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Direct Instruction Mathematics Programs: An Overview and Research Summary

Abstract: This paper provides an overview and research summary of Direct Instruction (DI) mathematics programs, specifically *DISTAR Arithmetic I* and *II* (Engelmann & Carnine, 1975, 1976), *Corrective Mathematics* (Engelmann & Carnine, 1982), and *Connecting Math Concepts* (*CMC*; Engelmann, Carnine, Kelly, & Engelmann, 1996a). A comparison of the constructivist approach to the direct or explicit approach to math instruction was conducted. Overviews and ways in which DI math programs meet the 6 principles for improving math instruction as provided by the National Council of Teachers of Mathematics (NCTM; 2000b) are noted. Finally, a research review and analysis of DI math programs published since 1990 (yielding 12 studies) was completed. Seven of the 12 studies compared DI math programs to other math programs. Four studies investigated the efficacy of DI math programs without comparison to other math programs. A meta-analysis conducted by Adams and Engelmann (1996) was also described. Study characteristics (i.e., reference, program or program comparison, participants, research design, dependent variable(s)/measures, and results) were examined for each of the 12 studies. Eleven of the 12 studies showed positive results for DI math programs. Eight areas for future research are included.

This paper provides a review of DI mathematics programs including *DISTAR Arithmetic I* and *II*, *Corrective Mathematics*, and *CMC*. In addition, the constructivist approach and the direct or explicit approach to math instruction are compared. Primary emphasis was placed on the direct approach and how DI math programs meet NCTM's six principles for improving math instruction. A research review of studies published after 1990 using these programs was also conducted. Finally, areas for future research on DI math programs are provided.

Overview of Math Statistics

In our rapidly changing and technologically dependent society, we are faced with the need for a solid understanding of mathematical skills and concepts. This need is no longer limited to scientific and technical fields. Virtually every type of employment requires a more sophisticated understanding of mathematics. For example, in a 1989 report by the National Research Council, over 75% of all jobs required proficiency in simple algebra and geometry, either as a prerequisite to a training program or as part of a licensure examination. Further, in a more recent report by the Bureau of Labor Statistics (2002), estimates indicate that four of the top five employment growth fields will require a bachelor's degree in technical studies such as mathematics or computer science. Given the emphasis of mathematical skills in our society, it seems critical that our students should demonstrate basic mathemati-

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cal and higher order thinking skills to be successful in present and future environments.

In 1995, the largest international study (Third International Mathematics and Science Study [TIMMS]) of academic achievement was conducted by the International Study Center (ISC) at Boston College. This study included over half a million students from 41 countries. According to the ISC's report (2001), when compared to other countries, math scores in the United States were ranked in the bottom half of the participating countries. American 4th graders ranked 12th out of 26, 8th graders ranked 28th out of 41, and 12th graders ranked 19th out of 21 countries who participated in the assessment.

The National Center for Education Statistics (2001) published its most recent results of the 2000 National Assessment of Educational Progress. In this report, known as *The Nation's Report Card*, the mathematics achievement levels of 4th-, 8th-, and 12th-grade students were assessed. The following three levels of performance were identified:

1. basic: this level denotes partial mastery of prerequisite knowledge and skills that are fundamental for proficient work at each grade.

2. proficient: the proficient level represents solid mathematical performance for each grade assessed. Students reaching this level have demonstrated competency over challenging subject matter, including mathematical knowledge, application of such knowledge to real-world situations, and analytical skills.

3. advanced: the advanced level signifies superior performance. (p. 9)

The *proficient* level is the overall performance goal for all students. Results indicated that only 26% of 4th-grade students, 27% of 8th-grade students, and 17% of 12th-grade students performed at the proficient level in math.

NCTM Principles

Given the mathematical performance of our students on various assessments and comparisons conducted within and beyond the U.S., it seems imperative to examine how best to teach math in our public schools. The NCTM is the world's largest mathematics education organization, founded in 1920. The mission of the NCTM (2000a) is "to provide the vision and leadership necessary to ensure a mathematics education of the highest quality for all students" (p. 1). In order to accomplish this mission, the NCTM (2000b) developed five overall curricular goals for student success in mathematics: (a) learning to value mathematics, (b) becoming confident in one's own mathematical ability, (c) becoming a mathematical problem solver, (d) learning to communicate mathematically, and (e) learning to reason mathematically. The NCTM (2000b) developed *Principles and Standards for School Mathematics* as a framework for guiding educational professionals in meeting these five goals. While the standards describe the mathematical *content* and *processes* that students should learn, the principles describe *features* of high quality mathematics education (2000b). In an earlier paper, Kelly (1994) provided examples from various levels of *CMC* to illustrate how these standards can be met through *CMC*. This paper focuses on how the principles (vs. standards) were met by *CMC*, *DISTAR I* and *II*, and *Corrective Mathematics*. According to the NCTM (2000b), the six principles should be used to influence the development and selection of curricula, instructional planning, assessment design, and establishment of professional development programs for educators (see Table 1). It is through these six principles that educators can begin to address the composite themes of high quality mathematics education.

Primary Approaches to Math Instruction

There are two primary approaches to mathematics instruction. These include the constructivist approach and the direct or explicit approach (see Table 2). According to Applefield, Huber, and Moallem (2000/2001), constructivism is based on a postulate that student learning is influenced by four primary factors: (a) learners construct their own learning, (b) new learning is dependent upon students' existing understanding of the world, (c) social interaction plays a critical role in that students work in heterogeneous cooperative learning groups, and (d) authentic learning tasks are used for meaningful learning. The constructivist approach is primarily an inquiry- or discovery-oriented approach. Students are put into learning situations that allow them to "discover" which problem solving strategies will be the most effective. Through exposure to real-life situations, students use inductive reasoning to make generalizations about mathematical concepts and problem solving strategies. The following is an example of a constructivist lesson taken from *Math Trailblazers* (TIMS Project: University of Illinois at Chicago, 1998, p. 61).

> Recycling 100 Cans. Have the children bring in aluminum cans for recycling. The first goal might be to collect 10 cans, then 50, and finally, 100. Of course, this can be a continuing project for your class. Have the class figure out how many cans would have to be brought in by each child to reach the goal of 100 cans, or if every child brings in a can every day, how many days will it take to reach 100 cans?

First, students are encouraged to brainstorm which problem solving strategies would be most effective in solving their problem. Then, through trial and error, a solution is reached.

Table 1

NCTM Principles for Improving Math Instruction

The Equity Principle	Excellence in mathematics education requires equity—high expectations and strong support for all students.
The Curriculum Principle	A curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well-articulated across the grades.
The Teaching Principle	Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
The Learning Principle	Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
The Assessment Principle	Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.
The Technology Principle	Technology is essential in teaching and learning mathematics: it influences the mathematics that is taught and enhances students' learning.

A second approach to mathematics instruction is known as an explicit or direct approach. In this approach, teachers help students acquire knowledge in the form of concepts, principles or rules, cognitive strategies, and physical operations (Kozloff, LaNunziata, Cowardin, & Bessellieu, 2000/2001). This knowledge is most effectively taught in the following manner: (a) teaching with clear objectives; (b) teaching concepts, principles, strategies, and operations explicitly and systematically; and (c) monitoring progress continually (Kozloff et al.). Stein, Silbert, and Carnine (1997) refer to explicit instruction as being clear, accurate, and unambiguous; therefore, the clearer the instruction, the more efficient it will be. This approach provides a comprehensive set of prescriptions for organizing instruction so that students acquire, retain, and generalize new learning in a manner that is as humane, efficient, and effective as possible. The following is an example of part of a lesson using an explicit or direct approach to instruction as provided by Stein et al. (p. 65).

Teacher	Students
1. *(Give students paper and pencil.)*	
2. You are going to write a problem. First, you'll say it. Listen: Six plus two equals how many? Say that.	Six plus two equals how many?
To correct: Respond with students until they can say the statement at the normal rate of speech.	
3. Now we'll say it the slow way. Every time I clap, we'll say a part of the statement.	
(Respond with the students.) Get ready.	
(Clap) Six. *(Pause two seconds; clap.)*	six
Plus *(Pause two seconds; clap.)* Two.	plus two
(Pause two seconds; clap.) Equals.	equals
(Pause two seconds; clap.) How many?	how many?
(Repeat step 3 until students appear able to respond on their own.)	
4. Now I'll clap and you say the statement by yourselves. *(Pause.)* Get ready. *(Clap at two-second intervals.)*	Six plus two equals how many?
To correct: Respond with students.	
5. Now write the problem.	Students write $6 + 2 =$ ___
6. *(Repeat steps 1–5 with three more equations.)*	

Efficacy of Direct Approach in Meeting the NCTM Principles for Improving Math Instruction

As shown in Table 1, the NCTM (2000b) recommended six principles to guide educators in making sound decisions about mathematics instruction. The direct approach to teaching mathematics is an effective and efficient way to meet these principles. Within this direct approach to teaching, Stein et al. (1997) identified three variables for effective instruction: (a) effective instructional design, (b) effective presentation techniques, and (c) logical organization of instruction. Descriptors of each of these variables follow.

Effective instructional design. Effective instructional design consists of nine elements. First, long- and short-term objectives must by specified. Both long- and short-term objectives should explicitly state observable behaviors, performance criteria, and the conditions under which the behavior will be performed. Long-term objectives should specify exactly what students should do at the end of an educational program. The following is an example of a long-term objective taken from Lignugaris/Kraft, Marchand-Martella, and Martella (2001): "Given a worksheet with 20 addition problems up to 3D + 3D + 3D with and without regrouping, Larry will write correct answers with 90% accuracy on three consecutive weekly classroom exercises" (p. 56). On the other hand, short-term objectives are based on the component skills needed to reach the long-term goal. The following is an example of a short-term objective taken from Lignugaris/Kraft et al.: "Given a worksheet with 10 addition problems with sums less than 19 and both addends less than 10, Larry will write correct answers

Table 2

Summary of Two Primary Approaches to Math Instruction

Constructivist approach	Explicit or direct approach
• Teacher presents real-life situations and facilitates inquiry- or discovery-based problem solving.	• Teacher directly teaches concepts, principles or rules, cognitive strategies, and physical operations.
• Students construct their own learning based on their current understanding of the world usually within heterogeneous cooperative learning groups.	• Comprehensive set of prescriptions for organizing instruction to guide students' acquisition, retention, and generalization of new knowledge.
• Steps in student learning process: 1. Presented with real-life situation. 2. Brainstorm possible problem solving strategies. 3. Solution reached through trial and error.	• Three variables for effective instruction (Stein et al., 1997): 1. Effective instructional design. 2. Effective presentation techniques. 3. Logical organization of instruction
• Spiral-based curriculum design.	• Strand design.

with 90% accuracy on three consecutive weekly classroom exercises" (p. 56).

Second, efficient procedural strategies must be designed. Kameenui and Carnine (1998) define a strategy as a set of skills used to acquire and use knowledge. To maximize student learning and instructional efficiency, it is imperative that strategies be taught to allow students to solve the greatest number of problems with the fewest possible number of steps (Kameenui & Carnine, 1998; Stein et al., 1997). The following is a number-family problem solving strategy as noted by Stein et al.:

> The number-family strategy is based on the concept that three numbers can be used to form four math statements. For example, the numbers 2, 5, and 7 yield 2 + 5 = 7, 5 + 2 = 7, 7 - 5 = 2, and 7 - 2 = 5. In a typical problem, two of the numbers in the family are provided. Students place these numbers where they belong in the family and then determine whether the missing number is obtained by adding or subtracting. The strategy is applied to word problems in that if the total number of a fact family is given, the problem requires subtraction. For example, "Kyle had two snakes. Now he has seven snakes. How many more snakes did he get?" The last sentence asks about how many more, not about the total. So one of the numbers in the problem, 2 or 7, must be the big number, the total. The phrase "Now he has 7" indicated that 7 is the total number. Students then subtract 2 from the total number; 7 - 2 = 5. Kyle got 5 more snakes. (p. 221)

Third, necessary preskills must be determined. Instruction should be sequenced so that the component skills of a strategy are taught before the strategy itself is introduced. For example, the strategy for solving addition problems and repeating addition statements should be taught before column addition problems are taught. Component skills must be mastered before students can be expected to use them as a part of a strategy.

Fourth, preskills should be logically sequenced to maximize student learning. Three sequencing guidelines are recommended when introducing new information to students. First, preskills of a strategy are taught before the strategy. For example, when teaching students to add single numbers to teen numbers with sums over 20, students must have the following preskills: symbol identification, place value, basic addition facts, and renaming. Along these lines, Carnine (1980) found that preteaching the component skills of a multiplication algorithm resulted in more rapid learning of the complex skill than teaching the components and the complex skill concurrently. Second, easy skills are taught before more difficult ones. For example, students should be taught the "regular" teen numbers 14, 16, 17, 18, and 19 before the "irregular" teen numbers 11, 12, 13, and 15. It is easier to learn the names and, therefore, the value of the number 17 ("*seven*teen"). Conversely, the number 11 is considered "irregular" and more difficult to learn ("eleven" not "one-teen"). Finally, information that is likely to be confused is not introduced consecutively. For example, students are likely to confuse the numerals *6* and *9*, so they should not be introduced consecutively.

Fifth, teaching procedures must be selected for three types of tasks: motor, labeling, and strategy tasks, because each type of task requires a different teaching procedure (Stein et al., 1997). Motor tasks, which require students to articulate a rule or to perform a precise movement, are taught using the following four-step teaching procedure: model, lead, test, and delayed test. An example of this procedure used to teach students to articulate the equality rule in addition is shown in Figure 1 (Engelmann, Carnine, Kelly, & Engelmann, 1996b, p. 50). Workbook practice provides the delayed test step in the motor task procedure.

Labeling tasks, which require students to say the word that correctly labels an object, are taught using the following three-step teaching procedure: model, alternating test, and delayed test. An example of this procedure, used to teach students how to read thousands numbers, follows (Stein et al., 1997, p. 76).

Figure 1

Example of a motor task used to teach students to articulate a rule.

EXERCISE 1 EQUALITY

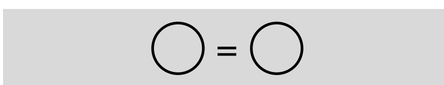
a. (Write on the board:)

=

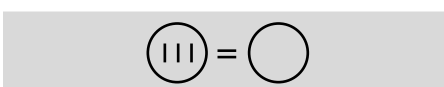
- This is a very important sign that we'll use to work on hard problems. This sign is called an equal sign.

b. What's it called? (Signal.) *An equal sign.*
(Repeat step b until firm.)

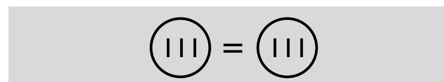
c. (Draw a circle on each side of the equal sign:)



- Here's a rule about the equal sign: You must end up with the same number on both sides of the equal sign. Listen again: You must end up with the same number on both sides of the equal sign. Watch.
- (Make 3 lines in the left circle:)

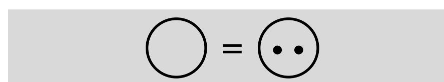


- I made lines on one side of the equal sign. Everybody, how many lines did I make? (Signal.) *3.*
- I must end up with the same number on both sides of the equal sign. So how many lines do I have to make on the other side of the equal sign? (Signal.) *3*
- (Make 3 lines in the right circle:)

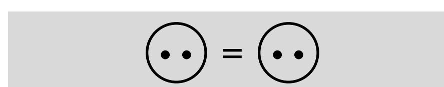


- I did it. I ended up with 3 on both sides of the equal sign. So it says **3 equals 3**. What does it say? (Signal.) *3 equals 3.*
- (Erase the lines.)

d. New problem: I'm going to make little marks on one side of the equal sign. Watch.
(Make 2 marks in the right circle:)



- How many marks did I make on one side of the equal sign? (Signal.) *2.*
- I must end up with the same number on both sides of the equal sign. So how many marks do I have to make on the other side? (Signal.) *2.*
- (Make 2 marks in the left circle:)



- I ended up with 2 on both sides of the equal sign. So it says **2 equals 2**. What does it say? (Signal.) *2 equals 2.*
- (Do **not** erase the board.)

Note. From Engelmann, S., Carnine, D., Kelly, B., & Engelmann, O. (1996b). *Connecting Math Concepts: Level A*, p. 50. Columbus, OH: SRA/McGraw-Hill. Reproduced with permission of The McGraw-Hill Companies.

Teacher	Students
1. When a big number has one comma, the comma tells about thousands. Here's the rule. The number in front of the comma tells how many thousands. What does the number in front of the comma tell?	how many thousands
(Write on board: 6,781.)	
2. What number comes in front of the comma?	6
So what is the first part of the number?	6 thousand
3. *(Point to 781.)* Get ready to read the rest of the number.	781
4. Now you are going to read the whole number. *(Point to 6, then comma, then 781.)*	6,781
5. *(Repeat steps 2–4 with these numbers: 2,145 3,150 5,820 6,423.)*	
6. *(Give individual turns to several students.)*	

Finally, strategy tasks, which require the integration of a series of sequential steps to form a generalizable strategy, are taught using modeling, guided practice, and supervised independent work. An example of a strategy task, used to teach students how to divide using the short-form algorithm, is shown in Figure 2 (Stein et al., 1997, p. 204).

Sixth, teaching formats are designed to specify what teachers will say and do. These formats allow teachers to focus more attention on student performance. Figure 3 shows a sample format for teaching students how to find volume (Engelmann, Carnine, Kelly, & Engelmann, 1996d, pp. 348–349).

Seventh, appropriate examples are chosen for motor, labeling, and strategy tasks. Stein et al. (1997) recommend the following for choosing these examples. Examples should involve the current strategy or a previously mastered strategy. In addition, examples of previously introduced problem types should be included. This aspect of instructional design allows students to practice the new strategy, review previous strategies, and learn to differentiate between when to use specific strategies for a variety of similar problems.

Eighth, guided practice and review are used to ensure mastery of skills. Long-term skill retention can be facilitated in two ways: (a) massed practice should be done until fluency and mastery are reached, and (b) systematic review should be incorporated. Dixon (1994) noted that systematic review should distribute review opportunities over time to contribute to long-term retention and automaticity of knowledge, accumulate information taught in review (after A and B are taught, A and B are reviewed together), and vary review items to promote generalization and transference.

Finally, progress monitoring procedures must take place at regular intervals. These procedures should focus on curricular objectives and should assess progress on what is actually being taught in the classroom. By knowing the specific skills that students need to master, strategies aimed at teaching those skills can be developed. An example of this type of procedure can be seen in Figure 4 (Engelmann, Carnine, Kelly, & Engelmann, 1996c, p. 22).

Effective presentation techniques. The second of three variables in effective instruction as noted by Stein et al. (1997) involves the use of effective teacher presentation techniques. These techniques involve maintaining student attention during group instruction and teaching to criterion. In order to maintain student attention, explanations should be brief and concise. Students should be given frequent opportunities to respond during instructional times (Paine, Radicchi, Rosellini, Deutchman, & Darch, 1983). Unison responding is one way to ensure all students are actively engaged in the learning process. This type of presentation technique requires the use of signals. To signal a unison response, the teacher gives directions; provides a thinking pause; and cues the response by pointing, tapping a pencil, or snapping her fingers, for example. Additionally, adequate pacing is needed. Pacing requires material to be presented in a lively manner and without hesitation by the teacher. Finally, seating arrangements should be considered.

Figure 2

Example of a strategy task.

Basic Steps in the Short-Form Algorithm

TEACHER	STUDENTS
1. Read the problem.	7 goes into 238.
2. Underline the part you work first.	Students underline $7\overline{)\underline{23}8}$.
3. Say the underlined part.	7 goes into 23.
4. Write the answer above the last underlined digit.	$\begin{array}{r}3\\7\overline{)\underline{23}8}\end{array}$
5. Multiply 3 × 7, subtract, and then bring down the next number.	$\begin{array}{r}3\\7\overline{)\underline{23}8}\\\underline{21}\\28\end{array}$
6. Read the new problem.	7 goes into 28.
7. Write the answer number above the digit you just brought down.	$\begin{array}{r}34\\7\overline{)\underline{23}8}\\\underline{21}\\28\end{array}$
8. Multiply and subtract to determine the remainder.	$\begin{array}{r}34\\7\overline{)\underline{23}8}\\\underline{21}\\28\\\underline{28}\\0\end{array}$
9. Say the answer.	7 goes into 238, 34 times.

Note. Stein/Silbert/Carnine, *DESIGNING EFFECTIVE MATHEMATICS INSTRUCTION 3/e, 1997, p. 204. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, New Jersey.*

During large-group instruction, lower performing students should be seated at the front of the room to allow teachers to monitor their behavior more effectively. During small-group instruction, students should be seated in a semicircle with lower performing or easily distracted students seated toward the middle of the group.

In addition to maintaining student attention, teachers should also teach to criterion. In order to ensure all students reach mastery,

Figure 3

Example of a format for teaching students how to find volume from Connecting Math Concepts: Level F.

EXERCISE 5 VOLUME
Mixed set

a. Find part 5.

- Some of these figures come to a point. Others don't. Remember, for figures that do not come to a point, you find the area of the base times the height. For figures that come to a point, you find the area of the base times the height. Then what do you divide by? **(Signal.)** *3.*

b. Find the volume of figure A. Start with the equation for volume. Raise your hand when you're finished.
(Observe students and give feedback.)

- **(Write on the board:)**

$$\text{a. } \underline{\text{Area of b}} \times \text{h} = \text{V}$$
$$51 \times 22 = \text{V}$$
$$\boxed{\text{V} = 1{,}122 \text{ cu m}}$$

- Figure A has a triangular base. The area of the base is 51 square meters. Times the height of 22. That's 1,122 cubic meters.

c. Figure B also has a triangular base, but it comes to a point.

- Find the volume of figure B. Raise your hand when you're finished.
(Observe students and give feedback.)
- **(Write on the board:)**

$$\text{b. } \frac{\text{Area of b} \times \text{h}}{3} = \text{V}$$
$$\frac{27.5 \times 16}{3} = \text{V}$$
$$\boxed{\text{V} = 146.67 \text{ cu in}}$$

- Here's what you should have. The area of the base is 27 and 5-tenths square inches. Times the height of 16. That's 440. Divided by 3. The volume is 146 and 67-hundredths cubic inches.

d. Your turn: Work the rest of the problems in part 5. Raise your hand when you're finished.
(Observe students and give feedback.)

Key:

$$\text{c. } \underline{\text{Area of b}} \times h = V$$
$$113.04 \times 6 = V$$
$$\boxed{V = 678.24 \text{ cu in}}$$

$$\text{d. } \frac{\text{Area of b} \times h}{3} = V$$
$$\frac{9.45 \times 3.3}{3} = V$$
$$\boxed{V = 10.40 \text{ cu ft}}$$

$$\text{e. } \frac{\text{Area of b} \times h}{3} = V$$
$$\frac{254.34 \times 28}{3} = V$$
$$\boxed{V = 2373.84 \text{ cu cm}}$$

e. Find part J on page 361 of your textbook. That shows what you should have for problems C, D, and E. Raise your hand if you got everything right.

Note. From Engelmann, S., Carnine, D., Kelly, B., & Engelmann, O. (1996d). *Connecting Math Concepts: Level F*, p. 348–349. Columbus, OH: SRA/McGraw-Hill. Reproduced with permission of The McGraw-Hill Companies.

Stein et al. (1997) note that teachers must present a particular format until students are able to respond to every question or example in the format correctly. This step involves effective monitoring, error correction, and appropriate diagnosis and remediation of problems. Teaching to criterion is consistent with the Teaching Principle noted by the NCTM (2000b).

Logical organization of instruction. The third variable in effective instruction as noted by Stein

Figure 4

Example of CBM from Connecting Math Concepts: Level C, Teacher's Guide.

a. Open your workbook and find test 6. This is a test of things you've studied. You can earn as many as 20 points for doing well on the test. So work carefully.

b. Find part 1.
You're going to write answers to problems. You'll have to move pretty fast.

c. Touch A.
Here's the problem: 47 plus 10. Write the answer. √

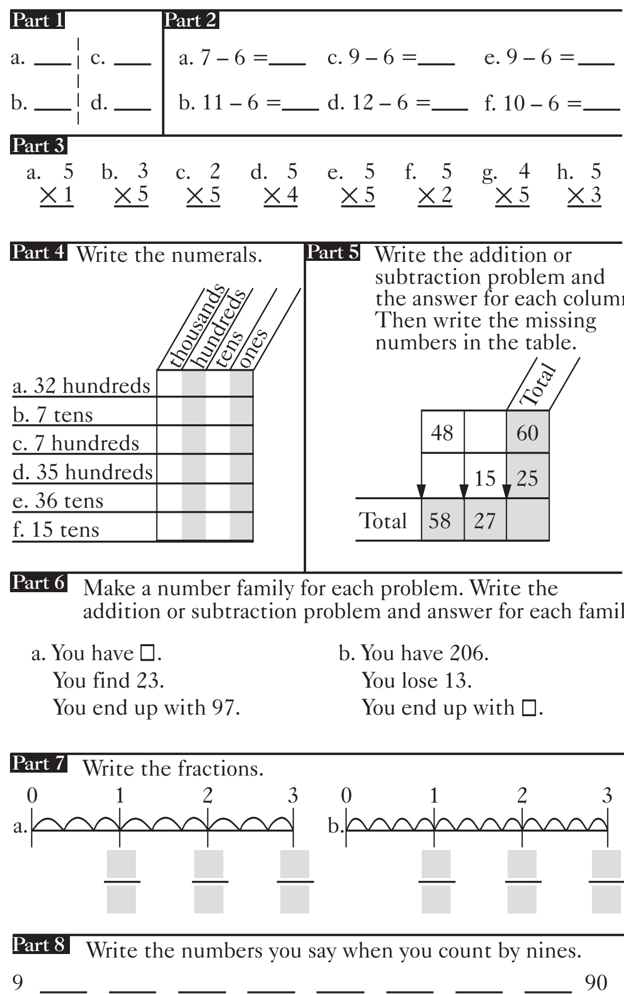
- Touch B.
Listen: 63 plus 5. Write the answer. √
- Touch C.
Listen: 52 plus 4. Write the answer. √
- Touch D.
Listen: 29 plus 10. Write the answer. √

d. Find parts 2 and 3.
You have 1 and a half minutes to write the answers for both parts 2 and 3. Get ready. Go.
(Observe students, but do not give feedback.)

- (After 1 1/2 minutes, say:) Stop. Cross out the problems you didn't finish. √

e. (If students have difficulty reading items of instructions, read the material to them.)

f. Finish the rest of the test on your own. Raise your hand when you're finished.



Note. From Engelmann, S., Carnine, D., Kelly, B., & Engelmann, O. (1996c). *Connecting Math Concepts: Level C, teacher's guide*, p. 22. Columbus, OH: SRA/McGraw-Hill. Reproduced with permission of The McGraw-Hill Companies.

et al. (1997) is the logical organization of instruction. There are two primary methods to organize math instruction. One way involves a spiral-based curriculum design present in many constructivist basal math programs today. In this design, lessons focus on a single topic for a number of days. Students then revisit these topics in each successive year with greater depth. This method of curriculum design, often referred to as "teaching for exposure," allows a large number of topics to be covered briefly each year. According to Carnine (1990), the intent of the spiral curriculum is to add depth each year, but the practical result is the rapid, superficial coverage of a large number of topics each year. In fact, Porter (1989) found as much as 70% of math topics are given less than 30 min of instructional time each year.

A second way to organize math instruction is through the strand design present in Direct Instruction programs. This design includes concepts or "big ideas" that are organized around skill development strands allowing a few important topics to be covered in 5- to 10-min segments within the context of 30-min lessons. Carnine (1990) cited a number of advantages for organizing curricula around strands: (a) students are more easily engaged with a variety of topics within a single lesson, (b) strands make the sequencing of component concepts more manageable, and (c) lessons composed of several segments make cumulative introduction feasible.

Direct Instruction Math Programs

Direct Instruction (DI) programs are a strand-based approach to math instruction. They are based on the explicit or direct approach to teaching that consists of effective instructional design, effective presentation techniques, and a logical organization of instruction (as previously noted by Stein et al., 1997). *DISTAR Arithmetic I* and *II* (Engelmann & Carnine, 1975), *Corrective Mathematics* (Engelmann & Carnine, 1982), and *Connecting Math Concepts* (Engelmann et al., 1996a) are the three research-validated math programs published by Science Research Associates (SRA).

DISTAR Arithmetic I. *DISTAR Arithmetic I* consists of an initial placement test, 160 lessons, 140 take-home assignments, and 72 in-program mastery tests. This program is effective for students of any skill level from preschool through the primary grades. Students complete a placement test before they start the program. They are then placed into flexible skill groups. Lower performing students can complete the program in fewer than 200 school days. Higher performing students may complete the program in fewer than 108 school days. By skipping specific lessons, these students may progress as quickly as they can.

DISTAR Arithmetic II. *DISTAR Arithmetic II* consists of 160 lessons, 160 take-home assignments, three placement tests, and 15 in-program mastery tests. According to Engelmann and Carnine (1976),

> Children who have had 100 or more lessons of *DISTAR Arithmetic I* or a beginning arithmetic program other than *DISTAR* can successfully complete level two since placement tests and procedures for reviewing *DISTAR Arithmetic I* are built into the program. (p. 1)

As in *DISTAR Arithmetic I*, students complete a placement test before they start the program. They are then placed into flexible skill groups. Group membership changes based on student behavior on individual tests within daily lessons and in-program mastery tests. Table 3 shows a summary of the skill development strands for *DISTAR Arithmetic I* and *II*.

Corrective Mathematics. *Corrective Mathematics* is designed for students in Grades 3 through postsecondary. The program may be used for remedial work or as a part of a developmental sequence. For example, students in Grades 4 through postsecondary can use the program for remediation if they have not yet mastered addition, subtraction, multiplication, and/or division. Students in Grades 3 through 6 who have mastered basic counting and symbol identification skills can use *Corrective Mathematics* to develop advanced addition, subtraction, multiplication, and/or division skills. The program consists of four basic modules (addition, subtraction, multiplication, division) and three supplemental modules (basic fractions; fractions, decimals, and percents; and ratios and equations). There are 65 lessons in the four basic modules, each with individual student worksheets. The supplemental basic fraction module includes 55 lessons; the fractions, decimals, and percents module contains 70 lessons; and the ratios and equations module includes 60 lessons. Each of the seven modules is accompanied by a minimum of 15 mastery tests as well as suggestions for remediation. Mastery tests measure students' acquisition of basic facts, operations, and story problems.

Generally, two modules may be taught per school year. The program also contains three provisions for accelerating higher performing students. First, each module contains a skipping schedule for students whose performance on mastery tests indicates accelerated progress. Second, teachers may also teach more than one lesson per day. Third, modules may be overlapped after students have completed Lessons 45 or 50 of their current module.

There are two placement methods in *Corrective Mathematics*. First, teachers may administer the preskill test and the placement test that are included in each specific module. Second, teachers may administer a comprehensive placement test that surveys skills across all module areas. Table 4 shows a summary of the skill development strands for the addition, subtraction, multiplication, and division modules.

The supplementary math modules are designed to teach advanced mathematical skills. They may be taught sequentially or independently. The basic fractions module may be added to the fourth-grade curricula. The fractions, decimals, and percents module

Table 3

Summary of Skill Development Strands for DISTAR Arithmetic I *and* II

Skill development strands	*DISTAR I*	*DISTAR II*
Rote counting	x	x
Matching	x	
Symbol identification	x	x
Cross-out game	x	
Symbol writing	x	
Pair relations	x	
Numerals and lines	x	
Equality	x	
Matching	x	
Addition	x	x
Algebra addition	x	x
Counting backward	x	x
Subtraction	x	
Dictation	x	
Facts	x	x
Story problems	x	x
Facts for symbol identification	x	
Problems in columns	x	x
Figuring out facts	x	
More or less	x	x
Written story problems	x	x
Ordinal counting	x	
Consolidation	x	
Fact derivation		x
Multiplication		x
Fraction operations		x
Length and weight measurement		x
Applications of operations		x
Negative numbers		x

and the ratios and equations module may be added to the fifth- or sixth-grade curricula. Table 5 shows a summary of the skill development strands for the supplementary modules.

Connecting Math Concepts. Connecting Math Concepts (*CMC*) consists of seven modules or levels (*A–F* and *Bridge*). Concepts covered in *CMC* are distributed across many successive lessons to allow important connections to be made and to provide ample time to become competent at each strategy. According to Engelmann et al. (1996c), *CMC* is particularly effective with students who are at risk in mathematics. *CMC Levels A–D* consist of 120 lessons, a placement test, and a mastery test every 10th lesson. *Level A* is designed for first grade and builds on counting experiences within a variety of contexts. *Level B* is designed for second grade and makes connections between mathematical concepts and real-life situations. *Level C* is designed for third grade and places a stronger emphasis on higher order thinking skills. *Level D* is designed for fourth grade and extends students' mathematical understanding by building on the foundation of *Levels A–C*.

CMC Level E contains 125 lessons, a placement test, and a mastery test every 10th lesson. It is designed for fifth grade. Extending the concepts and skills taught in earlier levels, students analyze and solve increasingly complex problems.

The Bridge module falls between *Level E* and *Level F* and can be used for older students performing at a fifth- or sixth-grade level who have not been through *CMC Level E* and who have passed *The Bridge* placement test. It may be used as a stand-alone course in preparation for a basic pre-algebra course or, preferably, in combination with *CMC Level F* for a more complete mathematical foundation. *The Bridge* contains 70 lessons and a mastery test every 10th lesson.

CMC Level F contains 100 lessons and a mastery test every 10th lesson. It is designed for sixth grade. A placement test is not included due to the assumption that students in *Level F* have either successfully completed *Level E* or have completed *The Bridge*. *Level F* prepares students for success in higher math. Table 6 shows a summary of the skill development strands for *CMC*.

In each of the seven levels of *CMC*, students are provided with independent work for each lesson. Teachers are provided with recommendations for remediation when students are found to be experiencing difficulties as indicated by results on mastery tests.

Table 4

Summary of Skill Development Strands for Corrective Mathematics *Basic Math Modules*

Skill development strands	Addition	Subtraction	Multiplication	Division
Facts	x	x	x	x
Place value	x	x	x	x
Operations	x	x	x	x
Story problems	x	x	x	x

Structure of DI Math Programs

DISTAR Arithmetic I and *II*, *Corrective Mathematics*, and *CMC* are structured through the use of tracks, formats, and tasks.

Tracks. Tracks (also called skill development strands) consist of major skills or strategies. An example of a track from *DISTAR Arithmetic I* is Written Story Problems (Lessons 140–159). According to Engelmann and Carnine (1975), the purpose of this track is to teach students to solve simple, written story problems independently.

In keeping with the belief that necessary preskills must be taught prior to their use in a composite strategy, the following prerequisite skills for the written story problem track

Table 5

Summary of Skill Development Strands for Corrective Mathematics *Supplementary Modules*

Skill development strands	Basic fractions	Fractions, decimals, and percents	Ratios and equations
Addition of fractions and whole numbers	x	x	
Subtraction of fractions and whole numbers	x	x	
Multiplication of fractions and whole numbers	x	x	
Write mixed numbers for fractions	x		
Find equivalent fractions	x		
Addition and subtraction of mixed numbers		x	
Multiplication and division of mixed numbers		x	
Reducing improper fractions		x	
Writing decimals or percents for fractions		x	
Writing fractions or percents for decimals		x	
Writing fractions or decimals for percents		x	
Finding ratios			x
Solving rate and distance problems			x
Using basic problem solving strategy for word problems			x
Using basic problem solving strategy for algebra problems			x

Table 6

Summary of Skill Development Strands for Connecting Math Concepts

Skill development strands	*Level A*	*Level B*	*Level C*	*Level D*	*Level E*	*Bridge*	*Level F*
Counting	x	x					
Symbols	x						
More/less/equal	x		x				
Addition/subtraction	x						
Place value	x	x	x	x	x		
Problem solving	x	x	x	x	x	x	x
Word problems	x	x	x	x	x	x	
Application: money	x						
Following directions		x					
Addition and subtraction facts							
Number relationships		x	x		x	x	x
Number family tables				x	x	x	
Measurement		x					x
Column addition		x	x				
Column subtraction		x	x	x			
Mental arithmetic		x	x	x	x		
Money		x	x				
Multiplication		x				x	
Geometry: identifying shapes, finding perimeter and area		x		x		x	
Tables		x					
Addition and subtraction number families			x				
Multiplication and division facts			x	x			
Column multiplication			x	x	x		
Division with remainders			x	x	x	x	
Estimation			x			x	
Calculator skills			x	x	x		
Equation concepts			x				
Analyzing data: tables			x				
Fractions			x	x	x	x	
Coordinate system			x	x	x	x	x
Graphs			x				x
Area			x				x
Volume			x	x			x
Time		x	x				

continued

are (a) applying the appropriate strategy and solving problems in addition (introduced in Lesson 51), algebra addition (introduced in Lesson 61), and subtraction (introduced in Lesson 83); (b) writing arithmetic statements that are dictated by the teacher (introduced in Lesson 84); and (c) translating verbal story problems into written arithmetic statements (introduced in Lesson 102). Throughout each track, focus changes from teacher modeling to guided practice to independent practice.

Formats. Engelmann and Carnine (1975) define formats as patterns of teaching steps repeated in a number of successive lessons. A format for Counting Events and Objects from *DISTAR Arithmetic I* appears in Figure 5. Formats are maintained for three or more lessons before the focus shifts from teacher modeling to guided practice.

Tasks. A task is created by inserting a new set of numbers into a format pattern in which the wording remains unchanged. For example, the format for teaching symbol identification of the number 4 is shown in Figure 6. Notice how the wording is changed within the same format pattern to teach symbol identification of the number 2 (also seen in Figure 6). Tasks are presented in the simplest manner possible to eliminate confusion and follow a specific

Table 6, continued

Summary of Skill Development Strands for Connecting Math Concepts

Skill development strands	*Level A*	*Level B*	*Level C*	*Level D*	*Level E*	*Bridge*	*Level F*
Statistics: range			x				
Whole number operations				x	x	x	x
Equations and relationships				x			
Decimals				x		x	x
Percents						x	
Ratios and proportions				x		x	
Ratio tables						x	x
Fraction number families				x	x	x	
Fraction operations							x
Probability				x			x
Probability geometry				x	x		
Operational relationships					x	x	x
Rounding					x		
Whole number properties					x		
Mixed number operations					x		x
Decimal operations					x		
Circles					x	x	
Angles and lines					x	x	
Geometry facts							x
Signed numbers							x
Exponents							x

sequence to ensure mastery of the five program objectives. Subsequent tasks requiring similar procedures are taught in order to encourage generalization.

How DI Math Programs Meet NCTM's Principles for Improving Math Instruction

NCTM's principles for improving math instruction (NCTM, 2000b) can be met through each of the DI math programs. First, NCTM's *Equity Principle* calls for excellence in mathematics education with equally high expectations and strong support for all students. NCTM discourages "tracking" which is defined as a long-term, often permanent placement within an academic track based on perceived mathematical abilities. DI math programs use flexible skill grouping based on current levels of performance as determined by daily progress monitoring. Such monitoring consists of observations during lessons, performance on take-home assignments, and performance on in-program mastery tests. Group membership changes as dictated by individual student performance. In the *Equity Principle*, NCTM strongly encourages that expectations be the same for all students. DI math programs are based on specific performance objectives which all students must master as they progress through the program.

Second, NCTM's *Curriculum Principle* states that a curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well articulated across the grades. DI math programs meet this principle by using a strand design. In this design, lessons are organized around concepts or "big ideas." According to Dixon (1994), big ideas make it possible for students to learn the most and learn it most

Figure 5

Example of a format from DISTAR Arithmetic I.

TASK 4 **COUNTING EVENTS AND OBJECTS Children Clap as You Count**

Emphasize words in **boldface**.

Group Activity

a. You will count and clap, pausing one second between numbers.

Let's play a clapping game. Every time I count, I'm going to clap. Get ready. One...two...three...four.

b. You will pause two seconds between numbers as you count.

Your turn. I'm going to count. You're going to clap.
(Pause.) **Get ready. One...two...three...four. Stop.**
(*The children clap as you count; they do not count.)*

To correct	If the children have trouble coordinating their clapping with your counting, physically guide their hands to help them clap.

c. Repeat *b* until the response is firm.

Individual Test

Call on several children for *b*.

Note. From Engelmann, S., & Carnine, D. (1975). *DISTAR Arithmetic I: Teacher's guide*, p. 22. Columbus, OH: SRA/Macmillan/McGraw-Hill. Reproduced with permission of The McGraw-Hill companies.

efficiently. Specifically, DI math programs are designed to guide students' learning of basic operations, strategies, and applications to more complex applications throughout each level and throughout each grade.

Furthermore, NCTM notes that extensive field-testing should be conducted before school districts select curricular mathematics materials. DI programs have been implemented and researched in a wide variety of

Figure 6

Example of two tasks illustrating how a new set of numbers is inserted into a format pattern, taken from DISTAR Arithmetic I.

TASK 2 SYMBOL IDENTIFICATION
Introducing a New Symbol

When you **point** to a symbol, hold your finger an inch or two above the page. **Touch** with a definite motion just below the symbol.
Emphasize words in **boldface**.

Group Activity

Do *a*, *b*, and *c*.

4

a. Point. **This is a four. What is this?** Touch 4. *4.* **Yes, this is a four.**

b. Point. **Is this a four?** Touch the dog. *No.* To correct: **This is not a four. Is this a four?** *No.*

4

c. Point. **Is this a four?** Touch 4. *Yes.* To correct: Repeat *a,* then *c.*

Repeat *a, b,* and *c* in random order until responses are firm.

d. When I touch it, tell me what it is.

e. Point to *a* or *c*. Pause. **Get ready.** Touch.
Touch *a* and *c* in random order until responses are firm.

f. Randomly touch *a*, *b*, and *c*.

Individual Test

Call on some children to identify two symbols.

TASK 2 SYMBOL IDENTIFICATION
Introducing a New Symbol

When you **point** to a symbol, hold your finger an inch or two above the page. **Touch** with a definite motion just below the symbol.
Emphasize words in **boldface**.

Group Activity

Do *a*, *b*, and *c*.

2 4

a. Point. **This is a two. What is this?** Touch 2. *2.* **Yes, this is a two.**

b. Point. **Is this a two?** Touch 4. *No.* To correct: **This is not a two. Is this a two?** *No.*

2

c. Point. **Is this a two?** Touch 2. *Yes.* To correct: Repeat *a,* then *c.*

Repeat *a, b,* and *c* in random order until responses are firm.

d. When I touch it, tell me what it is.

e. Point to *a* or *c*. (Pause.) **Get ready.** Touch.
Touch *a* and *c* in random order until responses are firm.

f. Randomly touch *a*, *b*, and *c*.

Individual Test

Call on some children to identify two symbols.

Note. From Engelmann, S., & Carnine, D. (1975). *DISTAR Arithmetic I: Teacher's guide*, pp. 108, 133. Columbus, OH: SRA/Macmillan/McGraw-Hill. Reproduced with permission of The McGraw-Hill Companies.

settings for over 30 years (Adams & Engelmann, 1996).

Third, NCTM's *Teaching Principle* states that teachers should understand what students know and need to learn and then challenge and support them to learn it well. NCTM encourages teachers to reflect on and improve their lessons with the support of colleagues within a peer coaching model. This model allows teachers to plan for maximum student success carefully. DI math programs meet this principle by providing extensive preservice training for teachers to ensure appropriate implementation and effective instructional delivery. After training, teachers are provided with coaches who conduct observations to determine areas for improvement. Coaches and teachers then work together to ensure maximum student and teacher success. According to NCTM, teachers should also be able to predict what students will do when presented with particular problems and tasks. DI preservice training, program guides, and teacher presentation books offer precorrective strategies to minimize commonly anticipated errors. DI math lesson formats also contain specifically prescribed error correction procedures; an example of this was shown in Figure 5.

Fourth, NCTM's *Learning Principle* states that students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. NCTM recommends that elementary students should study mathematics from well-prepared teachers for at least 1 hr a day. An example of how DI math programs meet this standard can be found in *DISTAR Arithmetic I* and *II*. The teacher presents daily group instruction for 30–35 min, providing both modeling and guided practice. Students spend 20–30 min (or more, if necessary) completing independent seatwork. In DI math programs, after necessary preskills have been mastered, students also complete take-home work for added independent practice. DI math programs answer NCTM's recommendation for well-prepared teachers by using predesigned instructional formats. Teacher preparation of lessons is kept to a minimum thereby reserving valuable time and energy for focusing on student performance. Another major concern of NCTM is that students may become increasingly disengaged in mathematics instruction. DI math programs address this concern in two ways. First, higher skilled students are allowed to move as quickly through each program as necessary. In fact, Vreeland et al. (1994) found that two groups of academically talented students, who were taught using *CMC*, made gains of approximately two grade levels in 1 year on the Kaufman Test of Educational Achievement—Comprehensive Form (KTEA—C). Second, because DI math programs are strand-based, the multitopic, fast-paced formats keep students focused and motivated.

Further, the NCTM encourages conceptual understanding and problem solving skills by actively building new knowledge from experience and prior knowledge. In DI math programs, strategies are taught, rather than rote skills, in a specified order to promote generalization from previously mastered skills to new situations. According to Carnine and Engelmann (1990), explicitly taught strategies prepare students to see the total structure of a problem. DI math lessons are organized around skill development tracks or big ideas. These big ideas are introduced in small steps from one lesson to the next within each track. As concepts are mastered through massed practice, they are continuously reviewed. Similarly, the strand design of DI math programs teaches students to differentiate between situations which may appear to require the same strategic solution, when in fact, they may not. This combination of massed practice and cumulative review allows students to maintain prior knowledge while actively building new knowledge of important math concepts and strategies.

Fifth, NCTM's *Assessment Principle* states that assessment should support the learning of important mathematics and furnish useful information to both teachers and students. DI math programs address this principle by providing frequent in-program mastery tests to allow teachers to make daily decisions about individual students' progress. Additionally, with students and educators being penalized for low test scores on nationally normed achievement tests, the issue of "teaching to the test" becomes a great concern. DI math programs have been shown to have positive effects on norm-referenced test scores. Specifically, Brent and DiObilda (1993) found similar scores between DI math students and students who were taught using a curriculum deliberately aligned with districtwide standardized assessments. NCTM also suggests that perhaps multiple forms of assessment may offer more useful information in monitoring student progress. DI math programs provide several different forms of assessment including in-program mastery tests at frequent intervals, take-home assignments, and fact games.

Finally, NCTM's *Technology Principle* states that technology should influence the skills taught and enhance students' learning. Therefore, technology should be used to support the learning of mathematics, not the learning of technology. In so doing, NCTM recommends that technology be embedded in the mathematics program, rather than provided as a supplemental element. Concurrently, the National Assessment of Educational Progress (2001) report stated that eighth graders whose teachers reported that they permitted unrestricted use of calculators in class had higher average scores in 2000 than did students whose teachers restricted calculator use. In *CMC Levels C–F* students learn to use calculators to solve increasingly complex operations. For example, in *CMC Level C*, students use calculators to solve addition, subtraction, multiplication, and fraction problems. In *CMC Level F*, students use calculators to solve division problems that do not have whole-number answers and problems that multiply a fraction by a whole number or decimal. The use of the calculator in DI math programs is to support students' learning of the fundamental operations (i.e., addition, subtraction, multiplication, division) and to support their skills in using these tools when solving word problems.

Research Synthesis on DI Mathematics Programs

The purpose of this synthesis was to survey the studies conducted using DI Mathematics Programs (SRA). Studies including *DISTAR I* and *II*, *Corrective Mathematics*, and *CMC* were selected using the First Search, ERIC, PsycINFO, Education ABS, and ProQuest databases. Descriptors included the following: Direct Instruction, *DISTAR Arithmetic*, *DISTAR Arithmetic I*, *DISTAR Arithmetic II*, direct instruction, direct teaching, direct verbal instruction, explicit instruction, mathematics instruction, *Corrective Mathematics*, and *Connecting Math Concepts*. Ancestral searches of reference lists were used to identify other possible research articles. Also, hand searches were done in the following peer-reviewed journals: *Effective School Practices*, *Journal of Direct Instruction*, and *Education and Treatment of Children*. Research articles in peer-reviewed journals were included for review. Articles published before 1990 were not included in this review. A total of 12 studies were analyzed in this review.

Direct Instruction meta-analysis. Adams and Engelmann (1996) conducted a meta-analysis of DI programs including *DISTAR Arithmetic I* and *II*, *Corrective Mathematics*, *CMC*, and other DI programs. Included studies were required to have the following elements: means and standard deviations of groups, the use of a suitable comparison group, and random selection of participants to groups. Thirty-four out of 37 studies involved the active intervention of DI programs. Three follow-up studies were not included in the statistical analysis but were reviewed in a separate chapter. In a sample polling of means, 87% of the studies favored DI programs, 12% favored non-DI programs, and 1% found scores to be the same. In a sample polling of statistically significant outcomes, 64% found statistically significant differences in favor of DI programs. Finally, in a summary of the statistical analysis of math results, an effect size of 1.11 in favor of DI math programs was found in 33 of the comparisons (those studies that included a math component).

DISTAR Arithmetic. Table 7 shows one study using *DISTAR Arithmetic I*. Young, Baker, and Martin (1990) compared *DISTAR Arithmetic I* to a teacher-developed discrimination learning theory (DLT) program based on the first 60 lessons of *DISTAR Arithmetic I*. Participants included five students with intellectual disabilities; each scored between 35–54 on the Wechsler Intelligence Scale for Children—Revised (WISC—R). All participants had articulation problems (responses were limited to two- to three-word utterances). During the baseline phase, *DISTAR Arithmetic I* was implemented according to the program script. During baseline, average performance on mastery tests ranged from 18% to 73%, while average academic engaged time ranged from 18% to 31%. During the DLT phase, average performance on mastery tests ranged from 69% to 96%, while average academic engaged time ranged from 56% to 84%. It was further determined over 5 days during a 5-week maintenance probe that both mastery scores and academic engaged time remained higher than baseline rates. As a result, the author concluded that the match-to-sample format of the DLT phase was an effective adaptation of *DISTAR Arithmetic I* in teaching math skills to students with articulation problems.

Corrective Mathematics. Three studies were found using the *Corrective Mathematics* (*CM*) program (see Table 7). First, Parsons, Marchand-Martella, Waldron-Soler, Martella, and Lignugaris/Kraft (2004) examined the use of *CM* in a secondary general education classroom for students struggling in math as delivered by peer tutors. Ten students were assigned to the learner group based on referrals by a school counselor. All had failed the lowest level of math available at that school. Nine students were recruited by the *CM* teacher, school counselors, and other high school math teachers to serve as peer tutors. All participants were pre- and posttested using the Calculation and Applied Problems subtest of the Woodcock Johnson—Revised: Test of Achievement (WJ—R). After 60 instructional days, the authors found that both learners and peer tutors experienced posttest gains in one or both areas of the WJ—R subtests.

Second, Glang, Singer, Cooley, and Tish (1991) assessed the efficacy of *CM* in teaching math skills to an 8-year-old student with traumatic brain injury. In this study, the student was also instructed using *Corrective Reading Comprehension* (*Level A*) to improve his reasoning skills. Instruction took place twice a week over a period of 6 weeks. After 12 hr of instruction, the authors found that the student's math fact rate and story problem accuracy improved.

Finally, Sommers (1991) examined the effects of a comprehensive DI program in improving the overall performance of at-risk middle school students over a 2-year period. *CM* multiplication, division, basic fractions, fractions-decimals-percents, and ratios and equations mathematics modules were used in conjunction with a variety of DI reading, spelling, and writing programs. Students also used a variety of supplemental material including: self-chosen reading materials, *Journeys* (Harcourt Brace Jovanovich), *Warriner's English Grammar and Composition*, *DLM Growth in Grammar* workbooks, and *Heath Mathematics*. In math measures, students made average gains of 1.2 months per month of instruction (as noted by the author).

All three studies examined students with very different characteristics across various settings. In each study, authors found *CM* to be effective in increasing math skills. However, it

Table 7

Program Comparison Summary Information for Investigations Involving DI Math

Reference	Program/comparison	Participants/ characteristics	Research design	Dependent variable(s)/measures	Results
Adams & Engelmann (1996)	DI meta-analysis	37 studies	Meta-analysis	Overall program effectiveness	64% of studies found statistically significant outcomes in favor of DI programs. Statistical analysis of studies including math found an effect size of 1.11 in favor of DI math programs.
Young, Baker, & Martin (1990)	*DISTAR Arithmetic I* vs. teacher-developed DLT program based on the first 60 lessons of the *DISTAR Arithmetic I* program	5 students with mild mental retardation and articulation problems	Multiple baseline across subjects	*DISTAR Arithmetic I* placement test, teacher-designed test (including: counting, matching, selecting, numerosity, writing, and equality), and academic engaged time	Math skill scores were higher on teacher-designed mastery tests during DLT phase and across 5 days at a 5-week maintenance probe. Academic engaged time higher during DLT phase.
Glang, Singer, Cooley, & Tish (1991)	*CM*	8-year-old with traumatic brain injury	Multiple baseline across content areas	Math facts and story problems	Math fact rate increased but remained lower than average third grader's. Story problem accuracy increased from 11.4% correct to 91.25% correct during instruction.
Parsons, Marchand-Martella, Waldron-Soler, Martella, & Lignugaris/Kraft (2004)	*CM*	19 students: 10 learners and 9 peer tutors	One-group pretest–posttest	WJ—R Math Calculation and Applied Math Problems subtests	Both learners and peer tutors improved in math calculation and applied math problems.
Sommers (1995)	*CM*	112 sixth, seventh, and eighth graders at risk for academic failure	One-group pretest–posttest	Pretest: Stanford Math Posttest: Key Math Diagnostic	Averaged 1.2 months gained per month of instruction.

continued

Table 7, continued

Program Comparison Summary Information for Investigations Involving DI Math

Reference	Program/ comparison	Participants/ characteristics	Research design	Dependent variable(s)/measures	Results
Brent & DiObilda (1993)	*CMC* vs. *Holt Math Series*	189 students entering first grade: experimental group consisted of 23 stable and 76 mobile students; control group consisted of 27 stable and 63 mobile students	Nonequivalent control group	CTBS and Metropolitan Achievement Test (math subtests included: concepts/applications, computation, and total math). MAT was given to stable students only (math subtests included: computation, concepts/applications, problem solving, and total math).	Stable and mobile DI groups received scores similar to the stable control group on the CTBS, while the mobile control group scored significantly lower on the concepts subtest. The DI group scored significantly higher than the control group on all areas of MAT (only administered to stable students). Mobility had a negative impact on all students. However, mobility was more detrimental to the control groups, as evidenced by CTBS scores.
Crawford & Snider (2000)	*CMC* vs. *Invitation to Mathematics*	15 fourth graders	Pretest–posttest control group	NAT: computation, concepts, and problem solving; CBM based on *CMC*; CBM based on *SF*; and multiplication facts	*CMC* group made greater gains than the previous year on both CBMs and on multiplication facts test. No significant NAT posttest results noted.
McKenzie, Marchand-Martella, Martella, & Moore (2004)	*CMC Level K*	16 preschool students with and without disabilities	One-group pretest–posttest	Pre- and posttest measures included the BDI and the *CMC Level A* placement test.	The BDI Total Cognitive score showed a combined effect size of .52 for students with and without disabilities. Combined scores on the *CMC Level A* placement test increased from a mean of 4.31 on the pretest to a mean of 7.69 on the posttest.
Snider & Crawford (1996)	*CMC* vs. *Invitation to Mathematics*	46 fourth graders	Pretest–posttest control group	NAT: computation, concepts, and problem solving; CBM based on *CMC*, CBM based on *SF*; and multiplication facts tests	*CMC* group scored higher than the *SF* group on NAT computation subtest. *CMC* group scored significantly higher than the *SF* group on all three curriculum-based tests.

continued

Table 7, continued

Program Comparison Summary Information for Investigations Involving DI Math

Reference	Program/ comparison	Participants/ characteristics	Research design	Dependent variable(s)/measures	Results
Tarver & Jung (1995)	*CMC* vs. *MTW*/CGI	119 students entering the first grade	Nonequivalent control group	CTBS mathematics concepts and applications, computation, and total math (averaged from the two subtests)	End of first grade CTBS results: *CMC* group scored higher on computation and total math but not on concepts and applications. End of second grade CTBS results: *CMC* group scored higher than the control group on all posttest measures.
Vreeland et al. (1994)	*CMC* vs. *Addison-Wesley Mathematics*	5 third-grade classrooms and 4 fifth-grade classrooms	Nonequivalent control group	ITBS Total Math (consisting of three subtests: computation, concepts, and problem solving); KTEA—C: calculation and applications subtests. (Applications subtest given to only six students.) CBM problem solving test based on *CMC* and *Addison-Wesley*.	No significant change in percentile rank for *CMC* groups on the ITBS. The percentile rank change for the control groups was -15%. *CMC* third graders made gains on the KTEA—C, while no gains were noted for *CMC* fifth graders. No KTEA—C scores were available for the control groups. Academically talented third and fifth graders made better than average gains on the KTEA—C.
Wellington (1994)	*CMC*	16 first-grade classrooms and 16 fourth-grade classrooms	Nonequivalent control group	Pretest: *CMC* placement test; posttests: CBM based on *CMC* and traditional basal; district-designed mastery test	One *CMC* first-grade group and one control group showed statistically significant improvement on the posttest. Six *CMC* fourth-grade groups showed statistically significant improvements on the posttest. District-designed mastery tests showed a decrease in the rate of mastery as grade levels increased.

should be noted that multiple treatment interference is a threat to the external validity of the study conducted by Sommers (1991). For this reason, it is difficult to clearly establish a causal relationship between DI programs and increases in student achievement.

Connecting Math Concepts. Seven studies were found using *CMC* (see Table 7). First, Snider and Crawford (1996) included 46 fourth graders who were randomly assigned to two general education classrooms. One teacher used *CMC*, *Level D*; the other teacher used *Invitation to Mathematics* (*SF*) by Scott Foresman. *CMC* students scored higher than the *SF* students on the Computation subtest of the National Achievement Test (NAT). In addition, *CMC* students scored significantly higher on both the multiplication facts test and on curriculum-based measures based on *CMC* and *SF*.

Second, in a follow-up study by Crawford and Snider (2000) both teachers used *CMC*. After 1 year of using *CMC*, the teacher who had used *SF* had students who made greater gains than the previous year on both the multiplication facts tests and on both curriculum-based measures. No significant posttest differences were noted on the NAT subtests or Total Test scores. The authors cited several possible reasons for the lack of significant pre- to posttest gains. Some of these included (a) less than optimal implementation of *CMC*, (b) lack of alignment between the NAT Concepts and Problem Solving subtests and either curriculum, and (c) performance on norm-referenced tests are more highly correlated to reading comprehension scores than with computation scores. Although the NAT results did not reach significance, the positive results shown by the remaining data prompted the districtwide implementation of *CMC*.

Third, Tarver and Jung (1995) compared *CMC* to a program that combined *Math Their Way* (*MTW*) and Cognitively Guided Instruction (CGI). One hundred nineteen students entering first grade were assigned to five classrooms. One experimental classroom used *CMC*, while four control classrooms used *MTW*/CGI. The study took place over 2 years. At the end of first grade, students were posttested using the Comprehensive Test of Basic Skills—Mathematics (CTBS—M). *CMC* students scored significantly higher than the control group on Computation and Total Math but not on the Concepts and Applications subtest. At the end of second grade, *CMC* students scored higher than the control group on all posttest measures as well as on an experimenter-constructed math attitudes survey. Tarver and Jung noted positive effects for both low and high performing students.

Fourth, Brent and DiObilda (1993) compared the effects of DI curricula to those of traditional basal curricula in Camden, New Jersey. At that time, Camden was considered to have the highest percentage of children who lived in poverty in the country. The mobility rate in Camden was also higher than the national average. For that reason, this study also examined the effects of each curriculum with stable and mobile urban children. In an attempt to improve their standardized test scores, school officials had previously aligned their schools' traditional basal programs to the Comprehensive Test of Basic Skills—Form U, Level D (CTBS). This study compared *CMC* with the *Holt Math Series*. Dependent measures included the CTBS and the Metropolitan Achievement Test (MAT). CTBS total math scores were similar among stable and mobile DI groups as well as stable control groups. Both stable and mobile DI groups scored higher than the control groups on the CTBS computation subtest, while the stable control group scored higher on the concepts subtest. On the MAT, administered to stable students only, the DI group scored significantly higher than the control group on all subtests. Overall, mobility was found to have a negative impact on student achievement in both the DI and the control groups. However, scores on the CTBS indicated that mobility was more detrimental to the control groups.

Fifth, McKenzie, Marchand-Martella, Martella, and Moore (2004) examined the effects of *CMC Level K* (prepublication copy; Engelmann & Becker, 1995) on preschoolers with and without developmental delays. Participants included 16 preschoolers who attended school 5 days per week. Each preschooler completed all 30 lessons contained in *CMC Level K*. Students were pre- and posttested using the Battelle Developmental Inventory (BDI) and a curriculum-based placement test (*CMC Level A*). Results showed an effect size of .61 for preschoolers without developmental delays and .54 for preschoolers with developmental delays on the BDI. The placement test for *CMC Level A* consists of 10 questions and was given to all preschoolers. Scores for preschoolers without developmental delays increased from a mean of 4.55 correct on the pretest to a mean of 7.9 on the posttest; scores for preschoolers with developmental delays increased from a mean of 3.8 correct on the pretest to a mean of 7.2 on the posttest.

Sixth, Vreeland et al. (1994) compared *CMC* to the *Addison-Wesley Mathematics* program. Participants included 5 third-grade classrooms and 4 fifth-grade classrooms. *CMC* third and fifth graders scored higher than the control group on CBM posttest measures, based on *CMC* and *Addison-Wesley*. *CMC* third graders showed little percentile rank changes on the ITBS, while the third-grade control group's percentile rank change was -15%. No percentile rank changes on the ITBS were noted for *CMC* fifth graders. No ITBS scores were available for fifth-grade control groups. Results of the KTEA—C posttest revealed that, at the end of third grade, *CMC* third graders scored at or above the fourth-grade level. No pre- to posttest gains were noted for *CMC* fifth graders. No KTEA—C posttest scores were available for the fifth-grade control groups. This study also examined the effects of *CMC* with academically talented students. For academically talented third graders, KTEA—C posttest scores showed mean grade equivalents of 5.7 on the math calculation subtest and 6.1 on the math applications subtest. For academically talented fifth graders, KTEA—C posttest scores showed mean grade equivalents of 8.0 on the math calculation subtest and 8.5 on the math applications subtest. Overall, results were positive enough to guide the school officials' decision to use *CMC* in many of its first- through sixth-grade classrooms. In a 1-year follow-up study, 12 classrooms, grades first through sixth, used *CMC Levels A–E*. Posttest results indicated that *CMC* students experienced gains, particularly in the higher levels of the program. These results, combined with positive teacher and parent reports, led to the use of *CMC* in nearly all first- through sixth-grade classrooms the following year.

Finally, Wellington (1994) examined the effectiveness of *CMC* for a period of 1 year in a socio-economically and ethnologically diverse school district. All eight of the district's elementary schools participated in the study. One first-grade classroom and 1 fourth-grade classroom per school served as experimental groups. First- and fourth-grade comparison groups were also chosen from each school. The pretest consisted of the *CMC* placement test, while the posttest consisted of a teacher-designed CBM based on *CMC* and the traditional basal used by the comparison groups. Results showed statistically significant (>.05 level) differences among posttest scores for two out of the eight first-grade groups: one in favor of the *CMC* group and one in favor of the comparison group. Fourth-grade results showed statistically significant differences in favor of six out of the eight *CMC* groups. The author stated that the narrower scope of material at the first-grade level as compared to the breadth of concepts at the fourth-grade level may account for the poor first-grade results. A district-designed mastery test was also administered to first through fifth graders at the end of the school year. The results indicated that the rate of mastery (defined as 70%) declined at the higher grade levels. The results of this test combined with posttest results compelled the school district to implement *CMC* districtwide in first through fifth grades.

All seven studies found positive results when *CMC* was used. Three of the seven studies (i.e., Brent & DiObilda, 1993; McKenzie et al., 2004; Vreeland et al., 1994) examined three varied populations. Brent and DiObilda specifically found *CMC* to have a positive effect on students from highly transient, low-income, minority families in an urban community. McKenzie et al. found *CMC Level K* to have positive effects on a diverse group of preschoolers that included those with and without developmental delays. Finally, Vreeland et al. found *CMC* to have positive effects for both general education and gifted students. It should also be noted that the results from three of the seven studies (i.e., Crawford & Snider, 2000; Snider & Crawford, 1996; Wellington, 1994) led to the large-scale adoption of *CMC*.

Summary. In all, 12 studies published since 1990 were found using DI math programs. The majority (11 out of 12) of these found DI math programs to be effective in improving math skills in a variety of settings with a variety of students. One study (Young et al., 1990) showed positive results for a DLT adaptation based on *DISTAR Arithmetic I* rather than *DISTAR Arithmetic I* in its original format.

Future Directions for Research

Recent national and international assessments have indicated the need to implement research validated math curricula in our schools. The NCTM responded to this need by publishing its *Principles and Standards for School Mathematics* (NCTM, 2000b). These principles and standards are recommended to influence the development and selection of high quality math curricula. The research included in this summary contributed positive evidence for the use of the direct or explicit approach to math instruction using these principles and standards. However, a number of implications for future research exist.

Populations. Direct Instruction curricula are often mistakenly associated for use primarily with students with special needs (i.e., Adams & Engelmann, 1996; Schieffer, Marchand-Martella, Martella, Simonsen, & Waldron-Soler, 2002). However, 7 out of 11 studies (meta-analysis excluded; i.e., Brent & DiObilda, 1993; Crawford & Snider, 2000; Parsons et al., 2004; Snider & Crawford, 1996; Sommers, 1991; Tarver & Jung, 1995; Vreeland et al., 1994; Wellington, 1994) examined the effectiveness of DI math programs on general education students. Two out of 11 studies (i.e., Glang et al., 1991; Young et al., 1990) examined the effectiveness of DI math programs with students with disabilities. One study (McKenzie et al., 2004) examined the effectiveness of DI math programs on a group of students that included those with and without developmental delays. In their DI meta-analysis, Adams and Engelmann (1996) calculated the average effect size per study according to general education and special education and found similar effect sizes for both groups (.82 and .90, respectively). Further, Vreeland et al. (1994) found grade-level gains of approximately 2 years for two groups of academically talented students using *CMC*. These results indicate the need for future examination of the effects of DI math programs based on specific learner characteristics (e.g., emotional and behavioral disabilities, attention-deficit disorder, at-risk or incarcerated youth, gifted learners).

Experimental analysis. In reviewing the research on DI math programs, several threats to internal and external validity were found. Selection is an issue in many of the studies due to a lack of random selection of participants (i.e., Brent & DiObilda, 1993; Glang et al., 1991; McKenzie et al., 2004; Parsons et al., 2004; Sommers, 1991; Tarver & Jung, 1995; Wellington, 1994; Young et al., 1990). Two studies, (i.e., Crawford & Snider, 2000; Snider & Crawford, 1996) attempted to assess group equivalence and randomly assign participants. Future studies should include random selection of participants from the target population, random assignment to groups, and determination of group equivalence.

Dependent variables and measures. A wide variety of norm-referenced tests were used to assess math skills (e.g., applied math, basic facts, computation, concepts, problem-solving). Six of the 12 studies (i.e., Crawford & Snider, 2000; Glang et al., 1991; Snider & Crawford, 1996; Vreeland et al., 1994; Wellington, 1994; Young et al., 1990) used CBM to determine students' level of math performance within the local curriculum. Only 4 of the 12 studies (i.e., Brent & DiObilda, 1993; Crawford & Snider, 2000; Snider & Crawford, 1996; Vreeland et al., 1994) reported using districtwide assessments as dependent measures. Our current standing on national and international assessments indicates that future research should include studies that use district and statewide assessments as dependent measures.

Two studies (i.e., Sommers, 1991; Vreeland et al., 1994) reported results as grade-level gains. According to Cohen and Spenciner (1998) and McLoughlin and Lewis (2001), such ordinal scale data should be interpreted with caution. Grade-level gains can be easily misinterpreted because the intervals in grade equivalents do not represent equal units of measurement. Therefore, according to Cohen and Spenciner, a grade-level gain of 1.0 is only representative of students who are in the average range for their grade (failing to account for individual differences). Future researchers should refrain from reporting ordinal scale data, such as age- and grade-equivalents; means cannot be calculated and, at best, only pretest medians and posttest medians (without mathematically manipulating differences) should be noted (if used at all).

Fidelity of implementation data. Inherent to the design of DI programs is the use of scripted formats and training opportunities. According to Adams and Engelmann (1996), the rationale for these scripted presentations is that if teachers present an adequate set of examples with clear, consistent wording, students will learn the material with less confusion. The delivery of these programs is a key factor in their effectiveness. However, in many of the studies investigated, verification of the independent variable and experimenter effects are concerns. Seven studies provided information describing program implementation (i.e., Crawford & Snider, 2000; Glang et al., 1991; McKenzie et al., 2004; Snider & Crawford, 1996; Tarver & Jung, 1995; Vreeland et al., 1994; Young et al., 1990). Future studies should monitor the implementation of DI curricula to limit experimenter effects and to increase our confidence in the fidelity of program implementation.

Implementation of DI math programs with other DI curricula. Three of the 12 studies (i.e., Brent & DiObilda, 1993; Glang et al., 1991; Sommers, 1991) investigated the use of DI math programs in conjunction with other DI programs (e.g., *Corrective Reading*, *Corrective Spelling Through Morphographs*, *DISTAR Language I*, *Expressive Writing*, *Reading Mastery*, *Reasoning and Writing*). Future studies should compare the effects of the implementation of DI math programs alone as compared to the implementation of DI math programs in conjunction with other DI curricula. On another hand, Sommers (1991) supplemented DI math curricula with other math curricula (i.e., *Heath Mathematics*), thereby making it difficult to claim that effects resulted from a single independent variable. Multiple treatment interference should be avoided by either describing the combined effects of multiple treatments or by providing only one independent variable.

Calculation of effect sizes. Tests of statistical significance are often relied upon to indicate the effectiveness of a given variable. However, statistical significance data are used to provide information about whether or not results are likely due to chance. An all too common practice in research consumerism is the misinterpretation of these results. Statistical significance does not necessarily mean educational significance. In our quest to find effective math curricula, educators must consider effect size when reviewing research data. According to Martella, Nelson, and Marchand-Martella (1999), an effect size is a standardized measure of the magnitude of the differences between groups. In other words, it measures how large the differences were and can be used as an indication of educational significance. Six of the 12 studies (i.e., Adams & Engelmann, 1996; Brent & DiObilda, 1993; McKenzie et al., 2004; Parsons et al., 2004; Tarver & Jung, 1995; Wellington, 1994) included measures of effect sizes. Future research on DI math programs should include measures of effect size to reflect the magnitude of change in educational programs.

Maintenance and generalization data. Two of the most important considerations in choosing math curricula are maintenance and generalization. Long-term retention of mathematical skills and strategies is critical not only for academic success, but also for future employment success. Having students show generalized skills in a wide-variety of subjects and real life situations is equally critical. Three of the 12 studies (i.e., Vreeland et al., 1994; Wellington, 1994; Young et al., 1990) included some measure of maintenance and generalization. Future DI math studies should include such data to afford educators the opportunity to examine this valuable information.

Social validity. While a great deal of emphasis is rightfully placed on quantitative measures, social validity measures are an important source of information regarding the social relevance of research questions and results. According to Wolf (1978), "a number of the most important concepts of our culture are subjective, perhaps even the most important" (p. 210). Five of the 12 studies (i.e., Crawford & Snider, 2000; Snider & Crawford, 1996; Tarver & Jung, 1995; Vreeland et al., 1994; Wellington, 1994) reported findings on students' and teachers' attitudes and opinions about DI math programs. Brent and DiObilda (1993) provided socially relevant information regarding the effectiveness of *CMC* with mobile urban children. However, given the need for effective math curricula in a variety of social contexts, future research should include measures of social validity.

Conclusion

The *National Assessment of Educational Progress: Mathematics Highlights*, as reported by the National Center for Education Statistics (2001), shows that 4th- and 8th-grade math scores were higher than in earlier national assessments. However, the average math scores for 12th graders declined. In addition, math scores of American students rank in the bottom half of the countries that participated in the 1995 TIMMS Project (International Study Center, 2001). Combined with the ever-increasing technological complexity of future employment, it is clear that our students are in dire need of effective math instruction.

In its *Principles and Standards for School Mathematics* (2000b), NCTM developed five overall curricular goals for ensuring student success in mathematics. First, students should learn to value mathematics. Experiencing success in mathematics helps students to value mathematics. DI math programs are designed to allow students to experience success and to know that they are experiencing success on a daily basis. Second, students should become confident in their own mathematical abilities. DI math programs are designed to develop and maintain knowledge and application of skills and concepts. As students encounter and successfully solve a wide variety of mathematical problems both in the classroom and in the real world, confidence in their own abilities increases. Third, students should become mathematical problem solvers. DI math programs provide students with the necessary tools to solve a broad spectrum of word problems and real life problems. Fourth, students should learn to communicate mathematically. DI math programs directly teach mathematical vocabulary and strategies thereby strengthening students' abilities to communicate effectively about mathematics. Finally, students should learn to reason mathematically. DI math programs teach students to discriminate between different types of problems at gradually increasing levels of complexity. The ability to discriminate between types of problems and required operations further develops students' mathematical reasoning ability. DI math programs meet NCTM's goals for student success by providing students with the confidence and skills to become effective mathematical problem solvers in both classroom and real life mathematics.

NCTM's Principles and Standards for Improving Mathematics (2000b) also provide educators with six principles for improving math instruction. As stated previously, DI math programs effectively meet these principles and result in positive academic outcomes as shown by a majority of studies included in this summary. We encourage public/private school educators and academicians to continue to investigate the effects of DI math programs in consideration of our recommendations. This line of research will continue to ensure we are using math curricula that best serve the needs of all students.

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