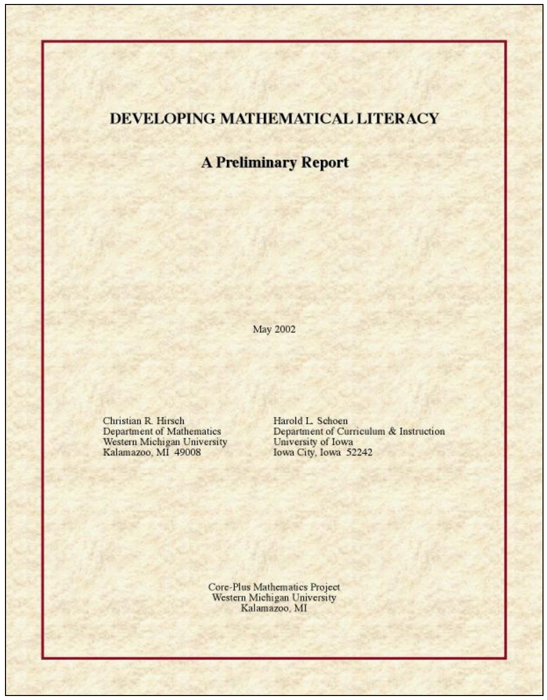

DEVELOPING MATHEMATICAL LITERACY

A Preliminary Report

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Christian R. Hirsch
Department of Mathematics
Western Michigan University
Kalamazoo, MI 49008

Harold L. Schoen
Department of Curriculum & Instruction
University of Iowa
Iowa City, Iowa 52242

Core-Plus Mathematics Project
Western Michigan University
Kalamazoo, MI

RESOURCE

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ABSTRACT

In 1997, the Core-Plus Mathematics Project began a five-year longitudinal study of students in three high schools in which the first edition of *Core-Plus Mathematics* was used with all students. Of special interest was the impact of the curriculum on students' mathematical literacy. This interim report begins by reviewing the literature that makes the case for the importance of mathematical (or quantitative) literacy for all adults in contemporary society and outlines the attributes of mathematical literacy.

Next a short summary is provided of the performance of Course 1, Course 2, and Course 3 field-test students on different versions of the *Ability to do Quantitative Thinking* subtest of the nationally-normed *Iowa Tests of Educational Development*.

Finally, development of a test of released items from the TIMSS assessment of general mathematical knowledge and its scoring are described. The test results of end-of-Course 3 students in the three longitudinal study schools are reported and compared to the performance of end-of-high school students in the U.S., the Netherlands, and the total International cohort. *Core-Plus Mathematics* students outperformed both the U.S. and International samples. Their performance was closest to that of the Netherlands, the top scoring country on the TIMSS general mathematical knowledge assessment.

Developing Mathematical Literacy

A Preliminary Report

In 1986 the Board of Directors of the National Council of Teachers of Mathematics (NCTM) established the Commission on Standards for School Mathematics to:

- Create a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied in diverse fields; and
- Create a set of standards to guide the revision of the school mathematics curriculum and its associated evaluation toward this vision (NCTM, 1989, p.1).

The products of this charge were NCTM's three Standards documents published in 1989, 1991, and 1995, and its recently published *Principles and Standards for School Mathematics* (2000).

The central tenet underlying this charge is that in today's information age and data-drenched society, to develop an informed citizenry and to support a democratic government, it is essential that *all* high school graduates be mathematically as well as verbally literate. To meet this challenge, over the last decade mathematics education has been negotiating several difficult transitions set out in *Everybody Counts* (MSEB 1989), including:

- Shifting the focus of school mathematics from a dualistic mission – minimal mathematics for the majority, advanced mathematics for a few – to a singular focus on a significant common core of mathematics for all students (p. 81); and
- Shifting the teaching of mathematics from [sole] emphasis on tools for future courses, to greater emphasis on topics that are relevant to students' present and future needs (p. 83).

The work reported in this paper was supported, in part, by grants from the National Science Foundation (MDR-9255257, ESI-9618193). Views and conclusions expressed are those of the authors and not necessarily those of the Foundation.

Since 1992, with support from the National Science Foundation, the Core-Plus Mathematics Project (CPMP) has been developing and evaluating a comprehensive high school mathematics curriculum intended to help U.S. schools in making these transitions as they work toward implementing the curriculum, instruction, and assessment recommendations in the *Standards* documents.

Elements of Mathematical (Quantitative) Literacy

In developing the CPMP curriculum we have taken the perspective that most mathematics in a high school program intended for all students should be developed in the context of its uses, with the abstraction and appreciation of mathematics as a deductive logical system building slowly across the four-course curriculum. Topics that have received increased attention in the CPMP curriculum include:

- Probability, which facilitates reasoning about uncertainty and assessment of risk;
- Exploratory data analysis and statistics, which facilitate reasoning about data;
- Model-building, which facilitates systematic, structured understanding of complex situations;
- Operations research, which facilitates planning of complex tasks and achieving performance objectives;
- Discrete mathematics, which facilitates understanding of most applications of computers.

However, mathematical literacy requires more than the acquisition of an expanded list of quantitative skills. As Price (1997) argues, logical thinking, analysis of evidence, and statistical reasoning are more important for engaged citizenship in the twenty-first century, than traditional mathematical skills. Perhaps this more comprehensive view of mathematical literacy is best captured in the International Life Skills Survey currently underway that defines *quantitative literacy* as reported in Steen (2001) as:

> An aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work.

Sometimes *mathematical literacy* and *quantitative literacy* are used interchangeably, but often they are used to denote important distinctions. For example, to distinguish between what is needed for life (quantitative) and what is needed for education (mathematics), or between what is needed for general school subjects (quantitative) and what is needed for engineering and physical science (mathematics) (Steen 2001). The CPMP curriculum has attempted to unify these orientations in an integrated, contextualized approach that is at once applied and theoretical, concrete and conceptual, practical and academic.

Assessing Mathematical Literacy

As part of the national field-testing of the CPMP curriculum, a standardized test called *Ability to Do Quantitative Thinking* (ITED-Q) (Feldt, Forsyth, Ansley & Alnot, 1993) was administered as a pretest to all CPMP students at the beginning of grade 9 and in alternative forms as posttests at the end of grade 9, grade 10, and grade 11. This test is a subtest of the *Iowa Tests of Educational Development* (ITED). The ITED-Q assesses high school students' ability to use appropriate mathematical reasoning in situations requiring the interpretation of data, charts, or graphs that represent information related to business, social and political issues, medicine, and science. It consists of three subtests:

- *Understanding Mathematical Concepts and Procedures*

  Items require students to select appropriate procedures, make connections among various concepts, and identify examples and counterexamples of concepts.
- *Interpreting Information*

  Items require students to make inferences or predictions based on data or information often given in graphs or tables.
- *Solving Problems*

  Items require students to apply quantitative procedures to relatively novel situations, reason quantitatively, and evaluate reasonableness of solutions.

The mathematical content includes whole numbers, exponents, fractions, decimals, percents, ratios, geometry, measurement, estimation, rounding, statistics, probability, tables, and graphs. Although very little symbolic algebra is required, the ITED-Q is quite demanding for the full range of high school students. For examples, on ITED-Q (Form K, Level 16) the median of the nationally representative norm group in Spring of Grade 10 is approximately 15 of 40 items correct and the 99th percentile is approximately 35 of 40 items correct.

The main field-test results related to ITED-Q are the following. For more details, see Schoen, Hirsch, and Ziebarth (1998) and Schoen and Hirsch (2003).

- At the end of Course 1, CPMP students' mean performance was significantly greater than that of ITED-Q pretest-matched Algebra students in more traditional curricula on the ITED-Q posttest.
- A similar pattern can be seen in Figure 1 where the means of the 1,457 students who completed the four ITED-Q test forms (pretest and end of Courses 1, 2, and 3) are compared to the growth of the ITED-Q's nationally representative norm group at the same pretest percentile (62nd) over the same test times. The growth of the CPMP students was significantly greater than that of the norm group during the first year of the field test and that improvement was maintained in the second and third years. This three-year pattern is consistent, on average, in rural, urban, and suburban schools, for males and females, for various minority groups, and for students for whom English was not their first language.

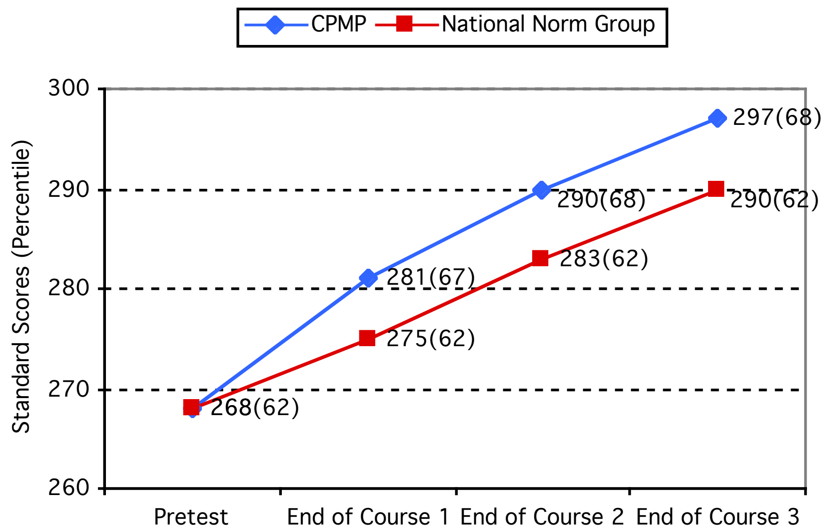

Figure 1. Three-year trends in ITED-Q standard scores and national student percentiles (based on all 1,457 CPMP students with complete data).

This report provides further data and analyses of the mathematical literacy achieved by students in CPMP classrooms. The preliminary results reported here are derived from a five-year longitudinal study of cohorts of students in three high schools that have been using the published *Core-Plus Mathematics* program, *Contemporary Mathematics in Context* (Coxford, Fey, Hirsch, Schoen et al., 1997, 1998, 1999, 2001), with all students.

International Comparisons

During 1995, the Third International Mathematics and Science Study (TIMSS) assessed the mathematics and science knowledge of a half-million students from 41 nations at three levels of schooling. In this report, our focus is on the TIMSS assessment of mathematics general knowledge of students in the last year of secondary school, including those who had taken advanced courses as well as those who had not. Mathematics general knowledge items were chosen by TIMSS "based on their likelihood of arising in real-life situations and not on their connection to a particular curriculum. However, they can be described in terms of common mathematics curriculum topics, such as number sense, including fractions, percentages, and proportionality; algebraic sense; measurement and estimation; and data representation." (USDE, 1998; p. 30). Clearly, mathematical literacy as discussed earlier has much in common with the

TIMSS mathematics general knowledge framework, and TIMSS documents seem to use the terms interchangeably as well (IEA TIMSS, 1998).

The TIMSS mathematics general knowledge assessment consisted of test booklets that contained 34 multiple-choice items and 10 free-response items. The assessment was administered to a representative sample of all students at the end of secondary school (twelfth grade in the U.S.) in 21 participating countries. U.S. twelfth graders performed below the international average, significantly below 14 countries, and significantly above just 2 countries. The mean performance of students in the Netherlands was the highest among the 21 countries, as seen in Table 1.

Table 1
TIMSS mathematics general knowledge achievement by country

Nations with Average Scores Significantly Higher than the U.S.

Nation	Average
Netherlands	560
Sweden	552
Denmark	547
Switzerland	540
Iceland	534
Norway	528
France	523
New Zealand	522
Australia	522
Canada	519
Austria	518
Slovenia	512
Germany	495
Hungary	483

Nations with Average Scores Not Significantly Different from the U.S.

Nation	Average
Italy	476
Russia Federation	471
Lithuania	469
Czech Republic	466
United States	461

Nation with Average Scores Significantly Lower than the U.S.

Nation	Average
Cyprus	446
South Africa	356

International Average = 500
U.S. Average = 461

Following the administration of the TIMSS assessments, many test items were released for use by researchers or school personnel. Scoring rubrics, answer keys, and detailed performance data by country were also made available for all released items (IEA TIMSS, 1998).

Procedures

In the present study, a test composed of TIMSS mathematical literacy assessment items was used to gather further evidence concerning how well *Standards*-oriented curricula prepare students in the mathematical competencies usually associated with quantitative literacy. Twelve released TIMSS mathematics general knowledge items were chosen, two or three from each of five content subtests (percent, ratio and proportion, numbers and operations, measurement, and graphical interpretation). To maintain a split between multiple-choice and free-response items proportional to that used by TIMSS, 10 of the items chosen were multiple-choice and two were free-response. In the following analysis, the items were separated into two 6-item subtests, Number Sense and Fluency (percent, ratio and proportion, and numbers and operations) and Measurement and Data Representation (geometric measurement and graphical interpretation). As in TIMSS, students had access to a calculator and were given a time limit that allowed an average of about two minutes per item.

This test was administered to all 371 students at the end of CPMP Course 3 in three high schools that use the CPMP curriculum as their only high school program. The students who had attended middle school in the same district as their high school (about 80% of tested students) also completed a *Standards*-oriented curriculum in middle school. Two of the participating school districts used the Connected Mathematics Project (CMP) curriculum and the other used the MATH Thematics (STEM) curriculum. The vast majority of the students were juniors, and less than 5% were either sophomores or seniors.

Two of the schools are in rural settings, and one is in a small city. The student population in one of the rural schools was about 28% under-represented minority, while the minority population of the other two schools was under 8%. Prior to the adoption of the CPMP curriculum, students in these three schools at the beginning of grade 9 scored on average at the 56th student percentile on the ITED-Q. On the whole, the communities these schools serve are working class and their students are not academically elite.

After testing these students, we scored their work using the scoring rubrics and answer key provided by TIMSS. Multiple-choice items were scored as correct (1 point) or incorrect and the two open-ended items were scored using the 2-point rubrics provided by TIMSS. We then compared the CPMP students' performance by content subtest and by item to the TIMSS performance data for the U.S., the international average, and the Netherlands.

Results

Means by content subtest of the 371 CPMP students and the TIMSS samples from the United States, the Netherlands, and the 21-country compilation are presented in Figure 2. The subtest means of the CPMP Course 3 students were consistently higher than the U.S. and international samples and were similar to the Netherlands, the highest performing country. Individual items together with performance statistics can be found in the Appendix.

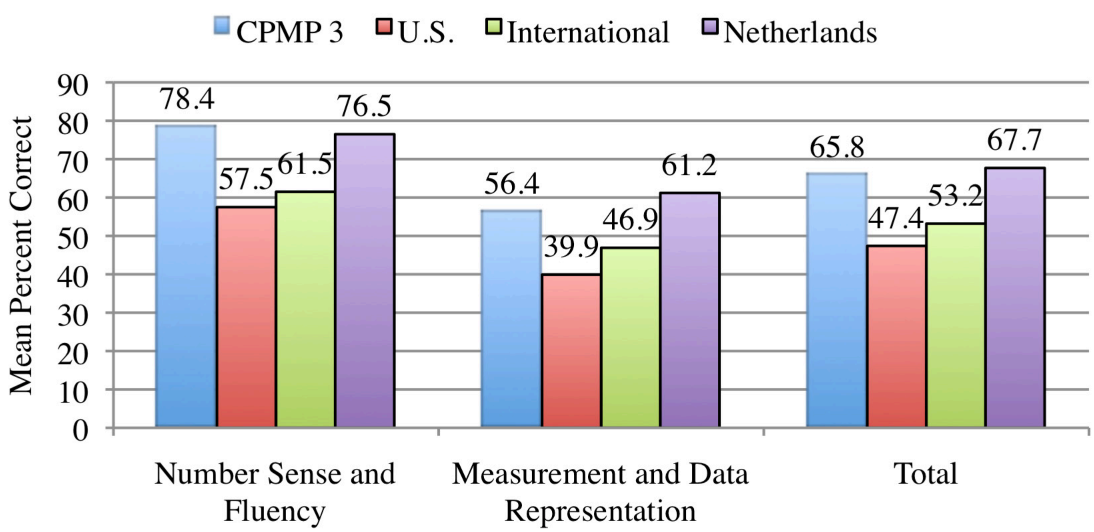

Figure 2. Mathematics General Knowledge Subtest Means

To further examine the group differences by subtest, we computed the effect size for each subtest and for each of the three comparison groups. See Table 2. The effect size is the number of standard deviations by which the CPMP subtest mean differed from each of the TIMSS comparison groups. Here the standard deviation is that of the CPMP group.

Effect size (ES) = (CPMP Course 3 mean – TIMSS group mean) / (Course 3 standard deviation)

Table 2

Subtest means and effect sizes for CPMP and TIMSS groups

	CPMP		U.S.		International		Netherlands	
Subtest	**Mean**	**SD**	**Mean**	**ES**	**Mean**	**ES**	**Mean**	**ES**
Number Sense and Fluency	78.4	21.1	57.5	0.99	61.5	0.80	76.5	0.09
Measurement and Data Representation	56.4	22.3	39.9	0.74	46.9	0.43	61.2	-0.22
Total	65.8	18.7	47.4	0.98	53.2	0.67	67.7	-0.10

As seen in Table 2, this study's sample means were similar to those of the Netherlands overall—slightly higher on the Number Sense and Fluency subtest and lower on the Measurement and Data Representation subtest. The means of this study's sample were much higher than those of the representative U.S. 12th-grade sample with effect sizes of .74 and .99. Students in this study also scored considerably better than the international average on all subtests, with effect sizes of .43 and .80. Compared to the subtest means of the U.S. TIMSS sample, both Course 3 subtest means are much higher, but the difference in Number Sense and Fluency means is relatively greater than the difference in Measurement and Data Representation means.

Performance of the CPMP students was generally consistent across the three schools as Table 3 shows. A MANOVA with the five subtest scores as dependent variables and school as independent variable resulted in no significant overall school differences ($p = 0.34$). With the exception of a difference of about one-third of a standard deviation between the means of School 1 and School 2 on the percent subtest, the school differences on individual subtests are well within chance variation.

Table 3

Subtest means and standard deviations for CPMP students by school

	CPMP (N = 371)		School 1 (N = 269)		School 2 (N = 56)		School 3 (N = 46)	
Subtest	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**
Number Sense and Fluency	78.4	21.1	77.4	21.4	81.0	17.5	80.8	23.3
Measurement and Data Representation	56.4	22.3	55.7	23.2	58.3	20.0	58.4	20.3

Discussion

There is a growing recognition of the importance of mathematical literacy for the general population. A summary of this study's main findings on the TIMSS-based mathematical literacy test follows. "Adjusted means" refer to means adjusted for variance on the *ITED-Q* pretest at the beginning of Grade 9.

- The mean of students at the end of Course 3 was nearly a full standard deviation above that of the U.S. TIMSS sample and about three-fourths of a standard deviation higher than the international mean.
- On the Number Sense and Fluency subtest, the mean of Course 3 students was slightly higher than that of the Netherlands, the highest scoring country, and on the Measurement and Data Representation subtest, their mean was about one-fifth of a standard deviation lower than that of the Netherlands.
- Adjusted means of schools did not differ significantly by school.

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APPENDIX

Mean Performance by Item

Number Sense and Fluency Subtest

1. Experts say that 25% of all serious bicycle accidents involve head injuries and that, of all head injuries, 80% are fatal. What percentage of all serious bicycle accidents involve fatal head injuries?

A. 16%	**CPMP**	**77%**
B. 20%	**U.S.**	**57%**
C. 55%	**International**	**65%**
D. 105%	**Netherlands**	**83%**

2. In a school election with three candidates, Joe received 120 votes, Mary received 50 votes, and George received 30 votes. What percentage of the total number of votes did Joe receive?

A. 60%	**CPMP**	**86%**
B. $66\frac{2}{3}\%$	**U.S.**	**62%**
C. 80%	**International**	**63%**
D. 120%	**Netherlands**	**80%**

3. If there are 300 calories in 100 grams of a certain food, how many calories are there in a 30-gram portion of that food?

A. 90	**CPMP**	**88%**
B. 100	**U.S.**	**68%**
C. 900	**International**	**71%**
D. 1000	**Netherlands**	**84%**
E. 9000		

4. From a batch of 3000 light bulbs, 100 were selected at random and tested. If 5 of the light bulbs in the sample were found to be defective, how many defective light bulbs would be expected in the entire batch?

A. 15	**CPMP**	**89%**
B. 60	**U.S.**	**62%**
C. 150	**International**	**66%**
D. 300	**Netherlands**	**85%**
E. 600		

5. In a vineyard there are 210 rows of vines. Each row is 192 m long and plants are planted 4 m apart. On average, each plant produces 9 kg of grapes each season. The total amount of grapes produced by the vineyard each season is closest to

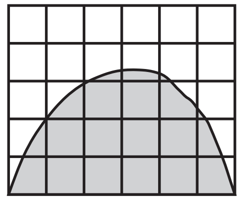
A. 10 000 kg	**CPMP**	**71%**
B. 100 000 kg	**U.S.**	**50%**
C. 400 000 kg	**International**	**55%**
D. 1 600 000 kg	**Netherlands**	**67%**

6. A school club is planning a bus trip to the wildlife park. A bus which will hold up to 45 people will cost $600 and admission tickets cost $30 each. If the cost of the trip, including bus and admission ticket, is set at $50 per person, what is the minimum number of people who must participate to ensure that these costs are covered?

A. 12	**CPMP**	**60%**
B. 20	**U.S.**	**45%**
C. 30	**International**	**50%**
D. 45	**Netherlands**	**61%**

Measurement and Data Representation Subtest

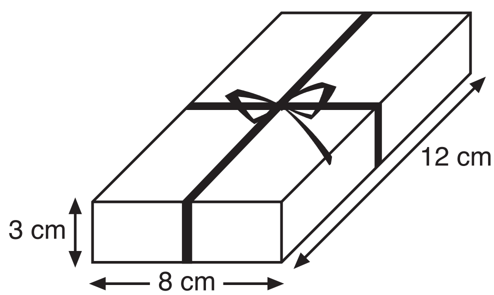
1.



Each of the small squares in the figure is 1 square unit. Which is the best estimate of the area of the shaded region?

A. 10 square units	**CPMP**	**81%**
B. 12 square units	**U.S.**	**54%**
C. 14 square units	**International**	**60%**
D. 16 square units	**Netherlands**	**75%**
E. 18 square units		

2. Stu wants to wrap some ribbon around a box as shown and have 25 cm left to tie a bow. How long a piece of ribbon does he need?

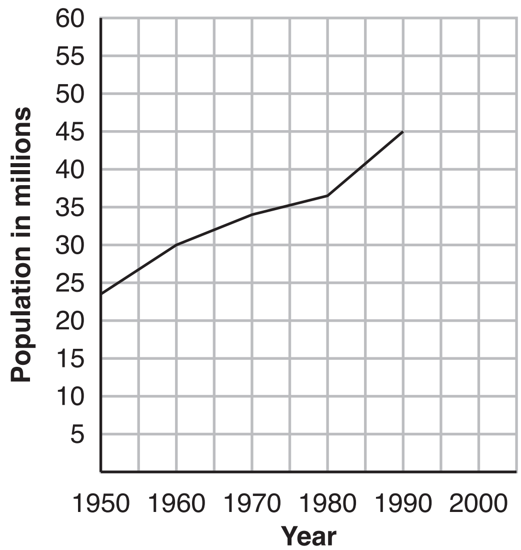


A. 46 cm	**CPMP**	**66%**
B. 52 cm	**U.S.**	**32%**
C. 65 cm	**International**	**45%**
D. 71 cm	**Netherlands**	**62%**
E. 77 cm		

3. Brighto soap powder is packed in cube-shaped cartons. A carton measures 10 cm on each side. The company decides to increase the length of each edge of the carton by 10 percent. How much does the volume increase?

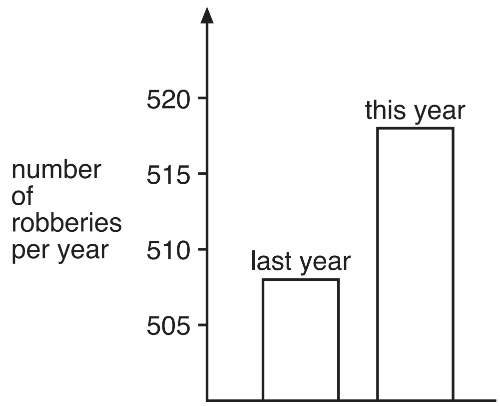
A. 10 cm^3	**CPMP 3**	**40%**
B. 21 cm^3	**U.S.**	**17%**
C. 100 cm^3	**International**	**31%**
D. 331 cm^3	**Netherlands**	**50%**

4. If the population increases by the same rate from the year 1990 to the year 2000 as in the years from 1980 to 1990, approximately what is the expected population by the year 2000?



A. 47 million	**CPMP**	**73%**
B. 50 million	**U.S.**	**69%**
C. 53 million	**International**	**71%**
D. 58 million	**Netherlands**	**85%**

5. A TV reporter showed this graph and said: “There’s been a huge increase in the number of robberies this year.”

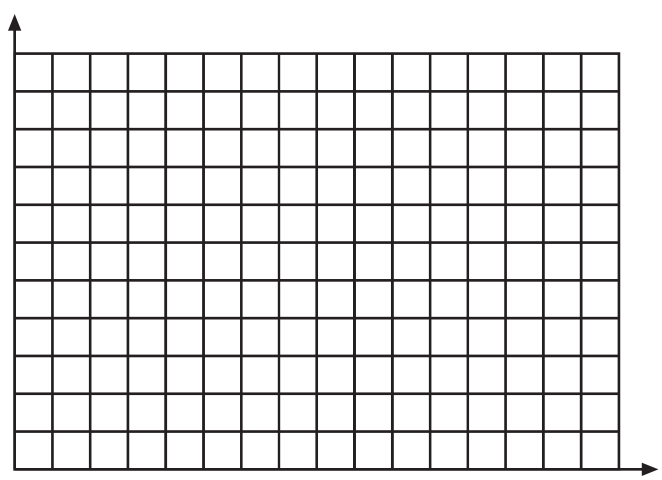


Do you consider the reporter’s statement to be a reasonable interpretation of the graph?
Yes ___ No ___

Briefly explain.

CPMP (51%) U.S. (42%) International (41%) Netherlands (58%)
Percents are of 2 possible points.

6. Using the set of axes below, sketch a graph which shows the relationship between the height of a person and his/her age from birth to 30 years. Be sure to label your graph, and include a realistic scale on each axis.



CPMP (45%) U.S. (32%) International (44%) Netherlands (51%)
Percents are of 2 possible points.