [pause], 6, 7, 8, 9, 10, 11, 12,” extending a finger with each count. “The answer is 12” [The counting sequence may also begin with the larger number]</td>
<td>“Take 1 from the 7 and give it to the 5. That makes 6 + 6, and that’s 12.”</td>
<td>5 plus 7 is 12.</td>
</tr>
<tr>
<td><strong>12 - 5 = ?</strong>&lt;br&gt;Separate Result Unknown</td>
<td>Makes a set of 12 counters and removes 5 of them. Then counts the remaining counters.</td>
<td>Counts back “12, 11, 10, 9, 8 [pause], 7. It’s 7.” Uses fingers to keep track of the numbers of steps in the counting sequence.</td>
<td>“12 take away 2 is 10, and take away 3 more is 7.”</td>
<td>12 take away 5 is 7.</td>
</tr>
<tr>
<td><strong>4 + ? = 11</strong>&lt;br&gt;Join Change Unknown</td>
<td>Makes a set of 4 counters. Makes a second set of counters, counting “5, 6, 7, 8, 9, 10, 11,” until there is a total of 11 counters. Counts the 7 counters in the second set.</td>
<td>Counts “4 [pause], 5, 6, 7, 8, 9, 10, 11,” extending a finger with each count. Counts the 7 extended fingers. “It’s 7.”</td>
<td>“4 + 6 is 10 and 1 more is 11. So it’s 7.”</td>
<td>4 and 7 make 11.</td>
</tr>
<tr>
<td><strong>5 x 7 = ?</strong></td>
<td>Makes 7 groups of 5 counters and counts them all.</td>
<td>5, 10, 15, 20, 25, 30, 35</td>
<td>5 times 5 is 25 and 10 more is 35.</td>
<td>5 times 7 is 35.</td>
</tr>
<tr>
<td><strong>56 ÷ 8 = ?</strong></td>
<td>Counts out 56 counters. Pulls out groups of 8 until 7 groups are made.</td>
<td>8, 16, 24, 32, 40, 48, 56</td>
<td>8 times 8 is 64. 8 less is 56. So that’s 7.</td>
<td>8 x 7 is 56.</td>
</tr>
</tbody>
</table>

## Appendix C: Reviewer Training Resources

### CHILDREN’S STRATEGIES FOR MULTI-DIGIT COMPUTATION

<table>
<thead>
<tr>
<th>PROBLEM</th>
<th><strong>DIRECT MODELING</strong></th>
<th><strong>COUNTING</strong></th>
<th><strong>ALTERNATIVE OR INVENTED ALGORITHMS</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>25 + 37 = ?</strong></td>
<td>Makes a set of 25 counting by ones and a set of 37 counting by ones and then counts them all beginning with 1.</td>
<td>Start with 25, counts on by 1s, keeping track of how many are added on until the total is reached. e.g., (25, 26, 27, 28, 29, 30, 31, \ldots) or begins with 37 and counts on to 62.</td>
<td>20 and 30 is 50, 5 and 7 is 12. 50 and 12 is 62. (combining 10s &amp; 1s) OR 25 and 30 is 55 and 7 more is 62. (incrementing) OR 25 and 40 is 65, less 3 is 62. (compensating)</td>
</tr>
<tr>
<td><strong>47 - 28 = ?</strong></td>
<td>Makes a set of 47 by ones and then takes away 28 by ones.</td>
<td>Counts back from 47 by ones or counts on from 28 until get to 47</td>
<td>40 take away 20 is 20. 8 take away 7 is 1. 20 take away 1 is 19 OR 47 take away 20 is 27. 27 take away 8 is 19. OR 47 take away 30 is 17 plus two is 19.</td>
</tr>
<tr>
<td><strong>12 x 15 = ?</strong></td>
<td>Makes a set of 12 by ones and repeats that 15 times. Counts everything up.</td>
<td>SKIP COUNTS 12, 24, 36, 48, \ldots) or adds 12, 15 times and figures out various ways of adding the list up.</td>
<td>12 times 12 is 144. 12 times 3 is 36. 144 and 36 is 180. OR 12 times 10 is 120. 12 times 5 is 60. 120 and 60 is 180. OR 12 times 5 is 60. 60 times 3 is 180.</td>
</tr>
<tr>
<td><strong>120 ÷ 15</strong></td>
<td>Makes a set of 120 by ones. Pulls out groups of 15 and counts how many groups are pulled out and how many are left over.</td>
<td>SKIP COUNTS 15, 30, 45, 60, 75, 90, 105, 120 or adds up 15 until get close to or to 120.</td>
<td>15 goes into 105 7 times and 15 more is 120. That’s 8. OR 15 times 4 is 60. 60 times 2 is 120. That’s 8.</td>
</tr>
</tbody>
</table>

### Appendix D: Program Review Document – EXAMPLE*

*GRADE LEVEL DOCUMENTS MAY BE DOWNLOADED FROM [http://www.k12.wa.us/CurriculumInstruct/Mathematics/default.aspx](http://www.k12.wa.us/CurriculumInstruct/Mathematics/default.aspx)

#### Grade 4

<table>
<thead>
<tr>
<th>Assessment</th>
<th>GLE 1.1.3 Evidence (e.g. p 5, 7)</th>
<th>Score 0 [1 2 3 4]</th>
<th>GLE 1.1.4 Evidence (e.g. p 5, 7)</th>
<th>Score 0 [1 2 3 4]</th>
<th>GLE 1.1.5 Evidence (e.g. p 5, 7)</th>
<th>Score 0 [1 2 3 4]</th>
<th>GLE 1.1.6 Evidence (e.g. p 5, 7)</th>
<th>Score 0 [1 2 3 4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Pre-/ Diagnostic (min. 1 ea. criteria)</td>
<td>Comm add</td>
<td></td>
<td></td>
<td></td>
<td>Associ add</td>
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<td></td>
<td>Comm mult</td>
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<td></td>
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<td>Associ mult</td>
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<tr>
<td>2. Formative/ Progress Monitoring (min. 2 ea. criteria)</td>
<td>Comm add</td>
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<td>Associ add</td>
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Program Number ___________ Reviewer Number ___________
Review Start ______________ Review Finish _____________
Total Time on Task __________}

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### Grade 4

<table>
<thead>
<tr>
<th>Assessment</th>
<th>GLE 1.1.3</th>
<th>GLE 1.1.4</th>
<th>GLE 1.1.5</th>
<th>GLE 1.1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Evidence</td>
<td>Score</td>
<td>Evidence</td>
<td>Score</td>
</tr>
<tr>
<td></td>
<td>(e.g. p 5, 7)</td>
<td>0 [1 2 3 4]</td>
<td>(e.g. p 5, 7)</td>
<td>0 [1 2 3 4]</td>
</tr>
<tr>
<td>3. Summative/Post-assessment</td>
<td>Comm add</td>
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<td></td>
<td>Comm mult</td>
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<td>Zero mult</td>
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</table>

### Evidence
- Comm add
- Comm mult
- Assoc add
- Assoc mult
- Identity add
- Identity mult
- Zero mult

### Score
- 0 [1 2 3 4]
## Appendix D: Program Review Document

### Grade 4

<table>
<thead>
<tr>
<th>Instruction</th>
<th>GLE 1.1.3</th>
<th>GLE 1.1.4</th>
<th>GLE 1.1.5</th>
<th>GLE 1.1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Evidence</strong></td>
<td><strong>Score</strong></td>
<td><strong>Evidence</strong></td>
<td><strong>Score</strong></td>
<td><strong>Evidence</strong></td>
</tr>
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<td>(e.g. p 5, 7)</td>
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<td>Comm add</td>
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<td>Zero mult</td>
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<tr>
<td><strong>1. Develop conceptual understanding using multiple representations</strong></td>
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<tr>
<td>(e.g. symbolically, with a model, or a real-world situation)</td>
<td>(min. 2 ea. criteria)</td>
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<tr>
<td><strong>Meaning of addition for like-denom fractions</strong></td>
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<td><strong>Multiply whole numbers</strong></td>
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<td><strong>Meaning of subtraction for like-denom fractions</strong></td>
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<td><strong>Divide whole numbers</strong></td>
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<td><strong>Meaning of subtraction for like-denom fractions</strong></td>
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<td><strong>Divide whole numbers</strong></td>
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<td><strong>Zero mult</strong></td>
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<tr>
<td><strong>2. Make connections explicit among representations</strong></td>
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<td>(min. 2 ea. criteria)</td>
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<td><strong>Divide whole numbers</strong></td>
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</tbody>
</table>
### Grade 4

<table>
<thead>
<tr>
<th>Instruction</th>
<th>GLE 1.1.3</th>
<th>Score</th>
<th>GLE 1.1.4</th>
<th>Score</th>
<th>GLE 1.1.5</th>
<th>Score</th>
<th>GLE 1.1.6</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Use a variety of problem types (e.g. start, change, and result unknown; compare; grouping, rate, price, multiplicative comparison; partitive and measurement division)</td>
<td>Comm add</td>
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<tr>
<td>4. Use computation in a variety of problem situations (contexts)</td>
<td>Comm add</td>
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<tr>
<td>(min. 2 ea. criteria)</td>
<td>Comm mult</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Assoc add</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Assoc mult</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Identity add</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Identity mult</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Zero mult</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Appendix D: Program Review Document

#### Grade 4

<table>
<thead>
<tr>
<th>Instruction</th>
<th>GLE 1.1.3</th>
<th>GLE 1.1.4</th>
<th>GLE 1.1.5</th>
<th>GLE 1.1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Evidence</td>
<td>Score</td>
<td>Evidence</td>
<td>Score</td>
</tr>
<tr>
<td></td>
<td>(e.g. p 5, 7)</td>
<td>[0 1 2 3 4]</td>
<td>(e.g. p 5, 7)</td>
<td>[0 1 2 3 4]</td>
</tr>
<tr>
<td>5. Develop efficient, accurate strategies</td>
<td>Comm add</td>
<td>___</td>
<td>__</td>
<td>Mean of addition for like-denom fractions</td>
</tr>
<tr>
<td>(min. 2 ea. criteria)</td>
<td>Comm mult</td>
<td>___</td>
<td>___</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Assoc add</td>
<td>__</td>
<td>__</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Assoc mult</td>
<td>___</td>
<td>___</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Identity add</td>
<td>___</td>
<td>___</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Identity mult</td>
<td>___</td>
<td>___</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Zero mult</td>
<td>___</td>
<td>___</td>
<td></td>
</tr>
<tr>
<td>6. Develop efficient, accurate procedures and algorithms</td>
<td>Comm add</td>
<td>___</td>
<td>___</td>
<td>Mean of addition for like-denom fractions</td>
</tr>
<tr>
<td>(min. 2 ea. criteria)</td>
<td>Comm mult</td>
<td>___</td>
<td>___</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Assoc add</td>
<td>___</td>
<td>___</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Assoc mult</td>
<td>___</td>
<td>___</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Identity add</td>
<td>___</td>
<td>___</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Identity mult</td>
<td>___</td>
<td>___</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Zero mult</td>
<td>___</td>
<td>___</td>
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</table>
### Appendix D: Program Review Document

#### Grade 4

<table>
<thead>
<tr>
<th>Practice</th>
<th>GLE 1.1.3</th>
<th>Score</th>
<th>GLE 1.1.4</th>
<th>Score</th>
<th>GLE 1.1.5</th>
<th>Score</th>
<th>GLE 1.1.6</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evidence (e.g. p 5, 7)</td>
<td></td>
<td></td>
<td>Evidence (e.g. p 5, 7)</td>
<td></td>
<td>Evidence (e.g. p 5, 7)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comm add</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Meaning of addition for like-denom fractions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comm mult</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Multiply whole numbers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assoc add</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Meaning of subtraction for like-denom fractions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assoc mult</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Divide whole numbers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Identity add</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Identity mult</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zero mult</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>1. Use efficient, accurate strategies</strong> (min. 2 ea. criteria)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2. Use efficient, accurate procedures and algorithms</strong> (min. 2 ea. criteria)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Grade 4

<table>
<thead>
<tr>
<th>Practice</th>
<th>GLE 1.1.3</th>
<th>Score</th>
<th>GLE 1.1.4</th>
<th>Score</th>
<th>GLE 1.1.5</th>
<th>Score</th>
<th>GLE 1.1.6</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Evidence</td>
<td>Score</td>
<td>Evidence</td>
<td>Score</td>
<td>Evidence</td>
<td>Score</td>
<td>Evidence</td>
<td>Score</td>
</tr>
<tr>
<td></td>
<td>(e.g. p 5, 7)</td>
<td></td>
<td>(e.g. p 5, 7)</td>
<td></td>
<td>(e.g. p 5, 7)</td>
<td></td>
<td>(e.g. p 5, 7)</td>
<td></td>
</tr>
<tr>
<td>3. Solve a variety of problem types</td>
<td>Comm add</td>
<td></td>
<td>Comm mult</td>
<td></td>
<td>Assoc add</td>
<td></td>
<td>Assoc mult</td>
<td></td>
</tr>
<tr>
<td>(e.g. start, change, and result unknown; compare; partitive and measurement division)</td>
<td>Identity add</td>
<td></td>
<td>Identity mult</td>
<td></td>
<td>Zero mult</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(min. 2 ea. criteria)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Solve problems in a variety of situations</td>
<td>Comm add</td>
<td></td>
<td>Comm mult</td>
<td></td>
<td>Assoc add</td>
<td></td>
<td>Assoc mult</td>
<td></td>
</tr>
<tr>
<td>(contexts)</td>
<td>Identity add</td>
<td></td>
<td>Identity mult</td>
<td></td>
<td>Zero mult</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(min. 2 ea. criteria)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix D: Program Review Document – Grade Level Review Rubric

The “Total Number of Criteria” refers to the number of specific criteria identified in each of GLEs 1.1.3–1.1.6 at each grade level.

### Grade Level Review Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>Indicator</th>
<th>Total Number of Criteria (x)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Meets or exceeds all criteria</td>
<td>≥2</td>
</tr>
<tr>
<td>3</td>
<td>Sufficiently meets criteria</td>
<td>&gt;1</td>
</tr>
<tr>
<td>2</td>
<td>Minimally meets criteria</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Meets some, but not all, criteria</td>
<td>Partially met 1</td>
</tr>
<tr>
<td>0</td>
<td>Meets no criteria</td>
<td>0</td>
</tr>
</tbody>
</table>

### Program Feature Review Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>Indicator</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No</td>
<td>There is inconsistent or no evidence that meets the required criteria.</td>
</tr>
<tr>
<td>1</td>
<td>Yes</td>
<td>There is consistent evidence that meets the required criteria.</td>
</tr>
</tbody>
</table>
## Appendix D: Program Review Document

<table>
<thead>
<tr>
<th>Assessment</th>
<th>Supporting Evidence</th>
<th>Consistent? 0 = No; 1 = Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Program has flexibility to use screening assessment to place students at an appropriate entry point.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Multiple assessments are provided: pre-/diagnostic assessment, formative assessment/progress monitoring, and summative assessment.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Program provides clear guidelines for using assessment information to differentiate instruction and report progress.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Assessments measure the understanding and application of concepts, strategies, procedures, and algorithms in the instructional program.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Program provides specific criteria for determining student’s recall/automaticity in computation (e.g.: number of correct digits per minute).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Assessments vary in administration (group and individual) and in types of tasks (e.g. multiple choice, short answer, extended response).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Appendix D: Program Review Document

<table>
<thead>
<tr>
<th>Explicit Instructional Guidance</th>
<th>Supporting Evidence</th>
<th>Consistent? 0 = No; 1 =Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Instructional objective(s) of lessons are explicitly stated and clearly noted.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Scope and sequence of instructional objectives is logical.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Materials needed to deliver the lesson are listed.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Suggested time frames are noted in lessons to assist with appropriate instructional pacing.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Answer keys are provided for all workbooks and other related student activities.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Instruction to students is clear, using mathematical vocabulary and language that students understand.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Instructional guidance identifies common student misconceptions and errors and provides teachers with suggestions as how to correct them.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Instruction includes differentiation/intervention designed to increase student recall/automaticity in computation.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. Instruction develops conceptual understanding before and/or with development of strategies, procedures, and algorithms.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. Instruction develops efficient/accurate strategies, procedures, and algorithms prior to independent student practice.</td>
<td></td>
<td>See Grade Level Review</td>
</tr>
<tr>
<td>11. Practice to develop recall/automaticity is brief (10–15 minutes at a time).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. Practice to develop recall/automaticity is frequent (minimum 3 times a week).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13. Program includes suggestions for parents on how to support student learning and achievement.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Appendix D: Program Review Document

<table>
<thead>
<tr>
<th>Review Start Date/Time</th>
<th>Program Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Review Finish Date/Time</td>
<td>Grade Level</td>
</tr>
<tr>
<td>Total Time on Task</td>
<td>Reviewer Number</td>
</tr>
</tbody>
</table>

#### Program Design

<table>
<thead>
<tr>
<th>Supporting Evidence</th>
<th>Consistent?</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 = No; 1 = Yes</td>
<td></td>
</tr>
</tbody>
</table>

1. The components within the program are organized to facilitate use with minimal handling of additional pieces and guides.
   - See Grade Level Review

2. Instructional components are clearly identified. (e.g.: assessment, instruction, guided practice, independent practice, homework, and intervention/differentiation)

3. Content is written in a manner that is mathematically correct.
   - Cite errors if “No”.

#### Universal Access/Cultural Relevance

<table>
<thead>
<tr>
<th>Supporting Evidence</th>
<th>Consistent?</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 = No; 1 = Yes</td>
<td></td>
</tr>
</tbody>
</table>

1. Instructional materials provide scaffolding to support student access to content.

2. The instructional materials address special needs of English Language Learners (ELL) by providing:
   - Exposure to increasingly challenging, interesting, and appropriate uses of mathematical language.
   - Extensive practice in the use of mathematical language.
   - Background knowledge.

3. Program uses age appropriate illustrations, examples, information, and story content that present all human beings with respect and dignity while avoiding images and roles that might be perceived as stereotypic or negative.

4. Program has data demonstrating its efficacy among all student groups. (e.g.: African American, Hispanic, ELL, Special Education, Low-Income, etc.)
Appendix E: Data Analysis Approach

Historically, OSPI has used several different approaches to analyze data collected from mathematics material reviews. In the K–12 Mathematics Diagnostic-Intervention Program Review the data was presented using medians. This is because the reports focused on thinly sliced data with 4 data elements per sample. In this instance, the median approach is warranted, to ensure that individual outliers in the 4-sample data set do not unduly influence the outcome. For example, consider the ratings (0, 4, 4, 4). The average of that data set is 3, while the median is 4. Using a median with very small samples reduces the influence of outliers.

Based on feedback from users of prior reports, OSPI approached the analysis a different way this year. OSPI recognized that school districts and curriculum specialists wanted a more global aggregate comparison of the available computational fluency curricula. Users were asking questions like how well does each program match Grade Level Expectations across all elementary grades, and which programs have the greatest emphasis on instruction at the high school level? With that in mind, we approached the data analysis in a different way. Shifting from collapsing the data from thousands of individual assessment criteria with 4 data points for each sample to aggregating the data for each program by Grade Level Expectation or Program Element (Assessment, Instruction, Practice) seemed to be a better approach. Then sample sets had hundreds to thousands of data points allowing the use of statistical methods more suited to the sample size and data characteristics.

The mathematics computational fluency review data is ordinal in nature. It consists of rankings 0–4. There were 32,000+ rankings, across grades K–12, in 40 programs. Given the number of rankings and the relative value of the ordinal data (e.g. a rating of 4 is better than a rating of 3) the data could be treated as either ordinal or interval.

Each program-grade sample was compared to its corresponding population of ratings for all programs within that grade. For example, the sample of ratings from fourth grade Math Expeditions was compared to the population of all ratings for all programs for the fourth grade.

Three statistical tests were compared to determine which was the most robust to use with the data. The statistical tests were:

- Student’s t-test (parametric)
- Mann-Whitney U Test (non-parametric)
- Mood Median Test (non-parametric)

To determine which test to use for subsequent analysis, six random samples were selected from all program-grade samples, and compared to their respective grades. The point of this initial “test” of the statistical tests was to determine if all the tests consistently resulted in the same outcome.

In the statistical world, parametric tests are generally considered more robust than non-parametric tests. However, parametric tests generally assume a normal distribution of data. If the data is not normally distributed, and is nominal or ordinal in nature, a non-parametric test may be warranted. If the sample size is sufficiently large, it is possible to use a t-test even if the data is not normally distributed. The sample sets and the population sets in the Mathematics Computational Fluency project were sufficiently large to allow consideration of the t-test, even though the data was not normally distributed.
Appendix E: Data Analysis Approach

Findings

The Student’s t-test allowed the most robust analysis of the difference of two sample means. The Mann-Whitney U Test had consistent results with the Student’s t-test. The Mood Median test, considered the weakest of the three, produced inconsistent results compared to the other two tests.

Given the equivalent outcomes for the Student’s t-test and the Mann-Whitney U test, and the general propensity to use a parametric test where possible, the project team decided to use the Student’s t-test as the de facto choice of statistical analysis tests for all program-grade combinations.

A sample was defined as all the ratings for a particular program-grade combination. The population it was compared to was all the ratings for the specific grade. The null hypothesis was that the sample mean was no different than the population mean. The alternate hypothesis is that a difference of means existed. In all six random samples shown, the Student’s t-test showed that a statistically significant difference of means existed between the sample and the population. In other words, we can demonstrate that the random programs selected did indeed have a difference in average ratings that could not be attributed to chance alone.

The results from the three statistical tests for six random program-grade samples are shown.
Appendix E: Data Analysis Approach

Grade 3, Program A

t-test Result for Datasets:
Set 1 Range = Grade 3
Set 2 Range = Gr 3, Program A

Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Std Err</th>
<th>Lower 95% CL</th>
<th>Upper 95% CL</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column I</td>
<td>1.243</td>
<td>1.656</td>
<td>0.026</td>
<td>1.192</td>
<td>1.295</td>
<td>4033</td>
</tr>
<tr>
<td>Column I</td>
<td>0.820</td>
<td>1.462</td>
<td>0.052</td>
<td>0.717</td>
<td>0.923</td>
<td>778</td>
</tr>
</tbody>
</table>

2-tailed t-test

<table>
<thead>
<tr>
<th>Ho. Diff</th>
<th>Mean Diff.</th>
<th>SE Diff.</th>
<th>T</th>
<th>DF</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.423</td>
<td>0.059</td>
<td>7.233</td>
<td>1195.013</td>
<td>0.000</td>
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</tbody>
</table>

F-Test for Equality of Variances

<table>
<thead>
<tr>
<th>Variable</th>
<th>Variance</th>
<th>F</th>
<th>DF 1</th>
<th>DF 2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column I</td>
<td>2.742</td>
<td>4032</td>
<td>777</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Column I</td>
<td>2.137</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sample variances differ at the specified alpha of 0.0500 so the individual variances are used in the t-test.

Mann-Whitney Test Results for:
Set 1 Range = Grade 3
Set 2 Range = Gr 3, Program A

2-tailed Test

<table>
<thead>
<tr>
<th>U</th>
<th>DF 1</th>
<th>DF 2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>1785416.000</td>
<td>4033</td>
<td>778</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Mood Median Test Results for:
Grade 3, Program A
Grand Median = 0

Counts

<table>
<thead>
<tr>
<th>&gt; Grand Median</th>
<th>Grade 3</th>
<th>Program A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1626</td>
<td>35</td>
</tr>
<tr>
<td>&lt;= Grand Median</td>
<td>2407</td>
<td>113</td>
</tr>
</tbody>
</table>

Test Results

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
<th>DF</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-Square</td>
<td>16.566</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>Yates Correction</td>
<td>15.877</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>Cochran Correction</td>
<td>16.155</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>17.716</td>
<td>1</td>
<td>0.000</td>
</tr>
</tbody>
</table>
### Appendix E: Data Analysis Approach

#### Grade 5, Program B

- **t-test Result for Datasets:**
  - Set 1 Range = Grade 5
  - Set 2 Range = Grade 5, Program B

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Std Err</th>
<th>Lower 95% CL</th>
<th>Upper 95% CL</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column I</td>
<td>0.753</td>
<td>1.223</td>
<td>0.019</td>
<td>0.715</td>
<td>0.790</td>
<td>4033</td>
</tr>
<tr>
<td>Column I</td>
<td>0.419</td>
<td>0.617</td>
<td>0.051</td>
<td>0.319</td>
<td>0.519</td>
<td>148</td>
</tr>
</tbody>
</table>

- **2-tailed t-test**

<table>
<thead>
<tr>
<th></th>
<th>Ho. Diff</th>
<th>Mean Diff.</th>
<th>SE Diff.</th>
<th>T</th>
<th>DF</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.000</td>
<td>0.334</td>
<td>0.054</td>
<td>6.147</td>
<td>192.214</td>
<td>0.000</td>
</tr>
</tbody>
</table>

- **F-Test for Equality of Variances**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Variance</th>
<th>F</th>
<th>DF 1</th>
<th>DF 2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column I</td>
<td>1.495</td>
<td>3.922</td>
<td>4032</td>
<td>147</td>
<td>0.000</td>
</tr>
<tr>
<td>Column I</td>
<td>0.381</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sample variances differ at the specified alpha of 0.0500 so the individual variances are used in the t-test.

- **Mann-Whitney Test Results for:**
  - Set 1 Range = Grade 5
  - Set 2 Range = Gr 5, Program B

<table>
<thead>
<tr>
<th>2-tailed Test</th>
<th>U</th>
<th>DF 1</th>
<th>DF 2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>319144.000</td>
<td>4033</td>
<td>148</td>
<td>0.099</td>
</tr>
</tbody>
</table>

- **Mood Median Test Results for:**
  - Grade 5, Program B
  - Grand Median = 0

<table>
<thead>
<tr>
<th>Counts</th>
<th>Grade 5</th>
<th>Program B</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; Grand Median</td>
<td>1568</td>
<td>54</td>
</tr>
<tr>
<td>&lt;= Grand Median</td>
<td>2465</td>
<td>94</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test Results</th>
<th>Statistic</th>
<th>Value</th>
<th>DF</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Chi-Square</td>
<td>0.344</td>
<td>1</td>
<td>0.557</td>
</tr>
<tr>
<td></td>
<td>Yates Corr</td>
<td>0.251</td>
<td>1</td>
<td>0.616</td>
</tr>
<tr>
<td></td>
<td>Cochran Corr</td>
<td>0.266</td>
<td>1</td>
<td>0.606</td>
</tr>
<tr>
<td></td>
<td>Log-Liklih.</td>
<td>0.347</td>
<td>1</td>
<td>0.556</td>
</tr>
</tbody>
</table>
### Appendix E: Data Analysis Approach

**Grade 6, Program C**

#### t-test Result for Datasets:
Set 1 Range = Grade 6  
Set 2 Range = Gr 6 Program C

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Std Err</th>
<th>Lower 95% CL</th>
<th>Upper 95% CL</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column I</td>
<td>0.872</td>
<td>1.378</td>
<td>0.019</td>
<td>0.835</td>
<td>0.910</td>
<td>5136</td>
</tr>
<tr>
<td>Column I</td>
<td>3.076</td>
<td>1.487</td>
<td>0.124</td>
<td>2.831</td>
<td>3.321</td>
<td>144</td>
</tr>
</tbody>
</table>

2-tailed t-test

<table>
<thead>
<tr>
<th>Ho. Diff</th>
<th>Mean</th>
<th>SE Diff.</th>
<th>T</th>
<th>DF</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>-2.204</td>
<td>0.117</td>
<td>-18.887</td>
<td>5278.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

#### F-Test for Equality of Variances

<table>
<thead>
<tr>
<th>Variable</th>
<th>Variance</th>
<th>F</th>
<th>DF 1</th>
<th>DF 2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column I</td>
<td>1.899</td>
<td>143</td>
<td>1</td>
<td>5135</td>
<td>0.091</td>
</tr>
<tr>
<td>Column I</td>
<td>2.211</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sample variances don't differ at the specified alpha of 0.0500 so the following pooled variance was used in the t-test.
Pooled Variance = 1.908

#### Mann-Whitney Test Results for:
Set 1 Range = Grade 6  
Set 2 Range = Gr 6, Program C

2-tailed Test

<table>
<thead>
<tr>
<th>U</th>
<th>DF 1</th>
<th>DF 2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>613980.500</td>
<td>5136</td>
<td>144</td>
<td>0.000</td>
</tr>
</tbody>
</table>

#### Mood Median Test Results for:
Grade 6, Program C

**Grand Median = 0**

<table>
<thead>
<tr>
<th>Counts</th>
<th>Grade 6</th>
<th>Program C</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; Grand Median</td>
<td>1894</td>
<td>125</td>
</tr>
<tr>
<td>&lt;= Grand Median</td>
<td>3242</td>
<td>19</td>
</tr>
</tbody>
</table>

**Test Results**

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
<th>DF</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-Square</td>
<td>147.854</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>Yates Correction</td>
<td>145.747</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>Cochran Correction</td>
<td>145.747</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>150.355</td>
<td>1</td>
<td>0.000</td>
</tr>
</tbody>
</table>
## Appendix E: Data Analysis Approach

### Grade 9/10, Program D

#### t-test Result for Datasets:
- Set 1 Range = Grade 9/10
- Set 2 Range = Gr 9/10, Program D

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Std Err</th>
<th>95% Lower CL</th>
<th>95% Upper CL</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column I</td>
<td>0.548</td>
<td>0.988</td>
<td>0.022</td>
<td>0.504</td>
<td>0.592</td>
<td>1950</td>
</tr>
<tr>
<td>Column I</td>
<td>0.838</td>
<td>0.965</td>
<td>0.089</td>
<td>0.661</td>
<td>1.014</td>
<td>117</td>
</tr>
</tbody>
</table>

#### 2-tailed t-test

<table>
<thead>
<tr>
<th>Ho. Diff</th>
<th>Mean Diff.</th>
<th>SE Diff.</th>
<th>T</th>
<th>DF</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>-0.289</td>
<td>0.094</td>
<td>-3.081</td>
<td>2065.000</td>
<td>0.002</td>
</tr>
</tbody>
</table>

#### F-Test for Equality of Variances

<table>
<thead>
<tr>
<th>Variable</th>
<th>Variance</th>
<th>F</th>
<th>DF 1</th>
<th>DF 2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column I</td>
<td>0.976</td>
<td>1.050</td>
<td>1949</td>
<td>116</td>
<td>0.376</td>
</tr>
<tr>
<td>Column I</td>
<td>0.930</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sample variances don't differ at the specified alpha of 0.0500 so the following pooled variance was used in the t-test.

Pooled Variance = 0.974

#### Mann-Whitney Test Results for:
- Set 1 Range = Grade 9/10
- Set 2 Range = Gr 9/10, Program D

<table>
<thead>
<tr>
<th>2-tailed Test</th>
<th>U</th>
<th>DF 1</th>
<th>DF 2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>140176.000</td>
<td>1950</td>
<td>117</td>
<td>0.000</td>
</tr>
</tbody>
</table>

#### Mood Median Test Results for:
- Grade 9/10 Program D

<table>
<thead>
<tr>
<th>Grand Median = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counts</td>
</tr>
<tr>
<td>Grade 9/10</td>
</tr>
<tr>
<td>&gt; Grand Median</td>
</tr>
<tr>
<td>&lt;= Grand Median</td>
</tr>
</tbody>
</table>

#### Test Results

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
<th>DF</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-Square</td>
<td>29.593</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>Yates Correction</td>
<td>28.511</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>Cochran Correction</td>
<td>29.387</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>27.821</td>
<td>1</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Appendix E: Data Analysis Approach

Grade 11/12, Program E

t-test Result for Datasets:
Set 1 Range = Grade 11/12
Set 2 Range = Gr 11/12, Program E

Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Std Err</th>
<th>95% CL Lower</th>
<th>95% CL Upper</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column I</td>
<td>0.720</td>
<td>1.213</td>
<td>0.038</td>
<td>0.645</td>
<td>0.795</td>
<td>1014</td>
</tr>
<tr>
<td>Column I</td>
<td>1.128</td>
<td>1.303</td>
<td>0.148</td>
<td>0.834</td>
<td>1.422</td>
<td>78</td>
</tr>
</tbody>
</table>

2-tailed t-test

<table>
<thead>
<tr>
<th>Ho. Diff</th>
<th>Mean Diff.</th>
<th>SE Diff.</th>
<th>T</th>
<th>DF</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>-0.408</td>
<td>0.143</td>
<td>-2.849</td>
<td>1090.000</td>
<td>0.004</td>
</tr>
</tbody>
</table>

F-Test for Equality of Variances

<table>
<thead>
<tr>
<th>Variable</th>
<th>Variance</th>
<th>F</th>
<th>DF 1</th>
<th>DF 2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column I</td>
<td>1.471</td>
<td>77</td>
<td>1013</td>
<td>0.179</td>
<td></td>
</tr>
<tr>
<td>Column I</td>
<td>1.698</td>
<td>1</td>
<td>78</td>
<td>0.900</td>
<td></td>
</tr>
</tbody>
</table>

Sample variances don't differ at the specified alpha of 0.0500 so the following pooled variance was used in the t-test.
Pooled Variance = 1.487

Mann-Whitney Test Results for:
Set 1 Range = Grade 11/12
Set 2 Range = Gr 11/12, Program E

2-tailed Test

<table>
<thead>
<tr>
<th>U</th>
<th>DF 1</th>
<th>DF 2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>47398.000</td>
<td>1014</td>
<td>78</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Mood Median Test Results for:
Grade 11/12, Program E

Grand Median = 0

Counts

<table>
<thead>
<tr>
<th>Grade 11/12</th>
<th>Program E</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; Grand Median</td>
<td>317</td>
</tr>
<tr>
<td>&lt;= Grand Median</td>
<td>697</td>
</tr>
</tbody>
</table>

Test Results

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
<th>DF</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-Square</td>
<td>14.914</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>Yates Correction</td>
<td>13.963</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>Cochran Correction</td>
<td>14.097</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>13.994</td>
<td>1</td>
<td>0.000</td>
</tr>
</tbody>
</table>
### Appendix E: Data Analysis Approach

#### Grade 11/12, Program F

### t-test Result for Datasets:
Set 1 Range = Grade 11/12  
Set 2 Range = Gr 11/12, Program F

#### Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Std Err</th>
<th>Lower 95% CL</th>
<th>Upper 95% CL</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column I</td>
<td>0.720</td>
<td>1.213</td>
<td>0.038</td>
<td>0.645</td>
<td>0.795</td>
<td>1014</td>
</tr>
<tr>
<td>Column I</td>
<td>0.321</td>
<td>0.781</td>
<td>0.088</td>
<td>0.144</td>
<td>0.497</td>
<td>78</td>
</tr>
</tbody>
</table>

#### 2-tailed t-test

<table>
<thead>
<tr>
<th>Ho. Diff</th>
<th>Mean Diff.</th>
<th>SE Diff.</th>
<th>T</th>
<th>DF</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.399</td>
<td>0.096</td>
<td>4.147</td>
<td>107.929</td>
<td>0.000</td>
</tr>
</tbody>
</table>

#### F-Test for Equality of Variances

<table>
<thead>
<tr>
<th>Variable</th>
<th>Variance</th>
<th>F</th>
<th>DF 1</th>
<th>DF 2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column I</td>
<td>1.471</td>
<td>2.411</td>
<td>1013</td>
<td>77</td>
<td>0.000</td>
</tr>
<tr>
<td>Column I</td>
<td>0.610</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sample variances differ at the specified alpha of 0.0500 so the individual variances are used in the t-test.

### Mann-Whitney Test Results for:
Set 1 Range = Grade 11/12  
Set 2 Range = Gr 11/12, Program F

#### 2-tailed Test

<table>
<thead>
<tr>
<th>U</th>
<th>DF 1</th>
<th>DF 2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>45711.500</td>
<td>1014</td>
<td>78</td>
<td>0.005</td>
</tr>
</tbody>
</table>

### Mood Median Test Results for:
Grade 11/12, Program F

#### Grand Median = 0

<table>
<thead>
<tr>
<th>Counts</th>
<th>Grade 11/12</th>
<th>Program F</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; Grand Median</td>
<td>317</td>
<td>13</td>
</tr>
<tr>
<td>&lt;= Grand Median</td>
<td>697</td>
<td>65</td>
</tr>
</tbody>
</table>

#### Test Results

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
<th>DF</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-Square</td>
<td>7.317</td>
<td>1</td>
<td>0.007</td>
</tr>
<tr>
<td>Yates Correction</td>
<td>6.641</td>
<td>1</td>
<td>0.010</td>
</tr>
<tr>
<td>Cochran Correction</td>
<td>7.218</td>
<td>1</td>
<td>0.007</td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>8.122</td>
<td>1</td>
<td>0.004</td>
</tr>
</tbody>
</table>
Acknowledgments

OSPI would like to thank the reviewers for their extraordinary commitment of time and expertise to this review. Their professionalism and thorough review of each program was invaluable. Our deep appreciation goes to each one.

K–12 COMPUTATIONAL FLUENCY SUPPLEMENTAL PROGRAM REVIEW PANEL

Debbie Aldous  Heidi Cowen  Jean Kilian  Mary Jo Ormsby
Kimberlee Armstrong  Jenneifer DeLashmutt  Keri Klayum  May Ovalles
Sharron Aune-Ruland  Tammy Droppo  Anton Kramer  Jeremy Rogers
Ida Baird  Kathleen Gibson  Lillian Lahiri  Judith Serrano
Catherine Ball  Carolyn Gordon  Linda Larson  Tara StaeheI
Jennifer Beymer  Shari Hartwig  Carole Leininger  Pamela Stephens
Steve Blum-Anderson  Shawna Hoggard  Nancy Lenihan  Beth Stoebuck
Brian Brockmueller  Tricia Hukee  Deanna Lomax  Marge Van Tress
Krista Canterbury  Paulette Johnson  Kristen Maxwell  Sherry Vandeventer
Kyung Chung  Char Kendall  Annette Moir  Gloria Williamson

OSPI STAFF

Mickey Venn Lahmann, Assistant Superintendent, Curriculum and Instruction
Jessica Vavrus, Program Manager, Curriculum and Instruction
Beverly Neitzel, Director, Mathematics Initiative
Kathy Dornhecker, Mathematics Initiative Specialist
Dorian “Boo” Drury, Mathematics Initiative Specialist
Lynda Eich, Mathematics Initiative Specialist
Karen Hall, Mathematics Initiative Specialist
Robert Hodgman, Mathematics Initiative Specialist
Mary Holmberg, Teacher on Special Assignment
Karrin Lewis, Mathematics Initiative Specialist
Gloria Hong, Mathematics Initiative Support
Lee Ann Mills, Mathematics Initiative Support
Kristina Quimby, Mathematics Initiative Support

EXTERNAL CONSULTANTS

Porsche Everson, Relevant Strategies
Eugene Ryser, Consultant