The Research Base for

*Pre-Algebra*
*Algebra 1*
*Geometry*
*Algebra 2*

Lois Edwards, Mathematics Education Consultant
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The Research Base for *Pre-Algebra, Algebra 1, Geometry, and Algebra 2*
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**Introduction**

Extensive efforts during development ensure that results of the latest research in mathematics education form the basis for Glencoe’s *Pre-Algebra, Algebra 1, Geometry, and Algebra 2*.

Educational research provides information about what students are likely to learn in specific content areas at specific levels and under specific pedagogical conditions. It provides guidance for effective curriculum and instruction. Glencoe’s authors have incorporated the results of educational research in mathematics into this curriculum, enabling teachers to provide their students with excellent learning experiences.

This white paper describes research results in several key areas of mathematics education: curriculum, educational principles, instructional strategies, and mathematical concepts and skills. It points out the Glencoe features that exemplify these research results and provides page numbers of selected examples. A comprehensive list of research references is included.

**Curriculum**

*NCTM Principles and Standards for School Mathematics*

The past two decades have seen an increased recognition of the importance of mathematics for every student and accompanying need for creating uniform national standards in mathematics education. The National Council of Teachers of Mathematics (NCTM) has led this movement.

Beginning in 1989, NCTM released publications on curriculum and evaluation, assessment, and professional standards, which have given focus, organization, and fresh ideas to efforts to improve mathematics education. In 2000, NCTM published the *Principles and Standards for School Mathematics*, a vision to guide educators toward continual improvement of mathematics education in classrooms, schools, and educational systems. It set forth important overall characteristics of mathematics programs (principles) and described the mathematical content that students should learn (standards). The *Principles and Standards for School Mathematics* are consistent with the best evidence on teaching and learning mathematics. They were written through a complex process that involved professional judgment, societal expectations, and research results.
Glencoe’s mathematics series incorporates the key characteristics recommended by *Principles and Standards for School Mathematics* for effective curricula:

- Interconnected topical strands, such as algebra and geometry;
- Mathematical concepts that are organized and integrated so that students see how concepts are related;
- Foundational concepts such as equivalence, proportionality, function, and rate of change;
- Activities that develop mathematical thinking and reasoning skills, including making conjectures and developing deductive arguments;
- Experiences that demonstrate mathematics’ usefulness in modeling and predicting real-world phenomena; and
- Guidance for teachers on the depth of study warranted at specific levels and when closure is expected.

Glencoe’s *Pre-Algebra, Algebra 1, Geometry, and Algebra 2* series meets all six of the principles set forth in the *Principles and Standards for School Mathematics*.

- **Equity** Glencoe’s texts encourage high achievement at all levels. Numerous teacher support materials provide activities for differentiated instruction, promotion of reading and writing skills, pacing for individual levels of achievement, and daily intervention opportunities.
- **Curriculum** Glencoe’s curriculum is based on a detailed scope and sequence that ensures a continuum of mathematical learning that builds on prior knowledge.
- **Teaching** The comprehensive Teacher Editions provide mathematical background, teaching tips, resource management guides, and tips for new teachers.
- **Learning** The Teacher Editions include ways to build on students’ prior knowledge and suggestions for dealing with common misconceptions. Students engage in both practice and exploratory activities.
- **Assessment** A full array of diagnostic, formative, and summative assessment is included.
- **Technology** The Student Editions offer opportunities to use graphing calculators, spreadsheets, and geometry software. Glencoe’s Web site is constantly updated to meet the needs of students and teachers.

Glencoe’s series also meets all of the *Principles and Standards for School Mathematics*’ Content Standards. (For details, see the sections in this document on Instructional Strategies and Mathematical Concepts and Skills.)
Curriculum Characteristics

Research shows that a curriculum that enables students to reach high mathematics standards has the following characteristics:

- **Balance**—focuses on both conceptual understanding and procedural fluency;
- **Comprehensiveness**—includes all the important content strands of mathematics, as well as computation and other procedural skills;
- **Alignment with standards**—includes content and strategies which align with state and national standards, external assessments and instruction; and
- **Coordination and coherence within and across grades**—presents well-developed concepts that build on and connect with other concepts, both within and across grades (Apthorp, Bodrova, Dean, & Florian, 2001).

Research shows that student learning is affected by the written curriculum but also by teacher beliefs, teacher knowledge, teacher interpretation of curriculum, and classroom structures and norms (Stein et al., 2007).

Providing opportunities both for discovery and for practice improves student achievement. Students can learn both concepts and skills by solving problems (Grouws & Cebulla, 2000).

Focusing instruction on the meaningful development of mathematical ideas increases the level of student learning. It is important to emphasize meaning and how the idea, concept, or skill is connected to other mathematical ideas (Grouws & Cebulla, 2000).

Glencoe’s *Pre-Algebra, Algebra 1, Geometry*, and *Algebra 2* exemplify these characteristics. Balanced instruction throughout the series emphasizes both understanding and fluency. Features include Key Concept boxes with multiple representations, a four-step problem solving plan, correlation between examples and exercises, word problems within each lesson, reading and vocabulary support, and student-centered activity labs.

The Glencoe series is a comprehensive program, including all of the major mathematical content strands and mathematical processes. Main Ideas and Key Concepts are highlighted in the Student Edition. The Teacher Edition includes Focus on Mathematical Content for each chapter. This section provides Big Ideas, which explains why the content is important in this
chapter and in later studies, and Lesson Summary features, which outline the mathematical ideas of each lesson in the chapter.

The Glencoe series is carefully aligned with national standards. Pre-Algebra is also correlated with the NCTM Curriculum Focal Points for Grades 7 and 8. Each curriculum contains Standardized Test Examples and practice problems like those found on standardized tests—within the lessons, the Mid-Chapter Quiz, and the Practice Chapter Test. A separate chapter in the Student Handbook, Preparing for Standardized Tests, provides comprehensive guidance.

The Glencoe curriculum is coherent and coordinated. It is vertically aligned between courses and vertically aligned at the lesson level. The Teacher Edition includes a detailed description of vertical alignment for each chapter. The Focus step in each lesson includes the vertical alignment for that lesson. In the Student Edition, Get Ready for the Chapter provides both a diagnostic quiz and a review of related concepts that were previously studied.

<table>
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**Educational Principles**

**Professional Development**

Research shows that to be effective, teachers need to fully understand the concepts that make up mathematics and underlie mathematical procedures. They need to understand the interconnections among topics (Mewborn, 2003; Swafford, Jones, & Thornton, 1997).
Many teachers lack a conceptual understanding of mathematics, although they have strong procedural knowledge (Mewborn, 2003; Even & Tirosh, 1995).

Teachers need to be actively engaged in learning new ideas and to translate what they learn into classroom practice (Mewborn, 2003; Shifter, 1998).

Teachers must listen to students’ reasoning and be able to make sense of students’ mathematical thinking (Mewborn, 2003; Thompson & Thompson, 1994, 1996).

Studies show that collegial support for teachers working to improve instructional practices is valuable, but it must be accompanied by a strong focus on mathematical content, students’ thinking, and curriculum (Mewborn, 2003).

Teachers need to learn and use three key classroom practices for doing and learning mathematics:

- Supporting discourse;
- Establishing norms, such as what counts as a mathematical explanation; and
- Building relationships, which includes understanding students’ identity and culture (Franke et al., 2007).

The Glencoe curriculum provides many opportunities for professional development. Learning opportunities through video, online activities, and on-site instructions are available and fully aligned for Pre-Algebra through Algebra 2.

This curriculum provides abundant support in clear, easy-to-use Teacher Editions with planning and pacing suggestions, background mathematics information, and vertical alignment charts. Features such as Teacher Tips, Tips for New Teachers, and Focus on Mathematical Content help teachers understand concepts and student thinking.

*What the Research Says* explains related research. *Reading and Writing in Mathematics* and *Project CRISS™* strategies guide teachers in helping students.

**Differentiated Instruction**

Research demonstrates that participation (what students do and become within the classroom community) and identity (students’ perception of themselves and others’ perception of them as doers of mathematics) are two critical aspects of classroom practices. (Diversity...., 2007).
Teachers need to know how children think in mathematics in order to make appropriate instructional decisions based on what each child knows and can do (Carey, Fennema, Carpenter, & Franke, 1995).

Studies show that students with learning disabilities benefit from activities-based, hands-on learning, as well as reasoning and problem-solving experiences. Strategies may need to be modified by providing explicit instruction, structuring activities, and stressing vocabulary acquisition (Goldman et al., 1998; Miller & Mercer, 1998).

Effective practices for teaching students with difficulties in mathematics (special education and low-achieving students) include systematic and explicit instruction, student think-alouds, visual and graphic depictions of problems, structured peer-assisted learning in heterogeneous ability groupings, and formative assessment data provided to teachers and to students (Gersten et al., 2006).

Effective strategies for students with learning disabilities include using advance organizers to clarify the lesson’s purpose; reviewing major concepts frequently; teaching generalization and application; modeling procedures at a slow pace with extra clues; presenting new skills using concrete materials, then pictures, and finally abstract explanations; providing extra practice in small steps with guidance; giving clear directions before practice; and checking for error patterns (Bishop & Forgasz, 2007).

Research has shown that specific strategies encourage girls in mathematics, including:

- “Teach students that academic abilities are expandable and improvable;
- Provide prescriptive, informational feedback;
- Expose girls to female role models who have succeeded in math and science;
- Create a classroom environment that sparks initial curiosity and fosters long-term interest in math and science; and
- Provide spatial-skills training” (Halpern et al., 2007).

The Glencoe curriculum includes extensive resources to help teachers provide differentiated instruction. Each chapter lists Teacher Resources in a chart with level codes: BL, OL, AL, ELL. Many blackline masters are also available in Spanish. Real-World Careers features include women who use mathematics.
Leveled Resources provide worksheets for each lesson to meet individual needs: Lesson Reading Guide, Study Guide and Intervention, Skills Practice, Practice, Word Problem Practice, Enrichment, and Graphing Calculator and Spreadsheet Activities. Each lesson provides teachers with a chart of Differentiated Homework Options, for BL, OL, and AL levels. Each chapter (Pre-Algebra, Algebra 1, and Geometry) includes Options for Differentiated Instruction, for ELL, AL, SS (Struggling Students), and SN (Special Needs) and provides teachers with information on the Quick Review Math Handbook for intensive intervention. Additional resources are available online and on CD-ROM (StudentWorks Plus™).

Assessment
Research has demonstrated that collecting assessment data from students, both formally and informally, is a key to shaping effective instruction (Brimijoin et al., 2003). This is called data-driven assessment.

An extensive research review of 250 journal articles concluded that stronger formative assessments produce significant learning gains (Wilson & Kenny, 2003; Black & Wiliam, 1998a; Black & Wiliam, 1998b).

The use of day-to-day formative assessment is one of the most powerful ways of improving learning in mathematics (Wiliam, 2007). Formative assessment includes determining where learners are in their learning, where they are going, and how they will get there (Wiliam & Thompson, 2007).

Research shows that to effectively use formative assessment, teachers should clarify, share, and understand goals and criteria for success; use classroom discussions, questions, activities, and tasks that elicit evidence of students’ learning; provide feedback that tells students how to improve; help students learn to help each other; and help students become responsible for their own learning (Wiliam, 2007).

Effective feedback is focused on the mastery of the task, given soon after the task, and specific and related to students’ needs (Marzano et al., 2001; Crooks, 1988).

Frequent, short tests are better than infrequent, long tests. New learning should first be tested within about a week of its first encounter (Black & Wiliam, 1998b).
Performance improves with frequent testing to a certain point, but additional tests have little extra impact (Wilson & Kenny, 2003; Bangert-Drowns, Kulik, & Kulik, 1991).

Throughout the Glencoe series, numerous Diagnostic, Formative, and Summative Assessments are offered. Data-driven decision making provides frequent and meaningful assessment of student progress within the curriculum structure and teacher support materials. Teachers can also create and customize their own assessments using ExamView Assessment Suite software.

Initial assessment at the beginning of the year using the Diagnostic and Placement Tests booklet assesses students’ prior knowledge. Get Ready for the Chapter and Get Ready for the Next Lesson in the Student Edition checks on students’ prior knowledge, while the Teacher Edition provides Intervention Options based on the results.

Opportunities for formative assessment are described in the Teacher Edition Assessment Planner for each chapter, listing formative and summative assessment options.

The Student Edition includes Check Your Progress, Check Your Understanding, Mid-Chapter Quiz, and Study Guide and Review for students to assess their progress. The Teacher Edition for each lesson includes Step 4 (Assess) of the Teaching Plan. Data-Driven Decision Making charts accompany the Mid-Chapter Quiz to help teachers modify instruction based on student performance. Self-check quizzes are available at the course Web site. Students learn to assess their own work in Find the Error exercises.

Summative assessments include Chapter Practice Test and Standardized Test Practice. The Teacher Edition includes Data-Driven Decision Making charts for the Chapter Tests. Alternate, leveled chapter tests in six forms are available in the Chapter Resource Masters (CRM). Six forms of Chapter Tests, four Quizzes, Vocabulary Test, Extended-Response Test, and Standardized Test Practice are also available.

**Technology**

Research shows that technology use increases representation fluency, the ability to move between representations while carrying the meaning from one representation to another. It does this by providing multiple representations that are linked and interactive (Zbiek et al., 2007).
When technology is used in classrooms, teachers more often tend to take on the role of counselor, providing mathematical assistance when students request it (Zbiek et al., 2007).

Students’ behaviors vary when using technological tools. Some students always use them judiciously; some use the tools capriciously and inappropriately. Most students can be educated to work judiciously with these tools (Zbiek et al., 2007; Ball & Stacey, 2005).

Research results on calculator use are generally positive (Apthorp et al., 2001). Research on graphing calculator use also shows positive effects on graphing ability, conceptual understanding of graphs, and relating graphs to other representations (Grouws & Cebulla, 2000).

Calculator use enhances learning arithmetical concepts and skills, problem solving, and attitudes of students (Hiebert, 2003). Most graphing calculator studies have not found negative effects on basic computation skills or factual knowledge (Grouws & Cebulla, 1999).

With calculators, teachers tend to ask more high-level questions; students are more actively involved in asking questions and conjecturing (Grouws & Cebulla, 1999).

Research on computer use indicates that application software, such as spreadsheets and graphing tools, can promote learning (Grouws & Cebulla, 2000). The most effective use of instructional software for mathematics occurs when meaning and understanding are emphasized, mathematical skills are embedded in context, and connections are made between subject areas and real-life experiences (Reyes et al., 1999). Software like Geometer’s Sketchpad, with its interactive, dynamic representations, can help students draw meaning about a mathematical entity if the students reflect on the connection between their actions and the change in representations (Zbiek et al., 2007).

The Glencoe curriculum offers fully integrated technology resources for teachers, students, and parents online and on CD-ROMs and DVDs, providing support for differentiated instruction, alternate teaching approaches, additional assessment opportunities, and motivating activities.

Students in Pre-Algebra are introduced to graphing calculators at the beginning of the year. The curriculum includes several Graphing Calculator Labs and Spreadsheet Labs.
Algebra 1 contains 14 Graphing Calculator Labs, ranging from linear functions to exponential functions to scatter plots and a Spreadsheet Lab on weighted averages.

In Geometry, Geometry Software Labs use Geometer’s Sketchpad software. Graphing Calculator Labs explore a variety of concepts and Spreadsheet Labs explore volume and angles in polygons.

In Pre-Algebra, Algebra 1, and Geometry, What’s Math Got to Do With It? Real Life Math Videos on DVD show students how math is used in everyday situations.

Algebra 2 contains 17 Graphing Calculator Labs, including function graphing, limits, solving equations, matrices, and modeling data. It offers three Spreadsheet Labs, including amortizing loans.

Throughout the series, Concepts in Motion, referenced in the Teacher Edition, provide online illustrations of key concepts. MindJogger Plus on DVD reviews concepts in a game-show format with student teams. Internet Resources for each chapter are listed in a table in the Teacher Edition.

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Instructional Strategies

Balanced Instruction

Research shows that teachers cannot simply transfer knowledge to students by lecturing. Students have to take an active role in their own learning. To accomplish this, mathematics programs must include ample opportunity to explore, question, discuss, and discover. The learning experience should include a balance of discovery learning and explicit instruction. Students need to learn how to grapple with problems and construct conceptual knowledge (Pressley, Harris, & Marks, 1992; Shulman & Keislar, 1996). Explicit instruction occurs when teachers and textbooks clearly explain problem-solving strategies to students (Duffy, 2002). In geometry instruction, a combination of expository and discovery methods has been found to be effective, especially when manipulatives are used to help students represent and comprehend geometric concepts (Klausmeier, 1992).

Glencoe’s curriculum offers a balanced approach of real-world applications, hands-on labs, direct instruction, writing exercises, and practice that enables students to develop both conceptual understanding and procedural knowledge. Lessons begin with real-world problems to solve, and then students use multiple representations to explore new concepts. Algebra Labs and Geometry Labs provide carefully designed opportunities for discovery. Graphing Calculator Labs and Spreadsheet Labs use technology to promote discovery of patterns and relationships.

Use of Prior Knowledge

Prior-knowledge strategies help students retrieve information stored in their long-term memories to learn new, related information. These strategies include recalling remembered information, asking questions, elaborating on textbook and teacher information, and referring students to other meaningful information. Asking students to use prior knowledge may remind them of information already in their long-term memory that, for some reason, is not easily remembered (Bransford, 1979; Pressley & McCormick, 1995).

Glencoe’s series of courses intertwine concepts and continuously refer to material in previous chapters and in students’ personal experiences. Spiral Review exercises are included in each lesson. Get Ready for the Next Lesson reminds students of prerequisite skills. The Teacher Edition includes a Vertical Alignment at the beginning of each lesson, so teachers are aware of previous and future related concepts. Vocabulary Link features in the Student Editions connect
everyday uses and math uses of words, for example, “distribute” and “distributive property.” Study Tips in the margin include Look Back features that help students review previously learned concepts at the point that they will be used.

Practice
Providing students with practice on important tasks has long been considered a successful strategy to improve understanding and memory. Giving students individual feedback on their practice helps them monitor and improve their mathematical learning. Practicing helps students acquire additional information as they work towards understanding and applying mathematical concepts. Research shows that mastering a skill requires focused practice. During practice, students adapt and shape what they have learned. In doing so, they increase their conceptual understanding (Clement, Lockhead, & Mink, 1979; Davis, 1984; Mathematical Science Education Board, 1990; Romberg & Carpenter, 1986).

Mastering a skill requires considerable focused practice. To reach a level of 80% of mastery usually takes about 24 practice sessions. However, the first four practice sessions lead to an approximately 48% level of proficiency (Marzano et al., 2001).

Check Your Understanding exercises in each lesson ensure that students understand the concepts and skills before they practice independently. Teachers can provide feedback before students work exercises on their own, either in class or as homework. Differentiated Homework Options in the Teacher Editions help teachers provide students with appropriate practice. H.O.T. Problems give practice in using higher order thinking skills. The Spiral Review section in every lesson includes cumulative practice. Each lesson also contains Standardized Test Practice. Each chapter provides two pages of Standardized Test Practice problems that enable students to practice skills and at the same time to gain experience in taking standardized tests. Additional worksheets (Skills Practice, Practice, Word Problem Practice) and online practice quizzes are also available, as is the Extra Practice section in the Student Handbook.

Note Taking
In the process of note-taking, students identify important items from reading and write them in an organized format. As they write and draw notes, students see relationships within the information. Notes should not be verbatim; note-taking is most valuable when students learn
to select important points (Marzano et al., 2001; Bretzing & Kulhary, 1979). Students benefit from learning various note-taking formats, since there is no single correct way to take notes (Marzano et al., 2001). Studies show that using notes as a study guide for tests is an effective form of review (Marzano et al., 2001).

When study skills, such as note-taking, are taught within the teaching of content, they promote learner activity and improve metacognition (Hattie et al., 1996; Robinson & Kiewra, 1995).

Each chapter in the Glencoe mathematics series includes instructions for study organizers, called Foldables®, created by Dinah Zike. Study organizers are handmade paper booklets. The Foldable, designed to fit each chapter’s content, guides students in choosing the important concepts and recording them in an organized format. Since students make their own three-dimensional Foldables as well as enter the notes, they feel a sense of ownership. Foldables also help students review the chapter’s concepts.

The Study Guide and Review feature at the end of each chapter consists of notes plus worked examples and practice exercises. The Lesson-by-Lesson Review presents a clear picture of the important concepts in each lesson and provides students with a model for taking notes. Project CRISS Study Skills include many note-taking strategies, such as Venn diagrams. The Notables™ Interactive Study Notebook with Foldables for each chapter helps students take and organize notes.

**Cooperative Learning**

Cooperative learning occurs when students work in pairs or groups of three or four to complete tasks. Research shows that cooperative learning provides practice at valuable skills, such as positive interdependence, face-to-face interactions, individual and group accountability, interpersonal skills, and group processing (Johnson & Johnson, 1999). Cooperative learning has a highly positive effect when compared with strategies in which students compete with each other and strategies in which students work on tasks individually (Marzano et al., 2001; Johnson, Maruyama, Johnson, Nelson, & Skon, 1981). A balance of cooperative learning and individual learning is needed, however, because students need time to practice skills independently (Anderson, Keder, & Simon, 1997).

Glencoe’s mathematics series provides a mix of individual and cooperative learning opportunities. The Teacher Editions include a detailed section on Cooperative Group
Strategies, helping teachers implement this strategy. A variety of labs—Algebra Labs, Graphing Calculator Labs, Geometry Labs, and Spreadsheet Labs—located throughout the texts, direct students to work with other students in carefully structured activities. In Algebra Labs, students work together in small groups to collect data, analyze data, and make conjectures. In lessons where it is appropriate, Tips for New Teachers: Working Together suggest cooperative learning opportunities. Differentiated Instruction: Interpersonal Learners describes additional learning activities where students work in pairs.

**Identifying Similarities and Differences**

Comparison and classification skills are vital in mathematics and science. Students identify similarities and differences to determine how the current problem and previously solved problems are alike and different. Research tells us that comparing and classifying are effective ways to identify similarities and differences (Chen, 1996; English, 1997; Newby et al., 1995). The most effective methods of working with similarities and differences are to have students identify similarities and differences on their own (Chen, 1996; Mason & Sorzio, 1996), use graphic and symbolic representations as well as words (Mason, 1994), and begin with concrete examples and then move towards abstract knowledge (Reeves & Weisberg, 1993).

In Glencoe’s mathematics series, students often explain the difference between two related concepts, such as a sequence and a series. They compare and contrast graphs or equations that belong to a family of graphs or equations, respectively. In H.O.T. Problems, Which One Doesn’t Belong? exercises are classification problems, and some Reasoning questions ask students to compare two items. Students also interpret and create Venn diagrams and other diagrams showing classification, such as various types of polygons.

**Reading and Writing in Mathematics**

Reading and writing are two of the major activities through which students acquire mathematical understanding. Their importance increases as grade level increases.

Reading mathematics requires unique knowledge and skills, since mathematical writing is dense, terse, and comprised of symbols as well as words. (Barton & Heidema, 2002). Reading helps students understand concepts, work problems, organize ideas, extend their thinking, and view mathematics as a valuable subject (Martinez & Martinez, 2001).
Teachers should model for students how to monitor and reflect on their reading. Modeling by thinking aloud is effective (Barton & Heidema, 2002). Teachers can model reading strategies by reading a problem aloud, paraphrasing it, and talking through how to figure out word meanings by asking if everyone is clear on a word’s meaning and by asking questions about the meaning of the problem (Kenney et al., 2005). Reading word problems requires special skills: understanding the problem situation, identifying the main idea, and extracting relevant information (Musthafa, 1996).

Mastering vocabulary is fundamental to mathematical understanding. Students struggle with mathematics vocabulary because of the overlap between mathematical and everyday English usage, the use of mathematical symbols (Fuentes, 1998), and concepts embedded within other concepts (Barton & Heidema, 2002). Direct instruction on vocabulary words increases student achievement (Stahl & Fairbanks, 1986). Graphic organizers, such as concept maps, help students learn vocabulary (Monroe, 1997). Research has shown that repetition and multiple exposures to vocabulary words are effective (National Reading Panel, 2000).

Textbook features—organization, presentation, coherence—have a direct impact on reading comprehension (Barton & Heidema, 2002). Characteristics that assist reading comprehension include a page layout that makes the organization of the content evident, a consistent pattern within each lesson or chapter, and explicit instruction on the text structure (Barton & Heidema, 2002; Dickson, Simmons, & Kameenui, 1995). Teachers should introduce students to the visual cues used in their textbook—colors, fonts, graphic symbols, text boxes—and model how to use features of their textbook (Barton & Heidema, 2002).

Writing in mathematics includes writing study notes, vocabulary definitions, explanations, descriptions, predictions, and justifications, along with attitudes, confusions, and questions. Writing helps students make sense of mathematics (Countryman, 1992). The thinking skills involved in writing are major aspects of doing mathematics. Through writing, students clarify and organize their thinking. Writing reinforces what students have learned, helping to retain it in long-term memory.

Research shows that frequent, regular writing in mathematics class leads to improved quality of writing, improved student attitudes toward mathematics, and teachers’ increased insight into
their teaching (Miller & England, 1989). Writing is effective in helping learn mathematical concepts. Writing by students gives teachers valuable information on student learning (Powell, 1997). When students write explanations about how to solve numerical problems, they are doing much more than acquiring content and demonstrating mastery of a benchmark. They are also communicating and problem solving (Urquhart & McIver, 2005).

The Student Editions of Glencoe’s mathematics series offer a consistent book layout and a clear, consistent lesson structure. The consistent page layout makes effective use of color, graphics, and fonts. Concepts are highlighted in shaded Key Concept and Theorem boxes, in words, symbols, and models and are summarized in the Study Guide and Review, with examples and exercises.

Reading Math margin notes in the Student Edition focus attention on reading mathematical symbols. Full-page Reading Math activities help students master mathematical language and symbols. Dinah Zike’s Foldables in each chapter help students organize their learning through reading and writing.

The Teacher Handbook includes Reading Strategies, which discusses activating prior knowledge, setting a purpose, developing vocabulary, taking notes, and summarizing.

Writing in Math questions in every lesson require students to use critical thinking skills. Project CRISS™ Study Skill features in each chapter of the Teacher Edition include writing skills, such as note taking or writing the steps in a mathematical process.

**Visual Literacy**

Visuals used in conjunction with verbal description increase student understanding and improve memory of relationships and properties. Visuals are often the only way to effectively communicate ideas that explain central concepts needed to understand algebra and geometry. Research shows that students are better able to organize and group concepts when visuals illustrate different and common characteristics (Hegarty, Carpenter, & Just, 1991). Also, the mental images that high-quality visuals encourage are an indispensable tool for recalling information, especially compared to information presented with only text or lower-quality visuals (Willows & Houghton, 1987).
Glencoe’s mathematics series includes high-quality charts, tables, graphs, drawings, and photographs throughout the text. The eye-catching photographs that begin each unit illustrate the Cross-Curricular Project. The photographs at the beginning of each chapter portray the Real World Link that relates to the main concepts in that chapter.

A visual accompanies each lesson’s Get Ready for the Lesson feature, drawing student attention and helping communicate the concept. Visuals in lab activities help students understand the activities and begin work. Drawings of algebra tiles in Algebra Labs help make connections between concrete tiles and abstract symbols. Photographs in Real-World Link margin features connect math concepts to real situations. Detailed images of graphing calculator screens enable students to use the tool efficiently so they can focus on the concepts.

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Mathematical Concepts and Skills

Algebra

Studies show that students have difficulty dealing with equations used in various ways—as a formula ($A = lw$), an equation to solve, ($4x = 20$), an identity ($\sin x = \cos x \cdot \tan x$), a property ($a + b = b + a$), or a function ($y = 3x$) (Chazan & Yerushalmy, 2003; Usiskin, 1988). Language plays a crucial role in algebra (RAND, 2003; Wheeler, 1996).

Learning rule-based procedures without creating meaning for the rules or learning when to use them leads to choosing inappropriate strategies (Kilpatrick, Swafford, & Findell, 2001). However, symbolic fluency is central to algebraic proficiency. Teachers need to motivate students’ interest to encourage persistence in practice (RAND, 2003).

Connecting algebra with arithmetic, geometry, and statistics can help build student understanding (RAND, 2003). Research into the use of concrete models for solving equations has had mixed results. Some studies have found balance scales and cups and counters to aid understanding, while others concluded that they were not helpful (Kieran, 2007).

Word problems are easier for students than solving the corresponding equations without context, contrary to what most teachers believe (Nathan & Koedinger, 2000). In solving word problems, students tend to revert to arithmetic problem-solving methods instead of using algebra (Stacy & MacGregor, 1999; Kieran 2007).

A function pairs each $x$-value with a corresponding $y$-value; this is called the correspondence approach. Alternately, a covariational perspective emphasizes that for a linear function the values in the $x$ column increase by constant, while values in the $y$ column increase by a
different constant. When students begin using function tables, they typically begin by noticing covariation, rather than ordered pairs (Smith, 2003; Confrey & Smith, 1995). The covariation approach is useful in modeling (Smith, 2003).

Teaching functions should build on students’ prior knowledge, starting with a simple familiar context and having students express concepts in their own language; develop conceptual understanding, procedural fluency, and connected knowledge; and help students become self-regulating problem solvers, using multiple strategies and representations (Kalchman & Koedinger, 2005).

Student understanding is increased when functions are defined as relationships between variables, introduced earlier, used to model concrete situations, and expressed in various representational systems (O’Callaghan, 1998). Students who work with multiple representations of functions develop a more comprehensive understanding of them (Leinhardt, Zaslavsky, & Stein, 1990).

Graphing calculators provide multiple representations. The use of graphing technology in algebra for solving problems and interpreting symbolic representations is an effective approach. However, the use of technology in algebra does not remove the need for paper-and-pencil techniques (Kieran, 2007). Many students confuse the algebraic and geometric aspects of slope, scale, and angle. Scale changes for linear graphs result in visual changes, while the slope remains constant (Zaslavsky, Sela, & Leron, 2002).

In the Glencoe mathematics series, functions are defined as relations of ordered pairs. Students explore functions as tables of values, mappings, and graphs. Writing and solving equations is emphasized with each lesson offering a balance of both extensive skills practice and H.O.T. Problems (Higher Order Thinking).

In Pre-Algebra, function notation is emphasized in a Reading Math feature. Slope is introduced in a lesson on rate of change, in a hands-on Algebra Lab, and in a Graphing Calculator Lab. Other Algebra Labs use algebra tiles to help students solve equations. Writing and solving equations is emphasized in Reading Math features, in lessons, and in word problems throughout the curriculum. Three chapters in Unit 4, Applying Algebra to Geometry, reinforce concepts from both topics and help students connect them.
In *Algebra I*, relations and functions are defined in terms of ordered pairs and represented as ordered pairs, tables, graphs, and mappings. *Graphing Calculator Labs* enable students to explore families of functions. Lines of fit connect algebra to statistics; equations of parallel and perpendicular lines tie algebra to geometry. Students graph ordered pairs of data to investigate real-world functions. *Multiple-Step Word Problems* throughout the lesson exercises provide practice in using algebra in realistic situations.

In *Geometry*, connections to algebra occur in many lessons, including the distance formula, coordinate geometry coordinate proofs, vectors, and trigonometry.

In *Algebra 2*, seventeen *Graphing Calculator Labs* explore function graphing, limits, solving equations, matrices, and modeling data. This course includes a wide array of connections to other areas of mathematics: scatter plots, linear programming, matrices, complex numbers, logarithmic functions, modeling motion, exponential functions, conic sections, sequences and series, mathematical induction, normal distribution, and trigonometric functions.

**Geometry and Measurement**

The van Hiele theory (van Hiele 1986; van Hiele, 1959/1985/2004), which defines sequential levels of reasoning in geometry, is valid and useful in describing students’ geometric-concept development (Clements & Batista, 1992; Battista, 2007).

Level 1 Visual – recognize shapes but not think about properties
Level 2 Descriptive/analytic – characterize shapes by their properties
Level 3 Abstract/relational – form definitions; understand and provide arguments
Level 4 Deduction – construct proofs; use definitions, axioms, theorems

Research indicates that the van Hiele levels may not be discrete; students often use more than one level of reasoning (Gutiérrez et al., 1991). The levels may develop simultaneously and at different rates, depending on maturation and instruction (Battista, 2007).

Students already have “fuzzy” concepts for geometric objects. Formal, mathematical definitions may differ from these fuzzy concepts. For example, students may not agree that a square is a rectangle; the fuzzy concept of rectangle does not match the mathematical definition (Battista, 2007). Careful use of geometry vocabulary is important in making clear the distinctions between common usage and mathematical usage (Clements, 2003; Fuys et al., 1988).
Students often confuse drawings with the theoretical geometric objects that the drawings represent (Battista, 2007). For example, they assume that lines that look parallel are parallel or that right triangles must have one horizontal side (Clements & Battista, 1992). Student difficulties in understanding the connection between linear equations and their graphs could be due to lack of understanding of how coordinates are related to length. Students may see coordinates as simply markers, like margin letters on a map (Battista, 2007; Knuth, 2000).

Students who receive training and practice in specific spatial skills (e.g., mental rotation, spatial perspective) improve their performance on mathematics tests (Halpern et al., 2007).

Understanding area and volume measure requires knowing: 1) what area/volume is and how it is conserved, 2) how to measure it by iterating units of area/volume, 3) numerical processes to determine measures for special shapes, and 4) how these processes are expressed in words and symbols (Battista, 2007).

In the Glencoe mathematics series, geometry and measurement concepts and skills are interwoven into each course. In Pre-Algebra, three chapters in Unit 4: Applying Algebra to Geometry focus on right triangles, and two- and three-dimensional figures. Geometry Labs enable students to work hands-on with geometric figures. Pre-Algebra students explore the Pythagorean Theorem and distance formula; they use similar figures in indirect measurement. They work with formulas for area, surface area, and volume; explore similar solids; and draw three-dimensional figures from various perspectives.

In Algebra 1, students write monomials as formulas for area and volume of geometric shapes, apply the Pythagorean Theorem and the distance formula, and explore parallel and perpendicular lines in their study of slope.

In Geometry, hands-on activities in Geometry Labs help students connect concrete and abstract concepts. Geometry Labs also provide van Hiele Level 3 activities, such as conjecturing about the angles of triangles. Students use Geometer’s Sketchpad software in Geometry Software Labs for a variety of activities, including making and testing conjectures.

Geometric vocabulary is presented verbally, in symbols, and with figures. Vocabulary Links in the margins clarify differences between everyday uses and mathematical uses of words, like
median. Reading Math features in each chapter focus on geometry vocabulary. The symbols and diagram markings used to indicate relationships, such as parallel lines, are clearly stated and emphasized. Definitions, postulates, theorems, and proofs are emphasized and used throughout the course.

Linear equations and slopes of lines are reviewed and extended to equations of perpendicular bisectors and determining that lines are parallel using coordinate geometry.

Right triangle trigonometry provides connections to many real-world problems. Transformations combine coordinate geometry, symmetry, nature, and art. Surface area and volume of solids is explored using various representations: models, nets, isometric views, orthographic drawings, cross sections of solids.

In Algebra 2, Chapters 13 and 14 on trigonometric functions and graphs are based on right-triangle geometry. A full chapter on conic sections combines geometry and algebra. Throughout the exercises, geometric problem situations, such as surface area and volume, relate geometry to algebra.

**Number and Operations**

Number and Operations at the high-school level includes understanding the rational and real number systems and their properties, representing numbers in a variety of ways, understanding how operations relate to each other, computing fluently, and making estimations (NCTM, 2000). It includes the effective organization of this knowledge and the ability to apply it (Donovan & Bransford, 2005).

Research studies show that many students in grades 9 and 10 are unable to classify real numbers as rational or irrational (Fischbein et al., 1995). Other studies demonstrate that understanding fundamental structural properties of a mathematical system improves retention of the system and application to new situations (Fey, 1990).

In Pre-Algebra, Algebra 1, and Algebra 2, students use a Venn diagram to understand the real number system and classify real numbers.

Students use number lines to compare rational numbers and real numbers. Students identify and apply properties of number systems. They learn and apply properties of square roots.
In Algebra 2, students extend the number system to include imaginary numbers, the complex number system. The chapter on matrices includes properties of matrix multiplication, identity and inverse matrices, and applications.

**Statistics, Data Analysis, and Probability**

Studies show that the concept of *mean* develops over many years. The mean should be introduced as a *typical value* or a *fair share*, to build on students’ intuitions. Later, the mean should be viewed as a *data reducer*, that is, a summary of data and a *signal amid noise*, or a tool to compare data sets (Shaughnessy, 2007; Watson & Moritz, 2000).

Many students use the mode when the median or mean is more appropriate. Few students understand anything about the median and mean other than how to compute them (Konold & Higgins, 2003; Russell et al., 2002; Cai, 1998).

Research indicates that effective statistics teachers emphasize variability, introduce the comparison of two data sets, build on students’ intuitive notions of centers and variation, use proportional reasoning to connect samples and populations, and keep in mind that statistics is different from mathematics; the context of the data is critical (Shaughnessy, 2007).

Concepts of chance and randomness challenge learners of all ages. Students fail to interpret uncertainty in patterns that emerge from repetitions, believe that a person or device (for example, a spinner) can control outcomes, and believe that some sort of order or reason underlies events. Students do not consider sample spaces when determining probabilities, even if they can list the outcomes in the sample space (Jones, Langrall, & Mooney, 2007).

Probability instruction should emphasize building sample spaces (all possible outcomes), connect probability and statistics, introduce probability through data, and use a problem-solving approach (Shaughnessy, 2003). Teachers should take into account student preconceptions and misconceptions, provide instruction in probability reasoning to challenge misconceptions, and use an experimental approach (Jones, Langrall, & Mooney, 2007; Castro, 1998).
The curriculum should include the concepts of chance variation, randomness, independence, calculating probabilities, terminology and language use, critical reasoning and questioning, and interpretation of the context, or the situation (Jones, Langrall, & Mooney, 2007; Gal, 2005).

In Pre-Algebra, students compare two data sets using histograms, stem-and-leaf plots, and box-and-whisker plots. Students explore experimental probability using graphing calculators for simulations.

In Algebra 1, Chapter 12 offers statistics and probability topics, including sampling, permutations and combinations, probability of compound events, probability distributions, and probability simulations. A Reading Math activity focuses on Survey Questions. Algebra Lab: Simulations involves students in experimenting with probability situations. Sample space is the focus of Lesson 12-2, Counting Outcomes.

In Geometry, students relate area to probability and explore circle graphs.

In Algebra 2, Chapter 12, Probability and Statistics, includes multiplying and adding probabilities, statistical measures, the normal distribution, exponential and binomial distribution, binomial experiments, and sampling and error. Word problems focus on choosing among median, mean, and mode to justify a claim. Algebra Labs on Simulations and on testing hypotheses provide explorations. A Reading Math activity on permutations and combinations is included.

Reasoning

Research tells us that students can show reasoning ability when: 1) they have sufficient knowledge, 2) the task is understandable and motivating, and 3) the context is familiar and comfortable (Kilpatrick et al., 2001; Alexander et al., 1997). Students engage in reasoning, explanations, and justification when they are in a classroom environment that supports these activities. Creating this classroom atmosphere requires explicit effort by the teacher (Yackel & Hanna, 2003; Lampert, 1990).

Explaining and justifying ideas not only helps students clarify and improve reasoning skills, it also increases their conceptual understanding (Kilpatrick et al., 2001; Marzano et al., 2001; Maher & Martino, 1996).
Mathematical language is the foundation of mathematical reasoning (Ball & Bass, 2003). Teachers should model the process of reasoning—building on others’ ideas, using the common knowledge base, and using mathematical language carefully (Ball & Bass, 2003).

Students need to learn what constitutes justification in mathematics (Yackel & Hanna, 2003). They need to explain their hypotheses and conclusions, preferably in writing, to deepen their understanding (Marzano et al., 2001).

Students need to justify procedures repeatedly over time, relating new concepts and procedures to those they already understand (Kilpatrick et al., 2001). They develop their concepts of justification and proof over many years, beginning with “making sense.” Mathematical reasoning is a process rather than a skill (Yackel & Hanna, 2003; Maher & Martino, 1996).

Deductive reasoning in geometry occurs at Level 3 of the van Hiele model when students can logically connect properties of concepts. The truth of a statement is connected to the accepted truth of other statements (Clements, 2003; Senk 1989). Students should first learn to create informal proofs, chains of informal correct conclusions starting from valid premises (Yackel & Hanna, 2003; Blum & Kirsch, 1991). Formal, mathematical proofs differ in several ways from reasoning in everyday life, which makes it difficult to learn to create proofs (Yackel & Hanna, 2003; Fishbein 1999).

Interactive geometry software can help motivate and enable students to investigate, generalize, and conjecture. It may in some cases blur the distinction between empirical evidence and proof (Yackel & Hanna, 2003; Dreyfus & Hadas, 1996; Chazan, 1993).

Throughout Glencoe’s mathematic series, reasoning problems in H.O.T. Problems (Higher Order Thinking) occur in each chapter. These include compare and contrast exercises and tell whether a statement is always, sometimes, or never true. Writing in Math exercises in H.O.T. Problems include many opportunities for students to explain their reasoning.

In Pre-Algebra and Algebra 1, students explore deductive reasoning, counterexamples, conjectures, and inductive reasoning. In Algebra 1, a lesson on Logical Reasoning and Counterexamples is included in Chapter 1. Several Reading Math activities involve reasoning and proof. Graphing Calculator Labs involve predicting the appearance of graphs of equations and describing how changes in equations affect graphs.
In Geometry, Chapter 2, **Reasoning and Proof**, provides the introduction to methods of proof used throughout the curriculum. Each of these proof topics is studied extensively: inductive reasoning and conjecture; conditional statements; deductive reasoning; postulates; paragraph proofs; algebraic proof; geometry proof; two-column, formal proofs; and coordinate proofs. Reasoning problems in **H.O.T. Problems** occur in each chapter. Some **Reading Math** activities relate to reasoning and proof. **Writing In Math** questions include many opportunities to explain, for example, how to balance a paper triangle on a pencil point. **Graphing Calculator Labs** involve making and testing conjectures.

In **Algebra 2**, proofs are frequently included in the lessons, as appropriate, especially in the trigonometry chapters. Many Reasoning exercises in **H.O.T. Problems** ask students to create proofs related to the lesson content. Additional **Reasoning** exercises and **Writing in Math** exercises focus on reasoning.

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