Producing Death:

Modeling and Estimating the Technology of Combat

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Abstract

What is the technology of attrition? And how has it changed over time? The technology of attrition is the function from forces to casualties. This is equivalent to Lanchester-type models. Using a set of battles from 1600-1983, I estimate a Bayesian hierarchical model of casualty rates. This provides estimates of the parameters of the Lanchester model for each war in the sample.

1 Introduction

How has the technology of combat changed over time? And prior to that question, how to define the technology of combat? I define the technology of attrition (or combat) as a production function that maps the forces of the belligerents to casualties. This definition is consistent with the Lanchester model of attrition. I estimate a Bayesian hierarchical model of casualties for a set of battles between 1600 and 1983. The hierarchical model provides estimates of the parameters of the Lanchester model for each war and battle, allowing for a comparison of the technology of attrition across wars.

Attrition can be considered a technology analogous to economic production.[1]. The technology of combat is a function between the inputs — the forces of the belligerents — and the outputs — the casualties of those forces. The technology of attrition at any point in time are the parameters of that function. And technological change is a change in the parameters of the function. At its most abstract, combat is a function that maps military forces into casualties, and the technology of combat is the shape of that function.

Representing combat with a production function is consistent with Lanchester-type models of war, which have been the foundation of combat models since the 1960's.[2] (Lepingwell 1987; Epstein 1985;

[1] A point made most eloquently by Hirshleifer (2000) and Hirshleifer (1991).

[2] The most extensive reference on the Lanchester Law's is Taylor (1983a) and Taylor (1983b). Other treatments include Taylor (1980), Karr (1983), Morse and Kimball (1951). Lepingwell (1987) is a non-mathematical introduction for political scientists. Wrigge, Fransén,

Anderton 1992)In Lanchester-type models, the casualty rates of combatants are represented as a pair of differential equations in which the casualty rates of each belligerent are a function of the size of its own and its opponent's forces. This system can be solved to find the evolution of the force sizes over time. The Lanchester model is not a complete model of battles. It does not account for geography, the termination rules, allocation of forces, etc. However, the attrition process is fundamental to war. The strategic and tactical decisions to start, continue, or terminate fighting and to retreat or advance of the belligerents are conditioned on the attrition that occurs when they do fight.

While there have been many works which estimate Lanchester-type models with casualty data from one or many battles, they do not account for the variation in parameters between battles (Engel 1954; Willard 1962; Weiss 1966; Dean S. Hartley 1989; Helmbold 1971; Helmbold 1961; Helmbold 1964; Hartley and Helmbold 1995; Helmbold 1995). Helmbold (1995) is the intellectual predecessor of this paper. It estimates a general Lanchester model using the CDB90 set of battles and attempts to address the "Constant Fallacy" which Helmbold coined. However, that paper only includes a linear time trend in its model. This is a restrictive assumption of how the attrition technology changes over time. It still assumes that the parameters relating the casualty rate of a belligerent and the forces remains constant across all wars. This paper allows all the parameters to vary between battles and wars to the extent that the data will allow such variation. Rather than trying to "test" the Lanchester model, this paper uses the Lanchester model as a description of the relationship between force sizes and casualties, and estimates how this relationship varies between battles.

2 The Lanchester Model

Lanchester-type combat models represent attrition in battle as a system of differential equations in which the rate of casualties for each belligerent is a function of its force size and the force size of its opponent.[3] Suppose that there are two belligerents, $I = \{a, d\}$, such that a is the attacker, and d is the defender. Each belligerent has a military force of size x_i. A Lanchester-type model is a pair of differential equations, in which each side's **casualty rate**, $\frac{dx_i}{dt}$, is a function of its own strength, x_i, and its opponent's strength x_{-i}. A commonly used general Lanchester-type model is the Taylor-Helmbold model (Taylor 1980, pp. 32-41),

$$\begin{aligned} \frac{dx_a}{dt} &= \alpha_a x_d^{\rho_a} x_a^{\beta_a} \\ \frac{dx_d}{dt} &= \alpha_d x_a^{\rho_d} x_d^{\beta_d} \end{aligned} \tag{1}$$

and Wigg (1995) and Helmbold (1993) are extensive bibliographies of all literature related to Lanchester up to the mid-1990s, i.e. the pre-internet age; for literature after that, see Google Scholar.

[3]For a more complete treatment of the Lanchester model wee Taylor (1980, Chapter 2).

The parameter α_i is called the **attrition-rate parameter**, β_i is the elasticity of i's casualties with respect to i's own size (x_i), and ρ_i is the elasticity of i's casualties with respect to the opponent's force size (x_{-i}).

The **state equation** (2) is an equality that gives the force levels of the belligerents for all times. Let $x_i(t)$ be the force size at time. Let $\lambda_i = \rho_{-i} - \beta_i + 1$. Then for any time t, the size of each force is given by the equation,

$$\frac{f(x_a(0), x_a(t), \lambda_a)}{\alpha_a} = \frac{f(x_d(0), x_d(t), \lambda_d)}{\alpha_d} \tag{2}$$

where

$$f(x_i(0), x_i(t), \lambda_i) = \begin{cases} x_i(0)^{\lambda_i} - x_i(t)^{\lambda_i + 1} & \text{if } \lambda_i \neq 0 \\ \log x_i(0) - \log x_i(t) & \text{if } \lambda_i = 0 \end{cases} \tag{3}$$

Several special cases of (1) are prominent in this literature (Helmbold 1994, p. 655; Taylor 1983a, p. 167).

	ρ_i	β_i	λ_i
Lanchester's Square Law	1	0	2
Lanchester's Linear Law	1	1	1
p-Linear Law	p	p	1
Logarithmic Law	0	1	0

Some of these special cases have been derived from micro-foundations of how each side target the other and the time to acquire targets Taylor (1983a, pp. 23-24). These special cases also have substantive interpretations. Lanchester (1916) derived the Square Law to represent "modern warfare" in which aimed weapons could be concentrated on the opponent, and the Linear Law to represent "ancient warfare". Non-combat operational losses can generate attrition according to the Logarithmic Law (Peterson 1967).

The **defensive advantage function** (u_0) is the force ratio at which the attacker and defender are evenly matched. The belligerents are evenly matched if in fighting to annihilation, they exhaust their forces at the same time, i.e. there is a t such $x_i(t) = 0$ for both i.[4] Suppose that $\lambda_a, \lambda_d \neq 0$. Take equation (2) and substitute $x_i(t) = 0$ for both i. Define the exchange ratio as $\epsilon = \frac{\alpha_a}{\alpha_d}$. The defensive-advantage function is,

$$u_0(\epsilon, \lambda_a, \lambda_d, y_0) = \left(\epsilon y_0^{\lambda_d - \lambda_a}\right)^{\frac{1}{\lambda_a}} \tag{4}$$

Suppose that the attrition-rate coefficient of the attacker is 3 times that of the defender ($\epsilon = 9$), and technology follows the Lanchester Square Law ($\lambda_a = \lambda_d = 2$), then the attacker would need to have 3 times the strength of the defender ($u_0 = 3$) be evenly matched.[5]

[4] If $\lambda_i = 0$, then that belligerent will not reach 0 in a finite time.

[5] Clearly, this is related to the $n : 1$ rules Mearsheimer (1989)epstein1989. However, the Lanchester model when solved in this manner only accounts for what would happen if the belligerents fought to annihilation. It can capture who has an attrition advantage,

The **elasticity of quality** (κ) is the log elasticity of the defensive-advantage function u_0 with respect to the exchange ratio ϵ,

$$\kappa = \frac{\partial \log u_0}{\partial \log \epsilon} = \frac{1}{\lambda_a} \tag{5}$$

The elasticity of quality represents the relative importance of "Quality" and "Quantity" in determining the outcome of the battle. When $\kappa < 1$, the needed change quantity of forces is less than the change in the attrition-rate ratio, thus quantity is more important than quality. For example, if $\kappa = 0.5$, then a one percent increase in the attrition rate ratio would require only a 0.5 percent increase in the force ratio to keep the forces evenly matched.

For estimation purposes, I will approximate the instantaneous casualty rate for a battle of duration T with the first-order Taylor series expansion at $t = 0$ (Helmbold 1995, pp. D-3),

$$x_i(0) - x_i(T) \approx x_i(0) - \left(x_i(0) + \left. \frac{dx}{dt} \right|_{t=0} T \right) \tag{6}$$

$$= \alpha_i x_{-i}^{\rho_i} x_i^{\beta_i} T \tag{7}$$

One more substantive is the **returns to scale**, defined for both belligerents and the battle,

$$\nu_i = \rho_i + \beta_i \tag{8}$$

$$\nu = \frac{1}{2}(\nu_a + \nu_d) \tag{9}$$

This returns to scale relate the casualty rate to the size of the forces. If the initial force sizes were multiplied by k, then the instantaneous casualty rate for belligerent i will increase by a factor of $k^{\rho_i + \beta_i}$. Thus, when $k = 1$, there are constant returns to scale; when $k < 1$, there are decreasing returns to scale; and when $k > 1$, there are increasing returns to scale.

3 Data

The U.S. Army's Concept Analysis Agency's CDB90 dataset is the source of data on battles in this paper Agency (1991).[6] The CDB90 dataset contains 660 battles from 1600-1984 (from the Dutch Revolt to the 1982 Lebanon War). The battles are from U.S., European European and Israeli Wars, the full list of wars appears and number of battles from each appears in Table **??**. This work uses a subset of 624 battles after dropping

but this is not necessarily the same as who will win the battle. For one thing, the outcome of the battle depends on the termination rules of the belligerents. Regardless of the attrition rates, the defender may be willing to suffer higher casualties.

[6] The CDB90 dataset is often called the HERO dataset. It is in fact a successor to the HERO dataset from 1984. CDB90 incorporates revisions to the HERO dataset based on independent reviews and incorporates an additional 60 battles from Anderson.

	N	Mean	Std. Dev.	Min	25%	50%	75%	Max
str	1238	71469.62	160311.14	188.00	8138.75	19872.00	60000.00	2200000.00
stro	1238	71475.13	160309.76	188.00	8138.75	19983.50	60000.00	2200000.00
duration	1238	3.10	8.54	0.01	0.33	0.99	2.34	129.00

Table 1: Summary statistics of battle data from CDB90

duplicate battles.[7]

The CDB90 database has been the target of criticism regarding its quality. However, it is the best data source on data on battles in interstate wars. [8] Biddle and Long (2004) summarizes this opinion

> The resulting data are not perfect — coding errors doubtless remain, HERO's selection rationale for the battles included is opaque, and the data set provides little information on the technical sophistication of the weapons it treats (weapons data are limited to the numbers of systems of each major type but not their particular makes, models, or performance). On the other hand, CDB90 offers the only meaningful data available on the outcomes of battles, as distinct from wars; it has already played a role in the democratic effectiveness literature; and the great majority of the spot-checked values were in reasonable consistency with the official historical record

A complaint of CDB90 is that the sample of wars is exclusively American, European, and Israeli (Biddle and Long 2004, p. 533; Biddle 2004). This is not a particularly troubling issue for the analysis conducted in this paper. I am primarily interested about the variation between and within wars. That these data come from a more homogenous sample of wars and battles than the overall population of wars, the results here will understate variation in the technology of combat. Alternatively, this sample of battles can be considered a better sample than choosing all wars because CDB90 has already pre-selected a set of wars with similar characteristics and participants, and thus the variation over time is more likely due to technological innovation rather than wars fought in different places at different times. The choice of this sample means that the extrapolation beyond the set of battles considered is not advisable, but that is always the case in inference.

A related but distinct complaint is that the CDB90 database is the variation in the level of detail in which wars are covered The hierarchical method used in this mitigates this issue by grouping observations at the war level. Wars with more battles will have more precise estimates, but by grouping wars, it mitigates the influence of wars with large numbers of battles on the overall estimate relative to a pooled estimate.

[7] Biddle and Long (2004) identified the duplicate battles in CDB90 among the 20th century. I drop all of these battles save one, which appeared to not be a duplicated. I checked battles before the 20th century and found no duplicates; which is not surprising. It is only in the 20th century that war becomes less discrete with the lines between battles, operations, and campaigns blurring.

[8] See Biddle (2001), Biddle and Long (2004), Brooks (2003), Mearsheimer (1989), and Ramsay (2008) for an overview of this debate.

Finally, Desch (2002) and Mearsheimer (1989) have criticized the quality of the CDB90 data. First, the major source of criticism in the reports that these sources cite was in subjective codings of concepts such as leadership and morale, rather than the casualty and strength values that are used in this work Epstein (1989). Second, while independent reviews disagreed with the HERO data, they independent reviews also disagreed with themselves, suggesting that the problem is one of measurement error and systematic bias (Epstein 1989). Third, the independent reviews used in the critiques were of the HERO data. CDB90 is a revision of the 1984 HERO which includes corrections to the problems that were identified in these reviews (Agency 1991, README.txt). Fourth, unlike many datasets CDB90 includes observation-level information on the measurement error in its casualties and strength data, something which few international relations datasets include. [9]

4 Statistical Model

The outcome variable to be modeled is the number of casualties for each belligerent for each battle in the data. Let y_i be the logarithm of the number of casualties suffered by belligerent i in a battle. Using the linear approximation in equation (6), a statistical model of casualties is,

$$y_i \sim N(\mu_i, \sigma^2) \tag{10}$$

$$\mu_i = \zeta_i + \rho_i \log \mathtt{stro}_i + \beta_i \log \mathtt{str}_i + \tau_i \log \mathtt{duration}_i \tag{11}$$

where $y_i = \log \mathtt{cas}_i = \log(x_i(0) - x_i(T))$ is the logarithm of casualties for belligerent i. $\zeta_i = \log \alpha_i$ is the logarithm of i's attrition-rate parameter, $\mathtt{str}_i = \log x_i$ is the logarithm of the number of personnel of the belligerent i, $\mathtt{stro}_i = \log x_{-i}$ is the number of personnel of the opponent, and $\mathtt{duration}_i$ is the duration of the battle in which belligerent i is fighting.

With only the casualties and initial force sizes in each battle, it is impossible to estimate ζ_i, β_i, ρ_i, and τ_i for each belligerent. However, since one of the goals of this work is to explore if and how this has changed over time, it does not make sense to estimate a pooled model in which the parameters are the same for all belligerents, e.g. a standard linear model. Instead, I will use a Bayesian hierarchical model to allow the parameters to vary by attacker/defender, war, war $\times$ attacker/defender, and battle. Both intercepts and slopes for all parameters are allowed to vary between groups. Interactions with `attacker` are modeled as

[9] Blackwell, Honaker, and King (2011) consider this best practice for reporting data.

fixed effects, while the interactions for `war`, `battle`, and `war : attacker` are modeled as random effects.

$$\zeta_i = \zeta_d + \Delta\zeta^w_{\texttt{war}_i} + \Delta\zeta^b_{\texttt{battle}_i} + (\Delta\zeta_a + \Delta\zeta^{w:a}_{\texttt{war:attacker}_i}) \times \texttt{attacker}_i \tag{12}$$

$$\rho_i = \rho_d + \Delta\rho^w_{\texttt{war}_i} + \Delta\rho^b_{\texttt{battle}_i} + (\Delta\rho_a + \Delta\rho^{w:a}_{\texttt{war:attacker}_i}) \times \texttt{attacker}_i \tag{13}$$

$$\beta_i = \beta_d + \Delta\beta^w_{\texttt{war}_i} + \Delta\beta^b_{\texttt{battle}_i} + (\Delta\beta_a + \Delta\beta^{w:a}_{\texttt{war:attacker}_i}) \times \texttt{attacker}_i \tag{14}$$

$$\tau_i = \tau_d + \Delta\tau^w_{\texttt{war}_i} + (\Delta\tau_a + \Delta\tau^{w:a}_{\texttt{war:attacker}_i}) \times \texttt{attacker}_i \tag{15}$$

Where $\texttt{attacker}_i$ is a binary variable indicating whether belligerent i is the attacker in that battle. $\texttt{battle}_i$ is the battle that belligerent i is fighting in, $\texttt{war}_i$ is the war of the battle that belligerent i is fighting in, and `war : attacker` is the interaction between $\texttt{war}_i$ and $\texttt{attacker}_i$. Parameters with the w superscript are war random effects; those with b are battle random effects; those with $w : a$ are war × attacker random effects.

The prefix Δ indicates that the parameter represents the additional effect relative to the baseline parameter. For example, $\Delta\zeta_a$ is the difference between the attacker's intercept and the defender's intercept. Thus $\zeta_a = \zeta_d + \Delta\zeta_a$ is the attacker's intercept. Likewise, $\Delta\zeta^w_{\texttt{war}_i}$ is the difference between the intercept for $\texttt{war}_i$ and the intercept ζ_d. Thus, the defender's intercept for $\texttt{war}_i$ is $\zeta^w_{\texttt{war}_i} = \Delta\zeta^w_{\texttt{war}_i} + \zeta_d$.

The random effects have the following priors, which allow the random effects in each level to be correlated.

$$(\Delta\zeta^w, \Delta\beta^w, \Delta\rho^w, \Delta\tau^w)_{\texttt{war}_i} \sim N(0, \Sigma^w) \tag{16}$$

$$(\Delta\zeta^{w:a}, \Delta\beta^{w:a}, \Delta\rho^{w:a}, \Delta\tau^{w:a})_{\texttt{war:attacker}_i} \sim N(0, \Sigma^{w:a}) \tag{17}$$

$$(\Delta\zeta^b, \Delta\beta^b, \Delta\rho^b, \Delta\tau^b)_{\texttt{attacker}_i} \sim N(0, \Sigma^b) \tag{18}$$

I set non-informative priors for all the other parameters. I estimate this model with the R package `MCMCglmm` (Hadfield 2010).

5 Results

6 Grand Means

The parameters ζ_i, β_i, ρ_i, and τ_i are the "fixed effects" or grand means. They represent the average parameters, i.e. the parameters in the average battle or war of the sample.

Table 2 shows summary statistics of the posterior distributions of the fixed effects, and derived param-

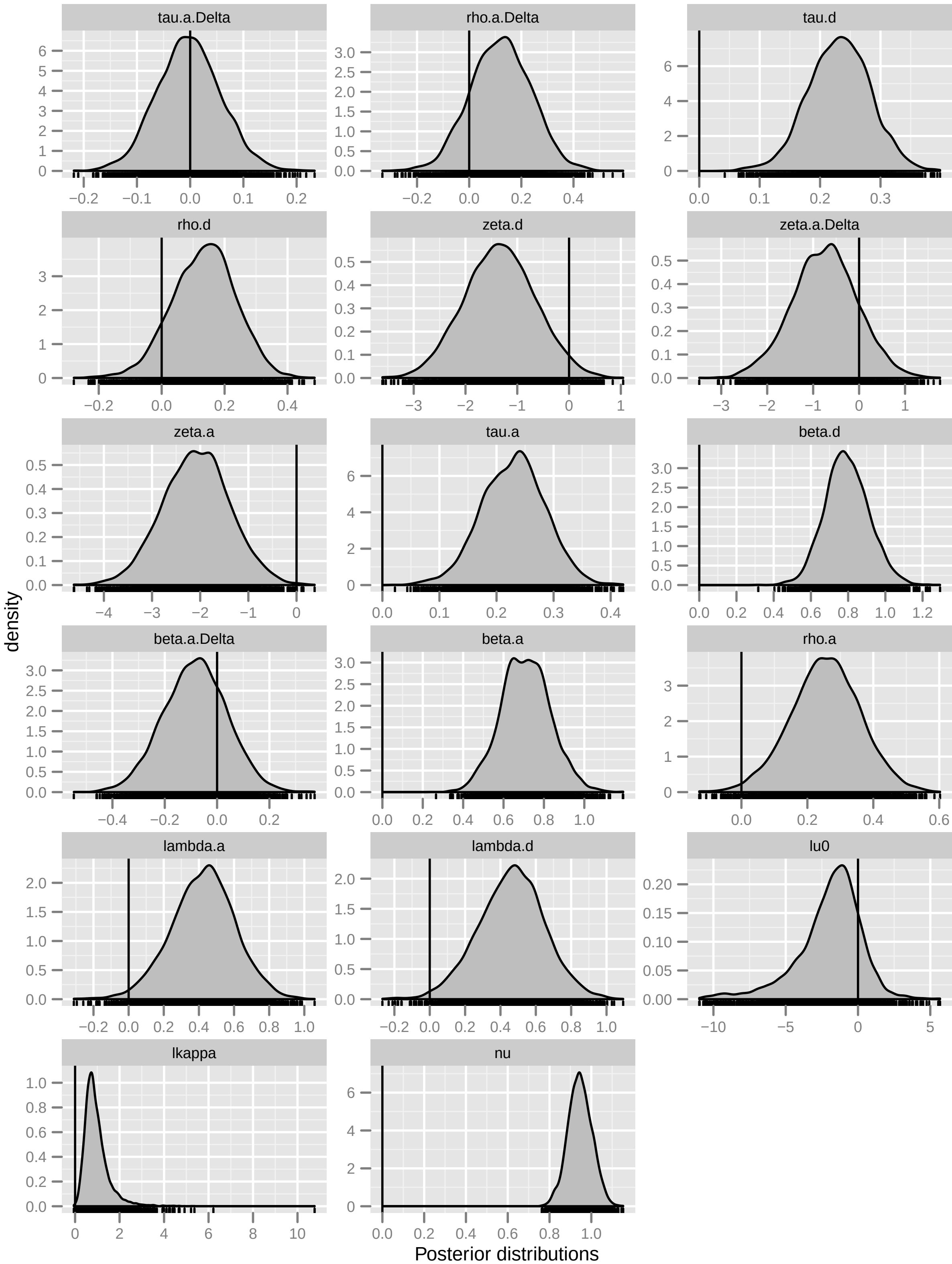

Figure 1: Posterior distributions

Parameter	Mean	Std. Dev.	2.5%	Median	97.5%
ζ_d	-1.335	0.683	-2.636	-1.341	0.029
ζ_a	-2.082	0.699	-3.450	-2.077	-0.725
$\Delta\zeta_a$	-0.747	0.710	-2.182	-0.738	0.625
ρ_d	0.134	0.101	-0.068	0.138	0.321
ρ_a	0.255	0.106	0.031	0.257	0.448
$\Delta\rho_a$	0.121	0.116	-0.106	0.121	0.337
β_d	0.793	0.118	0.578	0.788	1.038
β_a	0.710	0.122	0.469	0.710	0.947
$\Delta\beta_a$	-0.084	0.121	-0.321	-0.082	0.147
τ_d	0.232	0.051	0.128	0.232	0.326
τ_a	0.229	0.055	0.121	0.231	0.336
$\Delta\tau_a$	-0.002	0.060	-0.119	-0.003	0.116
λ_a	0.424	0.182	0.081	0.432	0.799
λ_d	0.461	0.187	0.091	0.468	0.826
$\log u_0$	-15.808	813.704	-10.105	-1.685	3.576
$\log\kappa$			0.132	0.834	2.021
ν	0.946	0.057	0.841	0.944	1.067

Table 2: Posterior estimates of a Bayesian hierarchical model of casualties in Equation (10).

eters from 2. [10]

The a ρ and β parameters of the average battle imply a Lanchester model that lies between the Logarithmic and Linear Laws. Plugging in the posterior means of these parameters, the implied Lanchester model is,

$$\frac{x_a}{dt} = e^{-2.1} x_d^{0.26} x_a^{0.71} \tag{19}$$

$$\frac{x_d}{dt} = e^{-1.3} x_a^{0.13} x_d^{0.79} \tag{20}$$

The 95 percent HPD intevals of both λ_a and λ_d are approximately 0.1 to 0.8. The values of β_a and ρ_d do are unlikely to be different from each other, the 95 percent HPD intervals of both $\Delta\rho_a$ and $\Delta\beta_a$ contain zero. This means that apart from the attrition rate coefficients, on average attackers and defenders have the same attrition process.

Rather than a defensive advantage in battles, on average, there appears to be an attacker advantage, at least in terms of attrition-rates. The 95 percent HPD interval of the log difference between the attrition rate coefficients ($\Delta\zeta_a$) contains 0. However, the probability that $\zeta_a < \zeta_d$ is 0.85. These results are similar to Helmbold (1995) in that I do not find a defensive advantage. This does not mean that there is not a defensive advantage, it means that there is little evidence of a defensive advantage in terms of casualty rate differential. Defenders may have a higher willingness to incur casualties than attacker. An alternative

[10] These results are similar to those in Helmbold (1995). This is good since I am using the same data and the same model, but generalizing to a hierarchical model with war and battle specific effects.

explanation for this is that attackers only select themselves into battles in which they have an attrition-rate advantage; thus I do not observe many battles when the defense has the advantage.

On average, the casualty rates have close to constant returns to scale with respect to the force sizes. The mean of the posterior distribution of ν is approximately 1, with a 95 percent HPD interval of 0.94 to 1.1. This means that in this sample, battles with larger forces have proportionally less casualties as battles with smaller forces. This is likely a feature local to the range of force sizes in the data. The data does not include many battles with less than division sized forces: the inter-quartile range of total forces is 20,500 to 120,000. This may suggest that it is more appropriate to use production functions with variable returns to scale when modeling battles. This is contradicts the conclusion of Helmbold (1995) that smaller forces inflict more casualties, perhaps due to our different definitions of returns to scale.

On average, longer battles have lower casualty rates. If the casualty rates for longer battles were directly proportional to the length of the battle, then $\tau = 1$, see equation 6. However, the mean of the posterior distribution of τ is 0.23; each additional day of battle reduces the baseline attrition rate by 100 - 23 = 87 percent. The effect of increased duration of battles on attackers and defenders is not asymmetric; the mean of the posterior distribution of $\Delta\tau_a$ is approximately zero.

7 War Level Parameters

The the posterior distributions of several war-level parameters are plotted in Figures 2 (λ_a^w), 3 (λ_d^w), 4 ($\Delta\zeta_a^w$), 5 (ν^w), and 6 (τ_d).

- The parameters appear to be fairly constant between wars or there is insufficient data to estimate individual war-level parameters.
- In λ_a^w and λ_d^w, there appears to be more between war variance before the Napoleonic Wars.
- In λ_a^w and λ_d^w, there are several wars that are estimated to have attrition laws that are between the Linear and Square Laws: the American Revolution, the American Civil War, World War I (Western Front), World War II (Okinawa). In those wars, There is one war in which is estimated to be below the logarithmic attrition law: The English Civil War.
- There does not appear to be much difference between the defensive advantage ($\Delta\zeta_a$) across wars. However, the estimates are relatively imprecise.
- Most wars have constant returns to scale. The exceptions are that the English Civil War had increasing returns to scale (battles with larger forces had proportionally more casualties), and the American Civil

War and World War II (European Theater) had decreasing returns to scale.

- There appears to be a time trend in τ_d. Over time τ_d has increased from about 0 to about 0.5. This means that earlier in this sample, longer battles had proportionally less casualties per day than they do now.
- I did not impose any constraints during the estimation, and many of these parameters fall outside of reasonable bounds. E.g. in some cases β and ρ are estimated to be negative, meaning that force sizes increase casualties decrease; this may be plausible in the case of β, but not with respect to the size of the opponent's force ρ.

Comparing the DIC of this hierarchical model with the DIC of simple linear normal model with and without time trends favor the more complex hierarchical model. Thus, it appears to be a situation such that the differences in parameters between the groups (wars, battles) is important, but there is not enough data to get precise estimates of the group-level parameters.

8 Appendix

war	end year	start year	number of battles
NETHERLAND'S WAR OF INDEPENDENCE	1600	1600	2
THIRTY YEAR'S WAR	1620	1648	36
ENGLISH CIVIL WAR	1642	1645	12
SECOND ENGLISH CIVIL WAR	1648	1651	6
THE FRONDE	1652	1652	2
FRANCO-SPANISH WAR	1658	1658	2
POLISH-TURKISH WAR	1673	1673	2
DUTCH WAR	1674	1675	10
MONMOUTH'S REBELLION	1685	1685	2
AUSTRO-TURKISH WAR	1664	1716	8
KING WILLIAM'S WAR	1689	1693	16
WAR OF THE SPANISH SUCCESSION	1704	1709	8
GREAT NORTHERN WAR	1709	1709	2
WAR OF THE AUSTRIAN SUCCESSION	1741	1745	14
JACOBITE REBELLION	1745	1746	4

THE SEVEN YEAR'S WAR	1756	1760	36
AMERICAN REVOLUTION	1775	1781	28
WAR OF THE FIRST COALITION	1792	1799	28
WAR OF THE SECOND COALITION	1799	1800	14
THE NAPOLEONIC WARS	1805	1815	58
WAR OF 1812	1813	1815	8
LATIN AMERICAN WARS OF INDEPENDENCE	1819	1824	12
WAR OF TEXAN INDEPENDENCE	1836	1836	2
US-MEXICAN WAR	1846	1847	16
CRIMEAN WAR	1854	1854	4
WAR OF AUSTRIA WITH FRANCE AND PIEDMONT	1859	1859	4
AMERICAN CIVIL WAR	1861	1865	98
AUSTRO-ITALIAN WAR	1866	1866	2
AUSTRO-PRUSSIAN (SEVEN WEEK'S) WAR	1866	1866	2
FRANCO-PRUSSIAN WAR	1870	1871	20
ZULU WAR	1879	1879	4
TRANSVAAL REVOLT	1881	1881	2
EGYPT AND THE SUDAN	1882	1898	4
ITALO-ETHIOPIAN WAR	1896	1896	2
SPANISH-AMERICAN WAR	1898	1898	2
BOER WAR	1899	1900	10
RUSSO-JAPANESE WAR	1904	1905	12
THE BALKAN WARS	1912	1913	10
WORLD WAR I (WESTERN FRONT 1914)	1914	1914	32
WORLD WAR I (EASTERN FRONT 1914)	1914	1914	18
WORLD WAR I (SERBIAN FRONT 1914)	1914	1914	4
WORLD WAR I (TURKISH FRONTS 1915)	1915	1915	8
WORLD WAR I (ITALIAN FRONT 1915)	1915	1915	8
WORLD WAR I	1915	1918	52
WORLD WAR I (TURKISH FRONTS 1917)	1917	1917	10
WORLD WAR I (WESTERN FRONT 1918)	1918	1918	72
RUSSO-POLISH WAR (1920)	1920	1920	4

THE SPANISH CIVIL WAR	1937	1937	2
THE MANCHURIAN INCIDENT (1938-1939)	1938	1939	10
RUSSO-FINISH WAR (1939-1940)	1939	1939	2
WORLD WAR II (NORTH AFRICA 1942-1943)	1942	1943	10
WORLD WAR II	1940	1944	29
WORLD WAR II (ITALY 1943-1944)	1943	1944	86
WORLD WAR II (EASTERN FRONT)	1941	1945	56
WORLD WAR II (ITALY 1944)	1944	1944	48
WORLD WAR II (EUROPEAN THEATER)	1944	1944	56
WORLD WAR II (PACIFIC, 1943-1945)	1943	1945	8
WORLD WAR II (OKINAWA)	1945	1945	56
ARAB-ISRAELI, 1948	1948	1948	18
KOREAN WAR	1950	1951	22
ARAB-ISRAELI, 1956	1956	1956	8
ARAB-ISRAELI WAR, 1967 (SIX-DAY WAR)	1967	1967	44
ARAB-ISRAELI WAR, 1968	1968	1968	2
VIET NAM	1972	1972	1
ARAB-ISRAELI WAR, 1973 (OCTOBER WAR)	1973	1973	66
ISRAEL-LEBANON, 1982	1982	1982	2

Table 3: List of Wars in CDB90. World War I and World War II are split into multiple wars by theater. The "WORLD WAR I" and "WORLD WAR II" wars contain battles from theaters in those wars which did not contain enough battles to warrant inclusion as a separate war.

9 Conclusion

The current state of this paper is preliminary. Comments and suggestions are welcome.

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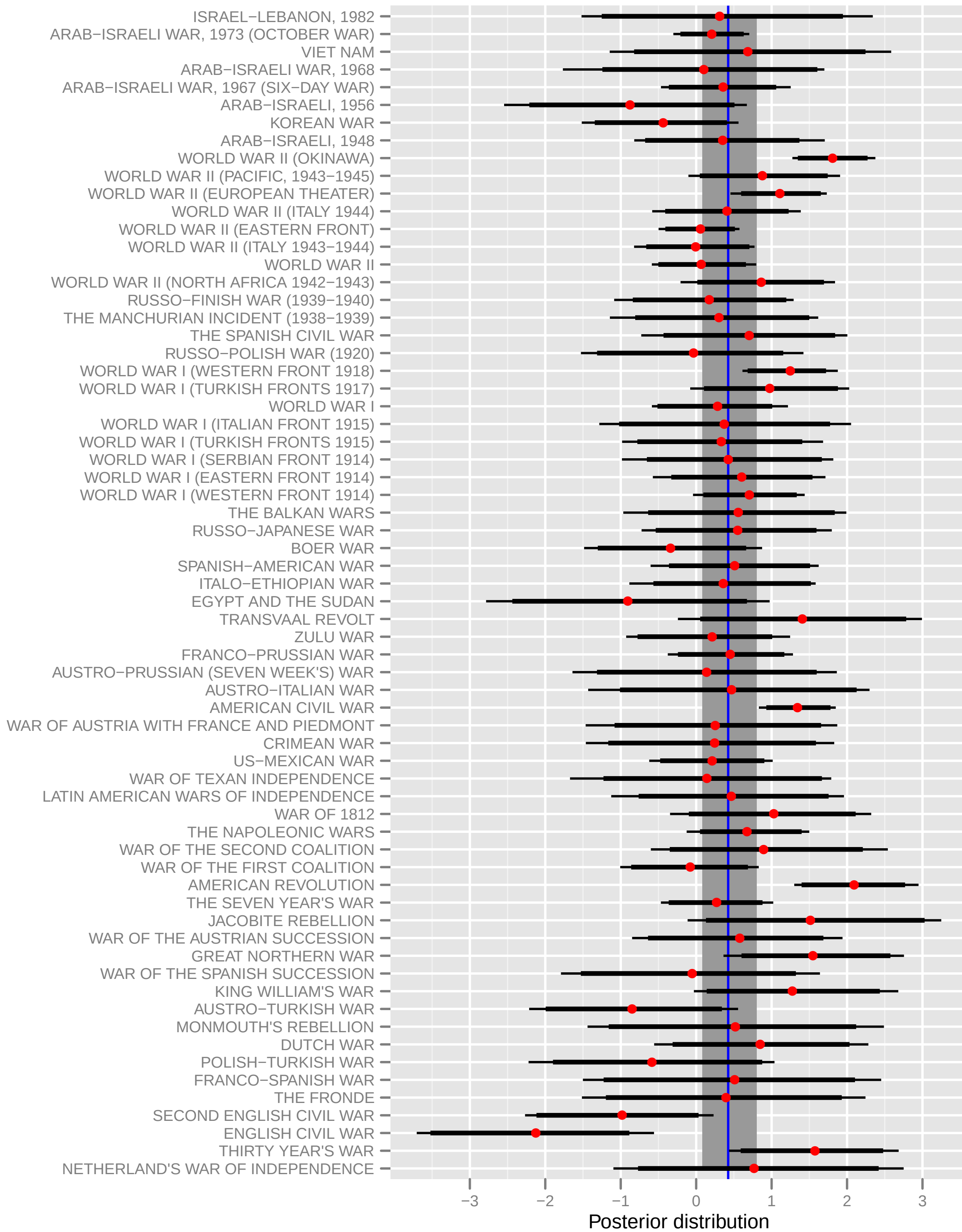

Figure 2: Posterior distributions of λ_d^w for each war. Points are the posterior mean. The thick lineranges are 75 percent HPD intervals. The thin lineranges are 95 HPD intervals. The vertical blue line and gray area are the mean of the posterior of the grand mean and its 95 percent HPD interval

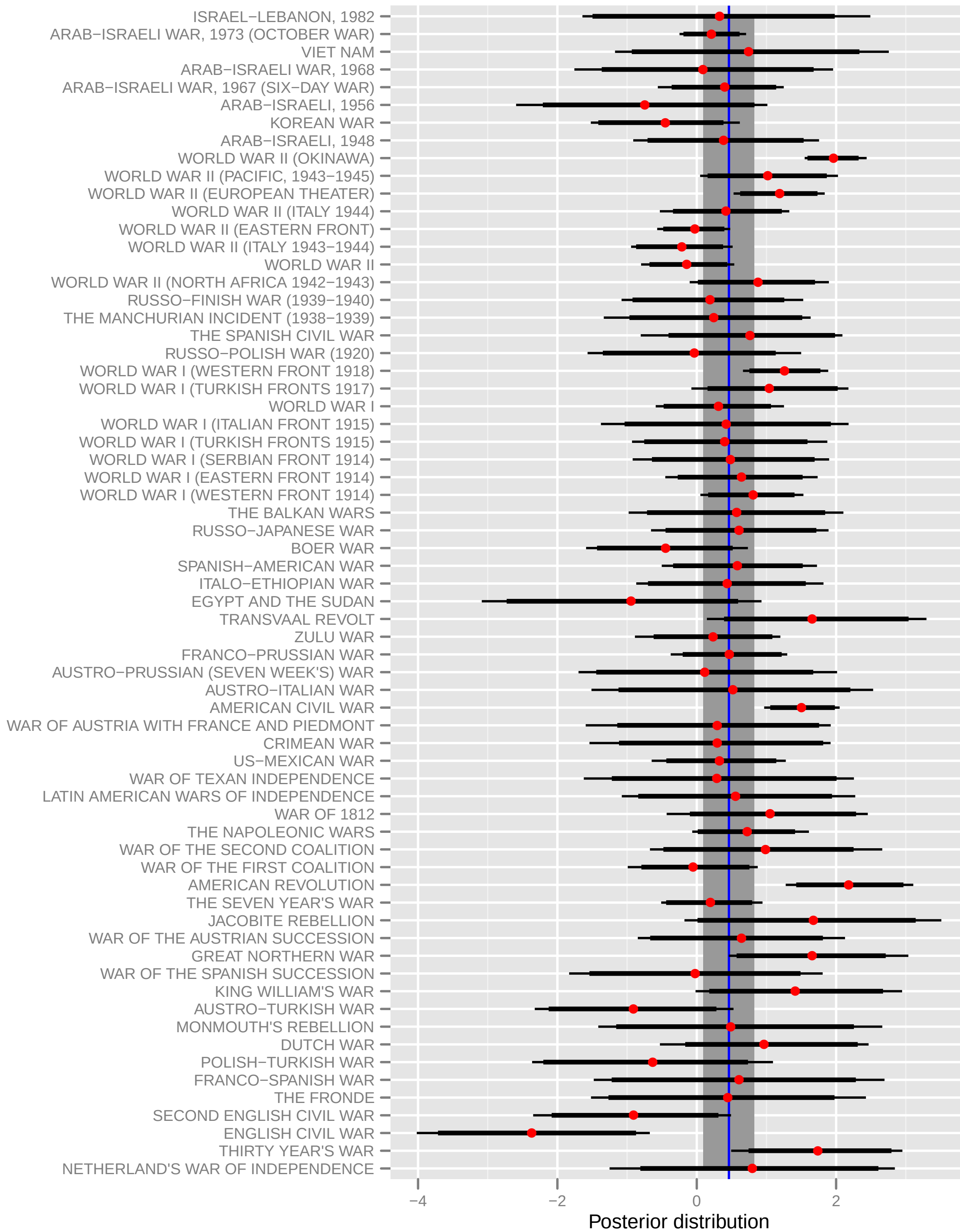

Figure 3: Posterior distributions of λ_d^w for each war. Points are the posterior mean. The thick lineranges are 75 percent HPD intervals. The thin lineranges are 95 HPD intervals. The vertical blue line and gray area are the mean of the posterior of the grand mean and its 95 percent HPD interval

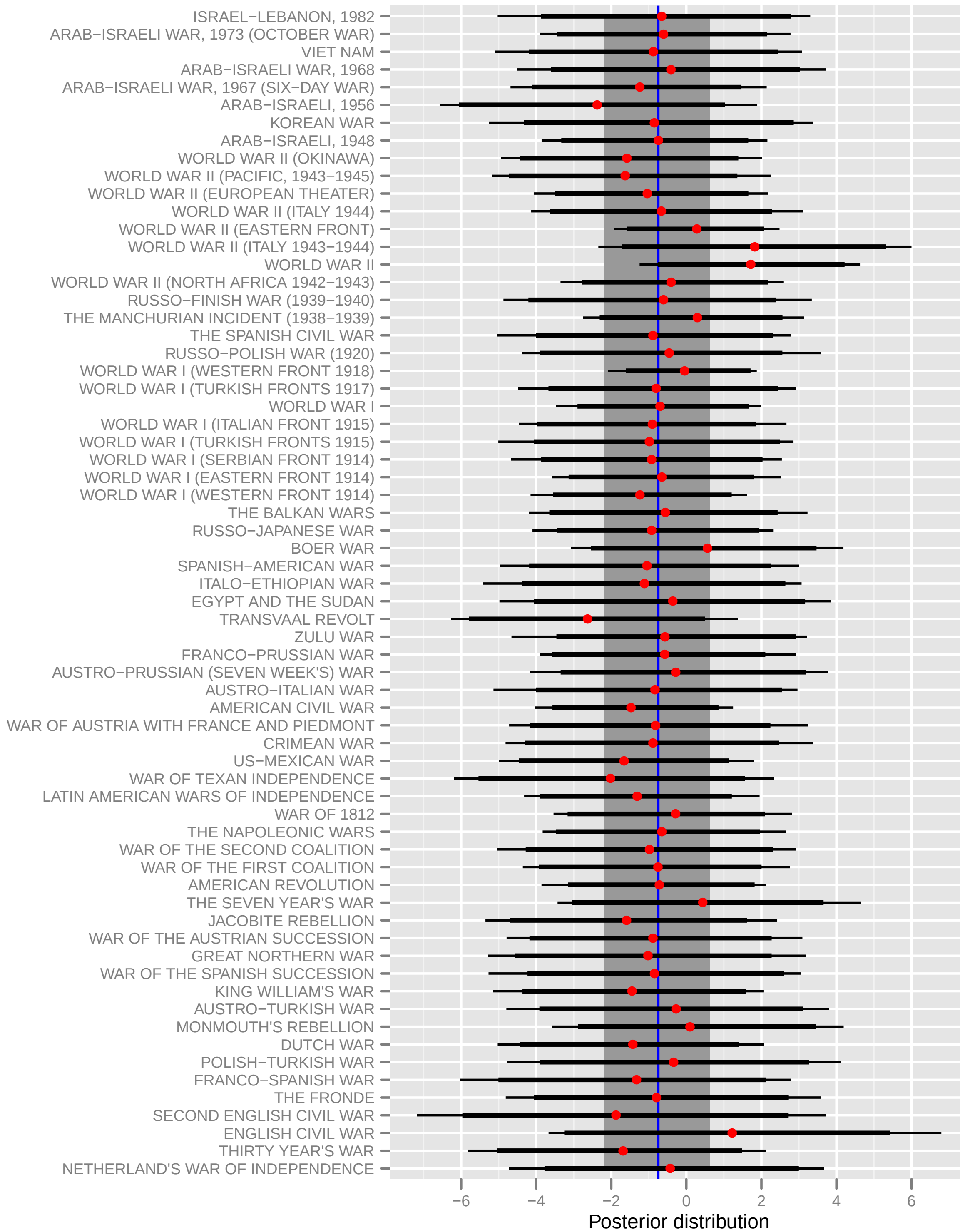

Figure 4: Posterior distributions of $\Delta\zeta_a^w = \log \epsilon^w$ for each war. Points are the posterior mean. The thick lineranges are 75 percent HPD intervals. The thin lineranges are 95 HPD intervals. The vertical blue line and gray area are the mean of the posterior of the grand mean and its 95 percent HPD interval

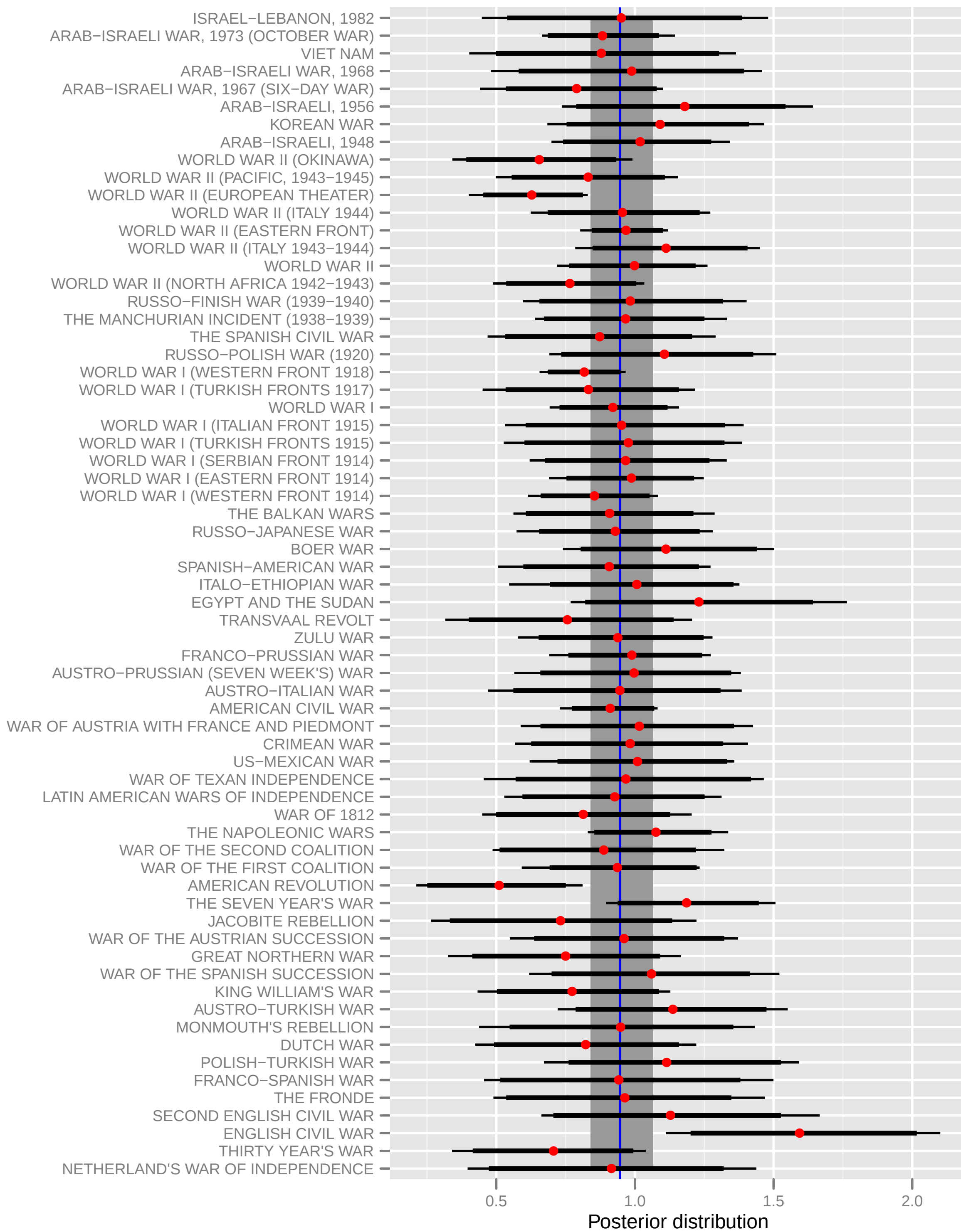

Figure 5: Posterior distributions of ν^w for each war. Points are the posterior mean. The thick lineranges are 75 percent HPD intervals. The thin lineranges are 95 HPD intervals. The vertical blue line and gray area are the mean of the posterior of the grand mean and its 95 percent HPD interval

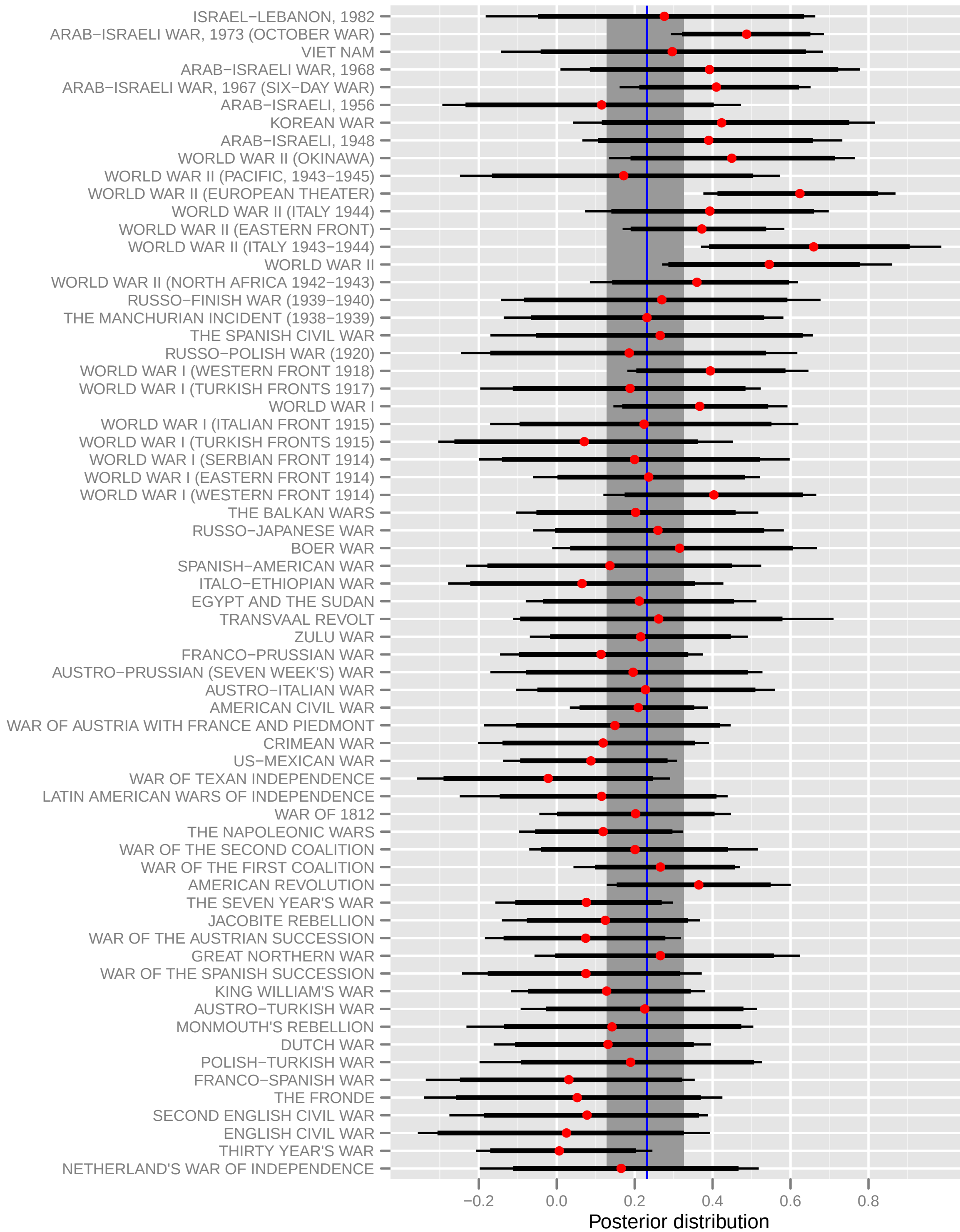

Figure 6: Posterior distributions of τ_d^w for each war. Points are the posterior mean. The thick lineranges are 75 percent HPD intervals. The thin lineranges are 95 HPD intervals. The vertical blue line and gray area are the mean of the posterior of the grand mean and its 95 percent HPD interval

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