LESSON 17

Measures of Dispersion

Key Concept

Measures of central tendency (Lesson 16) allow you to describe a data set with a single value. Although these measures give a sense of a "typical" data value, they do not describe how the data are spread out. In this lesson, students learn to calculate several *measures of dispersion*, including the range, the interquartile range, the variance, and the standard deviation.

Key Question: Example 1

What conclusions can you make about the data just by looking at the double box-and-whisker plot?

The data for subcompact cars are much more spread out than the data for midsize cars. The data for midsize cars are clustered more closely around the median.

Key Question: Example 2

How many of the data values are within one standard deviation of the mean? How many of the data values are within two standard deviations of the mean?

Four of the data values (29, 31, 33, 37) are within one standard deviation of the mean. All of the data values are within two standard deviations of the mean.

Differentiated Instruction

Example 2 Some students may get lost in the calculations for finding the variance and the standard deviation. A table like the one below can help students organize their work.

Data, x	23	37	33	39	29	31
Mean, $\overline{x}$	32	32	32	32	32	32
$x - \overline{x}$	−9	5	1	7	−3	−1
$(x - \overline{x})^2$	81	25	1	49	9	1

Point out to students that the variance is simply the mean of the values in the last row of the table.

$$\sigma^2 = \frac{81 + 25 + 1 + 49 + 9 + 1}{6} = 27.\overline{6}$$

The standard deviation is the square root of the variance.

$$\sigma = \sqrt{27.\overline{6}} \approx 5.26$$

Avoiding Common Errors

Example 2 When students calculate the variance and the standard deviation by hand, they might forget to square the difference of each data value and the mean. To help students understand why this step is important, have them add the unsquared differences. Students will discover that this sum is always equal to zero.

Key Question: Example 3

If you made a line plot for each of the data sets, what would you expect to see when you compared the line plots?

The data values for Machine 2 would be more spread out than those for Machine 1.

Closing the Lesson

Have students answer the following question: What are the steps for finding the standard deviation of a set of data?

Find the mean of the values. Then find the difference between each data value and the mean, and square the difference. Find the sum of the squared differences and divide by the number of data values. Finally, take the square root of the result.

Teaching Strategy

Exercises 21–24 A *control chart* is a way of graphically displaying a set of data so that the distance of each data value from the mean is apparent. Explain to students that control charts are often used to determine whether a manufacturing or business process is "under control" (that is, whether the future performance of the process can be predicted with confidence). If the process is not under control, any patterns that are revealed by the control chart can help determine possible sources of variation.

Homework Help

Example 1: Exs. 1–6
Example 2: Exs. 7–10
Example 3: Ex. 11–19
Enrichment: Exs. 20–25

Homework Check

To quickly check student understanding of key concepts, go over the following exercises: 3, 7, 13–15.

ANSWERS

Check Answers

1. 24 mi/gal; 6.5 mi/gal

2. 9 mi/gal; 4 mi/gal

3. 32.33; 5.69

4. 76; 8.72

5. 19.33; 6.18

6. 12.53; 0.56

Exercise Answers

1. 11 min; 5.5 min

2. 7; 3

3. $13.8 million; $6.6 million

4. $37.36; $22.33

5. Los Angeles: 16.9°F, 11.9°F; Buffalo: 46.3°F, 31.6°F

6. The average monthly temperatures for Buffalo have more variation than those for Los Angeles.

7. 1737.5; 41.68

8. 11.6; 3.41

9. 28.5; 5.34

10. 92.2; 9.60

11. Line Plot 1: 5, 2.58; Line Plot 2: 5, 3.59; Line Plot 3: 5, 0.94

12. The farther away more of the data are from the mean, the greater the standard deviation.

13. 3.175 in.; 1.73 in.

14. 3.525 in.; 1.03 in.

15. The mean precipitation in Galveston is greater than the mean precipitation in Seattle, while the standard deviation for Galveston is less than that for Seattle. In general, Galveston gets more precipitation, but Seattle has more variability in monthly precipitation.

16. See the line plot below.

17. 80.68

18. 18.95; 99.63; 61.73

19. 72%

20. North America and Europe: range = 6.4%, IQR = 2.5%, $\bar{x} \approx 17.74\%$, $\sigma \approx 1.73\%$; South America and the Caribbean: range = 14.8%, IQR = 7.1%, $\bar{x} \approx 30.51\%$, $\sigma \approx 4.43\%$. A greater percent of the population under 15 is in South America and the Caribbean than in North America and Europe. Also, the spread of the data for North America and Europe is less than that for South America and the Caribbean, indicating that the percent of the population under 15 in North American and European countries varies less than the percent in South American and Caribbean countries.

21. 12.54 mm; 12.42 mm

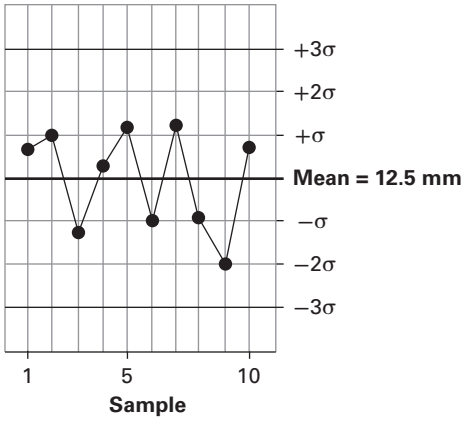
22. 11; 21

23. a. 17

b. 13

c. 22

24. *Sample answer:*



25. a. 17

b. absolute deviations (in same order as data values in exercise): 7, 5, 13, 9, 10, 2, 1, 2, 6, 13

c. 6.8

d. If a data value that is greater than or less than all the other data values is added to the data set, the mean absolute deviation will increase. If the new data value is equal to the mean of the original data, the mean absolute deviation will decrease.