LESSON 16

Measures of Central Tendency

Key Concept

Given a data set, *measures of central tendency,* also known as measures of location, provide an answer to the question "What is a typical value for the data set?" The three most common measures of central tendency are the mean, median, and mode. In addition to finding these measures, students learn to choose the measure that is most appropriate in a given situation.

Key Question: Example 1

Is the median always one of the values in the data set? Explain.

No; if there are an even number of values, the median is the average of the two middle values. In this case, the median may or may not be one of the values in the data set.

Teaching Strategy

Example 1 Students can use their graphing calculators to check their calculations and experiment with the data. Have students enter the given data values in list L1. Then have them press the STAT key, scroll to the CALC menu, and select 1-Var Stats. Point out that $\bar{x}$ is the mean. Students should use the arrow keys to scroll down to see the median ("Med"). Ask students to predict how the mean, median, and mode would change if another movie that is 102 minutes long were added to the data set. Then have them use their calculators to check their predictions.

Activity

- **Materials** cardboard, scissors, ruler, hole punch, string, tape, paper clips
- **Goal** Students make a simple balance scale to investigate the relationship between the mean of a set of values and the point at which the values balance.
- **Teaching Strategy** In Step 4, students will find that parts b and c have multiple answers. For each part, ask students to identify several possible answers and encourage them to consider what the answers have in common. You may want to point out that the missing values are not necessarily greater than or equal to the greatest value appearing in each list. Then prompt students to describe what must be true about any pair of numbers that works as an answer to part b or any pair of numbers that works as an answer to part c.
- **Key Discovery** The mean of a set of values is the point on the number line at which the values balance.

Key Question: Example 2

In part a, why might someone choose to describe the data set using the mode, even though the median is more typical of the values in the data set?

The mode makes the typical waiting time seem shorter. Someone who works at the doctor's office might use the mode to describe the data set to convince patients that the typical waiting time is not too long.

Closing the Lesson

Have students answer the following question: How do you find the mean, median, and mode of a data set?

To find the mean, add all the values in the data set and divide by the total number of values. To find the median, write the values in numerical order and choose the middle value (or the average of the two middle values if there is an even number of values in the data set). To find the mode, find the value or values that occur most frequently.

Avoiding Common Errors

Exercise 11 Students may find the mean of the values that appear in the histogram without taking their frequencies into account. Point out that the value 10 occurs 3 times, 12 occurs 8 times, and so on.

Teaching Strategy

Exercise 22 In this exercise, students use measures of central tendency to compare two data sets. Encourage students to organize their work in a table, like the one shown below.

	Roseville	Summerton
Median	80	83
Mean	82	77
Mode	75	84

The table makes it clear that only the mean gives the impression that Roseville is warmer than Summerton.

Homework Help

Example 1: Exs. 1–10

Example 2: Exs. 14–18

Enrichment: Exs. 11–13, 19–24

Homework Check

To quickly check student understanding of key concepts, go over the following exercises: 3, 9, 14, 15.

ANSWERS

Check Answers

1. 48; 45; no mode

2. 34; 39; 40

3. 75.$\overline{4}$; 79; 75 and 93

4. mean: 16.75; the mode, 30, is too high and the median, 13, is too low.

5. median: 3.45 or mean: 3.525; the mode, 3.25, is too low.

6. median or mode: 6; the mean, 7.3, is too high.

7. Only the mode would have to be a whole number. The mean could be a decimal if the sum of the data values is not evenly divisible by the number of data values. The median could be a decimal if there are an even number of data values.

Activity Answers

1. Check students' work.

2. Paper clip should be hung on the hole corresponding to 21.

3. 16

4. a. 18

b. Answers may vary. The sum of the two numbers must equal 32.

c. Answers may vary. The sum of the two numbers must equal 33.

5. They are equal.

6. No; adding the mean to a set of data gives the new set of data the same mean as the original set of data.

7. when the median is also the mean of the data

8. when the mode is also the mean of the data

Exercise Answers

1. 10; 12; 12

2. 12; 12; 16

3. 27; 26; no mode

4. 28; 25; 25 and 39

5. 14.75; 14.5; 14 and 15

6. 12.9; 13.5; no mode

7. 78.5; 78; 78

8. 65; 62; 57

9. 6.9; 6.4; no mode

10. 7.8; 7.8; 7.8

11. about 14.6 years; 15 years; 12 years and 15 years

12. about 73.8°F; 74°F; 72°F

13. You can find only the median, 55; to find the mean you need to know the sum of the data values and the total number of data values. To find the mode(s) you need to know the individual data values. A box-and-whisker plot does not give you this information.

14. median: 2.5 days; the mean, 3.55 days, is too high and the mode, 1 day, is too low.

15. mean: 30 minutes; both the median, 25 minutes, and the mode, 20 minutes, are too low.

16. modes: 32 fl oz and 128 fl oz; both the mean, about 66.7 fl oz, and the median, 48 fl oz, are not meaningful in the context of milk container sizes.

17. mode; the mean and the median are not meaningful in the context of radio station call numbers.

18. median; it is likely that the vice president and the president make a significant amount more than the rest of the workers, therefore pulling the mean higher than is representative of the salaries in general. The mode is likely to be too low or may not exist.

19. 5

20. a. 88

b. No; score less than or equal to 88 will result in a median of 88.

c. Yes; the only way the mode can be 88 is if the score on the last test is 88.

21. a. mean: about $58; median: $50; modes: $35 and $50

b. The mean will decrease, the median will not change, and there will be only one mode, 50. The new mean is 56.875, which is less than 58; when listed in order, the data values are 35, 35, 50, 50, 50, 65, 75, and 95, so 50 is still the median; 50 occurs the greatest number of times, so it is the mode.

22. a. Summerton

b. Roseville

c. Summerton

d. mean

23. Median (or mode); the median number of wins for the Dolphins is 9 (also the mode), compared to a median of 8 (also the mode) for the Saints.

24. Mean; the mean number of wins for the Saints is approximately 7.6, compared to a mean of approximately 7.1 for the Dolphins.